1.1 Express in SI and US Customary units the speeds of a) satellite, b) racing car, c) fighter plane, d) baseball, e) automobile piston.

a) satellite $\approx$ 20,000 mi/h

$$\frac{20{,}000\text{ mi}}{1\text{ hr}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{1\text{ h}}{3600\text{ s}} = \underline{\underline{29{,}333\text{ ft/s}}}$$

$$\frac{29{,}333\text{ ft}}{1\text{ s}} \times \frac{.3048\text{ m}}{1\text{ ft}} = \underline{\underline{8941\text{ m/s}}}$$

b) race car $\approx$ 180 mi/h

$$\frac{180\text{ mi}}{1\text{ hr}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{1\text{ h}}{3600\text{ s}} = \underline{\underline{264\text{ ft/s}}}$$

$$\frac{264\text{ ft}}{1\text{ s}} \times \frac{.3048\text{ m}}{1\text{ ft}} = \underline{\underline{80.47\text{ m/s}}}$$

c) plane $\approx$ 800 mi/h

$$\frac{800\text{ mi}}{1\text{ hr}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{1\text{ h}}{3600\text{ s}} = \underline{\underline{1173\text{ ft/s}}}$$

$$\frac{1173\text{ ft}}{1\text{ s}} \times \frac{.3048\text{ m}}{1\text{ ft}} = \underline{\underline{357.6\text{ m/s}}}$$

d) baseball $\approx$ 100 mi/h

$$\frac{100\text{ mi}}{1\text{ hr}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{1\text{ h}}{3600\text{ s}} = \underline{\underline{146.7\text{ ft/s}}}$$

$$\frac{146.7\text{ ft}}{1\text{ s}} \times \frac{.3048\text{ m}}{1\text{ ft}} = \underline{\underline{44.70\text{ m/s}}}$$

e) piston $\approx$ 5,000 rev/min

$$\frac{5000\text{ rev}}{1\text{ min}} \times \frac{2\text{ strokes}}{1\text{ rev}} \times \frac{3\text{ in}}{1\text{ stroke}} \times \frac{1\text{ ft}}{12\text{ in}} \times \frac{1\text{ min}}{60\text{ sec}} = \underline{\underline{41.67\text{ ft/s}}}$$

$$\frac{41.67\text{ ft}}{1\text{ s}} \times \frac{.3048\text{ m}}{1\text{ ft}} = \underline{\underline{12.7\text{ m/s}}}$$

1.2 a) Find % of mass a particle changes if $v =$ i) 3×10^4 m/s, ii) 3×10^6 m/s, iii) 3×10^7 m/s.

i) $\%\text{ change} = \dfrac{|m_o - m|}{m_o} \times 100$

$$m = m_o\left(1 - \frac{(3\times10^4)^2}{(3\times10^8)^2}\right)^{-1/2} = 1.000000\, m_o$$

$$\% = \frac{|m_o - 1.000\,000\, m_o|}{m_o} \times 100 = \underline{\underline{0\%}}$$

ii)

$$m = m_o\left(1 - \frac{(3\times10^6)^2}{(3\times10^8)^2}\right)^{-1/2} = 1.00005\, m_o$$

$$\% = \frac{|m_o - 1.00005\, m_o|}{m_o} \times 100 = \underline{\underline{.005\%}}$$

iii)

$$m = m_o\left(1 - \frac{(3\times10^7)^2}{(3\times10^8)^2}\right)^{-1/2} = 1.00504\, m_o$$

$$\% = \frac{|m_o - 1.00504\, m_o|}{m_o} \times 100 = \underline{\underline{.504\%}}$$

1.2 cont. b) Newtonian mechanics is basically exact for ordinary engineering speeds.

c) A particle becomes infinitely massive as it approaches the speed of light.

1.3 Evaluate the quality of data.

$$\vec{F} = m\vec{a} \rightarrow m = \frac{\vec{F}}{\vec{a}}$$

x-comp: $m = \dfrac{8}{2.5} = 3.2\text{ kg}$

y-comp: $m = \dfrac{10}{3.125} = 3.2\text{ kg}$

z-comp: $m = \dfrac{30}{10} = 3\text{ kg}$

There are errors in the data. The mass should be the same for all three projections.

1.4 Find x, y comp. of $\vec{F}$.

$m = 3\text{ kg}$ $\quad \vec{F} = m\vec{a}$

x-comp: $F_x = 3(2.5) = \underline{\underline{7.5\text{ N}}}$

y-comp: $F_y = 3(3.125) = \underline{\underline{9.375\text{ N}}}$

1.5 Assuming that the calibration of the scale was conducted where it was assembled, the difference in weight of the meteorite will be proportional to the difference in force due to gravity between the earth and the meteorite.

$W = mg$: $g_1 = g$ at elevation of assembly
$\qquad\qquad g_2 = g$ at Mt. McKinley

Since $g_2 < g_1$ due to elevation, $W_2 < W_1$. The scale will read less than the meteorite's actual weight.

1.6 a) 5 trillion $= 5\times10^{12} =$ 5 teradollars

b) Find # of times around the earth.

1 bill $\approx$ 6 in; circumference of earth = 25,000 mi

$$\frac{5\times10^{12}\text{ bills}}{25{,}000\text{ mi}} \times \frac{6\text{ in}}{1\text{ bill}} \times \frac{1\text{ ft}}{12\text{ in}} \times \frac{1\text{ mi}}{5280\text{ ft}} \approx \underline{\underline{18{,}939\text{ times}}}$$

1.7 Find # of transactions a) per day, b) per yr.

a) $$\frac{1{,}200\text{ hits}}{1\text{ sec}} \times \frac{3600\text{ s}}{1\text{ h}} \times \frac{24\text{ h}}{1\text{ day}} = 0.1037\times10^9\text{ hits/day} = \underline{0.1037\text{ gigahits/day}}$$

b) $$\frac{.1037\times10^9\text{ hits}}{1\text{ day}} \times \frac{365\text{ days}}{1\text{ yr}} = \underline{37.84\text{ gigahits/yr}}$$

1.8 Kinetic units in SI

Length: meter Mass: kilogram
Time: second Force: Newton

1.9 $MN\cdot m = 1000\, kg\cdot m^2/s^2$

MN·m is derived.

1.10 $N\cdot m/s = kg\cdot m^2/s^3$

N·m/s is derived.

1.11 Find height in in., ft, yd, m, and mm.

Answers may vary. If h = 5'6", then:

$5'6" = 5(12) + 6 = 66\text{ in}$
$= 5 + 6/12 = 5.5\text{ ft}$
$= 5.5/3 = 1.83\text{ yd}$
$= 5.5/3.281 = 1.676\text{ m}$
$= 1.676(1000) = 1676\text{ mm}$

$5'6" = 66\text{ in} = 5.5\text{ ft} = 1.83\text{ yd} = 1.676\text{ m} = 1676\text{ mm}$

1.12 Find my mass in slugs and kg.

Answers may vary. If weight = 125 lbs, then:

$125\text{ lbs} = W = mg$
$m = W/g = 125/32.2 = 3.88\text{ slugs}$
$= 3.88(14.59) = 56.6\text{ kg}$

1.13 Use prefixes to express exponents.

a) 10^{-6} phone = microphone
b) 10^{6} lopolis = megalopolis
c) 2×10^{1} cards = 2 dekacards
d) 10^{-6} scope = microscope
e) 10^{6} bucks = megabucks

1.14 a) Is A or B faster? By what %?

Runner A: $100\text{ m}\times\dfrac{1.094\text{ yd}}{1\text{ m}} = 109.4\text{ yd}$

109.4 yd > 100 yd, so Runner A is faster than B.

$\dfrac{109.4-100}{100} = 0.094 = 9.4\%$

b) Does A or B run farther? By what %?

Runner A: $10\text{ km}\times\dfrac{1\text{ mi}}{1.609\text{ km}} = 6.214\text{ mi}$

6.214 mi > 6 mi, so Runner A runs farther than B.

$\dfrac{6.214-6}{6} = 0.0356 = 3.56\%$

1.15 Find # of mi in 1 km.

$1\text{ km}\times\dfrac{10^6\text{ mm}}{1\text{ km}}\times\dfrac{1\text{ in}}{25.4\text{ mm}}\times\dfrac{1\text{ ft}}{12\text{ in}}\times\dfrac{1\text{ mi}}{5280\text{ ft}} = 0.6214\text{ mi}$

1.16 Find the density in a) slug/ft³, b) g/cm³.

a) $\dfrac{10^3\text{ kg}}{1\text{ m}^3}\times\dfrac{1\text{ slug}}{14.59\text{ kg}}\times\dfrac{1\text{ m}^3}{35.31\text{ ft}^3} = 1.941\text{ slug/ft}^3$

b) $\dfrac{10^3\text{ kg}}{1\text{ m}^3}\times\dfrac{1000\text{ g}}{1\text{ kg}}\times\dfrac{1\text{ m}^3}{(100\text{ cm})^3} = 1\text{ g/cm}^3$

1.17 Find the speed a) in mi/s, b) km/min.

a) $\dfrac{1450\text{ mi}}{1\text{ h}}\times\dfrac{1\text{ h}}{3600\text{ s}} = 0.4028\text{ mi/s}$

b) $\dfrac{1450\text{ mi}}{1\text{ h}}\times\dfrac{1.609\text{ km}}{1\text{ mi}}\times\dfrac{1\text{ h}}{60\text{ min}} = 38.88\text{ km/min}$

1.18 Find displacement in liters (L).

$350\text{ in}^3\times\dfrac{(25.4\text{ mm})^3}{1\text{ in}^3}\times\dfrac{1\text{ cm}^3}{10^3\text{ mm}^3}\times\dfrac{1\text{ L}}{1000\text{ cm}^3} = 5.735\text{ L}$

1.19 Find consumption in km/L.

$\dfrac{6\text{ mi}}{1\text{ gal}}\times\dfrac{1.609\text{ km}}{1\text{ mi}}\times\dfrac{1\text{ gal}}{3.785\text{ L}} = 2.551\text{ km/L}$

1.20 Find the relations between a) mi² and km², b) m³ and yd³, c) in² and yd².

a) $1\text{ mi}^2 = (1.609\text{ km})^2 = 2.589\text{ km}^2$

b) $1\text{ m}^3 = (1.094\text{ yd})^3 = 1.308\text{ yd}^3$

c) $1\text{ yd}^2 = (36\text{ in})^2 = 1296\text{ in}^2$

1.21 Find mass of earth in a) kip·yr²/mi, b) N·s²/m.

a) $\dfrac{4.08\times10^{23}\text{ lb}\cdot\text{s}^2}{1\text{ ft}}\times\dfrac{1\text{ kip}}{1000\text{ lb}}\times\dfrac{1\text{ day}^2}{(86{,}400\text{ s})^2}\times\dfrac{1\text{ yr}^2}{(365.25\text{ days})^2}\times\dfrac{5280\text{ ft}}{1\text{ mi}}$
$= 2.163\times10^{9}\text{ kip}\cdot\text{yr}^2/\text{mi}$

b) $\dfrac{4.08\times10^{23}\text{ lb}\cdot\text{s}^2}{1\text{ ft}}\times\dfrac{4.448\text{ N}}{1\text{ lb}}\times\dfrac{3.281\text{ ft}}{1\text{ m}}$
$= 5.954\times10^{24}\text{ N}\cdot\text{s}^2/\text{m}$

1.22 Find cost per mi², acre, m², and ft².

$A = (1/8\text{ mi})(1/2\text{ mi}) = 1/16\text{ mi}^2$
$\text{Cost} = \$32{,}000/\tfrac{1}{16}\text{ mi}^2 = \$512{,}000/\text{mi}^2$

$\dfrac{\$512{,}000}{1\text{ mi}^2}\times\dfrac{1\text{ mi}^2}{640\text{ acre}} = \$800/\text{acre}$

$\dfrac{\$512{,}000}{1\text{ mi}^2}\times\dfrac{1\text{ mi}^2}{(1609\text{ m})^2} = \$0.20/\text{m}^2$

$\dfrac{\$512{,}000}{1\text{ mi}^2}\times\dfrac{1\text{ mi}^2}{(5280\text{ ft})^2} = \$0.02/\text{ft}^2$

1.23 Find mag. of mass in lb·s²/in, g, slugs.

$$\frac{10^{-5}\ \text{kip}\cdot\text{h}^2}{1\ \text{mi}}\times\frac{1000\ \text{lb}}{1\ \text{kip}}\times\frac{(3600\text{s})^2}{1\ \text{h}^2}\times\frac{1\ \text{mi}}{5280\ \text{ft}}\times\frac{1\ \text{ft}}{12\ \text{in}}=\underline{\underline{2.045\ \text{lb}\cdot\text{s}^2/\text{in}}}$$

$$\frac{2.045\ \text{lb}\cdot\text{s}^2}{1\ \text{in}}\times\frac{1\ \text{slug}}{1\ \text{lb}\cdot\text{s}^2/\text{ft}}\times\frac{12\ \text{in}}{1\ \text{ft}}=\underline{\underline{24.55\ \text{slugs}}}$$

$$24.55\ \text{slug}\times\frac{14.59\ \text{kg}}{1\ \text{slug}}\times\frac{1000\ \text{g}}{1\ \text{kg}}=\underline{\underline{358.1\times10^3\ \text{g}}}$$

Find weight in lbs.

$$W=mg=(24.55\ \text{slugs})(32.2\ \text{ft/s}^2)=\underline{\underline{790.5\ \text{lbs}}}$$

1.24 a) Find dimensions of a and b.

$$y[L]=a[?]\times t^2[T^2]+b[?]\times t^4[T^4]$$
$$[L]=[a][T^2]+[b][T^4]$$
$$\underline{\underline{[a]=[L/T^2]\quad [b]=[L/T^4]}}$$

Find units for a and b using b) ft, s; c) mi, hr; d) km, s

b) $a\to \text{ft/s}^2\quad b\to\text{ft/s}^4$
c) $a\to \text{mi/h}^2\quad b\to\text{mi/h}^4$
d) $a\to \text{km/s}^2\quad b\to\text{km/s}^4$

1.25 Find c in min/m^{3/2}.

$$c=5.65\times10^{-6}\ \frac{\text{h}}{\text{mi}^{3/2}}\times\frac{60\ \text{min}}{1\ \text{h}}\times\frac{1\ \text{mi}^{3/2}}{(1609\ \text{m})^{3/2}}=\underline{\underline{5.252\times10^{-9}\ \frac{\text{min}}{\text{m}^{3/2}}}}$$

1.26 Are the relationships homogeneous?

a) $Fs=\tfrac{1}{2}mv^2$

$$[F][L]=[M]\left[\frac{L}{T}\right]^2\longrightarrow[F]=[M]\left[\frac{L}{T^2}\right]$$
$$[M]\left[\frac{L}{T^2}\right][L]=[M]\left[\frac{L^2}{T^2}\right]$$
$$\left[\frac{ML^2}{T^2}\right]=\left[\frac{ML^2}{T^2}\right]\qquad \underline{\underline{\text{Homogeneous}}}$$

b) $Fs=\dfrac{dA}{dt}=\dfrac{d(mvs)}{dt}$

$$\left[\frac{ML^2}{T^2}\right]=\frac{[M]\left[\frac{L}{T}\right][L]}{[T]}$$
$$\left[\frac{ML^2}{T^2}\right]=\left[\frac{ML^2}{T^2}\right]\qquad \underline{\underline{\text{Homogeneous}}}$$

c) $Ft=mv^2$

$$[M]\left[\frac{L}{T^2}\right][T]=[M]\left[\frac{L}{T}\right]^2$$
$$\left[\frac{ML}{T}\right]\neq\left[\frac{ML^2}{T^2}\right]\qquad \underline{\underline{\text{Not Homogeneous}}}$$

1.27 Find units for a and b.

$$\frac{d^2v}{dt^2}+a\frac{dv}{dt}+bv=0$$
$$\frac{\left[\frac{N\cdot m}{S\cdot A}\right]}{[s^2]}+[a]\frac{\left[\frac{N\cdot m}{S\cdot A}\right]}{[s]}+[b]\left[\frac{N\cdot m}{S\cdot A}\right]=0$$
$$\left[\frac{N\cdot m}{s^3\cdot A}\right]+[a]\left[\frac{N\cdot m}{s^2\cdot A}\right]+[b]\left[\frac{N\cdot m}{s\cdot A}\right]=0$$
$$[a]=\left[\frac{N\cdot m}{s^3\cdot A}\right]\times\left[\frac{s^2\cdot A}{N\cdot m}\right]=\underline{\underline{[s^{-1}]}}$$
$$[b]=\left[\frac{N\cdot m}{s^3\cdot A}\right]\times\left[\frac{s\cdot A}{N\cdot m}\right]=\underline{\underline{[s^{-2}]}}$$

1.28 Find dimensions of EI.

$$d=[L]\quad P=[F]\quad L=[L]\quad x=[L]$$
$$EI=\frac{P}{6d}(2L^3-3L^2x+x^3)$$
$$[EI]=\frac{[F]}{[L]}[L^3]=\underline{\underline{[F\cdot L^2]}}$$

1.29 Find numerical value if converted from $[\text{lb}^{1/2}\cdot\text{ft}^{-3}\cdot\text{min}^{-2}]$ to $[\text{N}^{1/2}\cdot\text{m}^{-3}\cdot\text{s}^{-2}]$.

$$\frac{\text{lb}^{1/2}}{\text{ft}^3\cdot\text{min}^2}\times\frac{(4.448\ \text{N})^{1/2}}{1\ \text{lb}^{1/2}}\times\frac{(3.281\ \text{ft})^3}{1\ \text{m}^3}\times\frac{1\ \text{min}^2}{(60\ \text{sec})^2}=\underline{\underline{0.0207\ \frac{\text{N}^{1/2}}{\text{m}^3\text{s}^2}}}$$

1.30 Find numerical value if converted from $[\text{slug}^3\cdot\text{ft}^{-1}\cdot\text{s}^2]$ to $[\text{kg}^3\cdot\text{m}^{-1}\cdot\text{s}^2]$.

$$\frac{\text{slug}^3\cdot\text{s}^2}{1\ \text{ft}}\times\frac{(14.59\ \text{kg})^3}{1\ \text{slug}^3}\times\frac{3.281\ \text{ft}}{1\ \text{m}}=\underline{\underline{10\,190\ \frac{\text{kg}^3\text{s}^2}{\text{m}}}}$$

1.31 Find % error of a) 22/7, b) 355/113 for π.

a) $$\frac{|22/7-3.141\,592\,654|}{3.141\,592\,654}\times100\%=\underline{\underline{0.0402\%}}$$

b) $$\frac{|355/113-3.141\,592\,654|}{3.141\,592\,654}\times100\%=\underline{\underline{8.478\times10^{-6}\%}}$$

1.32 Find σ in lb/in².

$$\sigma=\tfrac{1}{8}(3+0.3)(0.28\ \text{lb/in}^3)(200\ \text{rad/s})^2(12\ \text{in})^2(1\text{s}^2/32.18\text{ft})$$
$$=665{,}280\ \frac{\text{lb}}{\text{in}\cdot\text{s}^2}\left(\frac{1\ \text{s}^2}{32.18\ \text{ft}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$
$$=\underline{\underline{1{,}723\ \text{lb/in}^2}}$$

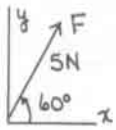
1.33 a) Find ρ of earth in slug/ft³.

$$\rho = \frac{4.08 \times 10^{23}\ \text{slug}}{4/3\ \pi (20.9 \times 10^{6}\ \text{ft})^3} = \underline{\underline{1.067 \times 10^{1}\ \text{slug/ft}^3}}$$

b) Find ρ in kg/m³.

$$\rho = \frac{10.67\ \text{slug}}{1\ \text{ft}^3} \times \frac{14.59\ \text{kg}}{1\ \text{slug}} \times \frac{(3.281\ \text{ft})^3}{1\ \text{m}^3} = \underline{\underline{5.50 \times 10^3\ \frac{\text{kg}}{\text{m}^3}}}$$

NOTE: Force notation in magnitude and direction - Positive angles are counterclockwise from the x-axis and negative angles are clockwise. For example,



$\vec{F} = 5\text{N},\ \theta = 60°$

2.1 List magnitude and direction of $\vec{A}, \vec{B}, \vec{C}, \vec{D}$.

$\vec{A} = 100\text{N},\ \theta = 45°$
$\vec{B} = 120\text{N},\ \theta = 75°$
$\vec{C} = 90\text{N},\ \theta = 135°$
$\vec{D} = 75\text{N},\ \theta = -135°$

2.2 Describe the characteristics of $\vec{A}$ and $\vec{B}$.

a) $\vec{A}$ and $\vec{B}$ are collinear.
b) $\vec{B}$ is a zero vector.
c) $\vec{A}$ and $\vec{B}$ are perpendicular.

2.3 Classify the force systems.

a) coplanar and concurrent
b) coplanar and collinear
c) concurrent and nonplanar
d) collinear and coplanar

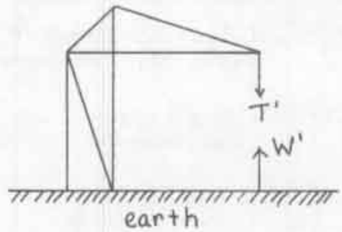
2.4 Describe a) direction line, b) magnitude, c) sense for each force.

$\vec{A}$: a) 30° clockwise from y-axis, b) 80N, c) from O to A
$\vec{B}$: a) 50° clockwise from y-axis, b) 120N, c) from O to B
$\vec{C}$: a) 30° counterclockwise from y-axis b) 100N, c) from O to C

2.5 Describe a) direction line, b) mag., c) sense.

$\vec{A}$: a) along x-axis, b) 3N, c) from O to A
$\vec{B}$: a) along y-axis, b) 4N, c) from O to B
$\vec{C}$: a) 53.13° counterclockwise from x-axis, b) 5N, c) from O to C

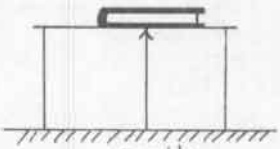
2.6



T' is the force exerted on the rope by the weight.
W' is the gravitational attraction exerted on the earth by the weight.

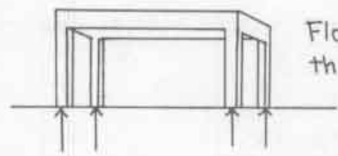
2.7 Describe and illustrate reaction forces.

a)



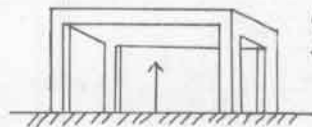
Gravitational force exerted on the earth by the book.

b)



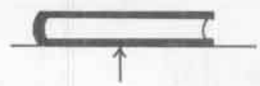
Floor pushes up on the legs.

c)



Gravitational pull on the earth by the desk.

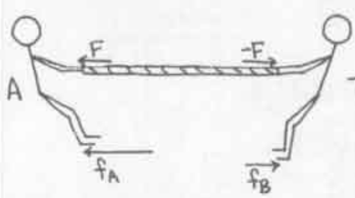
d)



Desk pushes up on the book.

2.8 How can either team win the tug of war?

Team A



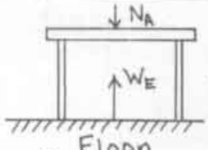
Team B

ΣForces = mass × acceleration
The forces on the rope are equal but opposite in direction. Team A has a greater friction force that allows the team members to pull back on the rope.

2.9 a) What forces act on the apple?

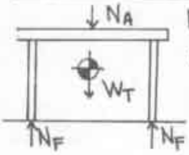
Weight W_A is the force exerted by the earth on the apple.
N_T is the force exerted by the table on the apple. *(continued)*

2.9 cont. b) What are the reaction forces?



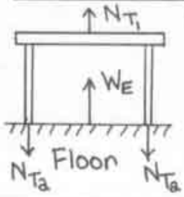
N_A is the force exerted by the apple on the table. W_E is the force exerted by the apple on the earth.

c) What forces act on the table?



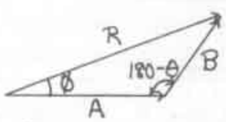
N_A is the force exerted by the apple on the table. W_T is the force exerted by the earth on the table. N_F is the force exerted by the floor on the table.

d) What are the reaction forces?



N_{T_1} is the force exerted by the table on the apple. W_E is the force exerted by the table on the earth. N_{T_2} is the force exerted by the table on the floor.

2.10 a) Derive formulas for R and ϕ.



Law of cosines: $R^2 = A^2 + B^2 - 2AB\cos(180-\theta)$

$$\underline{\underline{R = (A^2 + B^2 + 2AB\cos\theta)^{1/2}}}$$

Law of sines: $\dfrac{R}{\sin(180°-\theta)} = \dfrac{B}{\sin\phi}$

$$\underline{\underline{\phi = \sin^{-1}\left(\frac{B\sin\theta}{R}\right)}}$$

b) Check answers with $\theta = 0°, 90°, 180°$.

$\theta = 0°$ $\quad R = (A^2 + B^2 + 2AB)^{1/2} = \underline{\underline{A+B}}$

$\phi = \sin^{-1}\left(\frac{B}{R}\sin 0°\right) = \underline{\underline{0°}}$

$\theta = 90°$ $\quad R = (A^2 + B^2 + 2AB\cos(90°))^{1/2} = \underline{\underline{(A^2+B^2)^{1/2}}}$

$\phi = \sin^{-1}\left(\frac{B}{R}\sin 90°\right) = \underline{\underline{\sin^{-1}\left(\frac{B}{R}\right)}}$

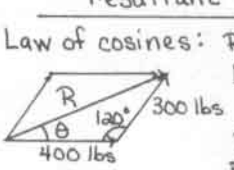
$\theta = 180°$ $\quad R = (A^2 + B^2 - 2AB)^{1/2} = \underline{\underline{A-B}}$

If $A>B$, $\phi = \sin^{-1}\left(\frac{B}{R}\sin 180°\right) = \underline{\underline{0°}}$

If $A<B$, $\phi = \underline{180°}$

2.11 b) Find magnitude and direction of resultant forces by trigonometry.

i) Law of cosines: $R^2 = 400^2 + 300^2 - 2(400)(300)\cos 120°$

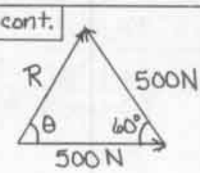


Law of sines: $\dfrac{\sin\theta}{300} = \dfrac{\sin 120°}{R}$

$$\underline{\underline{R = 608.3 \text{ lbs}, \theta = 25.3°}}$$

2.11 cont.

ii)

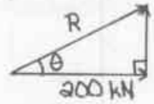


Law of cosines: $R^2 = 500^2 + 500^2 - 2(500)^2\cos 60°$

Law of sines: $\dfrac{\sin\theta}{500} = \dfrac{\sin 60°}{R}$

$$\underline{\underline{R = 500\text{ N}, \theta = 60°}}$$

iii)

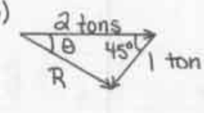


Law of cosines: $R^2 = 200^2 + 100^2 - 2(200)(100)\cos 90°$

Law of sines: $\dfrac{\sin\theta}{100} = \dfrac{\sin 90°}{R}$

$$\underline{\underline{R = 223.6 \text{ kN}, \theta = 26.6°}}$$

iv)

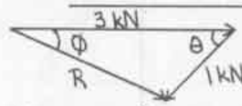


Law of cosines: $R^2 = 2^2 + 1^2 - 2(2)(1)\cos 45°$

Law of sines: $\dfrac{\sin\theta}{1} = \dfrac{\sin 45°}{R}$

$$\underline{\underline{R = 1.47 \text{ ton}, \theta = {}^-28.8°}}$$

2.12 a) Derive formula for magnitude of R.

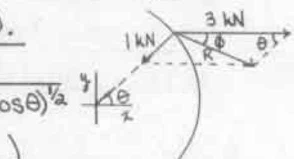


$R = (3^2 + 1^2 - 2(3)(1)\cos\theta)^{1/2}$

$$\underline{\underline{R = (10 - 6\cos\theta)^{1/2}}}$$

b) Derive formula for ϕ.

$\dfrac{\sin\phi}{1} = \dfrac{\sin\theta}{R} = \dfrac{\sin\theta}{(10-6\cos\theta)^{1/2}}$



$$\underline{\underline{\phi = \sin^{-1}\left(\frac{\sin\theta}{(10-6\cos\theta)^{1/2}}\right)}}$$

c) Evaluate for $\theta = 0°, 90°, 180°$.

$\theta = 0°$ $\quad R = (10 - 6\cos 0°)^{1/2}$

$\phi = \sin^{-1}\left(\dfrac{\sin 0°}{(10-6\cos 0°)^{1/2}}\right)$ $\quad \underline{\underline{R = 2\text{ kN}, \phi = 0°}}$

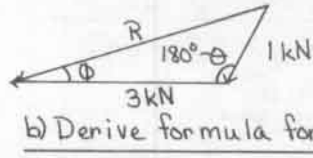
$\theta = 90°$ $\quad R = (10 - 6\cos 90°)^{1/2}$ $\quad \phi = \sin^{-1}\left(\dfrac{\sin 90°}{(10-6\cos 90°)^{1/2}}\right)$

$$\underline{\underline{R = 3.16\text{ kN}, \phi = {}^-18.4°}}$$

$\theta = 180°$ $\quad R = (10 - 6\cos 180)^{1/2}$ $\quad \phi = \sin^{-1}\left(\dfrac{\sin 180°}{(10-6\cos 180°)^{1/2}}\right)$

$$\underline{\underline{R = 4\text{ kN}, \phi = 0°}}$$

2.13 a) Derive formula for magnitude of $\vec{R}$.

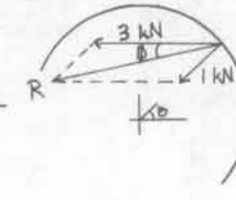


$R = (3^2 + 1^2 - 2(3)(1)\cos(180°-\theta))^{1/2}$

$$\underline{\underline{R = (10 + 6\cos\theta)^{1/2}}}$$

b) Derive formula for ϕ.

$\dfrac{\sin(180°-\theta)}{(10+6\cos\theta)^{1/2}} = \dfrac{\sin\phi}{1}$



$$\underline{\underline{\phi = \sin^{-1}\left(\frac{\sin\theta}{(10+6\cos\theta)^{1/2}}\right)}}$$

2.13 cont. c) Evaluate for $\theta = 0°, 90°, 180°$.

$\theta = 0°$ $R = (10 - 6\cos 180°)^{1/2}$ $\phi = \sin^{-1}\left(\frac{\sin 180°}{(10-6\cos 180°)^{1/2}}\right)$

$\underline{\underline{R = 4\ \text{kN},\ \phi = 0°}}$

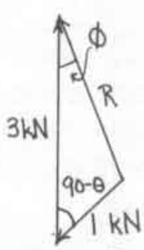
$\theta = 90°$ $R = (10 - 6\cos 90°)^{1/2}$ $\phi = \sin^{-1}\left(\frac{\sin 90°}{(10-6\cos 90°)^{1/2}}\right)$

$\underline{\underline{R = 3.16\ \text{kN},\ \phi = 18.4°}}$

$\theta = 180°$ $R = (10 - 6\cos 0°)^{1/2}$ $\phi = \sin^{-1}\left(\frac{\sin 0°}{(10-6\cos 0°)^{1/2}}\right)$

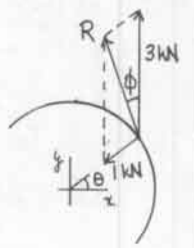
$\underline{\underline{R = 2\ \text{kN},\ \phi = 0°}}$

2.14 a) Derive formula for magnitude of $\vec{R}$.



$R = (3^2 + 1^2 - 2(3)(1)\cos(90° - \theta))^{1/2}$

$\underline{\underline{R = (10 - 6\sin\theta)^{1/2}}}$



b) Derive formula for ϕ.

$$\frac{\sin\phi}{1} = \frac{\sin(90° - \theta)}{(10 - 6\sin\theta)^{1/2}}$$

$$\underline{\underline{\phi = \sin^{-1}\left(\frac{\cos\theta}{(10-6\sin\theta)^{1/2}}\right)}}$$

c) Evaluate for $\theta = 0°, 90°, 180°$.

$\theta = 0°$ $R = (10 - 6\sin 0°)^{1/2}$ $\phi = \sin^{-1}\left(\frac{\cos 0°}{(10-6\sin 0°)^{1/2}}\right)$

$\underline{\underline{R = 3.16\ \text{kN},\ \phi = 18.43°}}$

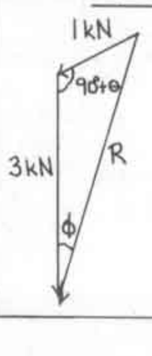
$\theta = 90°$ $R = (10 - 6\sin 90°)^{1/2}$ $\phi = \sin^{-1}\left(\frac{\cos 90°}{(10-6\sin 90°)^{1/2}}\right)$

$\underline{\underline{R = 2\ \text{kN},\ \phi = 0°}}$

$\theta = 180°$ $R = (10 - 6\sin 180°)^{1/2}$ $\phi = \sin^{-1}\left(\frac{\cos 180°}{(10-6\sin 180°)^{1/2}}\right)$

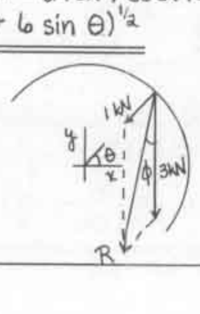
$\underline{\underline{R = 3.16\ \text{kN},\ \phi = -18.43°}}$

2.15 a) Derive formula for magnitude of $\vec{R}$.



$R = (3^2 + 1^2 - 2(3)(1)\cos(90° + \theta))^{1/2}$

$\underline{\underline{R = (10 + 6\sin\theta)^{1/2}}}$



2.15 cont. b) Derive formula for ϕ.

$$\frac{\sin\phi}{1} = \frac{\sin(90° + \theta)}{(10 - 6\cos(90° + \theta))^{1/2}} = \frac{\cos\theta}{(10 + 6\sin\theta)^{1/2}}$$

$$\underline{\underline{\phi = \sin^{-1}\left(\frac{\cos\theta}{(10+6\sin\theta)^{1/2}}\right)}}$$

c) Evaluate for $\theta = 0°, 90°, 180°$.

$\theta = 0°$ $R = (10 + 6\sin 0°)^{1/2}$

$\phi = \sin^{-1}\left(\frac{\cos 0°}{(10+6\sin 0°)^{1/2}}\right)$

$\underline{\underline{R = 3.16\ \text{kN},\ \phi = 18.43°}}$

$\theta = 90°$ $R = (10 + 6\sin 90°)^{1/2}$

$\phi = \sin^{-1}\left(\frac{\cos 90°}{(10+6\sin 90°)^{1/2}}\right)$

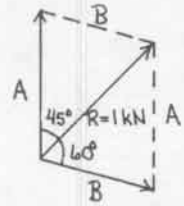
$\underline{\underline{R = 4\ \text{kN},\ \phi = 0°}}$

$\theta = 180°$ $R = (10 + 6\sin 180°)^{1/2}$

$\phi = \sin^{-1}\left(\frac{\cos 180°}{(10+6\sin 180°)^{1/2}}\right)$

$\underline{\underline{R = 3.16\ \text{kN},\ \phi = -18.43°}}$

2.16 b) Find magnitude of $\vec{A}, \vec{B}$ by trigonometry.

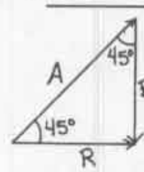


$$\frac{\sin 60°}{A} = \frac{\sin 45°}{B} = \frac{\sin(180° - 60° - 45°)}{1}$$

$A = 0.897\ \text{kN}$

$\underline{\underline{B = 0.732\ \text{kN}}}$

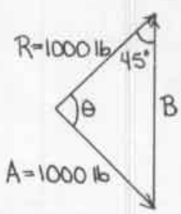
2.17 Find magnitudes of $\vec{A}, \vec{R}$.



$$\frac{\sin 45°}{10} = \frac{\sin 90°}{A} = \frac{\sin 45°}{R}$$

$\underline{\underline{A = 14.14\ \text{lb} \quad R = 10\ \text{lb}}}$

2.18 b) Find θ, B by trigonometry.



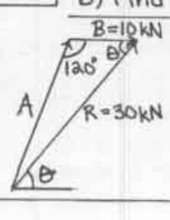
$A^2 = B^2 + R^2 - 2BR\cos 45°$

$1000^2 = B^2 + 1000^2 - 2000B(\cos 45°)$

$B^2 - 1414B = 0$ $\underline{\underline{B = 1414\ \text{lb}}}$

$\frac{\sin\theta}{1414} = \frac{\sin 45°}{1000}$ $\underline{\underline{\theta = 90°}}$

2.19 b) Find magnitude of $\vec{A}$ and direction of $\vec{R}$.



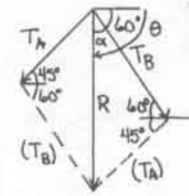
$R^2 = A^2 + B^2 - 2AB\cos 120°$

$30^2 = A^2 + 10^2 - 20A\cos 120°$

$A^2 + 10A - 800 = 0$ $\underline{\underline{A = 23.72\ \text{kN}}}$

$\frac{\sin 120°}{30} = \frac{\sin\theta}{23.72}$ $\underline{\underline{\theta = 43.22°}}$

2.20 b) Find magnitude and direction of $\vec{R}$ by trigonometry.



$R = (A^2 + B^2 - 2AB\cos 105°)^{1/2}$

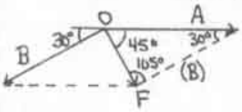
$R = (10^2 + 15^2 - 2(10)(15)\cos 105°)^{1/2}$

$\theta = 60° + \alpha$

$\dfrac{\sin\alpha}{10} = \dfrac{\sin 105°}{R} \qquad \alpha = 28.78°$

$R = 20.07\text{ kips},\ \theta = 88.78°$

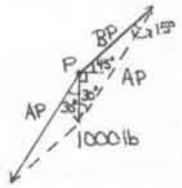
2.21 b) Find magnitudes of $\vec{A}, \vec{B}$ by trigonometry.



$\dfrac{\sin 105°}{A} = \dfrac{\sin 45°}{B} = \dfrac{\sin 30°}{F}$

$A = 1.93F \quad B = 1.41F$

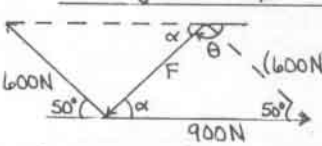
2.22 b) Find magnitudes of $\vec{AP}, \vec{BP}$ by trigonometry.



$\dfrac{\sin 15°}{1000} = \dfrac{\sin 135°}{AP} = \dfrac{\sin 30°}{BP}$

$AP = 1932\text{ lb} \quad BP = 2732\text{ lb}$

2.23 b) Find magnitude and direction of $\vec{F}$ by trigonometry.

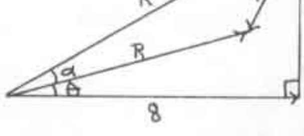


$F^2 = 900^2 + 600^2 - 2(900)(600)\cos 50°$

$\dfrac{\sin 50°}{F} = \dfrac{\sin\alpha}{600} \qquad \alpha = 41.78° \qquad \theta = -(180° - \alpha)$

$F = 689.8\text{ N},\ \theta = -138.2°$

2.24 b) Find $\vec{R}$ by trigonometry.



$(R')^2 = 8^2 + 5^2$

$R' = 9.43\text{ kips}$

$\phi = \tan^{-1}\left(\frac{8}{5}\right) - 30°$

$\phi = 28°$

$R^2 = (R')^2 + 4^2 - 2(R')(4)\cos\phi$

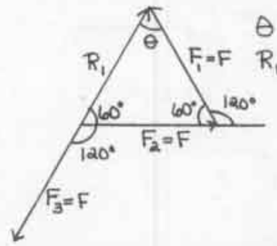
$R^2 = 9.43^2 + 4^2 - 2(9.43)(4)\cos 28° \quad R = 6.19\text{ kips}$

$\dfrac{\sin\alpha}{4} = \dfrac{\sin 28°}{6.19} \qquad \alpha = 17.7°$

$\theta = 90° - 30° - 28° - 17.7° = 14.3°$

$\vec{R} = 6.19\text{ kips},\ \theta = 14.3°$

2.25 Show that the resultant is 0.



$\theta = 180° - 60° - 60° = 60°$

$R_1 = (F^2 + F^2 - 2(F)(F)\cos 60)^{1/2} = F$

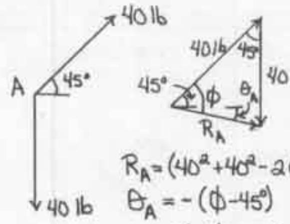
$R = \vec{F}_3 - \vec{R}_1$

$\vec{R}_1 = -\vec{F}_3$

$R = \vec{F}_3 - \vec{F}_3 = 0$

2.26 Find resultant forces exerted on A and B.

Peg A



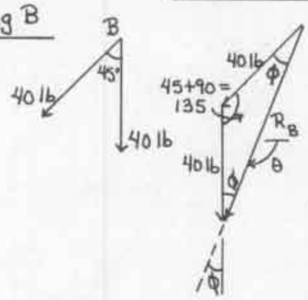
$R_A = (40^2 + 40^2 - 2(40)(40)\cos 45°)^{1/2}$

$\theta_A = -(\phi - 45°)$

$\dfrac{\sin\phi}{40} = \dfrac{\sin 45°}{R_A} \qquad \phi = 67.5°$

$\vec{R}_A = 30.6\text{ lb},\ \theta = -22.5°$

Peg B



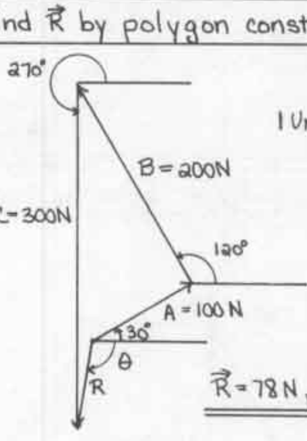
$R_B = (40^2 + 40^2 - 2(40^2)\cos 135)^{1/2}$

$\theta = -(90° + \phi);\ \phi = 22.5°$

$\dfrac{\sin\phi}{40} = \dfrac{\sin 135°}{R_B}$

$\vec{R}_B = 73.9\text{ lb},\ \theta = -112.5°$

2.27 Find $\vec{R}$ by polygon construction.

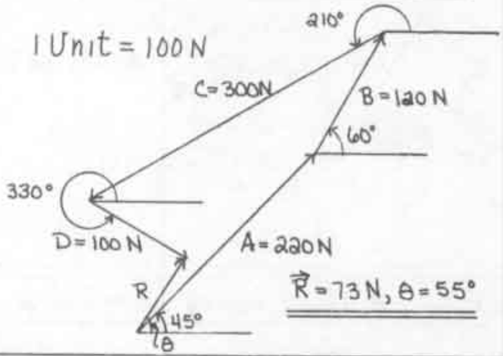


1 Unit = 100 N

$\vec{R} = 78\text{ N},\ \theta = -100°$

2.28 Find $\vec{R}$ by polygon construction.

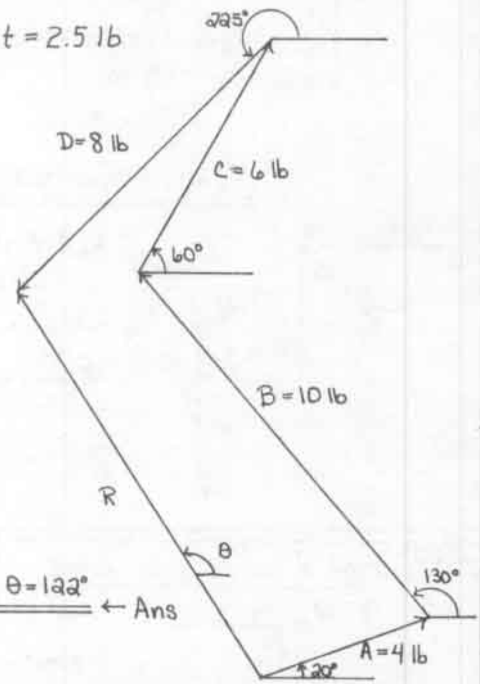
1 Unit = 100 N



$\vec{R}$ = 73 N, θ = 55°

2.29 Find $\vec{R}$ by polygon construction.

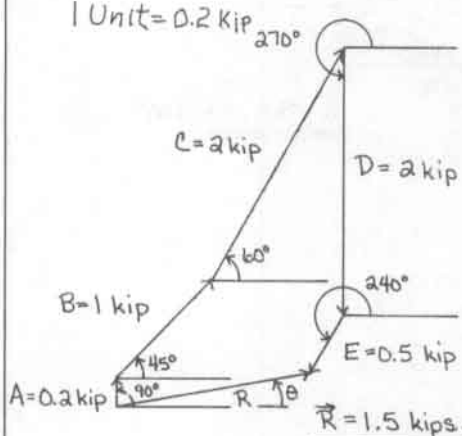
1 Unit = 2.5 lb



$\vec{R}$ = 10 lb, θ = 122° ← Ans

2.30 Find $\vec{R}$ by polygon construction.

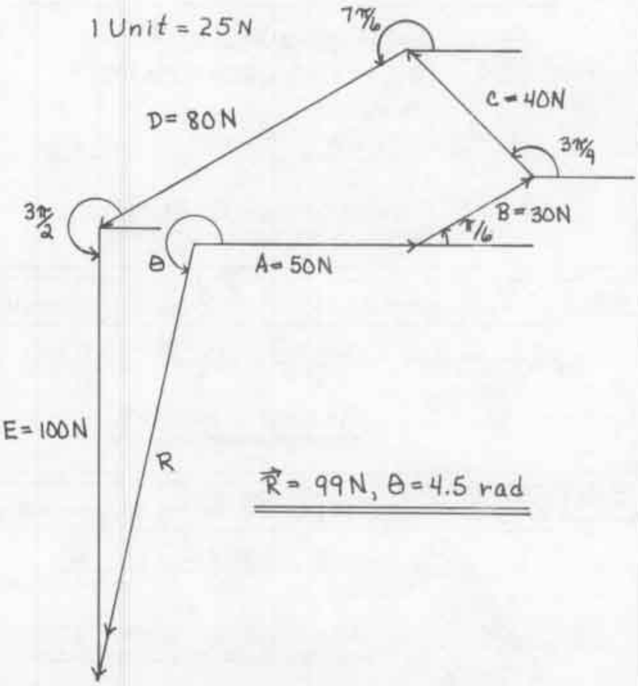
1 Unit = 0.2 kip



$\vec{R}$ = 1.5 kips, θ = 8°

2.31 Find $\vec{R}$ by polygon construction.

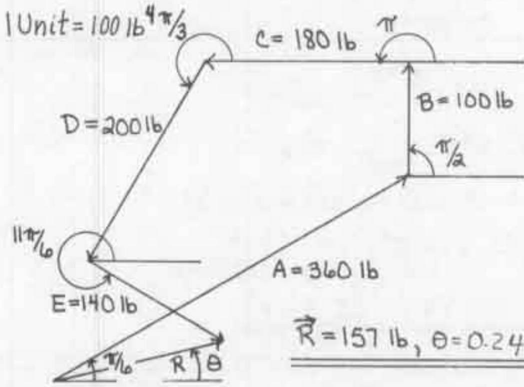
1 Unit = 25 N



$\vec{R}$ = 99 N, θ = 4.5 rad

2.32 Find $\vec{R}$ by polygon construction.

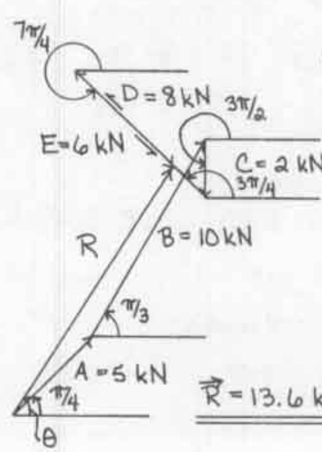
1 Unit = 100 lb



$\vec{R}$ = 157 lb, θ = 0.24 rad

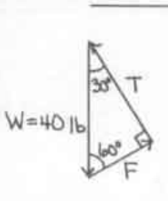
2.33 Find $\vec{R}$ by polygon construction.

1 Unit = 5 kN



$\vec{R}$ = 13.6 kN, θ = 1.0 rad

2.34 b) Find magnitudes of $\vec{F}, \vec{T}$ by trigonometry.



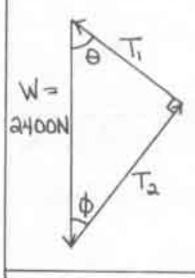
$T = W \sin 60° = 40 \sin 60°$

$\underline{\underline{T = 34.6 \text{ lb}}}$

$F = W \cos 60° = 40 \cos 60°$

$\underline{\underline{F = 20 \text{ lb}}}$

2.35 b) Find magnitudes of $\vec{T}_1, \vec{T}_2$ by trigonometry.



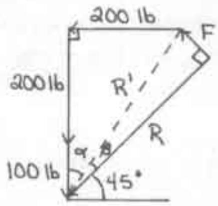
Since $10^2 = 8^2 + 6^2$

$\vec{T}_1$ is perpendicular to $\vec{T}_2$.

$T_1 = W \cos\theta = 2400\left(\frac{3}{5}\right) = \underline{1440 \text{ N}}$

$T_2 = W \cos\phi = 2400\left(\frac{4}{5}\right) = \underline{\underline{1920 \text{N}}}$

2.36 b) Find magnitudes of $\vec{R}, \vec{F}$ by trigonometry.



$R' = (200^2 + 300^2)^{1/2} = 360.56$

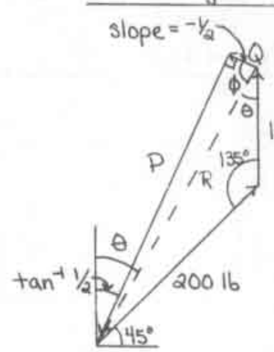
$\alpha = \tan^{-1}\left(\frac{200}{300}\right) = 33.69°$

$\beta = 90° - 33.69° - 45° = 11.31°$

$F = R' \sin 11.31° = \underline{\underline{70.71 \text{ lb}}}$

$R = R' \cos 11.31° = \underline{\underline{353.6 \text{ lb}}}$

2.37 Find magnitudes of $\vec{P}, \vec{Q}$.



$R = (100^2 + 200^2 - 2(100)(200)\cos 135)^{1/2}$

$R = 279.79 \text{ lb}$

$\frac{\sin\theta}{200} = \frac{\sin 135°}{279.79}$

$\theta = 30.4°$

$\phi = 90 - 30.4 + \tan^{-1}\left(\frac{1}{2}\right)$

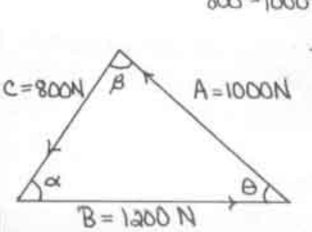
$\phi = 86.2°$

$P = R \sin 86.2° = \underline{\underline{279.2 \text{ lb}}}$

$Q = R \cos 86.2° = \underline{\underline{18.54 \text{ lb}}}$

2.38 b) Find interior angles by trigonometry.

$800^2 = 1000^2 + 1200^2 - 2(1000)(1200)\cos\theta$



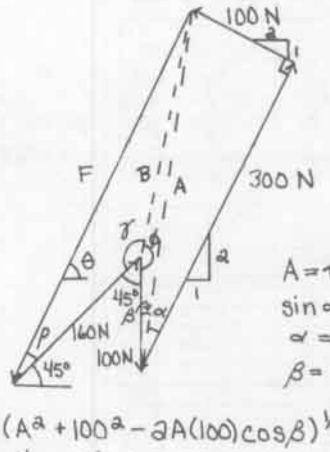
$\frac{\sin\alpha}{1000} = \frac{\sin\beta}{1200} = \frac{\sin\theta}{800}$

$\underline{\underline{\theta = 41.4°}}$

$\underline{\underline{\alpha = 55.8°}}$

$\underline{\underline{\beta = 82.8°}}$

2.39 Find $\vec{F}$ that makes resultant zero.



$A = \sqrt{300^2 + 100^2} = 316.23$

$\sin\alpha = 100/316.23$

$\alpha = 18.43°$

$\beta = \tan^{-1}\left(\frac{1}{2}\right) - \alpha = 8.13°$

$B = (A^2 + 100^2 - 2A(100)\cos\beta)^{1/2} = 217.69$

$\frac{\sin\phi}{A} = \frac{\sin\beta}{B} \qquad \phi = 180 - 11.85 = 168.15°$

$\gamma = 360 - 45 - \phi = 146.85°$

$F = (160^2 + B^2 - 2(160)(B)\cos\gamma)^{1/2} = 362.38$

$\frac{\sin\rho}{B} = \frac{\sin\gamma}{F} \qquad \rho = 19.18° \quad \theta = \rho + 45 = 64.18°$

$\underline{\underline{\vec{F} = 362.38\text{N}, \theta = 64.18°}}$

2.40 a) Develop spreadsheet to find resultant of concurrent, coplanar forces.

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2		magnitude	orientation, degrees	x projection	y projection
3	A			=B3*COS(C3*PI()/180)	=B3*SIN(C3*PI()/180)
4	B			=B4*COS(C4*PI()/180)	=B4*SIN(C4*PI()/180)
5	C			=B5*COS(C5*PI()/180)	=B5*SIN(C5*PI()/180)
6	D			=B6*COS(C6*PI()/180)	=B6*SIN(C6*PI()/180)
7	E			=B7*COS(C7*PI()/180)	=B7*SIN(C7*PI()/180)
8	R	=SQRT(D8^2+E8^2)	=ATAN(E8/D8)*180/PI()	=SUM(D3:D7)	=SUM(E3:E7)

b) Use spreadsheet to solve for 2.27-30.

Refer to problem solutions 2.27-30 for polygon construction.

NOTE: $\tan 60° = \tan 240° = \tan(60° + 180°)$

The computer will return values for orientation between -90° and 90°.

The actual orientation is either the returned value or the value + 180°.

2.27

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2		magnitude	orientation, degrees	x projection	y projection
3	A	100	30	86.60	50.00
4	B	200	120	-100.00	173.21
5	C	300	270	0	-300.00
6	D	0	0	0	0
7	E	0	0	0	0
8	R	77.95	80.10	-13.40	-76.79

$\theta = 80.10° + 180°$

$= 260.10°$

$= -99.9°$

2.28

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2		magnitude	orientation, degrees	x projection	y projection
3	A	220	45	155.56	155.56
4	B	120	60	60.00	103.92
5	C	300	210	-259.81	-150.00
6	D	100	330	86.60	-50.00
7	E	0	0	0	0
8	R	73.03	54.55	42.36	59.49

2.40 cont.

2.29

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2		magnitude	orientation, degrees	x projection	y projection
3	A	4	20	3.76	1.37
4	B	10	130	-6.43	7.66
5	C	6	60	3.00	5.20
6	D	8	225	-5.66	-5.66
7	E	0	0	0	0
8	R	10.09	-58.13	-5.33	8.57

$\theta = -58.13^\circ + 180^\circ$
$= 121.87^\circ$

2.30

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2		magnitude	orientation, degrees	x projection	y projection
3	A	0.2	90	0.000	0.200
4	B	1.0	45	0.707	0.707
5	C	2.0	60	1.000	1.732
6	D	2.0	270	0.000	-2.000
7	E	0.5	240	-0.250	-0.433
8	R	1.472	8.053	1.457	0.206

2.41 a) Develop spreadsheet to find resultant of concurrent, coplanar forces.

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2		magnitude	orientation, radians	x projection	y projection
3	A			=B3*COS(C3)	=B3*SIN(C3)
4	B			=B4*COS(C4)	=B4*SIN(C4)
5	C			=B5*COS(C5)	=B5*SIN(C5)
6	D			=B6*COS(C6)	=B6*SIN(C6)
7	E			=B7*COS(C7)	=B7*SIN(C7)
8	R	=SQRT(D8^2+E8^2)	=ATAN(E8/D8)	=SUM(D3:D7)	=SUM(E3:E7)

b) Use spreadsheet to solve for 2.31-33.

Refer to problem solutions 2.31-33 for polygon construction.

NOTE: $\tan \pi/3 = \tan 4\pi/3 = \tan(\pi/3 + \pi)$
The computer will return values for orientation between $-\pi/2$ and $\pi/2$.
The actual orientation is either the returned value or the value $+\pi$.

2.31

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2		magnitude	orientation, radians	x projection	y projection
3	A	50	0.00	50.00	0.00
4	B	30	0.52	25.98	15.00
5	C	40	2.36	-28.28	28.28
6	D	80	3.67	-69.28	-40.00
7	E	100	4.71	0.00	-100.00
8	R	99.10	1.35	-21.59	-96.72

$\theta = 1.35 + \pi$
$= 4.49$ rad

2.32

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2		magnitude	orientation, radians	x projection	y projection
3	A	360	0.52	311.77	180.00
4	B	100	1.57	0.00	100.00
5	C	180	3.14	-180.00	0.00
6	D	200	4.19	-100.00	-173.21
7	E	140	5.76	121.24	-70.00
8	R	157.37	0.24	153.01	36.79

2.33

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2		magnitude	orientation, radians	x projection	y projection
3	A	5	0.79	3.54	3.54
4	B	10	1.05	5.00	8.66
5	C	2	4.71	0.00	-2.00
6	D	8	2.36	-5.66	5.66
7	E	6	5.50	4.24	-4.24
8	R	13.62	1.02	7.12	11.61

2.42 Reduce vector equations to simplest form.

a) $2(3\vec{A} - \vec{B}) - \vec{C} + 6(\vec{B} - 4\vec{C}) = 0$
$6\vec{A} - 2\vec{B} - \vec{C} + 6\vec{B} - 24\vec{C} = 0$
$\underline{\underline{6\vec{A} + 4\vec{B} - 25\vec{C} = 0}}$

b) $\vec{F} = 3[4(\vec{A} + 2\vec{B} + 3\vec{C}) - (\vec{A} + 5\vec{C}) - 16\vec{B}]$
$\vec{F} = 12\vec{A} + 24\vec{B} + 36\vec{C} - 3\vec{A} - 15\vec{C} - 48\vec{B}$
$\underline{\underline{\vec{F} = 9\vec{A} - 24\vec{B} + 21\vec{C}}}$

c) $13\vec{A} + 7\vec{B} - 6(4\vec{B} - 3\vec{A}) = 0$
$13\vec{A} + 7\vec{B} - 24\vec{B} + 18\vec{A} = 0$
$\underline{\underline{31\vec{A} - 17\vec{B} = 0}}$

2.43 Find magnitudes and directions of a) $6.0\vec{A}$, b) $-5.0\vec{A}$.

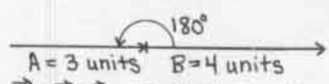
a) $\vec{F} = 6.0\vec{A}$ $\quad F = 6(5) = \underline{\underline{30,\ \theta = 90^\circ}}$

b) $\vec{F} = -5.0\vec{A}$ $\quad F = |-5|(5) = \underline{\underline{25,\ \theta = 270^\circ}}$

2.44 a) Yes. Two concurrent forces with equal magnitudes but opposite senses have a zero resultant.
b) No. It is not possible.
c) Yes. Polygon construction yeilds an equilateral triangle.
d) Yes. As long as the polygon construction yeilds a complete triangle, the resultant of the vectors is zero.

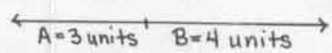
2.45 Show how $\vec{A}$ and $\vec{B}$ can combine to form $\vec{R}$ with magnitudes (units) a) 7, b) 1, c) 5, d) any magnitude between 1 and 7.

a)



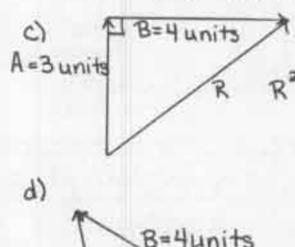
$\vec{R} = \vec{A} + \vec{B}$ $\quad R = 3 + 4 = 7$ units

b)



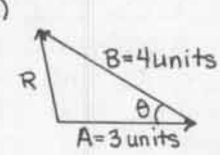
$R = -3 + 4 = 1$ unit

c)



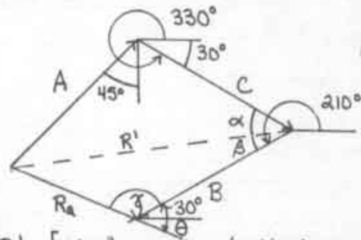
$R^2 = 3^2 + 4^2$ $\quad R = 5$ units

d)



$R = (3^2 + 4^2 - 2(3)(4)\cos\theta)^{1/2}$
$R = (25 - 24\cos\theta)^{1/2}$
$0 \le \theta \le 180^\circ$

2.46 a) Find $\vec{A}+\vec{B}+\vec{C}$.



A = B = C = 100 units

$R' = [100^2 + 100^2 - 2(100)(100)\cos(45° + (90°-30°))]^{1/2}$

$R' = 158.67$ units

$\frac{\sin\alpha}{100} = \frac{\sin(45°+60°)}{158.67}$ $\quad \alpha = 37.5°$

$\beta = 210° - (37.5° + 150°) = 22.5°$

$R_a = (158.67^2 + 100^2 - 2(158.67)(100)\cos 22.5)^{1/2}$

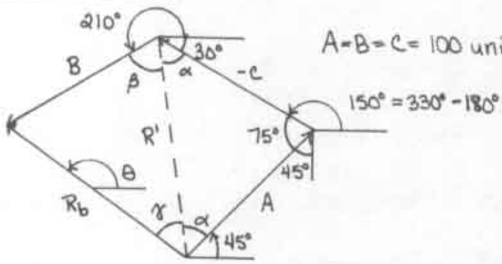
$R_a = 76.54$ units

$\frac{\sin\gamma}{158.67} = \frac{\sin 22.5°}{76.54}$ $\quad \gamma = 180° - 52.5° = 127.5°$

$\theta = -(180° - 127.5° - 30°) = -22.5°$

$\vec{R}_a = 76.54$ units, $\theta = -22.5°$

b) Find $\vec{A}+\vec{B}-\vec{C}$.



A = B = C = 100 units

$R' = (100^2 + 100^2 - 2(100)(100)\cos 75°)^{1/2} = 121.75$ units

$\frac{\sin\alpha}{100} = \frac{\sin 75°}{121.75}$ $\quad \alpha = 52.5°$

$\beta = 360° - 210° - 30° - 52.5° = 67.5°$

$R_b = (100^2 + 121.75^2 - 2(100)(121.75)\cos 67.5°)^{1/2}$

$R_b = 124.5$ units

$\frac{\sin\gamma}{100} = \frac{\sin 67.5}{124.5}$ $\quad \gamma = 47.9°$

$\theta = 45° + 52.5° + 47.9° = 145.4°$

$\vec{R}_b = 124.5$ units, $\theta = 145.4°$

c) Find $\vec{D}$ such that $\vec{A}+\vec{B}+\vec{C}+\vec{D} = 0$.

$\vec{D} = -\vec{R}_a$ from (a), therefore, $\vec{D}$ is 180° from $\vec{R}_a$

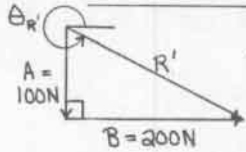
$\vec{D} = 76.54$ units, $\theta = -22.5° + 180° = 157.5°$

d) Find $\vec{D}$ such that $\vec{A}+\vec{B}-\vec{C}-\vec{D} = 0$.

$\vec{D} = \vec{R}_b$ from (b)

$\vec{D} = 124.5$ units, $\theta = 145.4°$

2.47 a) Find $(\vec{A}+\vec{B})+\vec{C}$.

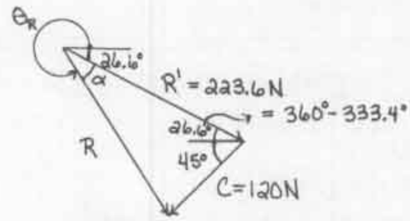


$\vec{R}' = \vec{A} + \vec{B}$

$R' = \sqrt{A^2 + B^2} = 223.6$ N

$\theta_{R'} = 270 + \tan^{-1}\left(\frac{B}{A}\right)$

$= 270 + \tan^{-1}\left(\frac{200}{100}\right) = 333.4°$



$R = (223.6^2 + 120^2 - 2(223.6)(120)\cos(45° + 26.6°))^{1/2}$

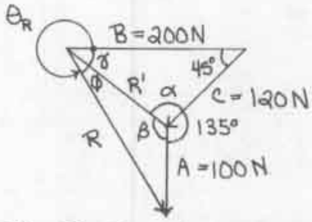
$R = 217.8$ N

$\frac{\sin\alpha}{120} = \frac{\sin 71.6°}{217.8}$ $\quad \alpha = 31.5°$

$\theta_R = 360° - 31.5° - 26.6° = 301.9°$

$\vec{R} = 217.8$ N, $\theta = 301.9°$

b) Find $(\vec{B}+\vec{C})+\vec{A}$.



$\vec{R}' = \vec{B} + \vec{C}$ $\quad R' = (200^2 + 120^2 - 2(200)(120)\cos 45°)^{1/2}$

$R' = 143.0$

$\frac{\sin\alpha}{200} = \frac{\sin 45°}{143}$ $\quad \alpha = 180° - 81.4°$, $\alpha = 98.6°$ $\quad \beta = 360° - 135° - 98.6°$, $\beta = 126.4°$

$\vec{R} = \vec{R}' + \vec{A}$ $\quad R = (143^2 + 100^2 - 2(143)(100)\cos 126.4°)^{1/2}$

$R = 217.8$ N

$\gamma = 180° - 45° - 98.6° = 36.4°$

$\frac{\sin\phi}{100} = \frac{\sin 126.4°}{217.8}$ $\quad \phi = 21.7°$

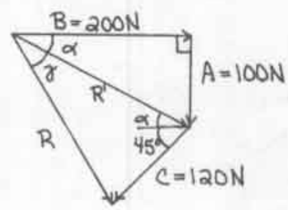
$\theta_R = 360° - 36.4° - 21.7° = 301.9°$

$\vec{R} = 217.8$ N, $\theta_R = 301.9°$

c) Find $\vec{A}+\vec{B}+\vec{C}$.

Same steps as part (a).

d) Find $\vec{B}+\vec{A}+\vec{C}$.



$\vec{R}' = \vec{B} + \vec{A}$

$R' = \sqrt{200^2 + 100^2}$

$= 223.6$ N

$\alpha = \tan^{-1}\left(\frac{100}{200}\right) = 26.56°$

2.47 cont.

$R = (223.6^2 + 120^2 - 2(223.6)(120)\cos(26.56^\circ + 45^\circ))^{1/2}$

$R = 217.8\text{ N}$

$\dfrac{\sin\gamma}{120} = \dfrac{\sin(26.56^\circ + 45^\circ)}{217.8}$ $\quad \gamma = 31.5^\circ$

$\theta_R = 360^\circ - 26.6^\circ - 31.5^\circ = 301.9^\circ$

$\vec{R} = 217.8\text{ N},\ \theta_R = 301.9^\circ$

2.48 a) Derive formulas for R and ϕ.

$A: (A, 0)\quad B: (B\cos\theta,\ B\sin\theta)$

$R: (A + B\cos\theta,\ B\sin\theta)$

$R = [(A + B\cos\theta)^2 + (B\sin\theta)^2]^{1/2}$

$R = [(A^2 + 2AB\cos\theta + B^2\cos^2\theta) + (B^2\sin^2\theta)]^{1/2}$

$\cos^2\theta + \sin^2\theta = 1$

$R = (A^2 + B^2 + 2AB\cos\theta)^{1/2}$

$\phi = \tan^{-1}\left(\dfrac{R_y}{R_x}\right) = \tan^{-1}\left(\dfrac{B\sin\theta}{A + B\cos\theta}\right)$

b) Check answers with $\theta = 0^\circ, 90^\circ, 180^\circ$.

$\theta = 0^\circ$ $\quad R = (A^2 + B^2 + 2AB\cos 0)^{1/2} = A + B$

$\phi = \tan^{-1}\left(\dfrac{B\sin 0^\circ}{A + B\cos 0^\circ}\right) = 0^\circ$

$\theta = 90^\circ$ $\quad R = (A^2 + B^2 + 2AB\cos 90^\circ)^{1/2} = (A^2 + B^2)^{1/2}$

$\phi = \tan^{-1}\left(\dfrac{B\sin 90^\circ}{A + B\cos 90^\circ}\right) = \tan^{-1}\left(\dfrac{B}{A}\right)$

$\theta = 180^\circ$ $\quad R = (A^2 + B^2 + 2AB\cos 180^\circ)^{1/2} = A - B$

$\phi = \tan^{-1}\left(\dfrac{B\sin 180^\circ}{A + B\cos 180^\circ}\right) = 0^\circ$

2.49 Find $\vec{R}$.

i) $R_x = 400 + 300\cos 60^\circ = 550$

$R_y = 300\sin 60^\circ = 259.8$

$R = (550^2 + 259.8^2)^{1/2} = 608.3\text{ lb}$

$\theta = \tan^{-1}\left(\dfrac{259.8}{550}\right) = 25.3^\circ$

$\vec{R} = 608.3\text{ lb},\ \theta = 25.3^\circ$

ii) $R_x = 500 + 500\cos 120^\circ = 250$

$R_y = 500\sin 120^\circ = 433$

$R = (250^2 + 433^2)^{1/2} = 500\text{ N}$

$\theta = \tan^{-1}\left(\dfrac{433}{250}\right) = 60^\circ$

$\vec{R} = 500\text{ N},\ \theta = 60^\circ$

iii) $R_x = 200 \quad R_y = 100$

$R = (200^2 + 100^2)^{1/2} = 223.6\text{ kN}$

$\theta = \tan^{-1}\left(\dfrac{100}{200}\right) = 26.6^\circ$

$\vec{R} = 223.6\text{ kN},\ \theta = 26.6^\circ$

2.49 cont.

iv) $R_x = 2 + \cos 135^\circ = 1.29 \quad R_y = -\sin 135^\circ = -0.71$

$R = (1.29^2 + (-0.71)^2)^{1/2} = 1.47\text{ tons}$

$\theta = \tan^{-1}\left(\dfrac{-0.71}{1.29}\right) = -28.8^\circ$

$\vec{R} = 1.47\text{ tons},\ \theta = -28.8^\circ$

2.50 a) Derive a formula for the magnitude of $\vec{R}$.

$R_x = 3 + \cos(180^\circ + \theta) \quad R_y = \sin(180^\circ + \theta)$

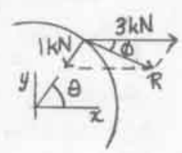
$R_x = 3 - \cos\theta \quad R_y = -\sin\theta$

$R = ((3 - \cos\theta)^2 + (-\sin\theta)^2)^{1/2}$

$R = (9 - 6\cos\theta + \cos^2\theta + \sin^2\theta)^{1/2}$

$\cos^2\theta + \sin^2\theta = 1$

$R = (10 - 6\cos\theta)^{1/2}$



b) Derive a formula for ϕ.

$\phi = \tan^{-1}\left(\dfrac{-\sin\theta}{3 - \cos\theta}\right)$

c) Evaluate for $\theta = 0^\circ, 90^\circ, 180^\circ$.

$\theta = 0^\circ$ $\quad R = (10 - 6\cos 0^\circ)^{1/2} = 2\text{ kN}$

$\phi = \tan^{-1}\left(\dfrac{-\sin 0^\circ}{3 - \cos 0^\circ}\right) = 0^\circ$

$\theta = 90^\circ$ $\quad R = (10 - 6\cos 90^\circ)^{1/2} = \sqrt{10}\text{ kN}$

$\phi = \tan^{-1}\left(\dfrac{-\sin 90^\circ}{3 - \cos 90^\circ}\right) = -18.43^\circ$

$\theta = 180^\circ$ $\quad R = (10 - 6\cos 180^\circ)^{1/2} = 4\text{ kN}$

$\phi = \tan^{-1}\left(\dfrac{-\sin 180^\circ}{3 - \cos 180^\circ}\right) = 0^\circ$

2.51 a) Derive formula for the magnitude of $\vec{R}$.

$R_x = -3 + \cos(180^\circ + \theta) \quad R_y = \sin(180^\circ + \theta)$

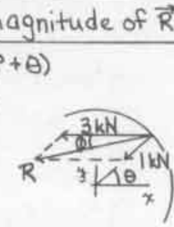
$R_x = -3 - \cos\theta \quad R_y = -\sin\theta$

$R = ((-3 - \cos\theta)^2 + (-\sin\theta)^2)^{1/2}$

$R = (9 + 6\cos\theta + \cos^2\theta + \sin^2\theta)^{1/2}$

$\cos^2\theta + \sin^2\theta = 1$

$R = (10 + 6\cos\theta)^{1/2}$



b) Derive formula for ϕ.

$\phi = \tan^{-1}\left(\dfrac{-\sin\theta}{-3 - \cos\theta}\right)$

c) Evaluate for $\theta = 0^\circ, 90^\circ, 180^\circ$.

$\theta = 0^\circ$ $\quad R = (10 + 6\cos 0^\circ)^{1/2} = 4\text{ kN}$

$\phi = \tan^{-1}\left(\dfrac{-\sin 0^\circ}{-3 - \cos 0^\circ}\right) = 0^\circ$

2.51 cont.

$\theta = 90^\circ \quad R = (10 + 6\cos 90^\circ)^{1/2} = \underline{\underline{3.162 \text{ kN}}}$

$$\phi = \tan^{-1}\left(\frac{-\sin 90^\circ}{-3 - \cos 90^\circ}\right) = \underline{\underline{18.43^\circ}}$$

$\theta = 180^\circ \quad R = (10 + 6\cos 180^\circ)^{1/2} = \underline{\underline{2 \text{ kN}}}$

$$\phi = \tan^{-1}\left(\frac{-\sin 180^\circ}{-3 - \cos 180^\circ}\right) = \underline{\underline{0^\circ}}$$

2.52 a) Derive formula for the magnitude of $\vec{R}$.

$R_x = \cos(180^\circ + \theta) \qquad R_y = \sin(180^\circ + \theta) + 3$

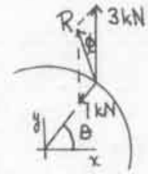
$R_x = -\cos\theta \qquad R_y = 3 - \sin\theta$

$R = ((-\cos\theta)^2 + (3 - \sin\theta)^2)^{1/2}$

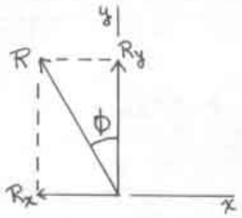
$R = (\cos^2\theta + 9 - 6\sin\theta + \sin^2\theta)^{1/2}$

$\cos^2\theta + \sin^2\theta = 1$

$\underline{\underline{R = (10 - 6\sin\theta)^{1/2}}} \leftarrow$ Ans



b) Derive formula for ϕ.



$$\phi = \sin^{-1}\left(\frac{|R_x|}{R}\right)$$

$$\underline{\underline{\phi = \sin^{-1}\left(\frac{\cos\theta}{\sqrt{10 - 6\sin\theta}}\right)}}$$

c) Evaluate for $\theta = 0^\circ, 90^\circ, 180^\circ$.

$\theta = 0^\circ \quad R = (10 - 6\sin 0^\circ)^{1/2} = \underline{\underline{3.162 \text{ kN}}}$

$$\phi = \sin^{-1}\left(\frac{\cos 0^\circ}{\sqrt{10 - 6\sin 0^\circ}}\right) = \underline{\underline{18.43^\circ}}$$

$\theta = 90^\circ \quad R = (10 - 6\sin 90^\circ)^{1/2} = \underline{\underline{2 \text{ kN}}}$

$$\phi = \sin^{-1}\left(\frac{\cos 90^\circ}{\sqrt{10 - 6\sin 90^\circ}}\right) = \underline{\underline{0^\circ}}$$

$\theta = 180^\circ \quad R = (10 - 6\sin 180^\circ)^{1/2} = \underline{\underline{3.162 \text{ kN}}}$

$$\phi = \sin^{-1}\left(\frac{\cos 180^\circ}{\sqrt{10 - 6\sin 180^\circ}}\right) = \underline{\underline{-18.43^\circ}}$$

2.53 a) Derive formula for the magnitude of $\vec{R}$.

$R_x = \cos(180^\circ + \theta) \qquad R_y = \sin(180^\circ + \theta) - 3$

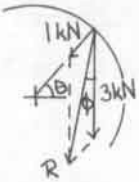
$R_x = -\cos\theta \qquad R_y = -\sin\theta - 3$

$R = ((-\cos\theta)^2 + (-\sin\theta - 3)^2)^{1/2}$

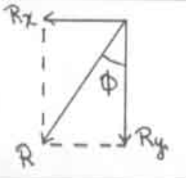
$R = (\cos^2\theta + \sin^2\theta + 6\sin\theta + 9)^{1/2}$

$\cos^2\theta + \sin^2\theta = 1$

$\underline{\underline{R = (10 + 6\sin\theta)^{1/2}}}$



b) Derive formula for ϕ.



$$\phi = \sin^{-1}\left(\frac{|R_x|}{R}\right)$$

$$\underline{\underline{\phi = \sin^{-1}\left(\frac{\cos\theta}{\sqrt{10 + 6\sin\theta}}\right)}}$$

2.53 cont. c) Evaluate for $\theta = 0^\circ, 90^\circ, 180^\circ$.

$\theta = 0^\circ \quad R = (10 + 6\sin 0^\circ)^{1/2} = \underline{\underline{3.162 \text{ kN}}}$

$$\phi = \sin^{-1}\left(\frac{\cos 0^\circ}{\sqrt{10 + 6\sin 0^\circ}}\right) = \underline{\underline{18.43^\circ}}$$

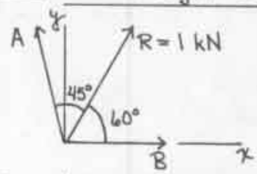
$\theta = 90^\circ \quad R = (10 + 6\sin 90^\circ)^{1/2} = \underline{\underline{4 \text{ kN}}} \leftarrow$ Ans

$$\phi = \sin^{-1}\left(\frac{\cos 90^\circ}{\sqrt{10 + 6\sin 90^\circ}}\right) = \underline{\underline{0^\circ}}$$

$\theta = 180^\circ \quad R = (10 + 6\sin 180^\circ)^{1/2} = \underline{\underline{3.162 \text{ kN}}}$

$$\phi = \sin^{-1}\left(\frac{\cos 180^\circ}{\sqrt{10 + 6\sin 180^\circ}}\right) = \underline{\underline{-18.43^\circ}}$$

2.54 Find magnitudes of $\vec{A}, \vec{B}$.



$R_x = A_x + B_x = A\cos 105^\circ + B$

$R_y = A_y + B_y = A\sin 105^\circ$

$R_x = 1\cos 60^\circ \quad R_y = 1\sin 60^\circ$

$A\cos 105^\circ + B = \cos 60^\circ$

$A\sin 105^\circ = \sin 60^\circ$

$\underline{\underline{A = 0.897 \text{ kN} \quad B = 0.732 \text{ kN}}}$

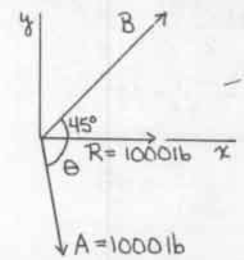
2.55 Find magnitudes of $\vec{A}, \vec{R}$.

$R_x = A_x + B_x = A\cos 45^\circ = R$

$R_y = A_y + B_y = A\sin 45 - 10 = 0$

$\underline{\underline{A = 14.14 \text{ lb} \quad R = 10 \text{ lb}}}$

2.56 Find θ and magnitude of $\vec{B}$.

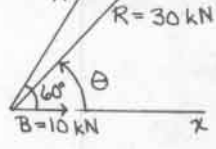


$R_x = 1000 = 1000\cos\theta + B\cos 45^\circ$

$R_y = 0 = -1000\sin\theta + B\sin 45^\circ$

$\underline{\underline{B = 1414 \text{ lb} \quad \theta = 90^\circ}}$

2.57 Find θ and magnitude of $\vec{A}$.

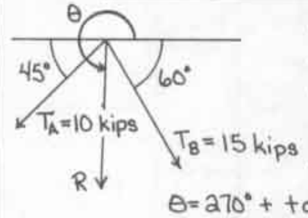


$R_x = 30\cos\theta = 10 + A\cos 60^\circ$

$R_y = 30\sin\theta = A\sin 60^\circ$

$\underline{\underline{A = 23.72 \text{ kN}, \quad \theta = 43.2^\circ}}$

2.58 Find magnitude and direction of $\vec{R}$.



$R_x = -10\cos 45^\circ + 15\cos 60^\circ$

$R_x = 0.429$

$R_y = -10\sin 45^\circ - 15\sin 60^\circ$

$R_y = -20.06$

$R = \sqrt{0.429^2 + (-20.06)^2} = 20.06$

$\theta = 270^\circ + \tan^{-1}(0.429/20.06) = 271.2^\circ$

$\vec{R} = 20.06$ kips, $\theta = 271.2^\circ$

2.59 Find $\vec{R}$.

$R_x = 100\cos 45^\circ + 200\cos 30^\circ - 150\cos 60^\circ - 100\cos 30^\circ$
$= 82.31$

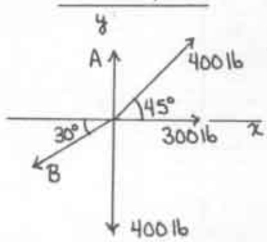
$R_y = -100\sin 45^\circ + 200\sin 30^\circ + 150\sin 60^\circ + 100\sin 30^\circ$
$= 209.2$

$R = \sqrt{82.31^2 + 209.2^2} = 224.8$

$\theta = \tan^{-1}(209.2/82.31) = 68.5^\circ$

$\vec{R} = 224.8$ lb, $\theta = 68.5^\circ$

2.60 Find $\vec{A}, \vec{B}$.

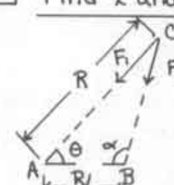


$R_x = 0 = 300 + 400\cos 45^\circ - B\cos 30^\circ$

$R_y = 0 = -400 + 400\sin 45^\circ + A - B\sin 30^\circ$

$A = 453.7$ lb $B = 673.0$ lb

2.61 Find x and y projections of $\vec{F}_1, \vec{F}_2$.



$m_A = m_B = m \quad m_C = m/2$

$F_1 = \dfrac{km(m/2)}{R^2} = \dfrac{km^2}{2R^2}$

$F_{1x} = \dfrac{-km^2}{2R^2}\cos\theta$

$F_{1y} = \dfrac{-km^2}{2R^2}\sin\theta$

$\overline{CB} = (R^2 + (R/2)^2 - 2R(R/2)\cos\theta)^{1/2}$
$= (R^2 + R^2/4 - R^2\cos\theta)^{1/2} = R(1.25 - \cos\theta)^{1/2}$

$F_2 = \dfrac{km(m/2)}{R^2(1.25-\cos\theta)} = \dfrac{km^2}{2R^2(1.25-\cos\theta)}$

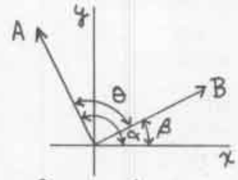
$\alpha: \dfrac{\sin\alpha}{R} = \dfrac{\sin\theta}{R(1.25-\cos\theta)^{1/2}}$

$\alpha = \sin^{-1}\left(\dfrac{\sin\theta}{(1.25-\cos\theta)^{1/2}}\right)$

$F_{2x} = \dfrac{-km^2}{2R^2(1.25-\cos\theta)^{1/2}}\cos\left[\sin^{-1}\left(\dfrac{\sin\theta}{(1.25-\cos\theta)^{1/2}}\right)\right]$

$F_{2y} = \dfrac{km^2}{2R^2(1.25-\cos\theta)^{1/2}}\left(\dfrac{\sin\theta}{(1.25-\cos\theta)^{1/2}}\right)$

2.62 Prove that $\vec{A}, \vec{B}$ are perpendicular to each other.



$\theta = \alpha - \beta$

$A_x = A\cos\alpha \quad A_y = A\sin\alpha$

$B_x = B\cos\beta \quad B_y = B\sin\beta$

$A_xB_x + A_yB_y = 0$

$AB\cos\alpha\cos\beta + AB\sin\alpha\sin\beta = 0$

$\cos\alpha\cos\beta + \sin\alpha\sin\beta = 0$

$\cos(\alpha-\beta) = 0$

$\therefore \cos\theta = 0 \quad \theta = \cos^{-1} 0$

or $\theta = 90^\circ, 270^\circ$

$\therefore \vec{A}, \vec{B}$ are perpendicular

2.63 Find θ.

$F_x = F\cos\theta \quad F_y = F\cos\theta \quad F_z = F\cos\theta$

$F = (F_x^2 + F_y^2 + F_z^2)^{1/2} = (3F^2\cos^2\theta)^{1/2} = \sqrt{3}F\cos\theta$

$1 = \sqrt{3}\cos\theta \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7^\circ$

2.64 Find $\vec{R} = (R_x, R_y, R_z)$.

$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x,\ A_y + B_y,\ A_z + B_z)$

$\vec{R} = (7-5, -2+6, -4+8) = (2, 4, 4)$

2.65 a) Find direction cosines of $\vec{R}$.

$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x,\ A_y + B_y,\ A_z + B_z)$
$= (6-4, -1+5, -3+7) = (2, 4, 4)$

$R = (2^2 + 4^2 + 4^2)^{1/2} = 6$

$\cos\theta_x = \dfrac{R_x}{R} = \dfrac{2}{6} = \dfrac{1}{3}$

$\cos\theta_y = \dfrac{R_y}{R} = \dfrac{4}{6} = \dfrac{2}{3}$

$\cos\theta_z = \dfrac{R_z}{R} = \dfrac{4}{6} = \dfrac{2}{3}$

b) Find angles that $\vec{R}$ forms with axes (x, y, z).

$\theta_x = \cos^{-1}(1/3) = 70.5^\circ \quad \theta_y = \theta_z = \cos^{-1}(2/3) = 48.2^\circ$

2.66 a) Find $\vec{R}$.

$\cos\theta_{Bx} = \dfrac{B_x}{100} = \dfrac{-4}{5};\ B_x = -80$ N

$\cos\theta_{By} = \dfrac{B_y}{100} = \dfrac{3}{5};\ B_y = 60$ N

$\cos\theta_{Bz} = \dfrac{B_z}{100} = \dfrac{0}{100};\ B_z = 0$

$\cos\theta_{Cx} = \dfrac{C_x}{100} = \dfrac{-4}{\sqrt{74}};\ C_x = -46.50$ N

$\cos\theta_{Cy} = \dfrac{C_y}{100} = \dfrac{3}{\sqrt{74}};\ C_y = 34.87$ N

$\cos\theta_{Cz} = \dfrac{C_z}{100} = \dfrac{7}{\sqrt{74}};\ C_z = 81.37$ N

2.66 cont. $\vec{R} = \vec{B} + \vec{C}$

$\vec{R} = (-80-46.50,\ 60+34.87,\ 0+81.37)$

$\vec{R} = (-126.50,\ 94.87,\ 81.37)$

b) Find direction angles of $\vec{R}$.

$R = ((-126.5)^2 + 94.87^2 + 81.37^2)^{1/2} = 177.8$

$\theta_x = \cos^{-1}\left(\frac{-126.5}{177.8}\right) = \underline{135.3^\circ}$

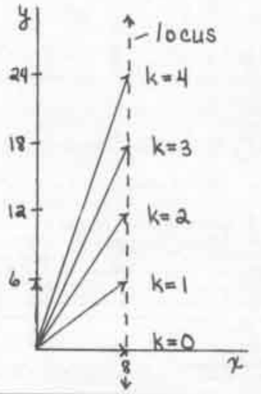
$\theta_y = \cos^{-1}\left(\frac{94.87}{177.8}\right) = \underline{57.76^\circ}$

$\theta_z = \cos^{-1}\left(\frac{81.37}{177.8}\right) = \underline{62.77^\circ}$

2.67 Find tip of $\vec{R}$ (point P) and plot location as k varies.

$\vec{R} = \vec{A} + k\vec{B} = (A_x + kB_x,\ A_y + kB_y) = (8,\ 6k)$

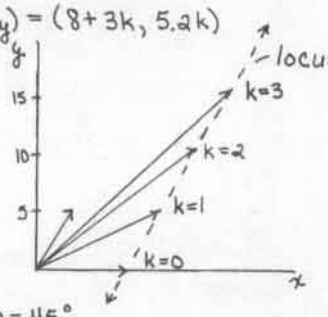
$k=0$ $\vec{R} = (8,0)$
$k=1$ $\vec{R} = (8,6)$
$k=2$ $\vec{R} = (8,12)$
$k=3$ $\vec{R} = (8,18)$
$k=4$ $\vec{R} = (8,24)$



2.68 a) Solve P2.67 for $\theta = 60^\circ$.

$\vec{R} = (A_x + kB_x,\ A_y + kB_y) = (8+3k,\ 5.2k)$

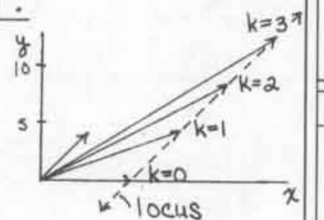
$k=0$ $\vec{R} = (8,0)$
$k=1$ $\vec{R} = (11,5.2)$
$k=2$ $\vec{R} = (14,10.4)$
$k=3$ $\vec{R} = (17,15.6)$



b) Solve P2.67 for $\theta = 45^\circ$.

$\vec{R} = (8+4.24k,\ 4.24k)$

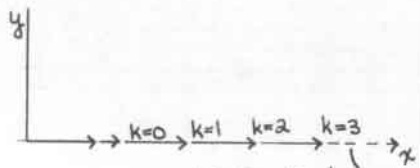
$k=0$ $\vec{R} = (8,0)$
$k=1$ $\vec{R} = (12.24,\ 4.24)$
$k=2$ $\vec{R} = (16.48,\ 8.48)$
$k=3$ $\vec{R} = (20.72,\ 12.72)$



c) Solve P2.67 for $\theta = 0^\circ$

$\vec{R} = (8+6k,\ 0)$

$k=0$ $\vec{R} = 8$
$k=1$ $\vec{R} = 14$
$k=2$ $\vec{R} = 20$
$k=3$ $\vec{R} = 26$



x-axis is locus

2.69 Find magnitudes of $\vec{u}, \vec{v}$.

$R_x = \frac{3}{5}(100) - 60 - u\cos 45^\circ - v\cos 45^\circ = 0$

$R_y = \frac{4}{5}(100) - 100 + u\sin 45^\circ - v\sin 45^\circ = 0$

$u = 14.14\ N \quad v = -14.14\ N$

2.70 a) Find $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$.

$F_x = 100\cos 45^\circ = 70.71\ N$

$F_y = 100\sin 45^\circ = 70.71\ N$

$\vec{F} = 70.71\hat{\imath} + 70.71\hat{\jmath}\ [N]$

b) Find $\vec{F} = F_x\hat{\imath} + F_u\hat{n}$.

$\hat{n} = -\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$

$\vec{F}_u = F_u\hat{n} = -\frac{F_u}{\sqrt{2}}\hat{\imath} + \frac{F_u}{\sqrt{2}}\hat{\jmath}$

$\vec{F} = \left(F_x - \frac{F_u}{\sqrt{2}}\right)\hat{\imath} + \frac{F_u}{\sqrt{2}}\hat{\jmath} = \frac{100}{\sqrt{2}}\hat{\imath} + \frac{100}{\sqrt{2}}\hat{\jmath}$

$F_x - \frac{F_u}{\sqrt{2}} = \frac{100}{\sqrt{2}} \qquad \frac{F_u}{\sqrt{2}} = \frac{100}{\sqrt{2}}$

$F_x = 100\sqrt{2} \quad F_u = 100$

$\vec{F} = 141.4\hat{\imath} + 100\hat{n}\ [N]$

2.71 a) Find magnitude of $\vec{F}$.

$F = (2^2 + (-3)^2 + 6^2)^{1/2} = \underline{7\ lb}$

b) Find $\theta_x, \theta_y, \theta_z$.

$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{2}{7}\right) = \underline{73.4^\circ}$

$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right) = \cos^{-1}\left(\frac{-3}{7}\right) = \underline{115.4^\circ}$

$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right) = \cos^{-1}\left(\frac{6}{7}\right) = \underline{31.0^\circ}$

2.72 Find $\vec{A}+\vec{B}$, $\vec{A}-\vec{B}$, $\vec{A}+3\vec{B}$, $3\vec{A}-\vec{B}$.

$\vec{A}+\vec{B} = (10\hat{\imath} + 20\hat{\jmath}) + (8\hat{\imath} - 6\hat{\jmath}) = \underline{18\hat{\imath} + 14\hat{\jmath}}$

$\vec{A}-\vec{B} = (10\hat{\imath} + 20\hat{\jmath}) - (8\hat{\imath} - 6\hat{\jmath}) = \underline{2\hat{\imath} + 26\hat{\jmath}}$

$\vec{A}+3\vec{B} = (10\hat{\imath} + 20\hat{\jmath}) + 3(8\hat{\imath} - 6\hat{\jmath}) = \underline{34\hat{\imath} + 2\hat{\jmath}}$

$3\vec{A}-\vec{B} = 3(10\hat{\imath} + 20\hat{\jmath}) - (8\hat{\imath} - 6\hat{\jmath}) = \underline{22\hat{\imath} - 66\hat{\jmath}}$

2.73 Find x, y, z projections of a) $\vec{A}$, b) $\vec{B}$.

a) $A_x = 3-(-4) = 7 \quad A_y = -2-6 = -8 \quad A_z = 1-(-2) = 3$

$\vec{A} = (7,\ -8,\ 3)$

b) $B_x = 5 - [3-(-4)] = -2 \quad B_y = 7 - [-2-3] = 12$

$B_z = 2 - [1-6] = 7$

$\vec{B} = (-2,\ 12,\ 7)$

2.74 Find $\vec{R}$.

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$\vec{A} = 100(\cos 0^\circ \hat{i} + \cos 90^\circ \hat{j} + \cos 90^\circ \hat{k}) = 100\hat{i}$

$\vec{B} = 200(\cos 90^\circ \hat{i} + \cos 90^\circ \hat{j} + \cos 180^\circ \hat{k}) = -200\hat{k}$

$\vec{C} = 160(\cos 90^\circ \hat{i} + \cos 30^\circ \hat{j} + \cos 60^\circ \hat{k})$

$= 138.6\hat{j} + 80\hat{k}$

$\vec{R} = (100\hat{i}) + (-200\hat{k}) + (138.6\hat{j} + 80\hat{k})$

$\vec{R} = 100\hat{i} + 138.6\hat{j} - 120\hat{k}$ N

2.75 Find $\vec{R}$.

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$\vec{A} = 5(\cos 90^\circ \hat{i} + \cos 180^\circ \hat{j} + \cos 90^\circ \hat{k}) = -5\hat{j}$

$\vec{B} = 4(\cos 120^\circ \hat{i} + \cos 45^\circ \hat{j} + \cos 120^\circ \hat{k})$

$= -2\hat{i} + 2.83\hat{j} - 2\hat{k}$

$\vec{C} = 3(\cos 54.74^\circ \hat{i} + \cos 54.74 \hat{j} + \cos 54.74^\circ \hat{k})$

$= 1.73\hat{i} + 1.73\hat{j} + 1.73\hat{k}$

$\vec{R} = (-5\hat{j}) + (-2\hat{i} + 2.83\hat{j} - 2\hat{k}) + (1.73\hat{i} + 1.73\hat{j} + 1.73\hat{k})$

$\vec{R} = -0.268\hat{i} - 0.440\hat{j} - 0.268\hat{k}$ kips

2.76 Find $\vec{R}$.

$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

$\vec{A} = 6(\cos 45^\circ \hat{i} + \cos 90^\circ \hat{j} + \cos 135^\circ \hat{k})$

$= 4.24\hat{i} - 4.24\hat{k}$

$\vec{B} = 4(\cos 90^\circ \hat{i} + \cos 45^\circ \hat{j} + \cos 45^\circ \hat{k})$

$= 2.83\hat{j} + 2.83\hat{k}$

$\vec{C} = 8(\cos 30^\circ \hat{i} + \cos 75^\circ \hat{j} + \cos 64.67^\circ \hat{k})$

$= 6.93\hat{i} + 2.07\hat{j} + 3.42\hat{k}$

$\vec{D} = 12(\cos 135^\circ \hat{i} + \cos 135^\circ \hat{j} + \cos 90^\circ \hat{k})$

$= -8.49\hat{i} - 8.49\hat{j}$

$\vec{R} = (4.24\hat{i} - 4.24\hat{k}) + (2.83\hat{j} + 2.83\hat{k})$
$+ (6.93\hat{i} + 2.07\hat{j} + 3.42\hat{k}) + (-8.49\hat{i} - 8.49\hat{j})$

$\vec{R} = 2.69\hat{i} - 3.59\hat{j} + 2.01\hat{k}$ kN

2.77 Find $\vec{R}$.

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$\vec{A} = 4(\cos \pi/4\, \hat{i} + \cos \pi/2\, \hat{j} + \cos \pi/4\, \hat{k}) = 2.83\hat{i} + 2.83\hat{k}$

$\vec{B} = 6(\cos 3\pi/4\, \hat{i} + \cos \pi/3\, \hat{j} + \cos \pi/3\, \hat{k})$

$= -4.24\hat{i} + 3.00\hat{j} + 3.00\hat{k}$

$\vec{C} = 3(\cos \pi/2\, \hat{i} + \cos \pi/2\, \hat{j} + \cos \pi\, \hat{k}) = -3\hat{k}$

$\vec{R} = (2.83\hat{i} + 2.83\hat{k}) + (-4.24\hat{i} + 3\hat{j} + 3\hat{k}) + (-3\hat{k})$

$\vec{R} = -1.41\hat{i} + 3\hat{j} + 2.83\hat{k}$ kN

2.78 Find $\vec{R}$.

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$\vec{A} = 16(\cos \pi/3\, \hat{i} + \cos 7\pi/6\, \hat{j} + \cos 3\pi/2\, \hat{k})$

$= 8\hat{i} - 13.86\hat{j}$

$\vec{B} = 20(\cos \pi/2\, \hat{i} + \cos 3\pi/4\, \hat{j} + \cos 3\pi/4\, \hat{k})$

$= -14.14\hat{j} - 14.14\hat{k}$

2.78 cont.

$\vec{C} = 10(\cos 2\pi/3\, \hat{i} + \cos 3\pi/4\, \hat{j} + \cos 3\pi/4\, \hat{k})$

$= -5\hat{i} - 7.07\hat{j} - 7.07\hat{k}$

$\vec{R} = (8\hat{i} - 13.86\hat{j}) + (-14.14\hat{j} - 14.14\hat{k}) + (-5\hat{i} - 7.07\hat{i} - 7.07\hat{k})$

$\vec{R} = 3\hat{i} - 35.1\hat{j} - 21.2\hat{k}$ lb

2.79 Find $\vec{R}$.

$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

$\vec{A} = 350(\cos \pi/2\, \hat{i} + \cos \pi/3\, \hat{j} + \cos \pi/6\, \hat{k})$

$= 175\hat{j} + 303.1\hat{k}$

$\vec{B} = 200(\cos 0\, \hat{i} + \cos \pi/2\, \hat{j} + \cos \pi/2\, \hat{k}) = 200\hat{i}$

$\vec{C} = 400(\cos \pi/3\, \hat{i} + \cos \pi/4\, \hat{j} + \cos 5\pi/6\, \hat{k})$

$= 200\hat{i} + 282.8\hat{j} - 346.4\hat{k}$

$\vec{D} = 300(\cos 3\pi/4\, \hat{i} + \cos \pi/3\, \hat{j} + \cos 2\pi/3\, \hat{k})$

$= -212.1\hat{i} + 150\hat{j} - 150\hat{k}$

$\vec{R} = (175\hat{j} + 303.1\hat{k}) + (200\hat{i}) + (200\hat{i} + 282.8\hat{j} - 346.4\hat{k})$
$+ (-212.1\hat{i} + 150\hat{j} - 150\hat{k})$

$\vec{R} = 187.9\hat{i} + 607.8\hat{j} - 193.3\hat{k}$ N

2.80 a) Develop spreadsheet to find resultant of concurrent, non-coplanar forces.

	A	B	C	D	E	F	G	H
1		Microsoft Excel Spreadsheet						
2				Angles in Degrees			Projections	
3		magnitude	θ_x	θ_y	θ_z	x	y	z
4	A					=$B4*COS(C4*PI()/180)	=$B4*COS(D4*PI()/180)	=$B4*COS(E4*PI()/180)
5	B					=$B5*COS(C5*PI()/180)	=$B5*COS(D5*PI()/180)	=$B5*COS(E5*PI()/180)
6	C					=$B6*COS(C6*PI()/180)	=$B6*COS(D6*PI()/180)	=$B6*COS(E6*PI()/180)
7	D					=$B7*COS(C7*PI()/180)	=$B7*COS(D7*PI()/180)	=$B7*COS(E7*PI()/180)
8	E					=$B8*COS(C8*PI()/180)	=$B8*COS(D8*PI()/180)	=$B8*COS(E8*PI()/180)
9	R	=SQRT(F9^2+G9^2+H9^2)	=(ACOS(F9/$B9))*180/PI()	=(ACOS(G9/$B9))*180/PI()	=(ACOS(H9/$B9))*180/PI()	=SUM(F4:F8)	=SUM(G4:G8)	=SUM(H4:H8)

b) Verify by solving Prob. 2.74, 2.75, 2.76.

2.74

	A	B	C	D	E	F	G	H
1		Microsoft Excel Spreadsheet						
2			Angles in Degrees			Projections		
3		magnitude	θ_x	θ_y	θ_z	x	y	z
4	A	100	0	90	90	100	0	0
5	B	200	90	90	180	0	0	-200
6	C	160	90	30	60	0	138.6	80
7	D	0	0	0	0	0	0	0
8	E	0	0	0	0	0	0	0
9	R	208.8	61.4	48.4	125.1	100	138.56	-120

2.75

	A	B	C	D	E	F	G	H
1		Microsoft Excel Spreadsheet						
2			Angles in Degrees			Projections		
3		magnitude	θ_x	θ_y	θ_z	x	y	z
4	A	5	90	180	90	0	-5	0
5	B	4	120	45	120	-2	2.828	-2
6	C	3	54.74	54.74	54.74	1.732	1.732	1.732
7	D	0	0	0	0	0	0	0
8	E	0	0	0	0	0	0	0
9	R	0.581	117.50	139.23	117.50	-0.268	-0.440	-0.268

2.80 cont.

2.76

	A	B	C	D	E	F	G	H
1	Microsoft Excel Spreadsheet							
2			Angles in Degrees			Projections		
3		magnitude	θ_x	θ_Y	θ_z	x	y	z
4	A	6	45	90	135	4.24	0	-4.24
5	B	4	90	45	45	0	2.83	2.83
6	C	8	30	75	64.67	6.93	2.07	3.42
7	D	12	135	135	90	-8.49	-8.49	0
8	E	0	0	0	0	0	0	0
9	R	4.91	56.84	136.92	65.85	2.69	-3.59	2.01

2.81 a) Develop spreadsheet to find resultant of concurrent, non-coplanar forces.

	A	B	C	D	E
1		Microsoft Excel Spreadsheet			
2				Angles in Radians	
3		magnitude	θ_x	θ_Y	θ_z
4	A				
5	B				
6	C				
7	D				
8	E				
9	R	=SQRT(F9^2+G9^2+H9^2)	=ACOS(F9/$B9)	=ACOS(G9/$B9)	=ACOS(H9/$B9)

F	G	H
	Projections	
x	y	z
=$B4*COS(C4)	=$B4*COS(D4)	=$B4*COS(E4)
=$B5*COS(C5)	=$B5*COS(D5)	=$B5*COS(E5)
=$B6*COS(C6)	=$B6*COS(D6)	=$B6*COS(E6)
=$B7*COS(C7)	=$B7*COS(D7)	=$B7*COS(E7)
=$B8*COS(C8)	=$B8*COS(D8)	=$B8*COS(E8)
=SUM(F4:F8)	=SUM(G4:G8)	=SUM(H4:H8)

b) Verify by solving Prob. 2.77-2.79.

2.77

	A	B	C	D	E	F	G	H
1	Microsoft Excel Spreadsheet							
2			Angles in Radians			Projections		
3		magnitude	θ_x	θ_Y	θ_z	x	y	z
4	A	4	0.785	1.571	0.785	2.828	0	2.828
5	B	6	2.356	1.047	1.047	-4.243	3	3
6	C	3	1.571	1.571	3.142	0	0	-3
7	D	0	0	0	0	0	0	0
8	E	0	0	0	0	0	0	0
9	R	4.359	1.901	0.812	0.865	-1.414	3	2.828

2.78

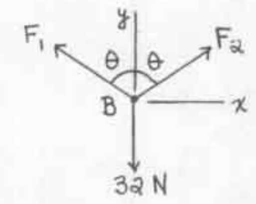
	A	B	C	D	E	F	G	H
1	Microsoft Excel Spreadsheet							
2			Angles in Radians			Projections		
3		magnitude	θ_x	θ_Y	θ_z	x	y	z
4	A	16	1.047	3.665	4.712	8	-13.856	0
5	B	20	1.571	2.356	2.356	0	-14.142	-14.142
6	C	10	2.094	2.356	2.356	-5	-7.071	-7.071
7	D	0	0	0	0	0	0	0
8	E	0	0	0	0	0	0	0
9	R	41.096	1.498	2.593	2.113	3	-35.070	-21.213

2.79

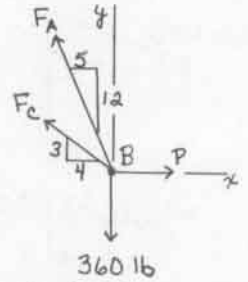
	A	B	C	D	E	F	G	H
1	Microsoft Excel Spreadsheet							
2			Angles in Radians			Projections		
3		magnitude	θ_x	θ_Y	θ_z	x	y	z
4	A	350	1.571	1.047	0.524	0	175	303.11
5	B	200	0	1.571	1.571	200	0	0
6	C	400	1.047	0.785	2.618	200	282.8	-346.41
7	D	300	2.356	1.047	2.094	-212.13	150	-150
8	E	0	0	0	0	0	0	0
9	R	664.93	1.28	0.42	1.87	187.87	607.84	-193.30

3.1 Draw free-body diagrams.

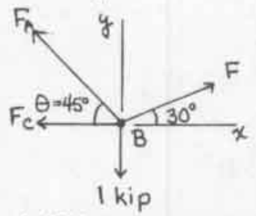
a) Fig. P3.2



b) Fig. P3.3

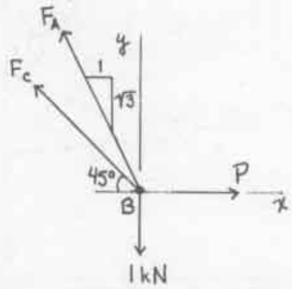


c) Fig. 3.4

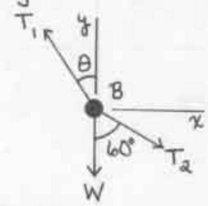


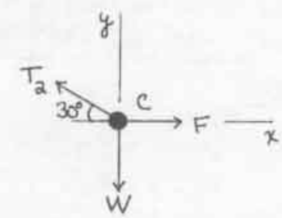
ΔABC is a right isoceles triangle. Therefore θ=45°.

d) Fig. P3.5

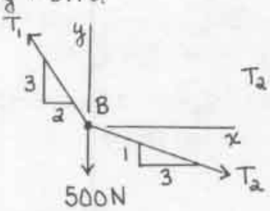


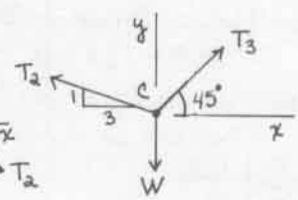
e) Fig. 3.10



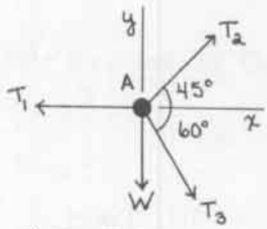


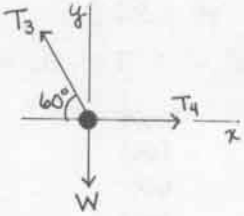
f) Fig P3.12



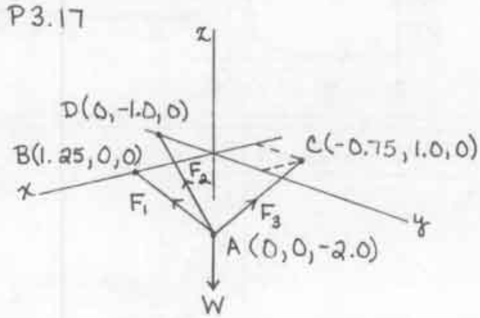


g) Fig P3.15



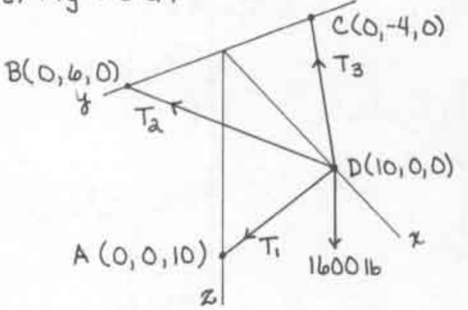


h) Fig P3.17

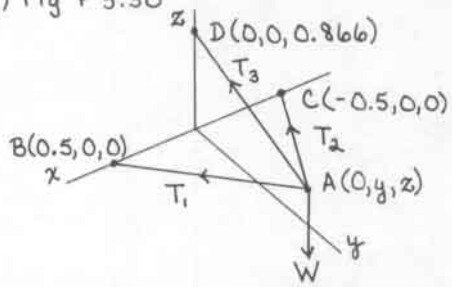


3.1 cont.

i) Fig P3.29

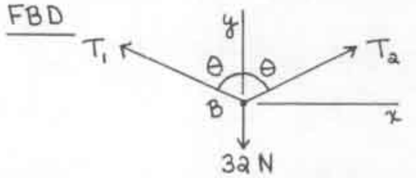


j) Fig P 3.30



3.2 a) Show that $T_1 = T_2$. Find T for $\theta = 0°$, $30°$, $60°$, $85°$, $89°$.

FBD



Equilibrium equation: $\Sigma F_x = 0 = -T_1 \sin\theta + T_2 \sin\theta$

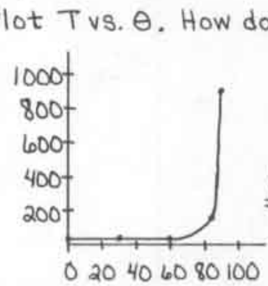
$T_1 \sin\theta = T_2 \sin\theta \longrightarrow T_1 = T_2 = T$

Equilibrium equation: $\Sigma F_y = 0 = 2T\cos\theta - 32$

$2T\cos\theta = 32 \longrightarrow T = 16/\cos\theta$

θ	T (N)
0°	16
30°	18.48
60°	32
85°	183.6
89°	916.8

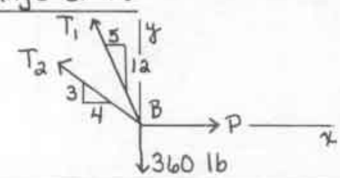
b) Plot T vs. θ. How does T behave as $\theta \to 90°$.



As $\theta \to 90°$, $T \to \infty$.

3.3 Find range of P.

FBD



3.3 cont. To find range of P, assume $T_1 = 0$ and solve for P. Repeat for $T_2 = 0$

Equilibrium equations

$\Sigma F_x = 0 = -5/13\, T_1 - 4/5\, T_2 + P = 0$

$\Sigma F_y = 0 = 12/13\, T_1 + 3/5\, T_2 - 360 = 0$

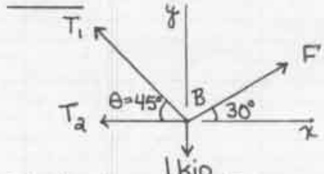
If $T_1 = 0$, then $T_2 = 600$ lb, $P = 480$ lb.

If $T_2 = 0$, then $T_1 = 390$ lb, $P = 150$ lb.

Range of P: 150 lb < P < 480 lb

3.4 Find range of F.

FBD



ΔABC is a right isoceles triangle, therefore $\theta = 45°$.

To find range of P, assume $T_1 = 0$ and solve for P. Repeat for $T_2 = 0$.

Equilibrium equations

$\Sigma F_x = 0 = -T_1 \cos 45° - T_2 + F\cos 30°$

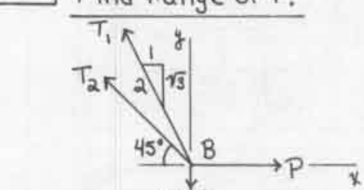
$\Sigma F_y = 0 = T_1 \sin 45° + F \sin 30° - 1$

If $T_1 = 0$, then $T_2 = 1.732$ kips, $F = 2$ kips.

If $T_2 = 0$, then $T_1 = 0.897$ kips, $F = 0.732$ kips.

Range of F: 0.732 kips < F < 2 kips

3.5 Find range of P.



To find range of P, assume $T_1 = 0$ and solve for P. Repeat for $T_2 = 0$.

Equilibrium equations

$\Sigma F_x = 0 = -\tfrac{1}{2} T_1 - T_2 \cos 45° + P$

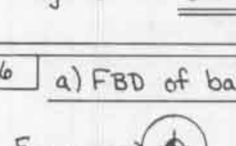
$\Sigma F_y = 0 = \sqrt{3}/2\, T_1 + T_2 \sin 45° - 1$

If $T_1 = 0$, then $T_2 = 1.414$ kN, $P = 1$ kN.

If $T_2 = 0$, then $T_1 = 1.155$ kN, $P = 0.577$ kN.

Range of P: 0.577 kN < P < 1 kN

3.6 a) FBD of ball when in contact with bat.



Neglect the air drag forces

b) FBD of ball after it leaves the bat.

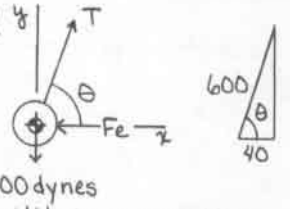
3.6 cont. c) Is the ball in equil. in (a)? Explain.

No. The resultant force is not equal to zero.

d) The ball is in equilibrium horizontally because the forces in the x-direction equal zero. The forces in the y-direction do not equal zero, therefore the ball is not in equilibrium vertically.

3.7 Find the charge on the spheres.

FBD



$\theta = \cos^{-1}\left(\frac{40}{600}\right) = 86.18°$

Equilibrium equations

$\Sigma F_x = 0 = -F_e + T\cos 86.18°$

$\Sigma F_y = 0 = -100 + T\sin 86.18°$

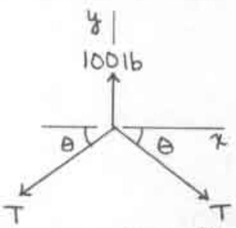
$F_e = 6.682$ dynes $\quad T = 100.2$ dynes

$F_e = \frac{ee'}{r^2} \quad e = e' \quad F_e = \frac{e^2}{r^2}$

$e = \sqrt{F_e r^2} = \sqrt{(6.682)(8^2)} = \underline{\underline{20.68 \text{ esu}}}$

3.8 a) Derive a formula for T.

FBD



Equilibrium equation: $\Sigma F_y = 0 = 100 - 2T\sin\theta$

$T = \frac{100}{2\sin\theta} = \underline{\underline{\frac{50}{\sin\theta}}} \leftarrow$ Ans

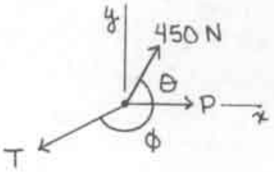
b) Find T for $\theta = 10°, 20°, 45°$. What can be concluded?

θ	T (lb)
10°	287.9
20°	146.2
45°	70.71

Conclusion: A larger pulling force results from a smaller angle.

3.9 a) Find θ to maximize T.

FBD



3.9 cont.

Equilibrium equation

$\Sigma F_x = 0 = -T\sin\phi + 450\sin\theta$

$T = \frac{450\sin\theta}{\sin\phi}$

To maximize T, set $\frac{dT}{d\theta} = 0$.

$\frac{dT}{d\theta} = \frac{-450\cos\theta}{\sin\phi} = 0 \qquad \cos\theta = 0 \rightarrow \underline{\underline{\theta = 90°}}$

b) Find T_{max} for $\phi = 170°, 160°, 150°$.

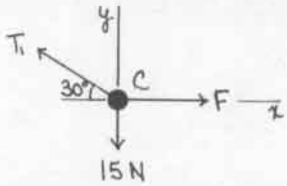
ϕ	T (N)
170°	2591
160°	1316
150°	900

$T_{max} = \frac{450}{\sin\phi}$

3.10 Find F, T_1, T_2, θ.

Divide system into 2 parts isolating B, C.

FBD: C



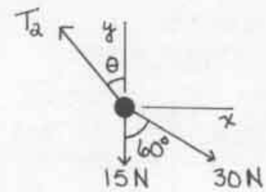
Equilibrium equations

$\Sigma F_x = -T_1\cos 30° + F = 0$

$\Sigma F_y = T_1\sin 30° - 15 = 0$

$\underline{\underline{T_1 = 30\text{ N} \quad F = 25.98\text{ N}}}$

FBD: B



Equilibrium equations

$\Sigma F_x = 0 = -T_2\sin\theta + 30\sin 60°$

$\Sigma F_y = 0 = T_2\cos\theta - 30\cos 60° - 15$

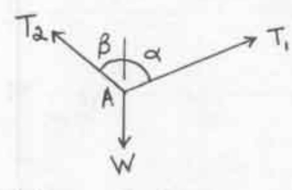
$T_2 = \frac{30\sin 60°}{\sin\theta} = \frac{30\cos 60° + 15}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta} = \frac{30\sin 60°}{30\cos 60° + 15} = \tan\theta$

$\underline{\underline{\theta = 40.9° \quad T_2 = 39.69\text{ N}}}$

3.11 Show that $T_1 = \frac{W\sin\beta}{\sin(\alpha+\beta)}$, $T_2 = \frac{W\sin\alpha}{\sin(\alpha+\beta)}$.

FBD



$\Sigma F_x = 0 = T_1\sin\alpha - T_2\sin\beta \qquad$ (Eq. 1)

$\Sigma F_y = 0 = T_1\cos\alpha + T_2\cos\beta - W \qquad$ (Eq. 2)

3.11 cont.

From Eq. 1 $T_2 = \dfrac{T_1 \sin\alpha}{\sin\beta}$

Substitute into Eq. 2 $W = \dfrac{T_1 \sin\alpha}{\sin\beta}\cos\beta + T_1\cos\alpha$

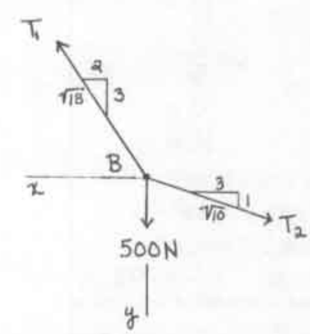
$$W\sin\beta = T_1 \sin\alpha\cos\beta + T_1\cos\alpha\sin\beta$$
$$W\sin\beta = T_1\sin(\alpha+\beta)$$
$$\underline{\underline{T_1 = \frac{W\sin\beta}{\sin(\alpha+\beta)}}}$$

Substitute into Eq. 1

$$T_2\sin\beta = \frac{W\sin\beta}{\sin(\alpha+\beta)}\sin\alpha$$
$$\underline{\underline{T_2 = \frac{W\sin\alpha}{\sin(\alpha+\beta)}}}$$

3.12 Find T_1, T_2, T_3, W.

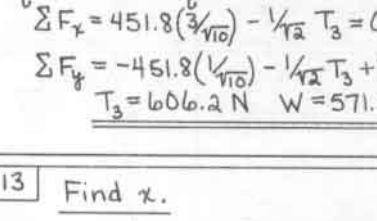
FBD: B



Equilibrium equations

$$\Sigma F_x = \tfrac{2}{\sqrt{13}}T_1 - \tfrac{3}{\sqrt{10}}T_2 = 0$$
$$\Sigma F_y = -\tfrac{3}{\sqrt{13}}T_1 + \tfrac{1}{\sqrt{10}}T_2 + 500 = 0$$
$$\underline{\underline{T_1 = 772.6\text{N}}} \quad \underline{\underline{T_2 = 451.8\text{ N}}}$$

FBD: C

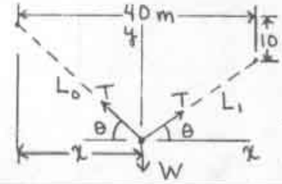


Equilibrium equations

$$\Sigma F_x = 451.8\left(\tfrac{3}{\sqrt{10}}\right) - \tfrac{1}{\sqrt{2}}T_3 = 0$$
$$\Sigma F_y = -451.8\left(\tfrac{1}{\sqrt{10}}\right) - \tfrac{1}{\sqrt{2}}T_3 + W = 0$$
$$\underline{\underline{T_3 = 606.2\text{ N} \quad W = 571.5\text{ N}}}$$

3.13 Find x.

FBD



If the tension is equal on both sides of the cable, the angles are equal.

3.13 cont. This problem can be solved with geometry alone.

$$\begin{cases} L_0 + L_1 = 50 \\ L_0\cos\theta + L_1\cos\theta = 40 \\ L_0\sin\theta - L_1\sin\theta = 10 \\ x = L_0\cos\theta \end{cases}$$

Alternatively, $(L_0+L_1) = 40/\cos\theta$. ∴ $\cos\theta = 0.8$; $\theta = 36.87°$. $(L_0 - L_1)\sin\theta = 10$; $L_0 + L_1 = 50$.

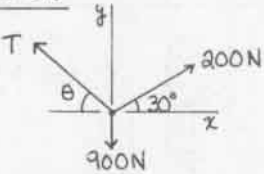
→ Solve using a computer equation solver.

$\theta = 36.87°$ $L_0 = 33.33$ m $L_1 = 16.67$ m

$$\underline{\underline{x = 26.67\text{ m}}}$$

3.14 Find θ.

FBD

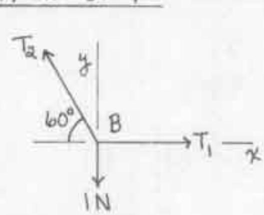


Equilibrium equations

$$\Sigma F_x = -T\cos\theta + 200\cos 30° = 0$$
$$\Sigma F_y = T\sin\theta + 200\sin 30° - 900 = 0$$
$$T = \frac{200\cos 30°}{\cos\theta} = \frac{900 - 200\sin 30°}{\sin\theta}$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{900 - 200\sin 30°}{200\cos 30°}$$
$$\underline{\underline{\theta = 77.8°}}$$

3.15 Find T_1, T_2, T_3, T_4.

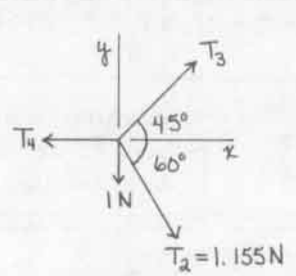
FBD: B



Equilibrium equations

$$\Sigma F_x = -T_2\cos 60° + T_1 = 0$$
$$\Sigma F_y = T_2\sin 60° - 1 = 0$$
$$\underline{\underline{T_1 = 0.577\text{ N} \quad T_2 = 1.155\text{ N}}}$$

FBD: A

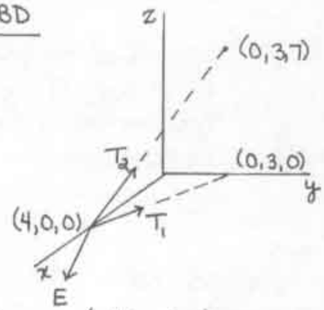


Equilibrium equations

$$\Sigma F_x = 1.155\cos 60° + T_3\cos 45° - T_4 = 0$$
$$\Sigma F_y = -1.155\sin 60° + T_3\sin 45° - 1 = 0$$
$$\underline{\underline{T_3 = 2.828\text{ N} \quad T_4 = 2.577\text{ N}}}$$

3.16 a) Find equilibrant at A.

FBD



$\vec{T}_1 = 100\left(\frac{-4\hat{\imath}+3\hat{\jmath}}{\sqrt{4^2+3^2}}\right) = -80\hat{\imath}+60\hat{\jmath}$

$\vec{T}_2 = 100\left(\frac{-4\hat{\imath}+3\hat{\jmath}+7\hat{k}}{\sqrt{4^2+3^2+7^2}}\right) = -46.5\hat{\imath}+34.87\hat{\jmath}+81.37\hat{k}$

For equilibrium $\vec{T}_1+\vec{T}_2+\vec{E}=0$

$0 = E_x - 80 - 46.5 \qquad E_x = 126.5 \text{ N}$

$0 = E_y + 60 + 34.87 \qquad E_y = -94.87 \text{N}$

$0 = E_z + 81.37 \qquad E_z = -81.37 \text{ N}$

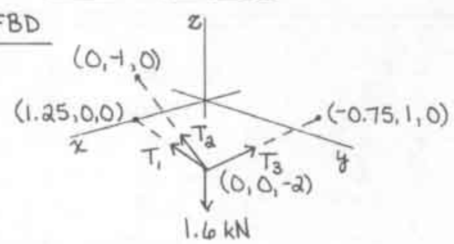
$\vec{E} = 126.5\hat{\imath} - 94.87\hat{\jmath} - 81.37\hat{k} \text{ N}$

b) Find θ_x.

$\theta_x = \cos^{-1}\left(\frac{126.5}{\sqrt{126.5^2+(-94.87)^2+(-81.37)^2}}\right) = 44.7°$

3.17 Find T_1, T_2, T_3.

FBD



Unit vectors

$\hat{n}_1 = \frac{1.25\hat{\imath}+2\hat{k}}{\sqrt{1.25^2+2^2}} = 0.53\hat{\imath}+0.848\hat{k}$

$\hat{n}_2 = \frac{-\hat{\jmath}+2\hat{k}}{\sqrt{(-1)^2+2^2}} = -0.447\hat{\jmath}+0.894\hat{k}$

$\hat{n}_3 = \frac{-0.75\hat{\imath}+\hat{\jmath}+2\hat{k}}{\sqrt{(-0.75)^2+1^2+2^2}} = -0.318\hat{\imath}+0.424\hat{\jmath}+0.848\hat{k}$

$\vec{T}_1 = T_1\hat{n}_1 = T_1(0.53\hat{\imath}+0.848\hat{k})$

$\vec{T}_2 = T_2\hat{n}_2 = T_2(-0.447\hat{\jmath}+0.894\hat{k})$

$\vec{T}_3 = T_3\hat{n}_3 = T_3(-0.318\hat{\imath}+0.424\hat{\jmath}+0.848\hat{k})$

Equilibrium equations

$\Sigma F_x = 0 = 0.53T_1 - 0.318T_3$

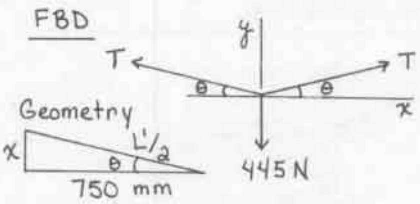
$\Sigma F_y = 0 = -0.447T_2 + 0.424T_3$

$\Sigma F_z = 0 = 0.848T_1 + 0.894T_2 + 0.848T_3 - 1.6$

$T_1 = 0.435 \text{ kN} \quad T_2 = 0.688 \text{ kN} \quad T_3 = 0.726 \text{ kN}$

3.18 a) Find x ; b) Find T.

FBD



$\Sigma F_y = 0 = 2T\sin\theta - 445$

$\sin\theta = x/0.5L'$ (from Geometry)

$L' = 1500 + 0.0114\,T$

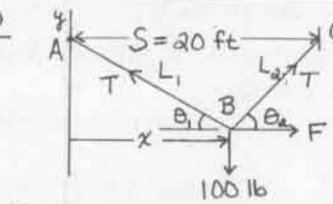
$L' = 1500/\cos\theta$

Using a computer equation solver.

$x = 113.2 \text{ mm} \quad T = 1491 \text{ N}$

3.19 Plot F as a function of x for $0 \le x \le S$.

FBD



Equilibrium equations

$\Sigma F_x = 0 = -T\cos\theta_1 + T\cos\theta_2 + F$

$\Sigma F_y = 0 = T\sin\theta_1 + T\sin\theta_2 - 100$

Geometry

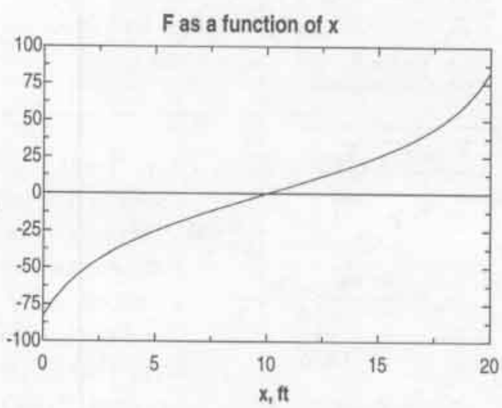
$L_1\cos\theta_1 + L_2\cos\theta_2 = 20$

$L_1\sin\theta_1 = L_2\sin\theta_2$

$L_1 + L_2 = 24$

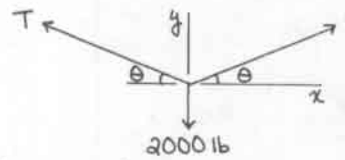
$x = L_1\cos\theta_1$

Using computer software that can solve simultaneous equations and graph variables, such as Engineering Equation Solver, plot F as a function of x.



3.20 Find length of cable and θ or recommend alternative solutions.

FBD



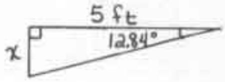
Equilibrium equation

$\Sigma F_y = 0 = 2T\sin\theta - 2000$

If $T = 4500$ lb, $\theta = \sin^{-1}\left(\dfrac{2000}{2(4500)}\right)$

$\theta = 12.84°$

Use geometry to check x.



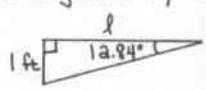
$\tan 12.84° = \dfrac{x}{5}$

$x = 1.14$ ft > 12 in.

The design fails.

One possible solution: A shorter distance between A and C.

Use geometry to find a suitable length AC.



$l = 1/\tan 12.84° = 4.387$ ft

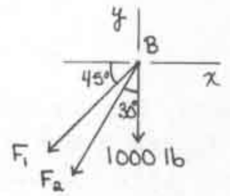
$AC = 2l = 8.775$ ft

New cable length $= 2\sqrt{1^2 + 4.387^2} = 9$ ft

Design: Cable length = 9 ft with AC = 8.775 ft

3.21 Find F_1, F_2.

FBD



Assume members are in tension. If negative, then member is in compression.

Equilibrium equations

$\Sigma F_x = 0 = -F_1\cos 45° - F_2\sin 30°$

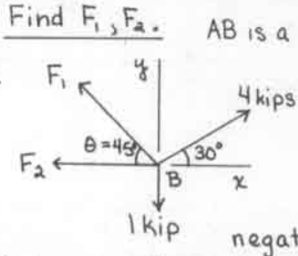
$\Sigma F_y = 0 = -F_1\sin 45° - F_2\cos 30° - 1000$

$F_1 = 1932$ lb (T) $F_2 = 2732$ lb (C)

3.22 Find F_1, F_2.

AB is a two-force member. ΔABC is a right isoceles triangle, therefore $\theta = 45°$. Assume all members in tension. If negative, then member is in compression.

FBD



Equilibrium equations

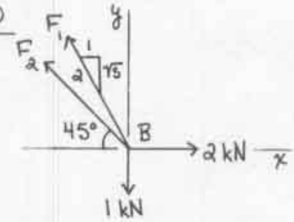
$\Sigma F_x = 0 = -F_1\cos 45° - F_2 + 4\cos 30°$

$\Sigma F_y = 0 = F_1\sin 45° + 4\sin 30° - 1$

$F_1 = 1.414$ kip (C) $F_2 = 4.464$ kips (T)

3.23 Find F_1, F_2.

FBD



Assume all members are in tension. If negative then member is in compression.

Equilibrium equations

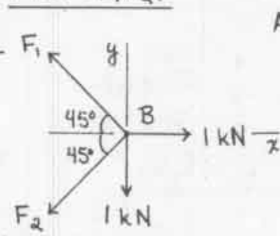
$\Sigma F_x = 0 = -1/2\,F_1 - F_2\cos 45° + 2$

$\Sigma F_y = 0 = \sqrt{3}/2\,F_1 + F_2\sin 45° - 1$

$F_1 = 2.732$ kN (C) $F_2 = 4.760$ kN (T)

3.24 Find F_1, F_2.

FBD



AB, BC are two-force members. Assume forces F_1, F_2 exerted by members AB, BC pull on B. If F_1, F_2 are negative, then they push on B.

Equilibrium equations

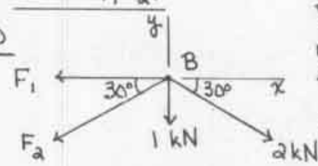
$\Sigma F_x = 0 = -F_1\cos 45° - F_2\cos 45° + 1$

$\Sigma F_y = 0 = F_1\sin 45° - F_2\sin 45° - 1$

$F_1 = 1.414$ kN (pulls on B) $F_2 = 0$ kN

3.25 Find F_1, F_2.

FBD



BC is a two-force member. Assume forces F_1, F_2 exerted on B by AB, BC pull on B. If F_1, F_2 are negative, they push on B.

Equilibrium equations

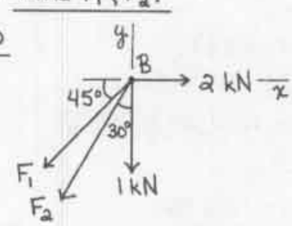
$\Sigma F_x = 0 = -F_1 - F_2\cos 30° + 2\cos 30°$

$\Sigma F_y = 0 = -F_2\sin 30° - 1 - 2\sin 30°$

$F_1 = 5.196$ kN (pulls on B) $F_2 = -4$ kN (pushes on B)

3.26 Find F_1, F_2.

FBD



Assume members are in tension. If neg., the member is in compression.

Equilibrium equations

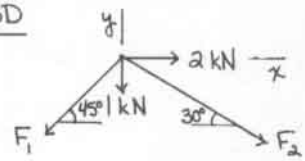
$\Sigma F_x = 0 = -F_1\cos 45° - F_2\sin 30° + 2$

$\Sigma F_y = 0 = -F_1\sin 45° - F_2\cos 30° - 1$

$F_1 = 8.624$ kN (T) $F_2 = 8.196$ kN (C)

3.27 Find F_1, F_2.

FBD



Assume members are in tension. If neg., then member is in compression.

Equilibrium equations

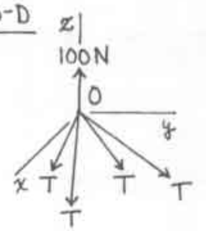
$\Sigma F_x = 0 = -F_1 \cos 45^\circ + F_2 \cos 30^\circ + 2$

$\Sigma F_y = 0 = -F_1 \sin 45^\circ - F_2 \sin 30^\circ - 1$

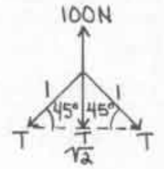
$\underline{F_1 = 0.139 \text{ kN (T)} \quad F_2 = 2.196 \text{ kN (C)}}$

3.28 Find T.

FBD: 3-D



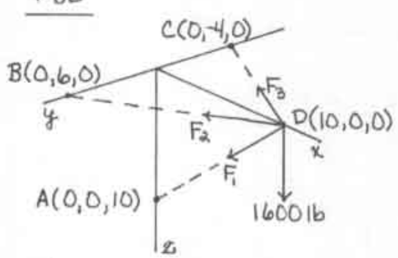
Side View



$\Sigma F_z = 0 = 100 - 4T \sin 45^\circ \longrightarrow \underline{T = 35.36 \text{ N}}$

3.29 Find F_1, F_2, F_3.

FBD



Assume members are in tension. If negative, member is in compression.

Unit vectors

$$\hat{n}_1 = \frac{-10\hat{\imath} + 10\hat{k}}{\sqrt{(-10)^2 + 10^2}} = -0.707\hat{\imath} + 0.707\hat{k}$$

$$\hat{n}_2 = \frac{-10\hat{\imath} + 6\hat{\jmath}}{\sqrt{(-10)^2 + 6^2}} = -0.857\hat{\imath} + 0.514\hat{\jmath}$$

$$\hat{n}_3 = \frac{-10\hat{\imath} - 4\hat{\jmath}}{\sqrt{(-10)^2 + (-4)^2}} = -0.928\hat{\imath} - 0.371\hat{\jmath}$$

$\vec{F}_1 = F_1\hat{n}_1 = F_1(-0.707\hat{\imath} + 0.707\hat{k})$

$\vec{F}_2 = F_2\hat{n}_2 = F_2(-0.857\hat{\imath} + 0.514\hat{\jmath})$

$\vec{F}_3 = F_3\hat{n}_3 = F_3(-0.928\hat{\imath} - 0.371\hat{\jmath})$

Equilibrium equations

$\Sigma F_x = 0 = -0.707F_1 - 0.857F_2 - 0.928F_3$

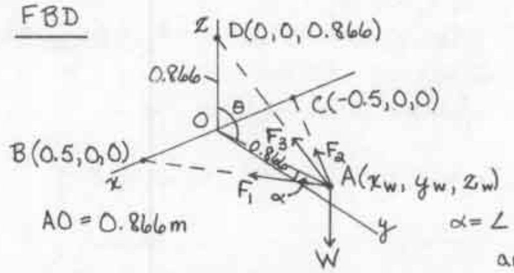
$\Sigma F_y = 0 = 0.514F_2 - 0.371F_3$

$\Sigma F_z = 0 = 0.707F_1 + 1600$

$\underline{F_1 = 2263 \text{ lb (C)} \quad F_2 = 746.4 \text{ lb (T)} \quad F_3 = 1034 \text{ lb (T)}}$

3.30 Find F_1, F_2, F_3.

FBD



AO = 0.866 m

α = ∠ between AO and y-axis

Use Law of Cosines to find θ.

$1^2 = 0.866^2 + 0.866^2 - 2(0.866)(0.866)\cos\theta$

$\theta = 70.53^\circ$

$\alpha = 90 - \theta = 19.47^\circ$

Location of A:

$x_w = 0$

$y_w = 0.866 \cos\alpha = 0.816$

$z_w = 0.866 \sin\alpha = 0.289$

$\vec{F}_1 = F_1(0.5\hat{\imath} - 0.816\hat{\jmath} - 0.289\hat{k})$

$\vec{F}_2 = F_2(-0.5\hat{\imath} - 0.816\hat{\jmath} - 0.289\hat{k})$

$\vec{F}_3 = F_3(-0.816\hat{\jmath} + 0.577\hat{k})$

Equilibrium equations

$\Sigma F_x = 0 = 0.5F_1 - 0.5F_2 \longrightarrow F_1 = F_2$

$\Sigma F_y = 0 = -0.816F_1 - 0.816F_2 - 0.816F_3 \longrightarrow F_3 = -(F_1 + F_2)$

$\Sigma F_z = 0 = -0.289F_1 - 0.289F_2 + 0.577F_3 - W$

$\longrightarrow -0.289(F_1 + F_2) - 0.577(F_1 + F_2) = W$

$-0.866(F_1 + F_2) = W$

$-0.866(2F_1) = W$

$\underline{F_1 = F_2 = -0.577\,W}$

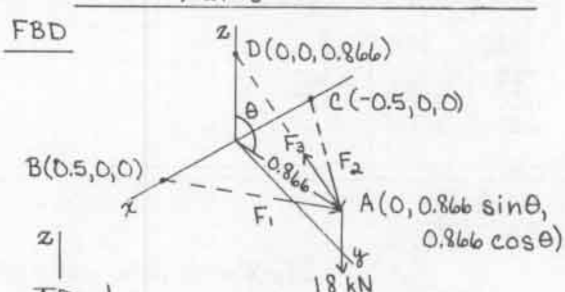
Substitute $F_3 = -(F_1 + F_2)$

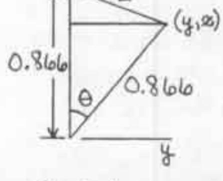
$-0.866(F_1 + F_2) = 0.866F_3 = W$

$\underline{F_3 = 1.155\,W}$

3.31 Plot F_1, F_2, F_3 for $0 < L < 1.732$ m.

FBD





$L^2 = 0.866^2 + 0.866^2 - 2(0.866^2)\cos\theta$

$L^2 = 1.5 - 1.5\cos\theta$

$\vec{F}_1 = F_1(-0.5\hat{\imath} + 0.866\sin\theta\,\hat{\jmath} + 0.866\cos\theta\,\hat{k})$

$\vec{F}_2 = F_2(0.5\hat{\imath} + 0.866\sin\theta\,\hat{\jmath} + 0.866\cos\theta\,\hat{k})$

$\vec{F}_3 = F_3\left(\frac{-0.866\sin\theta}{L}\hat{\jmath} + \frac{0.866 - 0.866\cos\theta}{L}\hat{k}\right)$

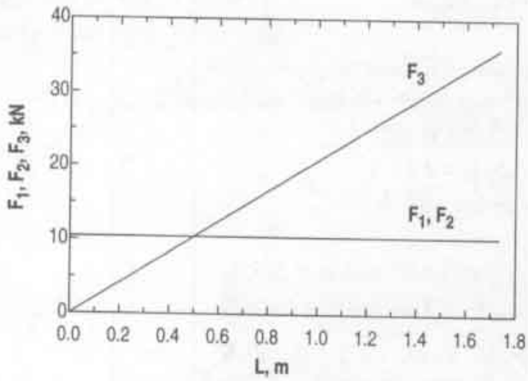
3.31 cont. Equilibrium equations

$\Sigma F_x = 0 = -0.5 F_1 + 0.5 F_2$

$\Sigma F_y = 0 = 0.866 \sin\theta F_1 + 0.866 \sin\theta F_2 - \dfrac{0.866 \sin\theta}{L} F_3$

$\Sigma F_z = 0 = 0.866 \cos\theta F_1 + 0.866 \cos\theta F_2 + \left(\dfrac{0.866 - 0.866\cos\theta}{L}\right) F_3 - 18$

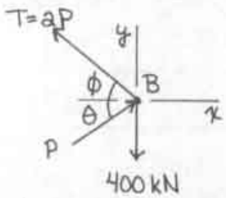
Plot F_1, F_2, F_3 as a function of L, by computer.



Note: Algebraically solving for $F_1 = F_2 = F$ and F_3, $F_3 = 20.78\,L$ km, and $F = 10.39$ km, agrees with plot.

3.32 Find θ, ϕ for P = a) 150 kN, b) 200 kN, c) 250 kN.

FBD



Equilibrium equations

$\Sigma F_x = 0 = P\cos\theta - 2P\cos\phi$

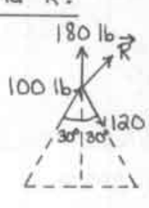
$\Sigma F_y = 0 = 2P\sin\phi + P\sin\theta - 400$

Using a computer equation solver.

	P(kN)	θ	ϕ
a.	150	50.43°	71.43°
b.	200	14.48°	61.05°
c.	250	-7.90°	60.31°

3.33 Find $\vec{R}$.

FBD



$\Sigma F_x = R_x = 100 \sin 30° + 120 \sin 30°$

$R_x = 110$ lb

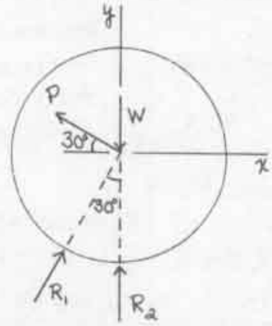
$\Sigma F_y = R_y = 100 \cos 30° + 180 - 120 \cos 30°$

$R_y = 162.7$ lb

$\vec{R} = 110\,\hat{i} + 162.7\,\hat{j}$ lb

3.34 Find F_x, F_y in terms of R_1, R_2, W, P.

FBD



Equilibrium equations

$\Sigma F_x = F_x = R_1 \sin 30° - P\cos 30°$

$\Sigma F_y = F_y = R_1 \cos 30° + P \sin 30° + R_2 - W$

$F_x = 0.5R_1 - 0.866P$; $F_y = 0.866R_1 + R_2 + 0.5P - W$

3.35 Find minimum force P.

When the roller is just beginning to roll, $R_2 = 0$.

$\Sigma F_x = 0 = 0.5R_1 - 0.866P$

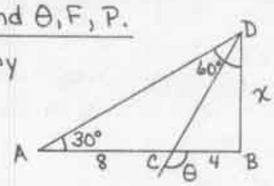
$\Sigma F_y = 0 = 0.866R_1 + 0.5P - W$

$R_1 = \dfrac{W - 0.5P}{0.866}$ $\quad P = \dfrac{0.5R_1}{0.866} = \dfrac{0.5W - 0.25P}{0.866^2}$

$P = 0.5W$

3.36 Find θ, F, P.

Geometry

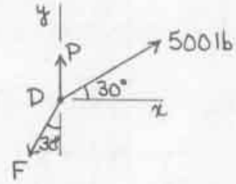


$\triangle ABD$ $\tan 30° = x/12$ $\quad x = 6.928$ ft

$\triangle BCD$ $\tan(180° - \theta) = 6.928/4$

$\theta = 120°$

FBD



Equilibrium equations

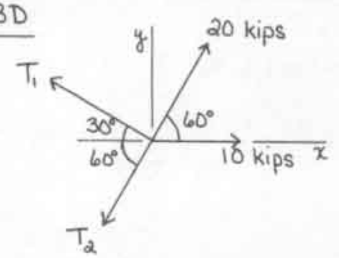
$\Sigma F_x = 0 = -F \sin 30° + 500 \cos 30°$

$\Sigma F_y = 0 = -F\cos 30° + 500 \sin 30° + P$

F = 866 lb $\quad$ P = 500 lb

3.37 a) Find T_1, T_2 by projecting on x, y axes.

FBD

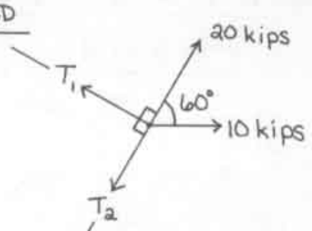


Equilibrium equations

$$\Sigma F_x = 0 = 10 + 20\cos 60° - T_1 \cos 30° - T_2 \cos 60°$$
$$\Sigma F_y = 0 = 20 \sin 60° + T_1 \sin 30° - T_2 \sin 60°$$
$$\underline{\underline{T_1 = 8.66 \text{ kips} \quad T_2 = 25 \text{ kips}}}$$

b) Find T_1, T_2 by projecting forces along T_1, T_2.

FBD

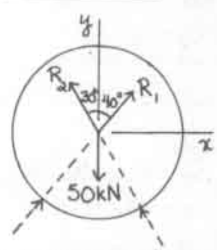


Equilibrium equations

$$\Sigma F_{T_1} = 0 = T_1 - 10 \sin 60°$$
$$\Sigma F_{T_2} = 0 = T_2 - 20 - 10 \cos 60°$$
$$\underline{\underline{T_1 = 8.66 \text{ kips} \quad T_2 = 25 \text{ kips}}}$$

3.38 Find R_1, R_2.

FBD

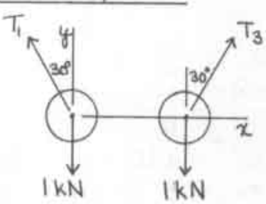


Equilibrium equations

$$\Sigma F_x = R_1 \sin 40° - R_2 \sin 30° = 0$$
$$\Sigma F_y = R_1 \cos 40° + R_2 \cos 30° - 50 = 0$$
$$\underline{\underline{R_1 = 26.6 \text{ kN} \quad R_2 = 34.2 \text{ kN}}}$$

3.39 Find T_1, T_2, T_3.

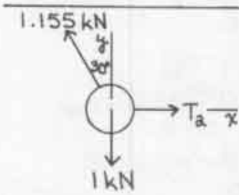
FBD: Whole system



3.39 cont. Equilibrium equations

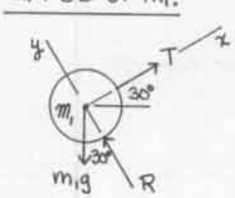
$$\Sigma F_x = T_3 \sin 30° - T_1 \sin 30° = 0$$
$$\Sigma F_y = T_1 \cos 30° + T_3 \cos 30° - 2 = 0$$
$$\underline{\underline{T_1 = 1.155 \text{ kN} \quad T_3 = 1.155 \text{ kN}}}$$

FBD: Isolate one sphere

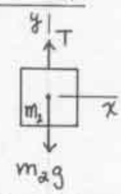


$$\Sigma F_x = 0 = T_2 - 1.155 \sin 30°$$
$$\underline{\underline{T_2 = 0.577 \text{ kN}}}$$

3.40 a) FBD of m_1.



b) FBD of m_2.



c) Find relationship between m_1, m_2.

m_1: $\Sigma F_x = 0 = T - m_1 g \sin 30° \longrightarrow T = m_1 g \sin 30°$

m_2: $\Sigma F_y = 0 = T - m_2 g \longrightarrow T = m_2 g$

Combine equations: $m_1 g \sin 30° = m_2 g$

$$\underline{\underline{m_1 = \frac{m_2}{\sin 30°}}}$$

d) Find T in terms of m_1.

From (c) $\underline{\underline{T = m_1 g \sin 30°}}$

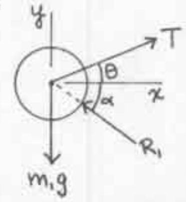
e) Find R in terms of m_1.

From (a) for m_1, $\Sigma F_y = 0 = R - m_1 g \cos 30°$

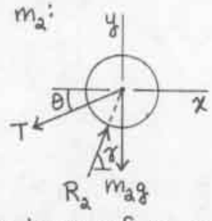
$$\underline{\underline{R = m_1 g \cos 30°}}$$

3.41 a) FBD of m_1, m_2.

m_1:



m_2:



b) Find T, R_1, R_2 in terms of $m_1, m_2, \theta, \alpha, \beta, \gamma$.

m_1: $\Sigma F_x = 0 = T\cos\theta - R_1 \cos\alpha$

$\Sigma F_y = 0 = T\sin\theta + R_1 \sin\alpha - m_1 g$

3.41 cont.

$T = R_1 \cos\alpha / \cos\theta \quad R_1 \sin\alpha + \dfrac{R_1 \cos\alpha \sin\theta}{\cos\theta} = m_1 g$

$R_1 = (\sin\alpha \cos\theta + \cos\alpha \sin\theta) = m_1 g \cos\theta$

$R_1 = \dfrac{m_1 g \cos\theta}{\sin(\alpha+\theta)} \quad T = \dfrac{m_1 g \cos\alpha}{\sin(\alpha+\theta)}$

m_2: $\Sigma F_x = 0 = R_2 \cos\gamma - T\cos\theta$

$\Sigma F_y = 0 = R_2 \sin\gamma - T \sin\theta - m_2 g$

$T = \dfrac{R_2 \cos\gamma}{\cos\theta} \quad R_2 \sin\alpha - \dfrac{R_2 \cos\gamma \sin\theta}{\cos\theta} = m_2 g$

$R_2(\sin\gamma\cos\theta - \cos\gamma\sin\theta) = m_2 g \cos\theta$

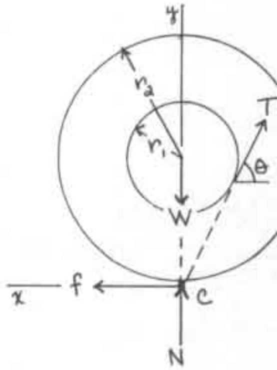
$R_2(\sin(\gamma-\theta)) = m_2 g \cos\theta$

$R_2 = \dfrac{m_2 g \cos\theta}{\sin(\gamma-\theta)}$

Note: T may also be written as:

$T = \dfrac{m_2 g \cos\gamma}{\sin(\gamma-\theta)}$

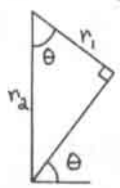
3.42 a) FBD of spool.



Forces are concurrent through C.

b) Find θ in terms of r_1, r_2.

Geometry

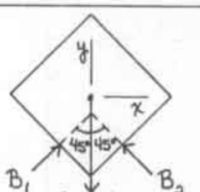


$\cos\theta = r_1/r_2$

$\theta = \cos^{-1}(r_1/r_2)$

3.43 Find all contact forces $(R_1, R_2, R_3, R_4, B_1, B_2)$.

FBD: Block



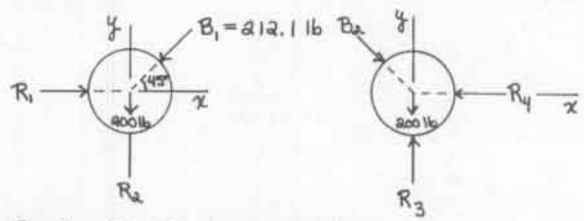
$\Sigma F_x = 0 = B_1 \sin 45° - B_2 \sin 45°$

$\Sigma F_y = 0 = B_1 \cos 45° + B_2 \cos 45° - 300$

$B_1 = B_2 = 212.1$ lb

3.43 cont.

FBD: Spheres



$R_1 = R_4$, $R_2 = R_3$ by symmetry.

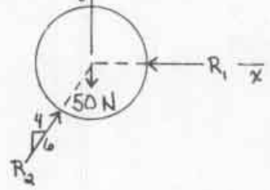
$\Sigma F_x = 0 = R_1 - 212.1 \cos 45°$

$\Sigma F_y = 0 = R_2 - 200 - 212.1 \sin 45°$

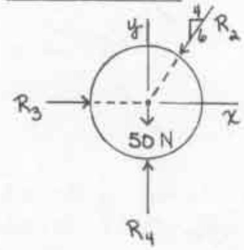
$R_1 = R_4 = 150$ lb $\quad R_2 = R_3 = 350$ lb

3.44 Find all contact forces (R_1, R_2, R_3, R_4).

FBD: Top



FBD: Bottom



Top: $\Sigma F_x = 0 = 4/6\, R_2 - R_1$

$\Sigma F_y = 0 = \sqrt{20}/6\, R_2 - 50$

$R_1 = 44.72$ N; $R_2 = 67.08$ N

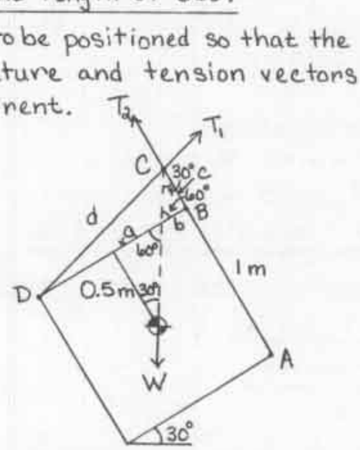
Bottom: $\Sigma F_x = 0 = R_3 - 4/6\,(67.08)$

$\Sigma F_y = 0 = R_4 - 50 - \sqrt{20}/6\,(67.08)$

$R_3 = 44.72$ N $\quad R_4 = 100$ N

3.45 Find length of BCD.

C has to be positioned so that the weight of the picture and tension vectors are concurrent.



Use Law of Sines to find a, c.

$\dfrac{a}{\sin 30°} = \dfrac{0.5}{\sin 60°} \quad \dfrac{c}{\sin 60°} = \dfrac{b}{\sin 30°}$

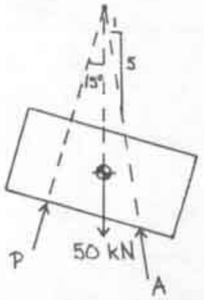
$a = 0.2887$ m $\quad b = 0.5 - a = 0.2113$ m $\quad c = 0.366$ m

Use Pythagorean theorem to find d.

$d = (0.366^2 + 1^2)^{1/2} = 1.065$ m; BCD $= c + d = 1.431$ m

3.46 Find A_x, A_y, P.

FBD



$\Sigma F_x = 0 = P \sin 15° - \frac{1}{\sqrt{26}} A$

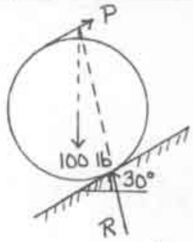
$\Sigma F_y = 0 = P \cos 15° + \frac{5}{\sqrt{26}} A - 50$

$A_x = -\frac{1}{\sqrt{26}} A \quad A_y = \frac{5}{\sqrt{26}} A \quad A = 29.2 \text{ kN}$

$P = 22.12 \text{ kN} \quad A_x = -5.726 \text{ kN} \quad A_y = 28.63 \text{ kN}$

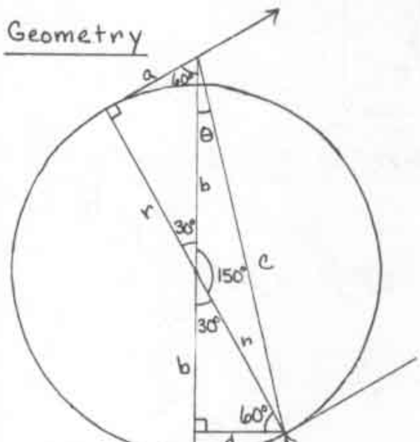
3.47 Find P, R by projecting forces onto a) x, y axes, b) lines parallel and perpendicular to inclined plane.

FBD



For equilibrium, all forces must be concurrent.

Geometry



Law of Sines: $\frac{r}{\sin 60°} = \frac{a}{\sin 30°} \quad a = 0.5774r$

Pythagorean Theorem: $b = \sqrt{(0.5774r)^2 + r^2} = 1.155r$

Law of Cosines: $c = (r^2 + (1.155r)^2 - 2(1.155r^2)\cos 150)^{1/2}$

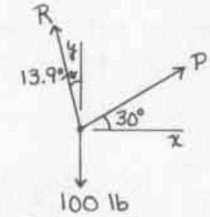
$c = 2.082r$

$d = r \sin 30° = 0.5r$

$\theta = \sin^{-1}\left(\frac{0.5r}{2.082r}\right) = 13.9°$

3.47 cont.

a) FBD: x, y axes

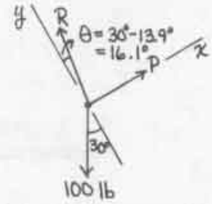


$\Sigma F_x = 0 = P \cos 30° - R \sin 13.9°$

$\Sigma F_y = 0 = P \sin 30° + R \cos 13.9° - 100$

$P = 25 \text{ lb} \quad R = 90.14 \text{ lb}$

b) FBD: parallel, perpendicular to plane



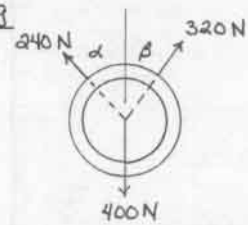
$\Sigma F_x = 0 = P + R \sin 16.1° - 100 \sin 30°$

$\Sigma F_y = 0 = R \cos 16.1° - 100 \cos 30°$

$R = 90.14 \text{ lb} \quad P = 25 \text{ lb}$

3.48 Find α, β.

FBD: Ring



$\Sigma F_x = 0 = 320 \sin\beta - 240 \sin\alpha$

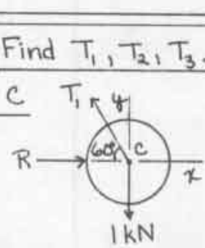
$\Sigma F_y = 0 = 320 \cos\beta + 240 \cos\alpha - 400$

Using a computer equation solver

$\alpha = 53.1° \quad \beta = 36.9°$

3.49 Find T_1, T_2, T_3.

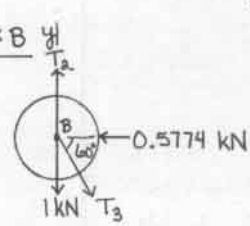
FBD: C



$\Sigma F_x = 0 = R - T_1 \cos 60°, \quad \Sigma F_y = T_1 \sin 60° - 1$

$R = 0.5774 \text{ kN} \quad T_1 = 1.155 \text{ kN}$

FBD: B



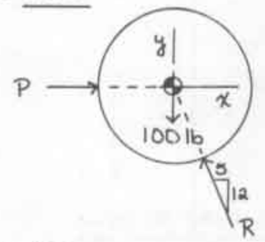
$\Sigma F_x = 0 = T_3 \cos 60° - 0.5774$

$\Sigma F_y = 0 = T_2 - T_3 \sin 60° - 1$

$T_2 = 2 \text{ kN} \quad T_3 = 1.155 \text{ kN}$

3.50 Find P, R by a) projecting on x, y axes, b) projecting onto lines parallel and perpendicular to the plane, c) graphical construction.

a) FBD

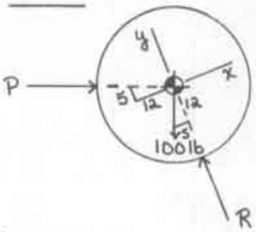


$\Sigma F_x = 0 = P - \frac{5}{13}R$

$\Sigma F_y = 0 = \frac{12}{13}R - 100$

$P = 41.67\text{ lb} \quad R = 108.3\text{ lb}$

b) FBD

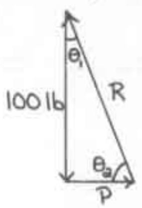


$\Sigma F_x = 0 = \frac{12}{13}P - \frac{5}{13}(100)$

$\Sigma F_y = 0 = R - \frac{5}{13}P - \frac{12}{13}(100)$

$P = 41.67\text{ lb} \quad R = 108.3\text{ lb}$

c) Geometry



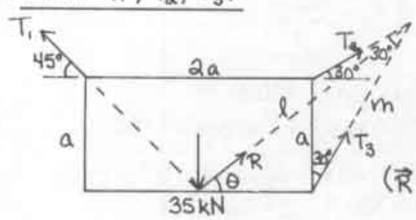
$\theta_1 = \sin^{-1}(\frac{5}{13}) = 22.62°$

$\theta_2 = \sin^{-1}(\frac{12}{13}) = 67.38°$

$R = \dfrac{100}{\sin 67.38°} = 108.3\text{ lb}$

$P = \dfrac{100}{\tan 67.38°} = 41.67\text{ lb}$

3.51 Find T_1, T_2, T_3.



$(\vec{R} = \vec{T}_2 + \vec{T}_3)$

Use geometry to find θ.

Law of Sines: $\dfrac{a}{\sin 30°} = \dfrac{m}{\sin 120°} \rightarrow m = a\sqrt{3}$

Law of Cosines:

$l^2 = a^2 + (a\sqrt{3})^2 - 2a(a\sqrt{3})\cos 120°$

$l^2 = 4a^2 + a^2\sqrt{3}$

$l = 2.394a$

Law of Sines: $\dfrac{a\sqrt{3}}{\sin\theta} = \dfrac{2.394a}{\sin 120°} \rightarrow \theta = 38.79°$

$\Sigma F_x = 0 = R\cos 38.79° - T_1\cos 45°$

$\Sigma F_y = 0 = R\sin 38.79° + T_1 \sin 45° - 35$

$R = 24.89\text{ kN} \quad T_1 = 27.44\text{ kN}$

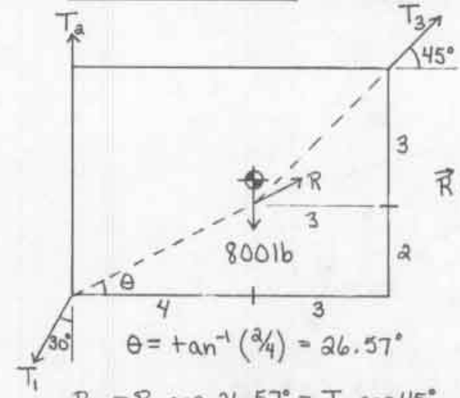
$\vec{R} = \vec{T}_2 + \vec{T}_3$:

$R_x = 24.89\cos 38.79° = T_2\cos 30° + T_3 \sin 30°$

$R_y = 24.89\sin 38.79° = T_2\sin 30° + T_3\cos 30°$

$T_2 = 18\text{ kN} \quad T_3 = 7.61\text{ kN}$

3.52 Find T_1, T_2, T_3.



$\vec{R} = \vec{T}_3 + \vec{W}$

$\theta = \tan^{-1}(\frac{2}{4}) = 26.57°$

$R_x = R\cos 26.57° = T_3\cos 45°$

$R_y = R\sin 26.57° = T_3\sin 45° - 800$

$R = 1789\text{ lb} \quad T_3 = 2263\text{ lb}$

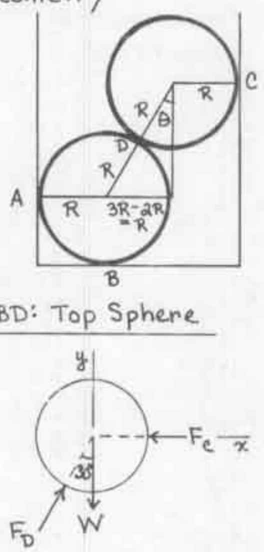
$\Sigma F_x = 0 = 1789\cos 26.57° - T_1\sin 30°$

$\Sigma F_y = 0 = 1789\sin 26.57° - T_1\cos 30° + T_2$

$T_1 = 3200\text{ lb} \quad T_2 = 1971\text{ lb}$

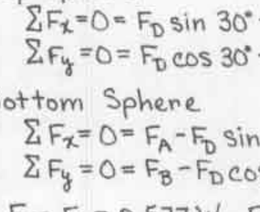
3.53 Find F_A, F_B, F_C, F_D.

Geometry

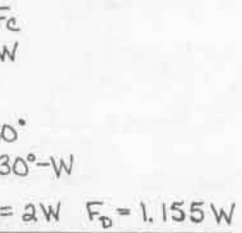


$\theta = \sin^{-1}\left(\frac{R}{2R}\right) = 30°$

FBD: Top Sphere



FBD: Bottom Sphere



Top Sphere

$\Sigma F_x = 0 = F_D\sin 30° - F_C$

$\Sigma F_y = 0 = F_D\cos 30° - W$

Bottom Sphere

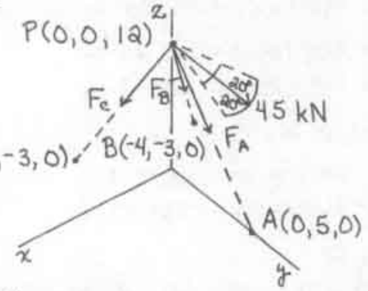
$\Sigma F_x = 0 = F_A - F_D\sin 30°$

$\Sigma F_y = 0 = F_B - F_D\cos 30° - W$

$F_A = F_C = 0.577W \quad F_B = 2W \quad F_D = 1.155W$

3.54 Find F_A, F_B, F_C as a function of θ.

FBD



Unit Vectors

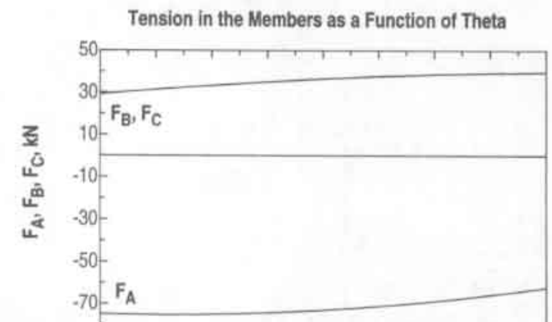
$$\hat{n}_A = \frac{5\hat{j} - 12\hat{k}}{\sqrt{(-5)^2 + 12^2}} = 5/13\,\hat{j} - 12/13\,\hat{k}$$

$$\hat{n}_B = \frac{-4\hat{i} - 3\hat{j} - 12\hat{k}}{\sqrt{(-4)^2 + (-3)^2 + (-12)^2}} = -4/13\,\hat{i} - 3/13\,\hat{j} - 12/13\,\hat{k}$$

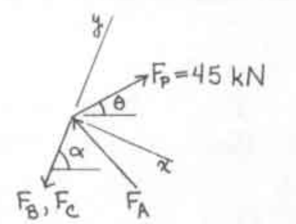
$$\hat{n}_C = \frac{4\hat{i} - 3\hat{j} - 12\hat{k}}{\sqrt{4^2 + (-3)^2 + (-12)^2}} = 4/13\,\hat{i} - 3/13\,\hat{j} - 12/13\,\hat{k}$$

$$\vec{F}_A = F_A\hat{n}_A = F_A\,(5/13\,\hat{j} - 12/13\,\hat{k})$$
$$\vec{F}_B = F_B\,\hat{n}_B = F_B\,(-4/13\,\hat{i} - 3/13\,\hat{j} - 12/13\,\hat{k})$$
$$\vec{F}_C = F_C\,\hat{n}_C = F_C\,(4/13\,\hat{i} - 3/13\,\hat{j} - 12/13\,\hat{k})$$
$$\vec{F}_P = 45\cos\theta\,\hat{j} + 45\sin\theta\,\hat{k}$$
$$\Sigma F_x = 0 = -4/13\,F_B + 4/13\,F_C$$
$$\Sigma F_y = 0 = 5/13\,F_A - 3/13\,F_B - 3/13\,F_C + 45\cos\theta$$
$$\Sigma F_z = 0 = -12/13\,F_A - 12/13\,F_B - 12/13\,F_C + 45\sin\theta$$

Where $-20° \le \theta \le 20°$



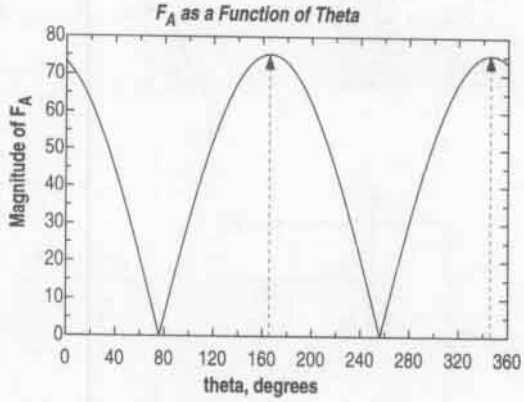
3.55 Find θ, F_A when F_A is at a minimum.



When $\vec{F}_P$ lies in the same plane as $\vec{F}_B, \vec{F}_C$, the magnitude of $\vec{F}_A$ is at a minimum.

When $\theta = \alpha$: $\Sigma F_x = 0$, $F_{Ax} = 0$.

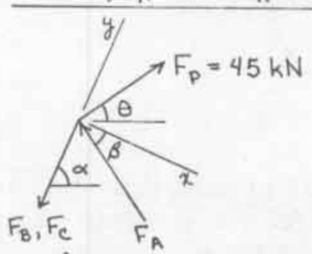
3.55 cont. Using the equations in Prob. 3.54, plot $|F_A|$ vs. θ for $0 \le \theta \le 360$.



F_A is at a minimum value of $\underline{0 \text{ kN}}$ when $\underline{\theta = 76°, 256°(-104°)}$

The applicable value is $\theta = 76°$.

3.56 Find θ, F_A when F_A is at a maximum.



When $\vec{F}_P$ lies perpendicular to the plane of $\vec{F}_B, \vec{F}_C$, the magnitude of $\vec{F}_A$ is at a maximum.

When $\theta = 90 - \alpha$: $\Sigma F_x = 0$,

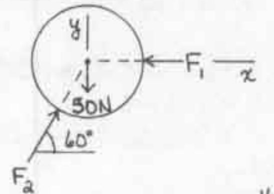
$$45\cos(90-\alpha) + F_A\cos\beta = 0$$

Using the same plot from Prob. 3.55,

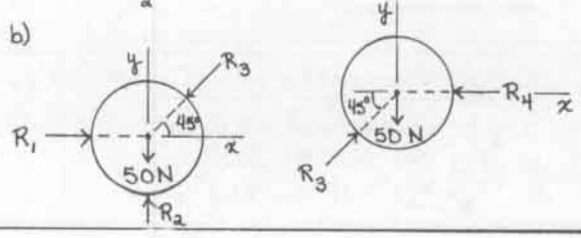
F_A is at a maximum value of $\underline{75.4 \text{ kN}}$ when $\underline{\theta = 166°, 346°(-14°)}$. F_A is never negative.

3.67 Draw FBD's.

a)

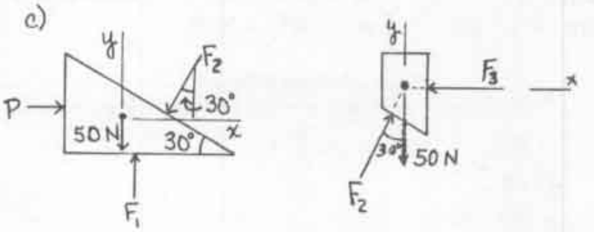


b)

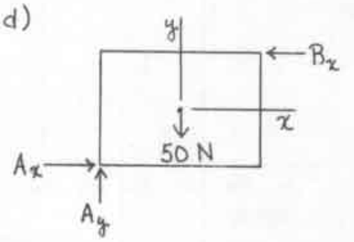


3.67 cont.

c)

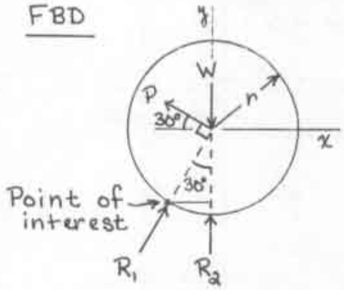


d)



4.1 Find the equation for the sum of the moments about the point at the curb.

FBD

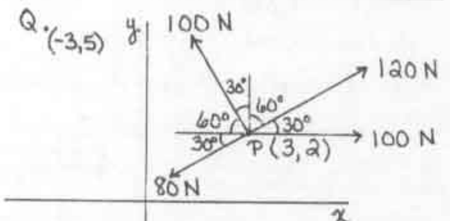


Moment = Σ(Forces × perpendicular distance)

$M = Pr - W(r\sin 30°) + R_2(r\sin 30°) + R_1(0)$

$\underline{\underline{M = Pr - 0.5Wr + 0.5R_2 r}}$

4.2 Find moment of forces about Q.



Find x, y projections of resultant force.

$F_x = -80\cos 30° - 100\cos 60° + 120\cos 30° + 100$
$= 84.64$ N

$F_y = -80\sin 30° + 100\sin 60° + 120\sin 30° = 106.60$ N

$r_x = 3-(-3) = 6 \quad r_y = 2-5 = -3$

$M = r_xF_y - r_yF_x = 6(106.60) - (-3)(84.64)$

$\underline{\underline{M = 893.5 \text{ N·m}}}$

4.3 a) Find moments about O. (See Fig. P4.3)

Find x, y projections of each force.

300 lb: $F_x = -3/\sqrt{13}\,(300) = -166.41$ lb
(0,2) $F_y = 3/\sqrt{13}\,(300) = 249.62$ lb

4.3 cont.

100 lb: $F_x = 1/\sqrt{5}\,(100) = 44.72$ lb
(2,2) $F_y = 2/\sqrt{5}\,(100) = 89.44$ lb

200 lb: $F_x = 2/\sqrt{8}\,(200) = 141.42$ lb
(4,2) $F_y = 2/\sqrt{8}\,(200) = 141.42$ lb

200 lb: $F_x = -200$ lb $F_y = 0$ lb
(1,-2)

300 lb: $F_x = 2/\sqrt{8}\,(300) = 212.13$ lb
(3,-1) $F_y = -2/\sqrt{8}\,(300) = -212.13$ lb

$M_o = xF_y - yF_x$

$\Sigma M_o = [0(249.62) + 2(89.44) + 4(141.42) + 1(0) + 3(-212.13)] - [2(-166.41) + 2(44.72) + 2(141.42) - 2(-200) - 1(212.13)]$

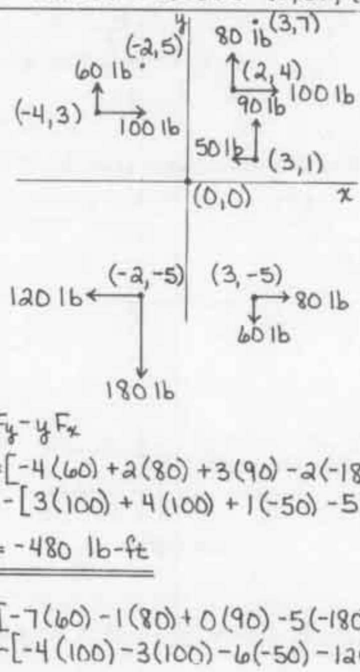
$\underline{\underline{\Sigma M_o = -119.16 \text{ lb-in}}}$

b) Find moments about (3,-1).

$\Sigma M_{(3,-1)} = [-3(249.62) - 1(89.44) + 1(141.42) - 2(0)] - [3(-166.41) + 3(44.72) + 3(141.42) - 1(-200)]$

$\underline{\underline{\Sigma M_{(3,-1)} = -956.07 \text{ lb-in}}}$

4.4 Find moments about (0,0), (3,7), (-2,5).



$M = xF_y - yF_x$

$\Sigma M_{(0,0)} = [-4(60) + 2(80) + 3(90) - 2(-180) + 3(-60)] - [3(100) + 4(100) + 1(-50) - 5(-120) - 5(80)]$

$\underline{\underline{\Sigma M_{(0,0)} = -480 \text{ lb-ft}}}$

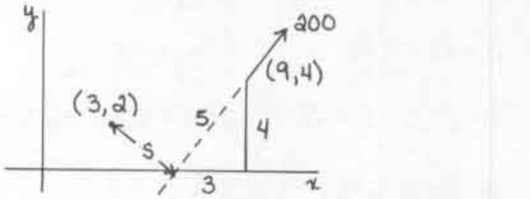
$\Sigma M_{(3,7)} = [-7(60) - 1(80) + 0(90) - 5(-180) + 0(60)] - [-4(100) - 3(100) - 6(-50) - 12(-120) - 12(80)]$

$\underline{\underline{\Sigma M_{(3,7)} = 320 \text{ lb-ft}}}$

$\Sigma M_{(-2,5)} = [-2(60) + 4(80) + 5(90) + 0(-180) + 5(-60)] - [-2(100) - 1(100) - 4(-50) - 10(-120) - 10(80)]$

$\underline{\underline{\Sigma M_{(-2,5)} = 50 \text{ lb-ft}}}$

4.5 Find moment about (3,2) by
a) calculating perpendicular distance,
b) $M = xF_y - yF_x$.



a) Line of action $y = \frac{4}{3}x + b$

Solve for b at (9,4): $4 = \frac{4}{3}(9) + b \rightarrow b = -8$

$$s = \frac{|mx_i - y_i + b|}{\sqrt{1+m^2}} = \frac{|\frac{4}{3}(3) - 2 - 8|}{\sqrt{1+(\frac{4}{3})^2}} = 3.6$$

$M = 200(3.6) = \underline{\underline{720}}$

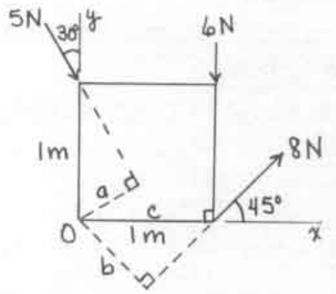
b) Resolve forces into x,y components

$F_x = \frac{3}{5}(200) = 120 \quad F_y = \frac{4}{5}(200) = 160$

$M = xF_y - yF_x = 6(160) - 2(120) = \underline{\underline{720}}$

4.6 Find moments about O by a) computing perpendicular distances, b) $M = xF_y - yF_x$

FBD



a) $a = 1\sin 30° = 0.5 \quad b = 1\sin 45° = 0.7071 \quad c = 1$

$M = -5(0.5) - 6(1) + 8(0.7071) = \underline{\underline{-2.843\ N\cdot m}}$

b) x, y components

5N: $F_x = 5\sin 30° = 2.5 \quad F_y = -5\cos 30° = -4.33$

8N: $F_x = 8\cos 45° = 5.657 \quad F_y = 8\cos 45° = 5.657$

6N: $F_x = 0 \quad F_y = -6$

$$M_o = xF_y - yF_x = [0(-4.33) + 1(-6) + 1(5.657)] - [1(2.5) + 1(0) + 0(5.657)]$$

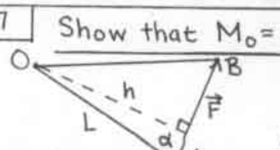
$\underline{\underline{M_o = -2.843\ N\cdot m}}$

SI units: $-2.843\ N\cdot m$

US Customary units:

$$-2.843\ N\cdot m \times \frac{1\ lb\cdot ft}{1.355818\ N\cdot m} = -2.097\ lb\text{-}ft$$

4.7 Show that $M_o = 2 \times$ Area of ΔOAB.



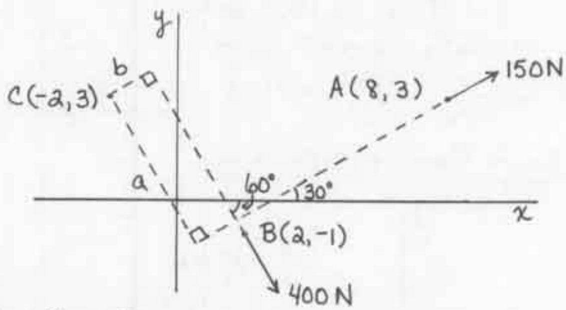
$$\text{Area} = \frac{Fh}{2} = \frac{FL\sin\alpha}{2}$$

$2 \times \text{Area} = FL\sin\alpha$

4.7 cont. $M_o = Fh = FL\sin\alpha$

$\therefore \underline{\underline{M_o = 2 \times \text{Area}}}$

4.8 Find moment about (-2,3) by a) computing perpendicular distances, b) computing perpendicular distance of resultant force, c) $M = xF_y - yF_x$.



a) a: line of 150 N

$y = mx + b \quad (8,3) \quad m = \tan 30° = 0.5774$

$3 = 0.5774(8) + b \rightarrow b = -1.619$

$(x_i, y_i) = (-2,3)$

$$a = \frac{|mx_i - y_i + b|}{\sqrt{1+m^2}} = \frac{|0.5774(-2) - 3 - 1.619|}{\sqrt{1+0.5774^2}}$$

$a = 5\ m$

b: line of 400N

$y = mx + b \quad (2,-1) \quad m = -\tan 60° = -1.732$

$-1 = -1.732(2) + b \rightarrow b = 2.464$

$(x_i, y_i) = (-2,3)$

$$b = \frac{|-1.732(-2) - 3 + 2.464|}{\sqrt{1+(-1.732)^2}} = 1.464\ m$$

$M_{(-2,3)} = 150(5) - 400(1.464) = \underline{\underline{164.4\ N\cdot m}}$

b) Resultant $\vec{R} = \vec{A} + \vec{B}$

$$\vec{R} = 150\cos 30°\hat{\imath} + 150\sin 30°\hat{\jmath} + 400\cos 60°\hat{\imath} - 400\sin 60°\hat{\jmath} = 329.9\hat{\imath} - 271.41\hat{\jmath}$$

$R = \sqrt{329.9^2 + (-271.41)^2} = 427.2\ N$

The resultant acts where $\vec{A}$ and $\vec{B}$ intersect.

$0.5774x - 1.619 = -1.732x + 2.464$

$(x, y) = (1.768, -0.598)$

Line of Resultant:

$m = -271.41 / 329.9 = -0.8227$

$-0.598 = -0.8227(1.768) + b \rightarrow b = 0.8565$

$$s = \frac{|-0.8227(-2) - 3 + 0.8565|}{\sqrt{1+(-0.8227)^2}} = 0.3847$$

$M_{(-2,3)} = Rs = 427.2(0.3847) = \underline{\underline{164.4\ N\cdot m}}$

c) $M = xF_y - yF_x$

$$M_{(-2,3)} = [10(150\sin 30°) + 4(-400\sin 60°)] - [0(150\cos 30°) + 4(400\cos 60°)]$$

$M_{(-2,3)} = \underline{\underline{164.4\ N\cdot m}}$

4.9 a) Find F, θ, b.

$\vec{F} = (300+400+600)\hat{\imath} + (500-100+800)\hat{\jmath}$

$\vec{F} = 1300\hat{\imath} + 1200\hat{\jmath}$

$F = \sqrt{1300^2 + 1200^2} = \underline{\underline{1769.2 \text{ lb}}}$

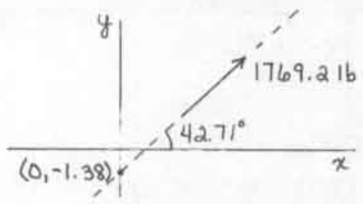
$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1200}{1300}\right) = \underline{\underline{42.71°}}$

$\Sigma M_o = [2(500) + 5(-100) - 1(800)] - [1(300) - 3(400) - 2(600)] = 1800 \text{ lb-ft}$

$M = -yF_x \quad @\, x = 0,\ y = b$

$y = \frac{-M}{F_x} = \frac{-1800}{1300} = -1.38 \qquad \underline{\underline{b = -1.38 \text{ ft}}}$

b) Draw $\vec{F}$ and resultant axis.



c) Write equation of resultant axis.

$y = mx + b = (1200/1300)x - 1.38$

$\underline{\underline{y = 0.92x - 1.38}}$

4.10 a) Find F, θ, b.

$\vec{F} = (200-800-1000)\hat{\imath} + (-100+500+700)\hat{\jmath}$

$\vec{F} = -1600\hat{\imath} + 1100\hat{\jmath}$

$F = \sqrt{(-1600)^2 + 1100^2} = \underline{\underline{1941.6 \text{ lb}}}$

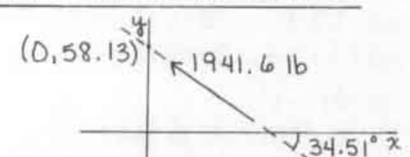
$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1100}{-1600}\right) = \underline{\underline{-34.51°}}$

$\Sigma M_o = [-20(-100) + 80(500) + 50(700)] - [20(200) + 100(-800) - 60(-1000)] = 93000 \text{ lb-in}$

$M = -yF_x \ @\, x = 0 \quad b = y$

$b = y = \frac{-M}{F_x} = \frac{-93{,}000}{-1600} = \underline{58.13 \text{ in}}$

b) Draw $\vec{F}$ and resultant axis.



c) Write equation of resultant axis.

$y = mx + b$

$y = \left(\frac{1100}{-1600}\right)x + 58.13$

$\underline{\underline{y = -0.688x + 58.13}}$

4.11 a) Find F, θ, b.

$\vec{F} = (3+2.5+7+3-6)\hat{\imath} + (-2+4-1+3)\hat{\jmath} = 9.5\hat{\imath} + 4\hat{\jmath}$

$F = \sqrt{9.5^2 + 4^2} = \underline{\underline{10.31 \text{ kips}}}$

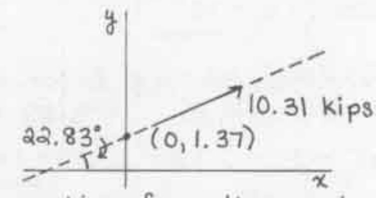
$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{4}{9.5}\right) = \underline{\underline{22.83°}}$

$\Sigma M_o = [5(-2) + 4(-1)] - [2(3) + 2(2.5) + 8(3) + 6(-6)]$
$= -13 \text{ kip-ft}$

$@\, x = 0,\ y = b,\ M = -yF_x$

$b = y = \frac{-M}{F_x} = \frac{13}{9.5} = \underline{\underline{1.37 \text{ ft}}}$

b) Draw $\vec{F}$ and resultant axis.



c) Write equation of resultant axis.

$y = mx + b = \left(\frac{4}{9.5}\right)x + 1.37$

$\underline{\underline{y = 0.421x - 1.37}}$

4.12 a) Find F, θ, b.

$\vec{F} = (40+25-80+30)\hat{\imath} + (10+45+50+35)\hat{\jmath}$

$\vec{F} = 15\hat{\imath} + 140\hat{\jmath}$

$F = \sqrt{15^2 + 140^2} = \underline{\underline{140.8 \text{ N}}}$

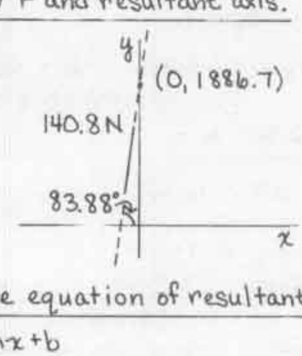
$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{140}{15}\right) = \underline{\underline{83.88°}}$

$\Sigma M_o = [80(10) + 100(45) - 200(50)] - [-60(40) - 60(25) - 400(-80) - 150(30)] = -28\,300 \text{ N}\cdot\text{mm}$

$@\, x = 0,\ y = b,\ M = -yF_x$

$b = y = \frac{-M}{F_x} = \frac{28\,300}{15} = \underline{\underline{1886.7 \text{ mm}}}$

b) Draw $\vec{F}$ and resultant axis.



c) Write equation of resultant axis.

$y = mx + b$

$y = \left(\frac{140}{15}\right)x + 1886.7$

$\underline{\underline{y = 9.33x + 1886.7}}$

4.13 a) Find F, θ, b.

$\vec{F} = (500 + 320 + 120 + 80 + 150 - 400)\hat{\imath} + (-600 - 100 + 100 + 300 + 125 - 600)\hat{\jmath} = 770\hat{\imath} - 775\hat{\jmath}$

$F = \sqrt{770^2 + (-775)^2} = \underline{\underline{1092.5\ \text{N}}}$

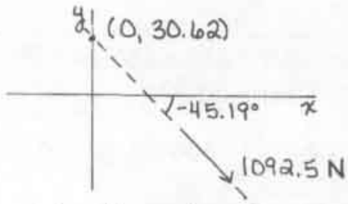
$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-775}{770}\right) = \underline{\underline{-45.19°}}$

$\Sigma M_o = [30(-600) + 6(-100) + 7(100) - 4(300) + 3(125) - 5(-600)] - [2(500) + 6(320) + 9(120) - 5(80) + 7(150) - 8(-400)] = -23\,575\ \text{N·m}$

@ $x = 0$, $y = b$, $M = -yF_x$

$b = y = \frac{-M}{F_x} = \frac{23\,575}{770} = \underline{\underline{30.62\,\text{m}}}$

b) Draw $\vec{F}$ and resultant axis.



c) Write equation of resultant axis.

$y = mx + b = \left(\frac{-775}{770}\right)x + 30.62$

$\underline{\underline{y = -1.006x + 30.62}}$

4.14 a) Find F, θ, b.

$\vec{F} = (300 - 200 - 300 + 100 - 500)\hat{\imath} + 0\hat{\jmath} = -600\hat{\imath}$

$\underline{\underline{F = 600\,\text{N}}}$

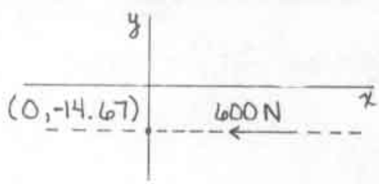
$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{0}{-600}\right) = \underline{\underline{0°}}$

$\Sigma M_o = -[6(300) + 7(-200) - 8(-300) - 12(-500)] = -8800\ \text{N·m}$

@ $x = 0$, $y = b$, $M = -yF_x$

$b = y = \frac{-M}{F_x} = \frac{8800}{-600} = \underline{\underline{-14.67\ \text{m}}}$

b) Draw $\vec{F}$ and resultant axis.



c) Write equation of resultant axis.

$y = mx + b$

$y = (0/_{-600})x - 14.67$

$\underline{\underline{y = -14.67\ \text{m}}}$

4.15 a) Find F, θ, b.

From Prob. 4.3

F_x (lb)	F_y (lb)	x(m)	y(m)
-166.41	249.62	0	2
44.72	89.44	2	2
141.42	141.42	4	2
-200	0	1	-2
212.13	-212.13	3	1

$\vec{F} = 31.86\hat{\imath} + 268.34\hat{\jmath}$

$F = \sqrt{31.86^2 + 268.34^2} = \underline{\underline{270.2\ \text{lb}}}$

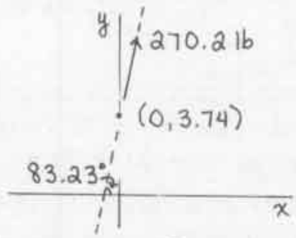
$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{268.34}{31.86}\right) = \underline{\underline{83.23°}}$

From Prob 4.3: $\Sigma M_o = -119.16$ lb·in

@ $x = 0$, $y = b$, $M = -yF_x$

$b = y = \frac{-M}{F_x} = \frac{119.16}{31.86} = \underline{\underline{3.74\ \text{in}}}$

b) Draw $\vec{F}$ and resultant axis.



c) Write equation of resultant axis.

$y = mx + b = \left(\frac{268.34}{31.86}\right)x + 3.74$

$\underline{\underline{y = 8.42x + 3.74}}$

4.16 a) Find $\vec{F}$. (See Fig. P4.16.)

$\vec{F} = (-60 + 100\sin 30°)\hat{\imath} + (100\cos 30° + 80)\hat{\jmath}$

$\underline{\underline{\vec{F} = -10\hat{\imath} + 166.7\hat{\jmath}}}$

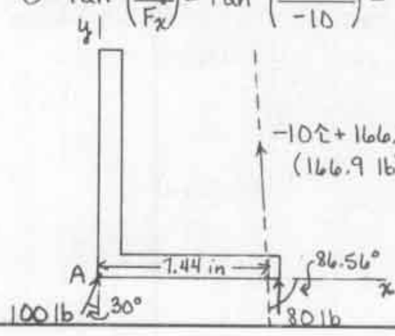
b) Find intersection of resultant axis and $\overline{AB}$. Draw $\vec{F}$ and resultant axis.

$\Sigma M_A = 60(10) + 80(8) = 1240$ lb·in

@ $y = 0$, $M_A = xF_y$

$x = \frac{M_A}{F_y} = \frac{1240}{166.6} = \underline{\underline{7.44\ \text{in from A}}}$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{166.6}{-10}\right) = -86.56°$



4.17 a) Find $\vec{F}$. (See Fig. P4.17 and Fig. a below.)

$\vec{F} = (-320\cos 45° + 400\sin 30°)\hat{\imath} + (-320\sin 45° + 240 + 400\cos 30°)\hat{\jmath} = \underline{\underline{-26.27\hat{\imath} + 360.14\hat{\jmath}\ \text{N}}}$

b) Find perpendicular distance between O and resultant axis. Draw $\vec{F}$ and resultant axis

$\Sigma M_o = 320\cos 45°(300) = 67\,882\ \text{N}\cdot\text{mm}$

@ $x=0$, $M_o = -yF_x$

$b = y = \dfrac{-M_o}{F_x} = \dfrac{-67\,882}{-26.27} = 2584\ \text{mm}$

$m = \dfrac{F_y}{F_x} = \dfrac{360.14}{-26.27} = -13.71 \qquad (x_i, y_i) = (0,0)$

$s = \dfrac{|mx_i - y_i + b|}{\sqrt{1+m^2}} = \dfrac{|-13.71(0) - 0 + 2584|}{\sqrt{1+(-13.71)^2}}$

$\underline{\underline{s = 187.97\ \text{mm}}}$

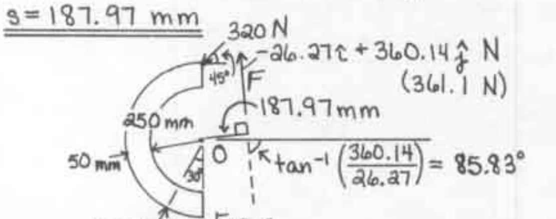


Fig. a

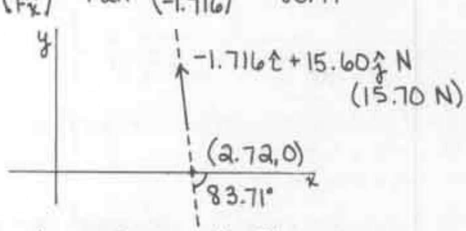
4.18 a) Find $\vec{F}$ and resultant axis. Show them on a figure. (Refer to Problem Statement.)

$\vec{F} = (-40 + 40\cos 45° + 20\cos 60°)\hat{\imath} + (-30 + 40\sin 45° + 20\sin 60°)\hat{\jmath} = \underline{\underline{-1.716\hat{\imath} + 15.60\hat{\jmath}\ \text{N}}}$

$M_o = 40\cos 45°(1.5) = 42.43\ \text{N}\cdot\text{m}$

@ $y=0$, $M = xF_y \rightarrow x = 42.43/15.6 = 2.72\ \text{m}$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{15.6}{-1.716}\right) = -83.71°$



b) Find $\vec{F}$ and resultant axis. Show them on a figure. (Refer to Problem Statement.)

$\vec{F} = (-40 + 40\cos 45°)\hat{\imath} + (-30 + 40\sin 45°)\hat{\jmath}$

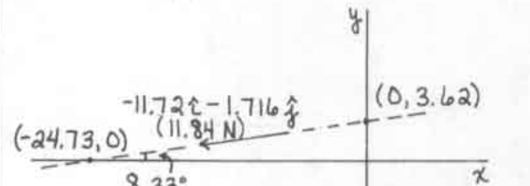
$\underline{\underline{\vec{F} = -11.72\hat{\imath} - 1.716\hat{\jmath}\ \text{N}}}$

$M_o = 42.43\ \text{N}\cdot\text{m} \qquad @\ y=0,\ M = xF_y$

$x = 42.43/-1.716 = -24.73\ \text{m}$

@ $x=0$, $M = -yF_x$, $y = -42.43/-11.72 = 3.62\ \text{m}$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-1.716}{-11.72}\right) = 8.33°$



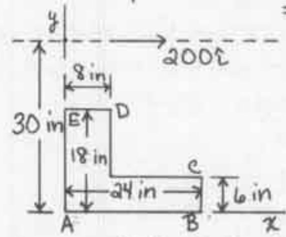
4.19 a) Find $\vec{F} = \vec{A} + \vec{B} + \vec{D}$ and intercept with $\overline{AE}$. Show in a figure.

$\vec{F} = (200)\hat{\imath} + (100 - 100)\hat{\jmath} = \underline{\underline{200\hat{\imath}\ \text{lb}}}$

$\Sigma M_E = -100(24) = -2400\ \text{lb}\cdot\text{in}$

@ $x=0$, $M_E = -yF_x \quad y = 2400/200 = 12\ \text{in}$

intercept $= 18 + 12 = \underline{\underline{30\ \text{in}}}$



b) Find $\vec{F} = \vec{B} + \vec{C} + \vec{D} + \vec{E}$ and intercept with $\overline{AE}$. Show in a figure.

$\vec{F} = (300\cos 45° + 200 + 250\sin 30°)\hat{\imath} + (300\sin 45° - 100 - 250\cos 30°)\hat{\jmath} = \underline{\underline{537.1\hat{\imath} - 104.4\hat{\jmath}\ \text{lb}}}$

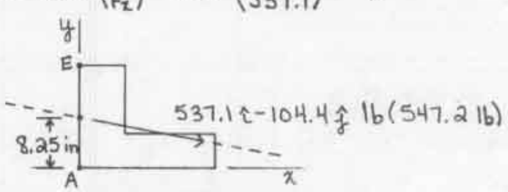
$\Sigma M_E = -100(24) + 300\cos 45°(12) + 300\sin 45°(24)$

$\Sigma M_E = 5236.75\ \text{lb}\cdot\text{in}$

Since $x=0$, $M_E = -yF_x \quad y = -5236.75/537.1 = -9.75\ \text{in}$

$\overline{AE}$ intercept $= 18 - 9.75 = \underline{\underline{8.25\ \text{in from A}}}$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-104.4}{537.1}\right) = -11°$



c) Find $\vec{F} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$ and intercept with $\overline{AE}$. Show in a figure.

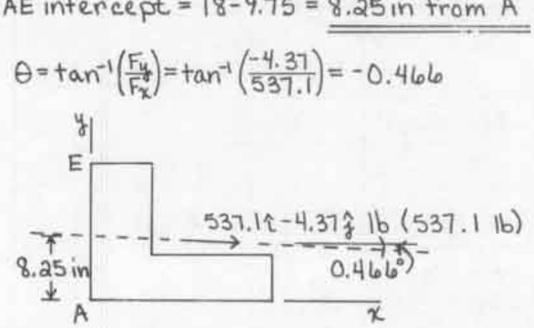
$\vec{F} = (300\cos 45° + 200 + 250\sin 30°)\hat{\imath} + 300\sin 45° + 100 - 100 - 250\cos 30°)\hat{\jmath} = \underline{\underline{537.1\hat{\imath} - 4.37\hat{\jmath}\ \text{lb}}}$

$\Sigma M_E = 5236.75\ \text{lb}\cdot\text{in}$ (from part b) : $\vec{A}$ passes through E.

Since $x=0$, $M_E = -yF_x \quad y = -5236.75/537.1 = -9.75\ \text{in}$

$\overline{AE}$ intercept $= 18 - 9.75 = \underline{\underline{8.25\ \text{in from A}}}$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-4.37}{537.1}\right) = -0.466$



4.20 a) Find $\vec{F}$.

$\vec{F} = (4 - 5\sin 30° + 3\cos 30°)\hat{\imath} + (-2 + 5\cos 30° + 3\sin 30°)\hat{\jmath}$

$\underline{\vec{F} = 4.10\hat{\imath} + 3.83\hat{\jmath}\ \text{kN}}$ (Continued)

4.20 cont. b) Find perpendicular distance of resultant axis to center.

$M_{center} = 3(1) = 3\ \text{kN}\cdot\text{m}$

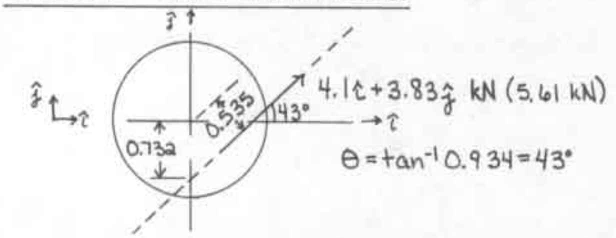
@ $x=0$, $y=b$, $M=-yF_x$

$b=y=-3/4.1=-0.732 \qquad m = 3.83/4.10 = 0.934$

$s = \dfrac{|mx_i - y_i + b|}{\sqrt{1+m^2}} = \dfrac{|0.934(0)-0-0.732|}{\sqrt{1+0.934^2}}$

$\underline{\underline{s = 0.535\ \text{m}}}$

c) Draw $\vec{F}$ and resultant axis.



4.21 a) Find $\vec{F}$ and elevation. (See Fig. P4.21.)

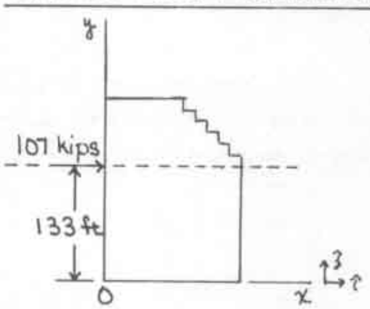
$\vec{F} = (5+11+8+7+7+11+10+10+9+9+8+12)\hat{i} + 0\hat{j}$

$\underline{\underline{\vec{F} = 107\hat{i}\ \text{kips}}}$

$\Sigma M_o = -[5(250)+11(230)+8(210)+7(190)+7(170)+11(150)+10(130)+10(110)+9(90)+9(70)+8(50)+12(30)] = -14,230\ \text{kip}\cdot\text{ft}$

@ $x=0$, $M_o=-yF_x \quad y = 14,230/107 = \underline{\underline{133\ \text{ft}}}$

b) Draw $\vec{F}$ and resultant axis.



4.22 Find $\vec{F}$ and x-intercept. (See Fig. P4.22)

$\vec{F} = (12\cos 10° - 125\sin 6°)\hat{i} + (12\sin 10° + 125\cos 6° - 120)\hat{j}$

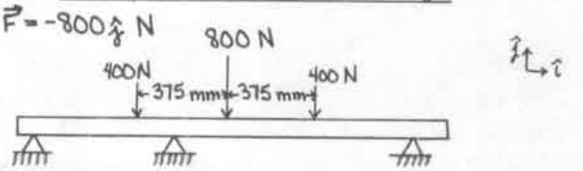
$\underline{\underline{\vec{F} = -1.25\hat{i} + 6.40\hat{j}\ \text{kips}}}$

$\Sigma M_o = -120(30) + 125\sin 6°(3) + 125\cos 6°(36) = 914.55\ \text{kip}\cdot\text{ft}$

@ $y=0$, $M=xF_y \quad x = 914.55/6.40 = \underline{\underline{142.9\ \text{ft}}}$

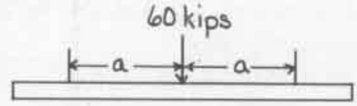
4.23 Find $\vec{F}$ and show on drawing.

$\vec{F} = -800\hat{j}$ N



4.24 Find $\vec{F}$ and show on drawing. (See Fig. P4.24)

$\vec{F} = -60\hat{j}$ kips

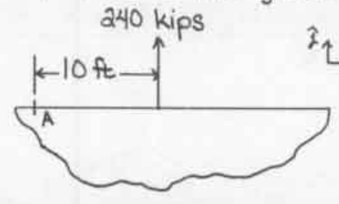


4.25 Find $\vec{F}$ and show on drawing. (See Fig. P4.25)

$\vec{F} = 240\hat{j}$ kips

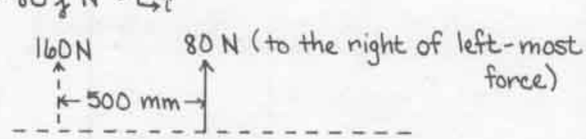
$\Sigma M_A = 60(4) + 60(14) + 60(22) = 240(r)$

$r = 10$ ft to the right of left-most force.



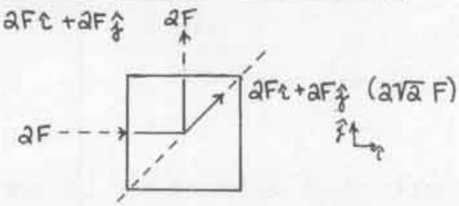
4.26 Find $\vec{F}$ and show on drawing. (See Fig. P4.26)

$\vec{F} = 80\hat{j}$ N

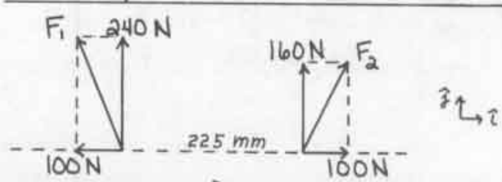


4.27 Find $\vec{F}$ and show on drawing.

$\vec{F} = 2F\hat{i} + 2F\hat{j}$

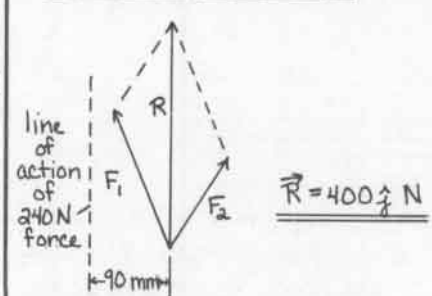


4.28 Find $\vec{F}_1$ and $\vec{F}_2$ that are dynamically equal to the system of parallel forces 240N, 160N.



$\underline{\underline{\vec{F}_1 = -100\hat{i} + 240\hat{j}\ \text{N} \quad \vec{F}_2 = 100\hat{i} + 160\hat{j}}}$

b) Find $\vec{R}$ graphically.



$\underline{\underline{\vec{R} = 400\hat{j}\ \text{N}}}$

(Continued)

4.28 cont. c) Use method of moments and compare with results of (b).

$M = rF$, where r is distance from F to 240 N.

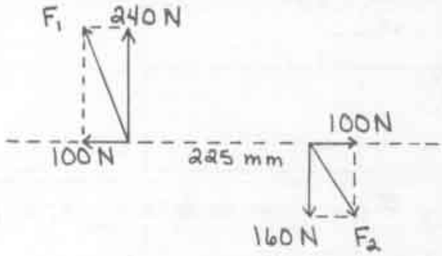
$M_{240N} = 0.225(160) = rR$

$M_{160N} = 0.225(240) = (0.225 - r)R$

$r = \dfrac{36}{R} = \dfrac{0.225R - 54}{R} \longrightarrow R = 400\text{ N}$

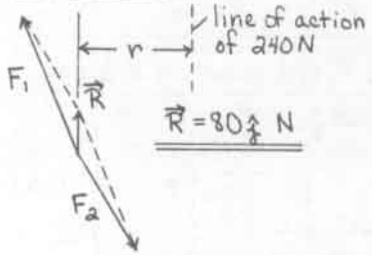
$\vec{R}$ is in the y-direction: $\underline{\underline{\vec{R} = 400\,\hat{j}\text{ N},\ r = 90\text{ mm}}}$

4.29 a) Find $\vec{F}_1$ and $\vec{F}_2$ that are dynamically equal to the system of *parallel forces 240N, 160N.*



$\underline{\underline{\vec{F}_1 = -100\,\hat{i} + 240\,\hat{j}\text{ N} \quad \vec{F}_2 = 100\,\hat{i} - 160\,\hat{j}\text{ N}}}$

b) Find $\vec{R}$ graphically.



$\underline{\underline{\vec{R} = 80\,\hat{j}\text{ N}}}$

c) Use method of moments and compare with results of (b).

$M = rF$, where r is distance from $\vec{R}$ to 240 N.

$M_{240N} = -0.225(160) = -rR$

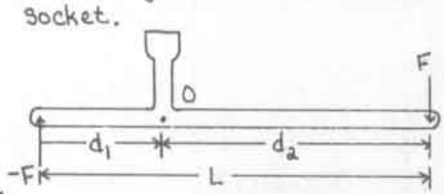
$M_{160N} = -0.225(240) = -(r + 0.225)R$

$r = \dfrac{36}{R} = \dfrac{54 - 0.225R}{R} \longrightarrow R = 80\text{ N}$

$\vec{R}$ is in the y-direction: $\underline{\underline{\vec{R} = 80\,\hat{j}\text{ N},\ r = 450\text{ mm}}}$

4.30 a) No.

b) No. The magnitude of the moment is the same regardless of the location of the socket.

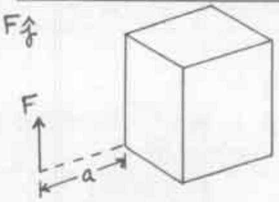


$\Sigma M_o = -Fd_1 - Fd_2 \quad L = d_1 + d_2 \longrightarrow M_o = -FL$

Forces create a couple of magnitude $-FL$ which is equal to ΣM_o ∴ distances d_1, d_2 are irrelevant.

4.31 Find $\vec{F}$ and show on drawing. *See Fig. P4.31.*

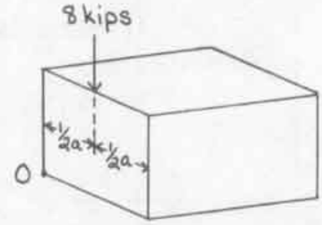
$\vec{F} = F\hat{j}$



4.32 Find $\vec{F}$ and show on drawing. *See Fig. P4.32.*

$\vec{F} = -8\,\hat{j}$ kips

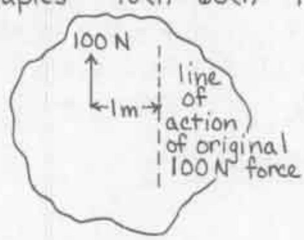
$\Sigma M_o = -6a - 3a + 5a = -8r \rightarrow r = \tfrac{1}{2}a$



4.33 Find $\vec{F}$ and show on drawing. *See Fig. P4.33.*

$\vec{F} = 100\,\hat{j}$ N

Σ Couples $= -40(1) - 60(1) = -100 = 100r \rightarrow r = -1$

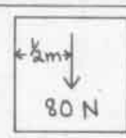


4.34 Find resultant force or couple for each case and show on drawing. *See Fig. P4.34*

a) $\vec{R} = (-P - P)\hat{j}$

$\vec{R} = (-2(40))\hat{j}$

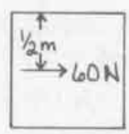
$\underline{\underline{\vec{R} = -80\,\hat{j}\text{ N}}}$



b) $\vec{R} = (F + F)\hat{i}$

$\vec{R} = (2(30))\hat{i}$

$\underline{\underline{\vec{R} = 60\,\hat{i}\text{ N}}}$

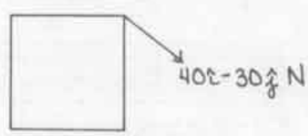


c) $\vec{R} = F\hat{i} - P\hat{j}$

$\underline{\underline{\vec{R} = 30\,\hat{i} - 40\,\hat{j}\text{ N}}}$

d) $\vec{R} = P\hat{i} - F\hat{j}$

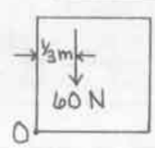
$\underline{\underline{\vec{R} = 40\,\hat{i} - 30\,\hat{j}\text{ N}}}$



e) $\vec{R} = (-P - Q)\hat{j}$

$\vec{R} = (-40 - 20)\hat{j}$

$\underline{\underline{\vec{R} = -60\,\hat{j}\text{ N}}}$



$\Sigma M_o = -20(1)$

$-20 = -60r$

$r = \tfrac{1}{3}$ m

(Continued)

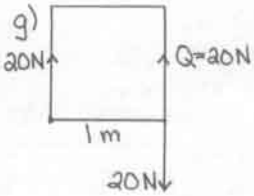
4.34 cont.

f) $\Sigma M = -C - C$

$\Sigma M = -2(20)$

$\underline{\underline{\Sigma M = -40 \text{ N·m}}}$

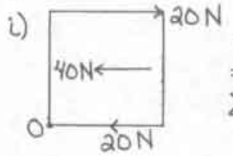
g)



$\underline{\underline{\vec{R} = 20\hat{j} \text{ N}}}$

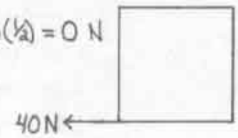
h) $\Sigma M = C - C = 0 \text{ N·m}$

i)



$\underline{\underline{\vec{R} = -40\hat{i} \text{ N}}}$

$\Sigma M_o = -20 + 40(\tfrac{1}{2}) = 0 \text{ N}$



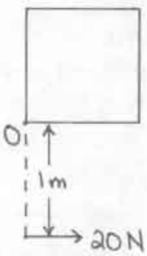
j) $\vec{R} = (P - Q - Q)\hat{i}$

$\vec{R} = (40 - 20 - 20)\hat{i} = \underline{\underline{0 \text{ N}}}$

k) $\vec{R} = (-Q + P)\hat{i}$

$\vec{R} = (-20 + 40)\hat{i}$

$\underline{\underline{\vec{R} = 20\hat{i} \text{ N}}}$

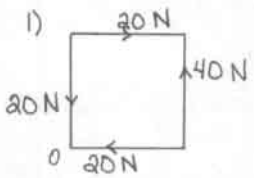


$\Sigma M_o = 20(1)$

$20 = 20r$

$r = 1 \text{ m}$

l)



$\vec{R} = (P - Q)\hat{j}$

$\vec{R} = (40 - 20)\hat{j}$

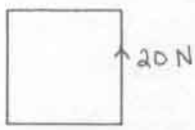
$\underline{\underline{\vec{R} = 20\hat{j} \text{ N}}}$

$\Sigma M_o = -C + P(1)$

$-20 + 40 = 20$

$20 = 20r$

$r = 1 \text{ m}$



m) $\vec{R} = (F - P)\hat{i} + (F - Q)\hat{j}$

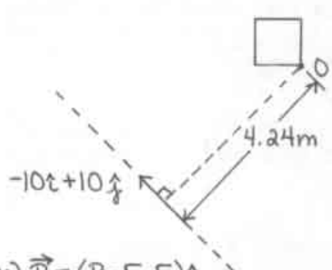
$\vec{R} = (30 - 40)\hat{i} + (30 - 20)\hat{j}$

$\underline{\underline{\vec{R} = -10\hat{i} + 10\hat{j} \text{ N}}}$

$\Sigma M_o = -F(1) - F(1) = -60$

$-60 = \sqrt{(-10)^2 + 10^2}\, r$

$r = -4.24 \text{ m}$



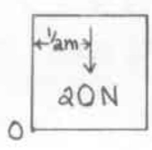
n) $\vec{R} = (P - F - F)\hat{j}$

$\vec{R} = (40 - 30 - 30)\hat{j}$

$\underline{\underline{\vec{R} = -20\hat{j} \text{ N}}}$

$\Sigma M_o = P(\tfrac{1}{2}) - F(1) = -10 \text{ N·m}$

$-10 = -20r \quad r = \tfrac{1}{2} \text{m}$



4.34 cont.

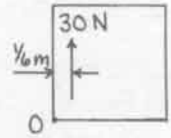
o) $\vec{R} = (P - F + Q)\hat{j}$

$\vec{R} = (40 - 30 + 20)\hat{j}$

$\underline{\underline{\vec{R} = 30\hat{j} \text{ N}}}$

$\Sigma M_o = -F(\tfrac{1}{2}) + Q(1) = 5 \text{ N·m}$

$5 = 30r \rightarrow r = \tfrac{1}{6} \text{ m}$



4.35 Find simplest resultant and show on drawing. See Fig. P4.35.

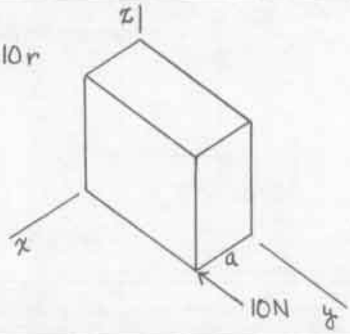
Since the couples are opposite and parallel, they cancel each other. $\vec{R} = 0$, $\Sigma M = 0$.

4.36 Find simplest resultant and show on drawing. See Fig. P4.36.

$\vec{R} = -10\hat{j} \text{ N}$

$\Sigma M_z = -10a = -10r$

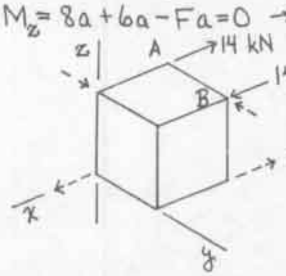
$r = a$



4.37 Find $\vec{A}$ and $\vec{B}$. *Refer to Problem Statement.*

$\vec{A}$ and $\vec{B}$ should be a couple to set the moments in equilibrium.

$\Sigma M_z = 8a + 6a - Fa = 0 \rightarrow F = 14 \text{ kN}$



$\underline{\underline{\vec{A} = -14\hat{i} \text{ kN} \quad \vec{B} = 14\hat{i} \text{ kN}}}$

4.38 Find $\vec{A}$ and $\vec{B}$ that make system dynamically equal. (See Fig. P4.38.)

$\vec{R} = 0$, $\Sigma M = -80(30) - 100(10) = -3400 \text{ lb·in}$

For equivalent system,

$-3400 = F(17) \quad F = -200 \text{ lb}$

For negative moment:

$\underline{\underline{\vec{A} = 200 \text{ lb} (\uparrow) \quad \vec{B} = 200 \text{ lb} (\downarrow)}}$

4.39 Find resultant moment. (See Fig. P4.39.)

$\Sigma M_o = 6(7) - 8(7)$

$= \underline{\underline{-14 \text{ kN·cm}}}$

4.40 Find resultant couple. See Fig. P4.40.

$\Sigma M = 200 - 300 + 100 = \underline{\underline{0 \text{ lb·ft}}}$ ← Ans

4.41 a) Find $\vec{R}$. See Fig. P4.41.

$\underline{\underline{\vec{R} = 300\hat{\imath} - 200\hat{\jmath} \text{ N}}}$ ← Ans

b) Find perpendicular distance of resultant axis to point O.

$\Sigma F_x = 300$ N $\quad \Sigma F_y = -200$ N

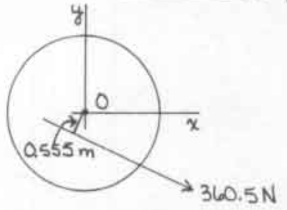
$R = \sqrt{300^2 + (-200)^2} = 360.5$ N

$\tan\theta = -200/300 \quad \theta = -33.69°$

$M_o = 400 - 200 = 200$ N·m

$Rr = 200$ N·m

$r = 200/360.5 = \underline{\underline{0.555 \text{ m}}}$ ← Ans

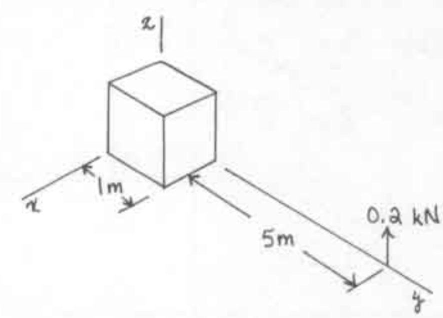


4.42 Find $\vec{R}$ for dynamically equal system of Fig. P4.42.

$\vec{R} = 1.2 - 1 = \underline{\underline{0.2\hat{k} \text{ kN}}}$ ← Ans

$\Sigma M_x = -1 - 2 + 3 + 1.2(1) = 1.2$ kN·m

$1.2 = 0.2r \rightarrow r = 6$ m



4.43 Find P. See Fig. P4.43.

$\Sigma M_o = -1(2) - 3(4) + P(2) = 0$

$\underline{\underline{P = 7 \text{ kips}}}$ ← Ans

4.44 The force on an infinitesimal arc length of the edge is cancelled by the force on the diametrically opposite infinitesimal arc. Hence, the system is dynamically equivalent to a couple. Therefore, it exerts the same moment about all points in the plane of the disk.

Refer to Fig. P4.44.

4.45 As in Prob. 4.44, the system is dynamically equivalent to a couple. Take moments about the center. The force on a length ds of the edge is 300 ds, and its moment about the center is:

$dM = 50 \times 300\, ds = 15\,000\, ds.$

Hence, $M = 15000 \int ds = 2\pi\, 50\, (15\,000)$

$= 471 \times 10^4$ N·mm $= \underline{\underline{4710 \text{ N·m}}}$ ← Ans

Refer to Fig. P4.45.

4.46 a) Find ΣM about (x_o, y_o). (See Table P4.46)

$M = xF_y - yF_x$

$\Sigma M_o = [500(4 - x_o) - 650(-7 - x_o) + 150(-5 - x_o) + 400(10 - x_o) - 400(-7 - x_o)] - [300(8 - y_o) + 700(3 - y_o) - 800(4 - y_o) - 200(-2 - y_o)]$

$\Sigma M_o = y_o(300 + 700 - 800 - 200) - x_o(500 - 650 + 150 + 400 - 400) + 2000 - 4550 - 750 + 4000 + 2800 - 2400 - 2100 + 3200 - 400$

$\underline{\underline{\Sigma M_o = 10{,}900 \text{ lb·in}}}$ ← Ans

b) The significance of the fact that the moment doesn't depend on (x_o, y_o) means that the system is dynamically equivalent to a couple. This is verified by the fact that $\Sigma F_x = \Sigma F_y = 0$.

4.47 Find ΣM. (See Fig. P4.47.)

$\Sigma F_y = 0 = 15 - 12 - F \rightarrow F = 3$ kips

$\Sigma M = -5(1) + 12(3) - 3(20) = \underline{\underline{-29 \text{ kip·ft}}}$ ← Ans

4.48 a) Replace forces with $\vec{R}$ and $\vec{M}_o$ (Fig. P4.48)

$\vec{R} = (-60 - 100 \sin 45°)\hat{\imath} + (-40 + 100\cos 45°)\hat{\jmath}$

$\underline{\underline{\vec{R} = -130.7\hat{\imath} + 30.71\hat{\jmath} \text{ lb}}}$ ← Ans

$\Sigma M_o = -40(16) - 100 \sin 45° (8) + 100 \cos 45° (16)$

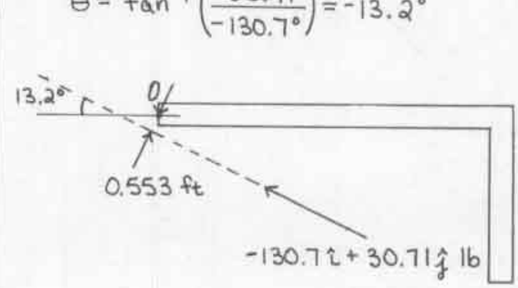
$\underline{\underline{\Sigma M_o = -74.31 \text{ lb·ft}}}$ ← Ans

b) Find resultant axis and show on drawing.

$-74.31 = \sqrt{(-130.7)^2 + 30.71^2}\ (r)$

$r = -0.553$ ft

$\theta = \tan^{-1}\left(\dfrac{30.71}{-130.7°}\right) = -13.2°$



4.49 a) Replace forces with $\vec{R}$ and $\vec{M}_o$. (Fig P4.49)

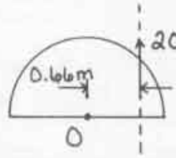
$\vec{R} = (-80-40+120+200)\hat{j} = \underline{200\hat{j}\ \text{N}}$

$\Sigma M_o = 80(0.9-0.3) - 120(0.3) + 200(0.6) = \underline{132\ \text{N·m}}$

b) Find resultant axis and show on drawing.

$M = rF$

$132 = 200r \rightarrow r = 0.66\text{m}$



4.50 a) Replace forces with $\vec{R}$ and $\vec{M}_o$. (Fig P4.50)

$\vec{R} = (40+60)\hat{i} + (-50)\hat{j} = \underline{100\hat{i} - 50\hat{j}\ \text{lb}}$

$\Sigma M_o = 60(4) + 40(2) - 100 = \underline{220\ \text{lb·ft}}$

b) Find resultant axis and show on drawing.

$R = \sqrt{100^2 + (-50)^2} = 111.8\ \text{lb}$

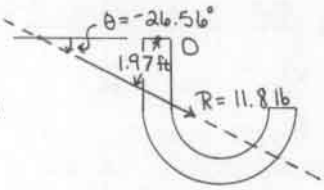
$M = rF$

$220 = 111.8r$

$r = 1.97\ \text{ft}$

$\tan\theta = -50/100$

$\theta = -26.56°$



4.51 Transfer force to A and find $\vec{M}_A$. (Fig. P4.51)

$F = 60\ \text{kN}$

$\vec{M}_A = 60(0.15) + 160 = \underline{169\ \text{kN·m}}$

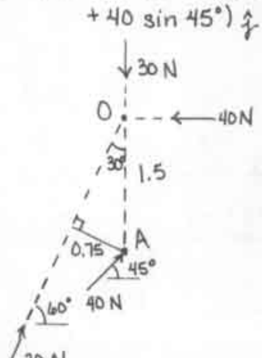
4.52 Replace forces with $\vec{R}$ and $\vec{M}_o$. (Fig. P4.52)

$\vec{R} = (100 + 100\cos 30° + 100\cos 60°)\hat{i} + (100\sin 30° - 100\sin 60°)\hat{j} = \underline{236.6\hat{i} - 36.60\hat{j}\ \text{lb}}$

$\Sigma M_o = 100\sin 30°(4) - 100\cos 30°(4) - 100\cos 60°(6) - 400 = \underline{-846.4\ \text{lb·ft}}$

4.53 Replace forces with $\vec{R}$ and $\vec{M}_A$. (See Fig. P4.18.)

$\vec{R} = (20\cos 60° + 40\cos 45° - 40)\hat{i} + (20\sin 60° - 30 + 40\sin 45°)\hat{j} = \underline{-1.72\hat{i} + 15.6\hat{j}\ \text{N}}$



$\Sigma M_A = 40(1.5) - 20(0.75)$

$\underline{\vec{M}_A = 45\ \text{N·m}}$

4.54 Find forces needed for equilibrium in P4.49.

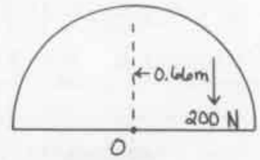
$\vec{F}$ is equilibriant of the system. So,

$\Sigma F_y = 0 = -80 + 120 - 40 + 200 + F$

$\underline{\vec{F} = -200\hat{j}\ \text{N}}$

$\Sigma M_o = 0 = -80(-0.6) + 120(-0.3) + 200(0.6) - 200(r)$

$\underline{r = 0.66\ \text{m}}$



4.55 Find force and couple at point O for equilibrium of Prob. 4.18.

$\vec{F}$ is the equilibriant force and $\vec{C}_o$ is the equilibriant couple of the system. So,

$\Sigma F_x = 0 = 20\cos 60° + 40\cos 45° - 40 + F_x$

$\Sigma F_y = 0 = 20\sin 60° + 40\sin 45° - 30 + F_y$

$\Sigma M_o = 0 = 40\cos 45°(1.5) + C_o$

$\underline{\vec{F} = 1.72\hat{i} - 15.6\hat{j}\ \text{N} \quad \vec{C}_o = -42.43\ \text{N·m}}$

4.56 Find $\vec{R}$ and $\vec{B}$. (See Figs. P4.56 + a below.)

FBD

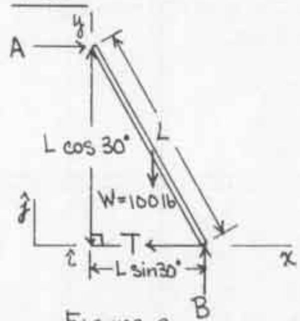


Figure a.

$\Sigma F_x = 0 = A - T$

$\Sigma F_y = 0 = B - 100$

$\Sigma M_B = 0 = 100(L\sin 30°/2) - A(L\cos 30°)$

$\vec{T} = -28.87\hat{i}\ \text{lb}$

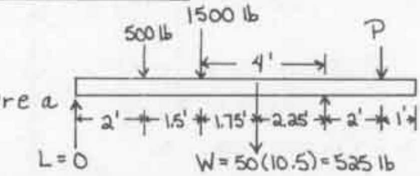
$\vec{A} = 28.87\hat{i}\ \text{lb}$

$\underline{\vec{B} = 100\hat{j}\ \text{lb}}$

4.57 Find minimum P. (See Figs. P4.57 + a below.)

FBD

Figure a



$\Sigma F_y = 0 = -500 - 1500 - 525 + R - P$

$\Sigma M_R = 0 = 500(5.5) + 1500(4) + 525(2.25) - P(2)$

$\underline{R = 7491\ \text{lb} \quad P = 4966\ \text{lb}}$

4.58 Find F and M for equilibrium. (See Fig. P4.58)

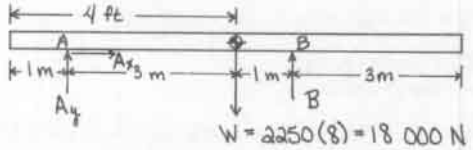
$\Sigma F_y = 0 = 500 - 600 + F$

$\Sigma M_F = 0 = 7500 - 500(120) + 600(70) + M$

$\underline{F = 100\ \text{lb} \quad M = 10{,}500\ \text{lb·in}}$

4.59 Find reactions at A and B.

FBD



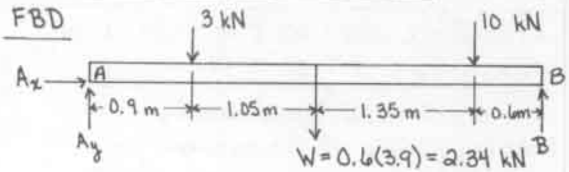
$\Sigma F_x = 0 = A_x \qquad \Sigma F_y = 0 = A_y - 18\,000 + B$

$\Sigma M_A = 0 = -18\,000(3) + B(4)$

$\vec{A} = 4500\,\hat{j}$ N $\quad \vec{B} = 13\,500\,\hat{j}$ N

4.60 Find reactions at A and B.

FBD



$\Sigma F_x = 0 = A_x \qquad \Sigma F_y = 0 = A_y - 3 - 2.34 - 10 + B$

$\Sigma M_A = -3(0.9) - 2.34(1.95) - 10(3.3) + B(3.9) = 0$

$\vec{A} = 5.02\,\hat{j}$ kN $\quad \vec{B} = 10.32\,\hat{j}$ kN

4.61 Find F and M. (See Fig. P4.61.)

$\Sigma F_y = 0 = 300 - 100 - 400 - W + F - 300$

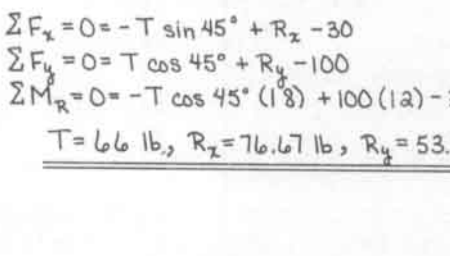
$\Sigma M_F = 0 = -300(2.5) + W(2.25) + 400(3) + 100(5) - 300(7) + M$

$W = 30(9.5) = 285$ lb

$F = 785$ lb $\quad M = 508.75$ lb·ft

4.62 Find T, R_x, R_y.

FBD



$\Sigma F_x = 0 = -T \sin 45° + R_x - 30$

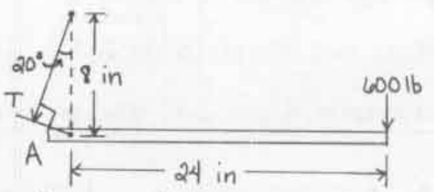
$\Sigma F_y = 0 = T \cos 45° + R_y - 100$

$\Sigma M_R = 0 = -T \cos 45° (18) + 100(12) - 30(12)$

$T = 66$ lb., $R_x = 76.67$ lb, $R_y = 53.33$ lb.

4.63 Find T. Refer to Problem 4.63.

FBD

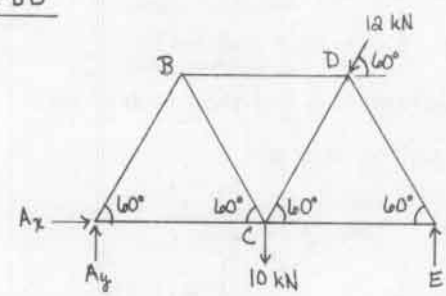


$\Sigma M_A = T(8 \sin 20°) - 600(24) = 0$

$T = 5263$ lb

4.64 Draw FBD and find reactions at the supports.

FBD



$\Sigma F_x = 0 = A_x - 12 \cos 60°$

$\Sigma F_y = 0 = A_y - 10 - 12 \sin 60° + E$

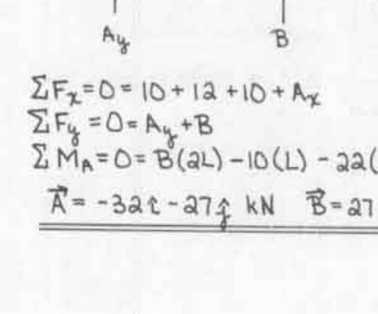
$\Sigma M_C = 0 = E(L) - A_y(L)$

$\vec{A} = 6\hat{i} + 10.2\,\hat{j}$ kN $\quad \vec{E} = 10.2\,\hat{j}$ kN

4.65 Draw FBD and find reactions at the supports.

Since all reactions and forces act on external joints, internal members may be ignored.

FBD



$\Sigma F_x = 0 = 10 + 12 + 10 + A_x$

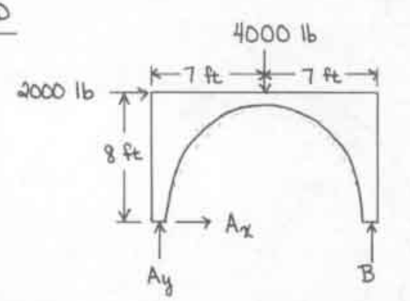
$\Sigma F_y = 0 = A_y + B$

$\Sigma M_A = 0 = B(2L) - 10(L) - 22(2L)$

$\vec{A} = -32\hat{i} - 27\hat{j}$ kN $\quad \vec{B} = 27$ kN

4.66 Draw FBD and find reactions at supports.

FBD



$\Sigma F_x = 0 = A_x + 2000$

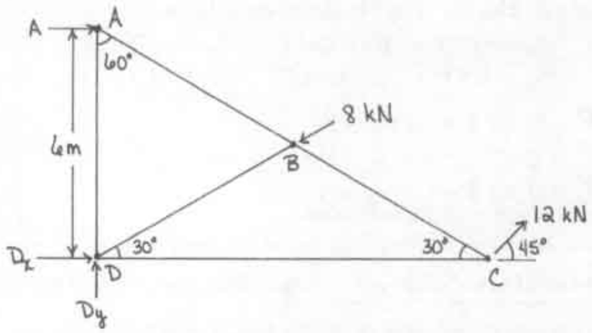
$\Sigma F_y = 0 = A_y + B - 4000$

$\Sigma M_A = 0 = -4000(7) + B(14) - 2000(8)$

$\vec{A} = -2000\,\hat{i} + 857.1\,\hat{j}$ lb $\quad \vec{B} = 3142.9\,\hat{j}$ lb

4.67 Draw FBD and find reactions at supports.

FBD



$\Sigma F_x = 0 = A + D_x - 8\cos 30° + 12\cos 45°$

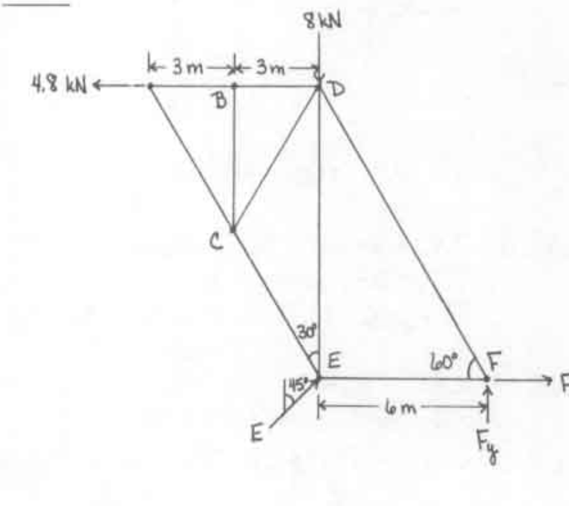
$\Sigma F_y = 0 = D_y - 8\sin 30° + 12\sin 45°$

$\Sigma M_D = 0 = -A(6) + 12\sin 45°(6/\tan 30°)$

$\vec{A} = 14.7\,\hat{i}$ kN $\quad \vec{D} = -16.25\,\hat{i} - 4.49\,\hat{j}$ kN

4.68 Draw FBD and find reactions at supports

FBD



4.68 cont'd

$\Sigma F_x = 0 = -4.8 + E\sin 45° + F_x$

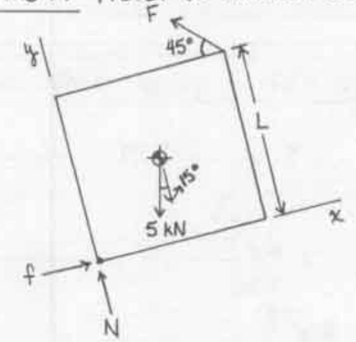
$\Sigma F_y = 0 = -8 + E\cos 45° + F_y$

$\Sigma M_F = 0 = 4.8(6/\tan 30°) + 8(6) - E\cos 45°(6)$

$E = 23.07$ kN $\quad \vec{E} = E\sin 45°\,\hat{i} + E\cos 45°\,\hat{j}$ kN

$\vec{E} = 16.31\,\hat{i} + 16.31\,\hat{j}$ kN $\quad \vec{F} = -11.51\,\hat{i} - 8.31\,\hat{j}$ kN

4.69 Find F. Refer to Problem Statement.



$\Sigma M_o = 0 = -5\cos 15°(L/2) + 5\sin 15°(L/2) + F(L/\sin 45°)$

$F = 1.25$ kN

4.70 Find reactions on wheels in terms of W and θ. See Figs. P4.70 and a (below).

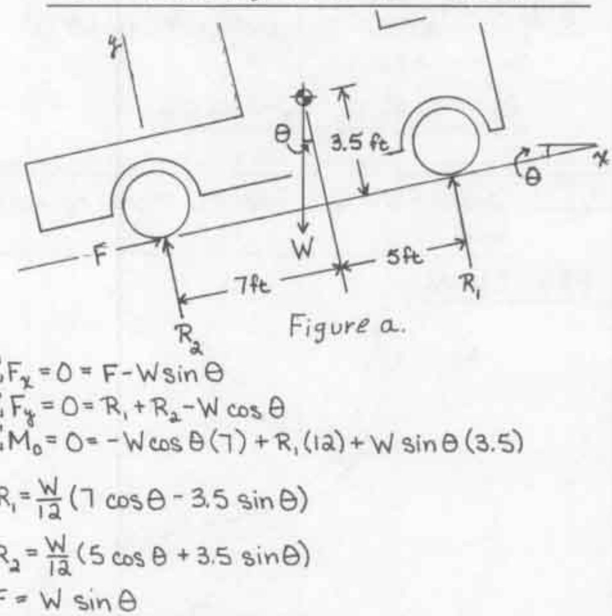


Figure a.

$\Sigma F_x = 0 = F - W\sin\theta$

$\Sigma F_y = 0 = R_1 + R_2 - W\cos\theta$

$\Sigma M_o = 0 = -W\cos\theta(7) + R_1(12) + W\sin\theta(3.5)$

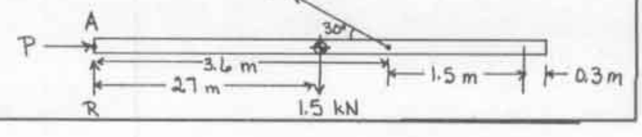
$R_1 = \frac{W}{12}(7\cos\theta - 3.5\sin\theta)$

$R_2 = \frac{W}{12}(5\cos\theta + 3.5\sin\theta)$

$F = W\sin\theta$

4.71 a) Draw FBD and calculate the tension T in tie rod.

FBD



4.71 cont'd

$\Sigma M_A = -1.5(2.7) - 6(5.1) + T\sin 30°(3.6) = 0$

$T = 19.25 \text{ kN}$

b) Find P and R.

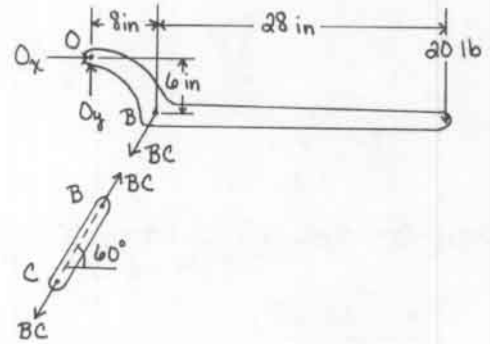
$\Sigma F_x = 0 = P - 19.25\cos 30°$

$\Sigma F_y = 0 = R - 1.5 + 19.25\sin 30° - 6$

$P = 16.67 \text{ kN} \quad R = -2.125 \text{ kN}$

4.72 a) Draw FBD of pump handle and link.

FBD



b) Find O_x, O_y.

$\Sigma F_x = 0 = O_x - BC\cos 60°$

$\Sigma F_y = 0 = O_y - BC\sin 60° - 20$

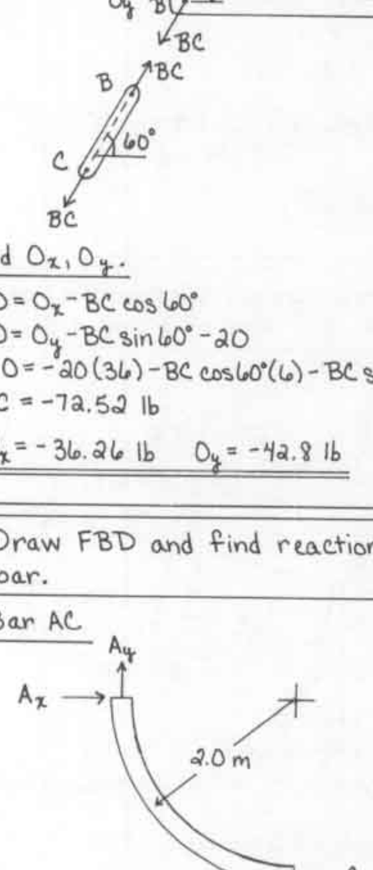
$\Sigma M_o = 0 = -20(36) - BC\cos 60°(6) - BC\sin 60°(8)$

$BC = -72.52 \text{ lb}$

$O_x = -36.26 \text{ lb} \quad O_y = -42.8 \text{ lb}$

4.73 Draw FBD and find reactions on each bar.

FBD: Bar AC



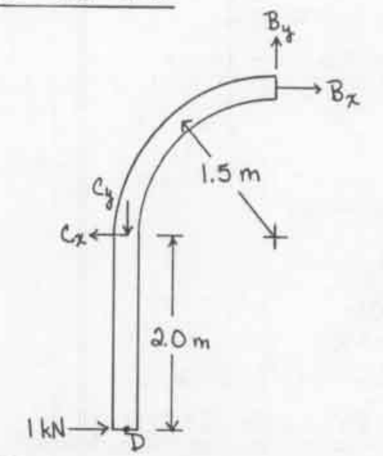
$\Sigma F_x = 0 = A_x + C_x$

$\Sigma F_y = 0 = A_y + C_y$

$\Sigma M_A = 0 = 2(C_x) + 2(C_y)$

4.73 cont'd

FBD: Bar BCD



$\Sigma F_x = 0 = B_x - C_x + 1$

$\Sigma F_y = 0 = B_y - C_y$

$\Sigma M_B = 0 = 1.5(-C_x) + 1.5(C_y) + 3.5(1)$

Solve the 6 equations simultaneously.

$A_x = -1.17 \quad B_x = 0.17 \quad C_x = 1.17 \text{ kN}$

$A_y = 1.17 \quad B_y = -1.17 \quad C_y = -1.17 \text{ kN}$

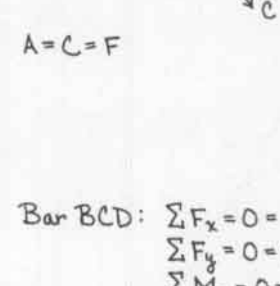
$\vec{A} = -1.17\hat{i} + 1.17\hat{j} \text{ kN}$

$\vec{B} = 0.17\hat{i} - 1.17\hat{j} \text{ kN}$

$\vec{C} = 1.17\hat{i} - 1.17\hat{j} \text{ kN}$

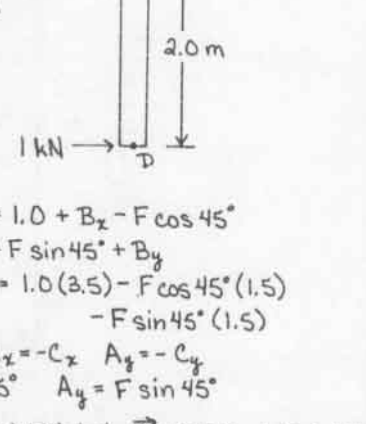
Alternative Solution: Use the concept that member AC is a two-force member.

FBD: Bar AC



$A = C = F$

FBD: Bar BCD



Bar BCD: $\Sigma F_x = 0 = 1.0 + B_x - F\cos 45°$

$\Sigma F_y = 0 = F\sin 45° + B_y$

$\Sigma M_B = 0 = 1.0(3.5) - F\cos 45°(1.5) - F\sin 45°(1.5)$

Bar AC: $A = C = F \quad A_x = -C_x \quad A_y = -C_y$

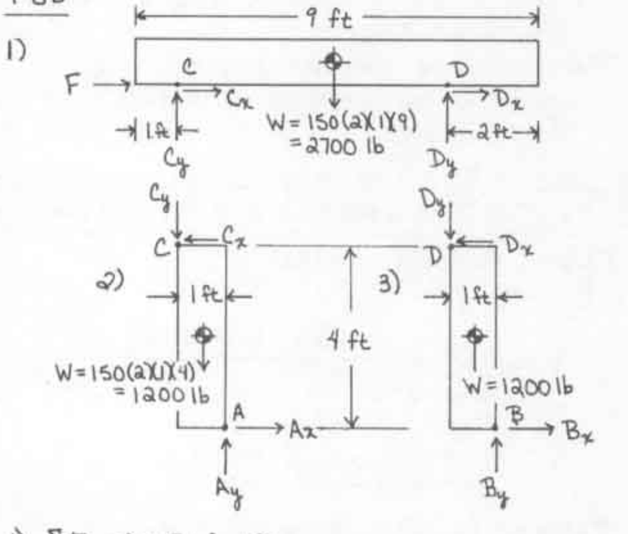
$A_x = -F\cos 45° \quad A_y = F\sin 45°$

Solve: $\vec{A} = -1.17\hat{i} + 1.17\hat{j} \text{ kN} \quad \vec{B} = 0.17\hat{i} - 1.17\hat{j} \text{ kN}$

$\vec{C} = 1.17\hat{i} - 1.17\hat{j} \text{ kN}$

4.74 Find $\vec{A}$, $\vec{B}$, and F. (See text Problem Statement)

FBD



1) $\Sigma F_x = 0 = F + C_x + D_x$

$\Sigma F_y = 0 = C_y + D_y - 2700$

$\Sigma M_c = 0 = -2700(3.5) + D_y(6)$

$D_y = 1575 \text{ lb} \quad C_y = 1125 \text{ lb}$

2) $\Sigma F_x = 0 = -C_x + A_x$

$\Sigma F_y = 0 = -C_y + A_y - 1200$

$\Sigma M_c = 0 = A_y(1) + A_x(4) - 1200(0.5)$

$A_y = 2325 \text{ lb} \quad A_x = -431.25 \text{ lb}$

$C_x = -431.25 \text{ lb}$

3) $\Sigma F_x = 0 = -D_x + B_x$

$\Sigma F_y = 0 = -D_y + B_y - 1200$

$\Sigma M_D = 0 = B_y(1) + B_x(4) - 1200(0.5)$

$B_y = 2775 \text{ lb} \quad B_x = -543.75 \text{ lb}$

$D_x = -543.75 \text{ lb} \quad F = 975 \text{ lb}$

$\vec{A} = -431.25\hat{\imath} + 2325\hat{\jmath} \text{ lb} \quad \vec{B} = -543.75\hat{\imath} + 2775\hat{\jmath} \text{ lb}$

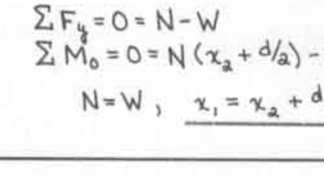
$F = 975 \text{ lb}$

4.75 Find k. (See text Problem Statement.)

W = weight of each penny

x_1, x_2, x_3, x_4, x_5 are x-coordinates of the center of each penny

Penny #1: Top penny



$\vec{N}$ acts at the end of the lower penny when penny #1 starts to tip.

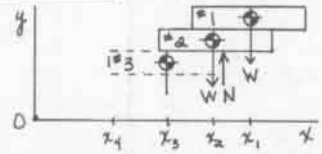
$\Sigma F_y = 0 = N - W$

$\Sigma M_o = 0 = N(x_2 + d/2) - W(x_1)$

$N = W, \quad \underline{x_1 = x_2 + d/2}$

4.75 cont'd

Penny #2:

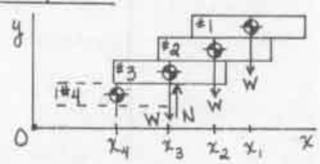


$\Sigma F_y = 0 = N - W - W$

$\Sigma M_o = 0 = N(x_3 + d/2) - W(x_2) - W(x_1)$

$N = 2W, \quad \underline{2x_3 + d = x_1 + x_2}$

Penny #3:

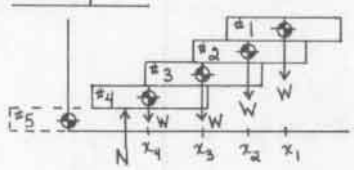


$\Sigma F_y = 0 = N - 3W$

$\Sigma M_o = 0 = N(x_4 + d/2) - W(x_3) - W(x_2) - W(x_1)$

$N = 3W, \quad \underline{3x_4 + 3/2\, d = x_1 + x_2 + x_3}$

Penny #4:



$\Sigma F_y = 0 = N - 4W$

$\Sigma M_o = 0 = N(x_5 + d/2) - W(x_4) - W(x_3) - W(x_2) - W(x_1)$

$N = 4W, \quad \underline{2d = x_1 + x_2 + x_3 + x_4}$

Gather equations:

1) $x_1 = x_2 + d/2$

2) $2x_3 + d = x_1 + x_2$

3) $3x_4 + 3/2\, d = x_1 + x_2 + x_3$

4) $2d = x_1 + x_2 + x_3 + x_4$

Solve equations 1-4 in terms of x_1 and d.

1) $x_2 = x_1 - d/2$

2) $x_3 = 1/2(x_1 + (x_1 - d/2) - d) = x_1 - 3/4\, d$

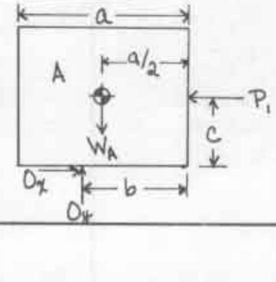
3) $x_4 = 1/3(x_1 + (x_1 - d/2 + (x_1 - 3/4\, d) = x_1 - 11/12\, d$

4) $2d = x_1 + (x_1 - d/2) + (x_1 - 3/4\, d) + (x_1 - 11/12\, d)$

$x_1 = 25/24\, d = kd \rightarrow \underline{\underline{k = 25/24}}$

4.76 Show that $a/b > 1 + \sqrt{2}/2$. (See Problem Statement.)

FBD: Block A



Let P_1 be the force required to tip block A alone.

(Continued)

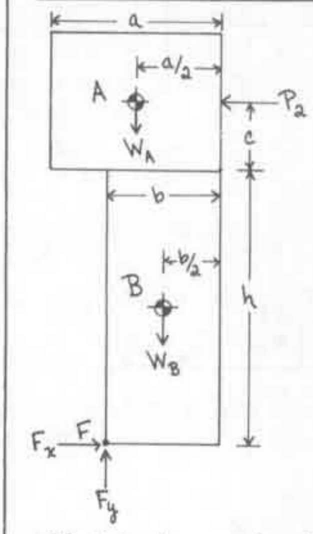
4.76 cont'd

$W_A = 2c(a)(w)(\rho)(g)$, where,
w = width
ρ = density
g = gravitational constant

$\Sigma M_0 = P_1 c - W_A(b - a/2) = 0$

$\therefore P_1 = aw\rho g(2b-a)$

FBD: Blocks A and B together



Let P_2 be the force required to topple the two blocks together.

$W_B = h(b)(w)(\rho)(g)$

$\Sigma M_F = P_2(c+h) - W_A(b - a/2) - W_B(b/2) = 0$

$(c+h)P_2 = caw\rho g(2b-a) + bhw\rho g(b/2)$

$\therefore P_2 = \dfrac{caw\rho g(2b-a)}{c+h} + \dfrac{b^2 hw\rho g}{2(c+h)}$

If A is to topple off without B moving, then $P_1 < P_2$.

$\therefore a(2b-a) < \dfrac{ac(2b-a)}{c+h} + \dfrac{b^2 h}{2(c+h)}$

or $\dfrac{a}{b}\left(2 - \dfrac{a}{b}\right) < \dfrac{a}{b}\dfrac{(2 - a/b)}{1 + h/c} + \dfrac{h/c}{2(1+h/c)}$

Let $a/b = x$, $h/c = y$.

$x(2-x) < \dfrac{x(2-x)}{1+y} + \dfrac{y}{2(1+y)}$

$x(2-x)(1+y) < x(2-x) + y/2$

$x(2-x) < 1/2$

$2x - x^2 < 1/2$

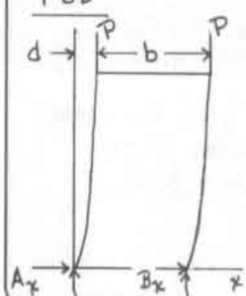
$1/2 < x^2 - 2x + 1 = (x-1)^2$

$\therefore x - 1 > \sqrt{2}/2 \rightarrow x > 1 + \sqrt{2}/2$

$a/b = x \rightarrow \underline{\underline{a/b > 1 + \sqrt{2}/2}}$

4.77 a) Find A_y, B_y in terms of P, b, d.

FBD



$\Sigma F_y = 0 = A_y + B_y - P - P$

$\Sigma M_A = 0 = b(B_y) - d(P) - (d+b)P$

$B_y = (2dP + bP)/b$

$A_y = 2P - B_y$

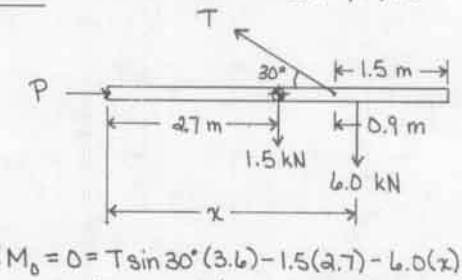
$\underline{\underline{A_y = P - 2Pd/b \quad B_y = P + 2dP/b}}$

4.77 cont'd b) Find A_x, B_x. Explain any difficulty.

The only remaining equation of equilibrium is
$\Sigma F_x = 0 = A_x + B_x$.
There are two unknowns, but only one equation. The structure is statically indeterminate.

4.78 Plot T as a function of x for $0 \le x \le 5.1$ m.

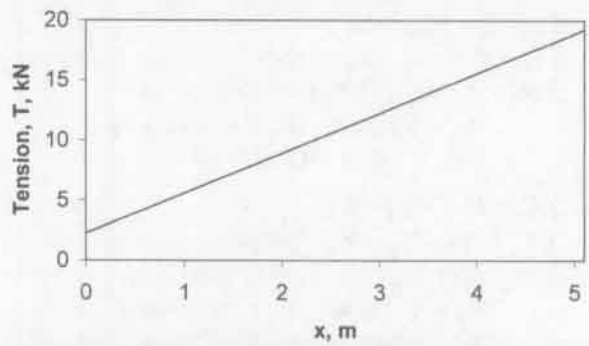
FBD Refer to Problem 4.71.



$\Sigma M_0 = 0 = T\sin 30°(3.6) - 1.5(2.7) - 6.0(x)$

$T = (6x + 4.05)/1.8 = \underline{3.33x + 2.25}$

Tension, T, as a Function of x

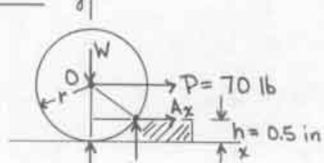


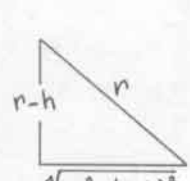
4.79 Find smallest wheel that meets the design requirements. (See Problem Statement)

Assumptions and Constraints:

1) Cart is designed to have 4 wheels of equal size distributed symetrically.
2) Load is equally distributed on the bed of the cart. Each wheel carries 1/4 of the load.
3) Cart is at a complete stop and 2 wheels are touching the riser (height h).
4) Average person can push or pull 70 lb parallel to the floor.
5) Design for maximum h (0.5 in) and maximum load (300 lb).

FBD





4.79 cont'd

Each wheel carries 1/4 of the load (75 lb). Two wheels hit the riser at the same time. Therefore, $W = 2(75) = 150$ lb. At the instant the wheels come off the floor, there is no normal force from the floor to the tire ($N=0$).

$\Sigma F_x = 0 = 70 + A_x \quad \Sigma F_y = 0 = A_y - 150$

$\Sigma M_o = A_y\sqrt{2rh-h^2} - A_x(r-h)$

$A_x = -70$ lb, $\quad A_y = 150$ lb

Solve moment equation to find r.

$$150\sqrt{2rh-h^2} = 70(r-h)$$
$$2rh - h^2 = 0.218(r^2 - 2rh + h^2)$$
$$h = 0.5 \text{ in} \quad \therefore\ r = 0.2623 \text{ in},\ 5.330 \text{ in}$$

Wheel diameter must be between 3 to 18 in. For this design, the wheel diameter $= 2(5.33) = 10.66 \approx 11$ in

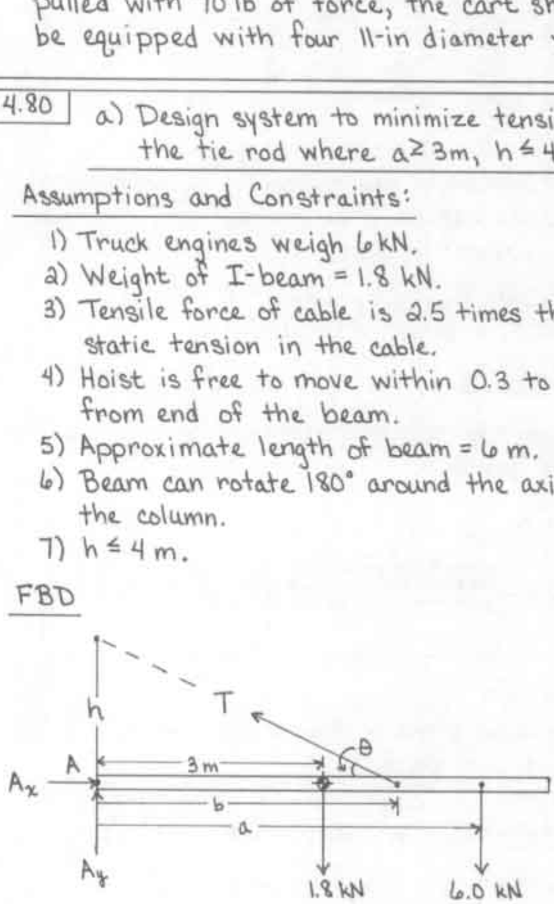
For a cart carrying a 300 lb load and being pulled with 70 lb of force, the cart should be equipped with four 11-in diameter wheels.

4.80 a) Design system to minimize tension in the tie rod where $a \geq 3$m, $h \leq 4$m.

Assumptions and Constraints:

1) Truck engines weigh 6 kN.
2) Weight of I-beam = 1.8 kN.
3) Tensile force of cable is 2.5 times the static tension in the cable.
4) Hoist is free to move within 0.3 to 3 m from end of the beam.
5) Approximate length of beam = 6 m.
6) Beam can rotate 180° around the axis of the column.
7) $h \leq 4$ m.

FBD



$\Sigma F_x = 0 = A_x - T\cos\theta$

$\Sigma F_y = 0 = A_y - 1.8 - 6 + T\sin\theta$

$\Sigma M_A = 0 = T\sin\theta(b) - 1.8(3) - 6(a)$

4.80 cont'd

$$T = \frac{5.4 + 6a}{b\sin\theta}, \quad \sin\theta = \frac{h}{\sqrt{h^2+b^2}}$$
$$\therefore\ T = \frac{5.4 + 6a}{bh/\sqrt{h^2+b^2}}$$

To minimize T: minimize a and maximize b, h

Max b = 6 m, Max h = 4 m, Min a = 3 m

$$\text{Minimum tension, } T = \frac{5.4 + 6(3)}{6(4)/\sqrt{4^2+6^2}} = 7.55 \text{ kN}$$

Minimum tension equals 7.55 kN when a is 3m, b is 6m, and h is 4 m.

b) Summarize results in a brief report.

The tension in the rod is minimized when "a" is minimized and "b" and "h" are maximized. Under the current restrictions with "b" and "h" maximized and "a" variable from 3 to 5.7 m from A, the required strength of the rod ranges between 17.6 to 29.7 kN; that is, 2.5 × 7.55 kN to 2.5 × 11.90 kN.

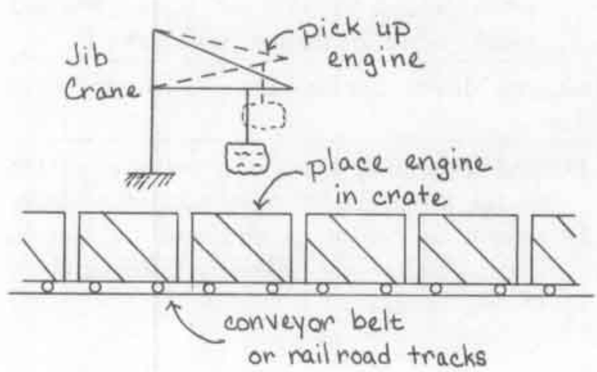
To further decrease the required tensile strength of the rod, increase "h".

4.81 Briefly discuss and illustrate different ways to crate truck engines. (See Problem 4.81)

NOTE: There is an unlimited number of possible answers. The following are just a few.

Design One:

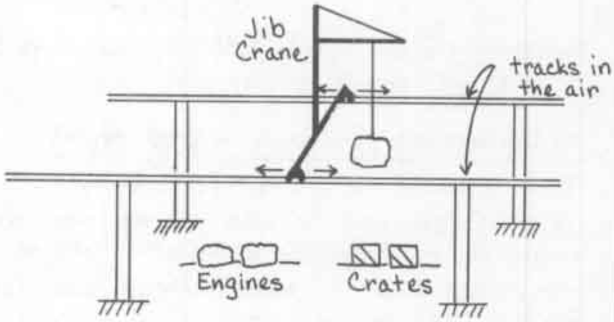
Jib crane is positioned above a conveyor belt or railroad cars on a track. The crates are moved along the belt or track as they are loaded with engines. The jib crane picks up an engine and rotates around the column axis placing the engine into the crate. As the crane rotates back for another engine, the crates move down one so an open crate is ready for the next engine.



4.81 cont'd

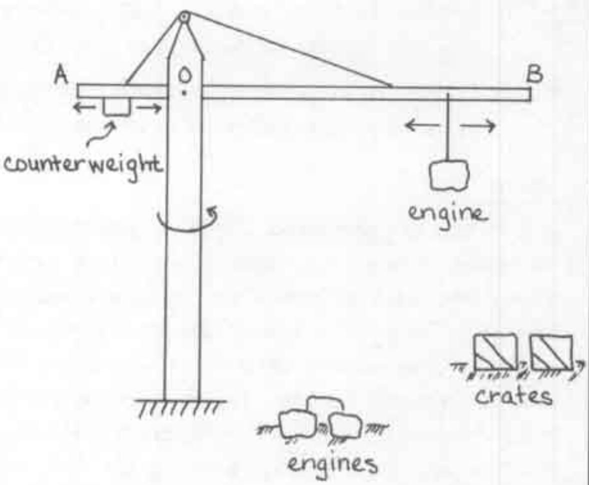
Design Two:

Jib crane is on rollers on a track so it moves back and forth along the track. The engines are at one end and the crates are at the other. The crane moves between the two areas collecting and depositing the engines in the crates.



Design Three:

Crane has a moving counterweight to balance the load at the other end. The counterweight moves along OA balancing the load moving along OB.



4.82 Find max. and min. reactions at supports when crossed by a) a car, b) an 18-wheel truck. (See Problem Statement.)

Necessary design parameters and how to obtain them.

1) Assume car and truck are entirely on the bridge when determining support reactions.
2) Weight of bridge is obtained by summing the weight of the individual materials of the bridge.

4.82 cont'd

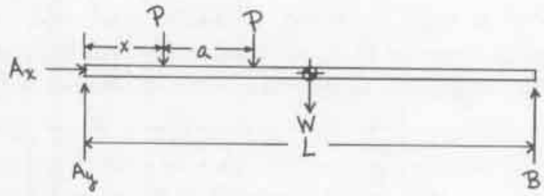
3) Call department of transportation to obtain the average weight of a car and a truck and the average distance between the axles.
4) Visit the bridge site and take measurements to obtain the specific dimensions.

Accuracy of these parameters depends on the statistics used in obtaining them. The larger the number of samples used to obtain the averages, the more accurate the estimate. The dimensions of the bridge depend on the accuracy of the measurements.

For this design, the parameters were left in variable form.

a) The Car

FBD



The maximum of one support and minimum of the other occurs when the car has one wheel on a support $(x=0)$.

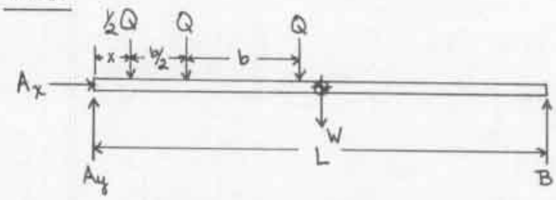
$\Sigma F_x = 0 = A_x \quad \Sigma F_y = 0 = A_y + B - 2P - W$

$\Sigma M_A = 0 = B(L) - W(L/2) - P(a)$

$$B = \frac{Pa}{L} + \tfrac{1}{2}W \qquad A_y = 2P - \frac{Pa}{L} + \tfrac{1}{2}W$$

b) The Truck

FBD



The maximum and minimum occur when the ½Q force is over point A.

$\Sigma F_x = 0 = A_x \quad \Sigma F_y = 0 = A_y + B - 2.5Q - W$

$\Sigma M_A = 0 = B(L) - W(L/2) - Q(3/2\,b) - Q(b/2)$

$$B = \frac{2Qb}{L} + \tfrac{1}{2}W \qquad A_y = \tfrac{3}{2}Q + \tfrac{1}{2}W - \frac{2Qb}{L}$$

To calculate the reactions for several vehicles on the bridge, you would need to know the weight of each vehicle and their axle locations on the bridge.

4.83 a) Find $\vec{R}$ and intersecting point with (x,y) plane. (See Table of forces in Problem 4.83)

$\Sigma F_z = R = -400 + 600 - 1200 + 1600 + 2400 - 2000$

$\vec{R} = 1000\,\hat{k}$ N

$\Sigma M_x = -400(75) + 600(125) - 1200(75) + 1600(-150) + 2400(0) - 2000(100) = -485\,000$ N·mm

$\Sigma M_y = -400(50) + 600(-25) - 1200(50) + 1600(-75) + 2400(0) - 2000(25) = 265\,000$ N·mm

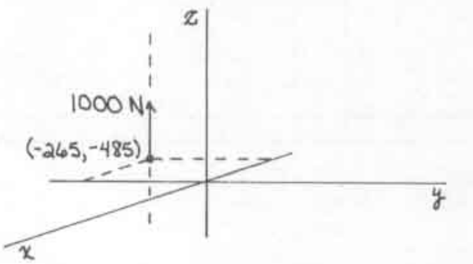
$M_x = y\Sigma F_z$ $\qquad$ $M_y = -x\Sigma F_z$

$-485\,000 = y(1000)$ $\qquad$ $265\,000 = -x(1000)$

$y = -485$ mm $\qquad$ $x = -265$ mm

$\vec{R} = 1000\,\hat{k}$ N @ $(-265, -485$ mm$)$

c) Draw a figure showing $\vec{R}$ and resultant axis.



4.84 a) Find $\vec{R}$ and intersecting point with (x,y) plane. (See Table of forces in Problem 4.84.)

$\Sigma F_z = R = 1 + 2.5 + 4 - 5 + 3 + 10 - 15.5$

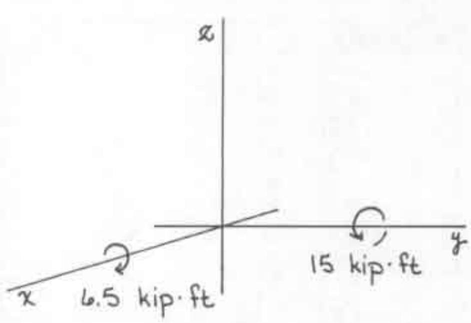
$\vec{R} = 0$ kips

b) Find resultant couples in (x,z) and (y,z) planes.

$\Sigma M_x = 1(1) + 2.5(2) + 4(-1) - 5(5) + 3(0) + 10(-3) - 15.5(-3) = \underline{\underline{-6.5 \text{ kip·ft}}}$

$\Sigma M_y = 1(1) + 2.5(0) + 4(-4) - 5(-1) + 3(2) + 10(2) - 15.5(2) = \underline{\underline{15 \text{ kip·ft}}}$

c) Draw a figure showing the resultant couples.



4.85 a) Find $\vec{R}$ and intersecting point with (x,y) plane. (See Table of forces in Problem 4.85)

$\Sigma F_z = R = 5 + 8 + 10 - 6 - 1.85 + 2.6 = 17.75$ kN

$\vec{R} = 17.75\,\hat{k}$ kN

$\Sigma M_x = 5(3) + 8(2) + 10(2) - 6(-4) - 1.85(4) + 2.6(-3) = 59.8$ kN·m

$\Sigma M_y = 5(1) + 8(2) + 10(-4) - 6(-2) - 1.85(0) + 2.6(5) = -6$ kN·m

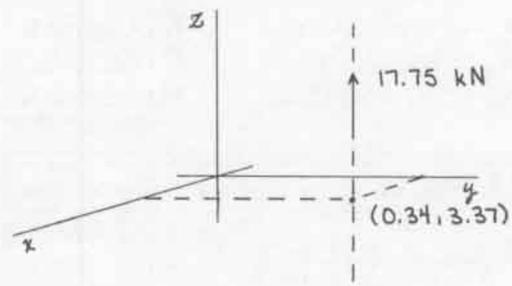
$M_x = y\Sigma F_z$ $\qquad$ $M_y = -x\Sigma F_z$

$59.8 = y(17.75)$ $\qquad$ $-6 = -x(17.75)$

$y = 3.37$ m $\qquad$ $x = 0.34$ m

$\vec{R} = 17.75\,\hat{k}$ kN @ $(0.34, 3.37$ m$)$

c) Draw a figure showing $\vec{R}$ and resultant axis.



4.86 a) Find $\vec{R}$ and intersecting point with (x,y) plane. (See Table of forces in Problem 4.86.)

$\Sigma F_z = R = 450 + 625 - 625 - 850 - 450 + 850$

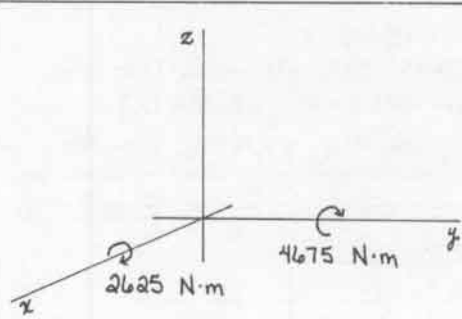
$\vec{R} = 0$ N

b) Find resultant couples in (x,z) and (y,z) planes.

$\Sigma M_x = 450(2) + 625(1) - 625(4) - 850(0) - 450(2) + 850(-3) = \underline{\underline{-2625 \text{ N·m}}}$

$\Sigma M_y = 450(2) + 625(-2) - 625(3) - 850(0) - 450(4) + 850(6) = \underline{\underline{-4675 \text{ N·m}}}$

c) Draw a figure showing the resultant couples.



4.87 Find resultant of forces (Fig. P 4.87.)

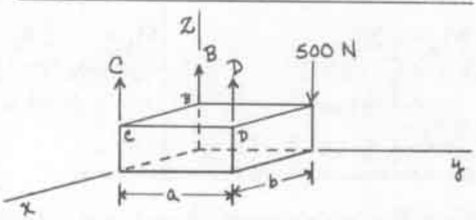
$\vec{R} = (-8+20-12+10)\hat{k} = 10\hat{k}$ k

$\vec{M}_x = -12(2) + 10(4) - 8(2) + 20(4) + 40 = 160$ k-ft

$\vec{M}_y = 12(1) - 10(1) + 8(4) - 20(4) = -46$ k·ft

$\underline{\vec{R} = 10\hat{k}\text{ k},\ \vec{M}_x = 160\text{ k·ft},\ \vec{M}_y = -46\text{ k·ft}}$

4.88 Replace 500 N force with $\vec{B}, \vec{C}, \vec{D}$.



$\Sigma F_z = -500 = B + C + D$

$\Sigma M_x = -500(a) = D(a)$

$\Sigma M_y = 0 = -C(b) - D(b)$

$\vec{B} = -500\hat{k}$ N

$\vec{C} = 500\hat{k}$ N

$\vec{D} = -500\hat{k}$ N

4.89 Find (x, y) of table's center of gravity.

(See Problem Statement.)

Force	x	y
F	0	0
5W/12	4	0
W/3	0	3
-W	x	y

$\Sigma F_z = 0 = F + 5W/12 + W/3 - W \rightarrow F = W/4$

$\Sigma M_x = 0 = 5W/12\,(0) + W/3\,(3) - W(y)$

$\Sigma M_y = 0 = -5W/12\,(4) - W/3\,(0) + W(x)$

$\underline{(x, y) = (5/3,\ 1)}$

4.90 Find A, B, C forces in terms of W.

(See Problem Statement.)

Force	x	y
-W	4	0.5
A	0	0
B	8	2
C	7	-1

$\Sigma F_z = 0 = A + B + C - W$

$\Sigma M_x = 0 = -W(0.5) + A(0) + B(2) + C(-1)$

$\Sigma M_y = 0 = -W(4) + A(0) + B(8) + C(7)$

$\underline{A = 21W/44 \quad B = 15W/44 \quad C = 2W/11}$

4.91 Find A, B, C.

Forces	x	y
-W	0	0
A	3	0
B	2.4 cos 75°	2.4 sin 75°
C	3 cos 135°	-3 sin 135°

4.91 cont'd

$W = 5 + 4(\pi 3^2)(10.68) = 1212.88$ kN

$\Sigma F_z = 0 = -1212.88 + A + B + C$

$\Sigma M_x = 0 = A(0) + B(2.4 \sin 75°) + C(-3 \sin 135°)$

$\Sigma M_y = 0 = A(3) + B(2.4 \cos 75°) + C(3 \cos 135°)$

$\underline{A = 258.1\text{ kN} \quad B = 456.2\text{ kN} \quad C = 498.6\text{ kN}}$

4.92 a) Develop spreadsheet to find R and intersecting point with (x, y) plane.

	A	B	C	D	E	F
1	Force	Magnitude	x	y	Mx	My
2	1				=B2*D2	=-B2*C2
3	2				=B3*D3	=-B3*C3
4	3				=B4*D4	=-B4*C4
5	4				=B5*D5	=-B5*C5
6	5				=B6*D6	=-B6*C6
7	6				=B7*D7	=-B7*C7
8	7				=B8*D8	=-B8*C8
9	8				=B9*D9	=-B9*C9
10	9				=B10*D10	=-B10*C10
11	10*				=B11*D11	=-B11*C11
12	Sum	=SUM(B2:B11)			=SUM(E2:E11)	=SUM(F2:F11)
13						
14	R= =B12					
15	x= =F12/B12		and	y= =E12/-B12		
16			*or*			
17	M_x= =E12			M_y= =F12		
18						
19	* May insert rows in the table to accommodate more than 10 forces.					

b) Use spreadsheet to solve Prob 4.83-86.

4.83

Force	Magnitude	x	y	Mx	My
1	-400	50	75	-30000	20000
2	600	-25	125	75000	15000
3	-1200	50	75	-90000	60000
4	1600	-75	-150	-240000	120000
5	2400	0	0	0	0
6	-2000	25	100	-200000	50000
7				0	0
8				0	0
9				0	0
10*				0	0
Sum	1000			-485000	265000

R= 1000

x = -265 and y = -485

4.84

Force	Magnitude	x	y	Mx	My
1	1.0	1	1	1	-1
2	2.5	0	2	5	0
3	4.0	-4	-1	-4	16
4	-5.0	-1	5	-25	-5
5	3.0	2	0	0	-6
6	10.0	2	-3	-30	-20
7	-15.5	2	-3	46.5	31
8				0	0
9				0	0
10*				0	0
Sum	0			-6.5	15

R= 0

M_x= -6.5 and M_y= 15

4.92 cont'd

4.85

Force	Magnitude	x	y	Mx	My
1	5	1	3	15	-5
2	8	2	2	16	-16
3	10	-4	2	20	40
4	-6	-2	-4	24	-12
5	-1.85	0	4	-7.4	0
6	2.6	5	-3	-7.8	-13
7				0	0
8				0	0
9				0	0
10*				0	0
Sum	17.75			59.8	-6

R= 17.75
x = 0.34 and y = 3.37

4.86

Force	Magnitude	x	y	Mx	My
1	450	2	2	900	-900
2	625	-2	1	625	1250
3	-625	3	4	-2500	1875
4	-850	0	0	0	0
5	-450	-4	-2	900	-1800
6	850	6	-3	-2550	-5100
7				0	0
8				0	0
9				0	0
10*				0	0
Sum	0			-2625	-4675

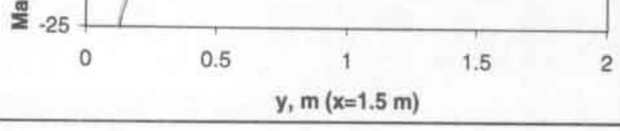
R= 0
M_x= -2625 and M_y= -4675

4.93 Plot magnitudes of $\vec{A}, \vec{B}, \vec{C}$ as they vary with (x,y) of C for a) $x = 1.5\,m$, $0 < y < 2m$, b) $y = 1.0\,m$, $0 < x < 3\,m$.

$\Sigma F_x = 0 = A + B + C - 2 - 4 - 6 - 1$
$\Sigma M_x = 0 = C(y) - 1(1) - 6(0.5) - 4(1)$
$\Sigma M_y = 0 = -A(3) - C(x) + 2(1.5) + 6(3) + 1(1.5)$

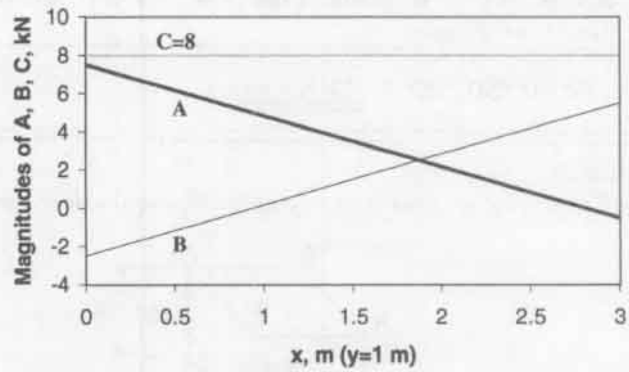
Equilibrium equations:
$A + B + C = 13$
$Cy = 8$
$3A + Cx = 22.5$

a) **Magnitudes of A, B, C as a Function of y**



4.93 cont'd

b) **Magnitudes of A, B, C as a Function of x**



4.94 Find range of values for x and y so that $A, B, C \leq 6\,kN$, where $0 < x < 3m$ and $0 < y < 2\,m$. (See Problem Statement.)

Using equations from Prob. 4.93, solve for A, B, C while varying x from 0 to 3m and y from 0 to 2 m.

There are many different solutions. If x and y are varied for the specified ranges by increments of 0.1 m, the following solution satisfies the requirements.

When $y = 1.4$, $0.8 \leq x < 3$
$y = 1.5$ or 1.6, $0.9 \leq x < 3$
$y = 1.7$, $1 \leq x < 3$
$y = 1.8$ or 1.9, $1.1 \leq x < 3$
$y \approx 2$, $1.2 \leq x < 3$

Find (x,y) when $A = B = C$.

Equations from 4.93:
$A + B + C = 13$
$Cy = 8$
$3A + Cx = 22.5$

$A = B = C$
$C + C + C = 13 \rightarrow C = 13/3$
$Cy = 8 \rightarrow y = 24/13 = 1.85$
$3C + Cx = 22.5 \rightarrow x = 9.5(3/13) = 2.9$

$\underline{\underline{x = 1.85\,m \quad y = 2.19\,m}}$

4.110

Three couples with moments 40, 50, and −20 N·mm act in the same plane. Find the moment of the resultant couple.

$M = 40 + 50 - 20 = \underline{\underline{70\ \text{N}\cdot\text{mm}}}$

4.111

From Fig. a find the dynamically equivalent force.

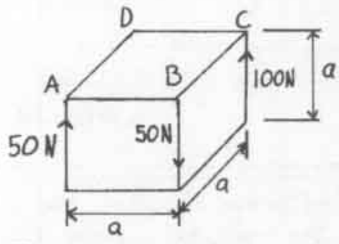


Figure a

Transfer couple from face AB to face DC (Fig. b)

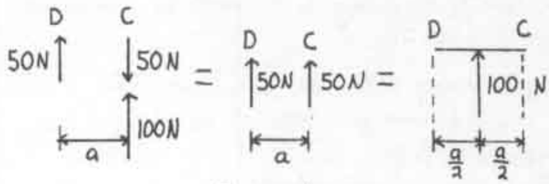


Figure b

The resultant acts on face DC midway between edges D and C.

4.112

A 100 N force F acts on the rigid body in Fig. a. Find the compensating couple required if F is displaced 75 mm.

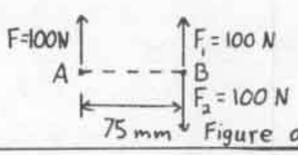


Figure a

In Fig. a, add self-equilibrating forces at B. The compensating couple is

$C = (100\ \text{N})(75\ \text{mm}) = \underline{\underline{7500\ \text{N}\cdot\text{mm}}}$ ↻

see Fig. b.

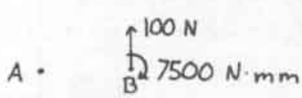


Figure b

4.114

The angle θ in Fig. a. is such that the vector sum of the forces is zero.

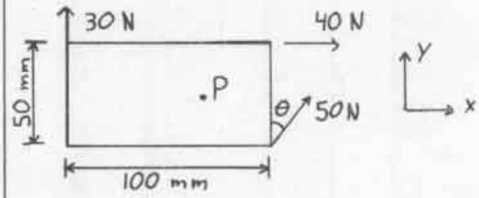


Figure a

Find the moment of the three forces about P.

By Fig. a,

$$\Sigma F_x = 40 + 50\sin\theta = 0$$
$$\Sigma F_y = 30 + 50\cos\theta = 0 \qquad (a)$$

The solution of Eqs (a) is

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-0.80}{-0.60}$$

or $\theta = 233.13°$

Therefore, the (x, y) projections of the 50 N force are $50\sin(233.13°) = -40$ N and $50\cos(233.13°) = -30$ N (see Fig. b)

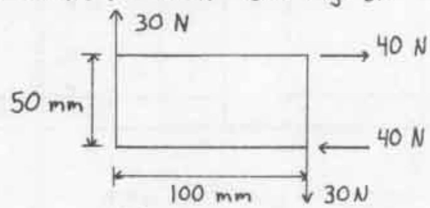


Figure b

The resultant of this force system is the resultant of two couples, namely,

$\vec{C}_1 = (40)(50) = 2000\ \text{N}\cdot\text{mm}$ ↻

$\vec{C}_2 = (30)(100) = 3000\ \text{N}\cdot\text{mm}$ ↻

or $\vec{C} = \vec{C}_1 + \vec{C}_2 = 5000\ \text{N}\cdot\text{mm}$ ↻

Since the resultant system is a couple in the plane, the moment about any point P in the plane is $M = 5000$ N·mm.

Alternatively, directly from Eqs. (a), the (x, y) projections of the 50 N force are

$50\sin\theta = -40$ N

$50\cos\theta = -30$ N

5.1 a) Find magnitude of $\vec{F} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$, b) angle between $\vec{F}$ and x-axis, c) direction cosines.

a) $F = (2^2 + (-3)^2 + 6^2)^{1/2} = \underline{\underline{7}}$

b) $\hat{x}$ is unit vector along x-axis
$\vec{F}\cdot\hat{x} = F\cos\theta_x = (1)(2) + (0)(-3) + (0)(6) = 2$

$\theta_x = \cos^{-1}(2/7) = \underline{\underline{73.4°}}$

c) $\underline{\underline{\cos\theta_x = 2/7 \quad \cos\theta_y = -3/7 \quad \cos\theta_z = 6/7}}$

5.2 Find projection of $\vec{F} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ N along axis with direction numbers a) (1,1,1), b) (−1,1,1).

a) $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$
Projection of $\vec{F}$ along $\vec{A} = F\cos\theta$
$A = \sqrt{1^2+1^2+1^2} = \sqrt{3}$
$\vec{A}\cdot\vec{F} = AF\cos\theta = A_xF_x + A_yF_y + A_zF_z$
$\sqrt{3}\,F\cos\theta = (1)(3) + (1)(4) + (1)(2) = 9$

$F\cos\theta = 9/\sqrt{3} = \underline{\underline{5.196 \text{ N}}}$

b) $\vec{A} = -\hat{\imath} + \hat{\jmath} + \hat{k} \quad A = \sqrt{(-1)^2+1^2+1^2} = \sqrt{3}$
$\sqrt{3}\,F\cos\theta = (-1)(3) + (1)(4) + (1)(2) = 3$

$F\cos\theta = 3/\sqrt{3} = \underline{\underline{1.73 \text{ N}}}$

5.3 Find a) magnitude of $\vec{F} = 4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, b) angle between $\vec{F}$ and y-axis, c) projection of $\vec{F}$ along line that passes through (1,2,3) and (7,4,0).

a) $F = \sqrt{4^2+(-2)^2+4^2} = \underline{\underline{6}}$

b) $\vec{F}\cdot\hat{\jmath} = F\cos\theta = 4(0) + (-2)(1) + (4)(0) = -2$
$6\cos\theta = -2$
$\theta = \cos^{-1}(-2/6) = \underline{\underline{109.5°}}$

c) $\vec{A} = (7-1)\hat{\imath} + (4-2)\hat{\jmath} + (0-3)\hat{k} = 6\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$
$A = \sqrt{6^2+2^2+(-3)^2} = 7$
$\hat{a} = \frac{\vec{A}}{A} = 1/7\,(6\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$
$\hat{a}\cdot\vec{F} = F\cos\theta = 1/7\,[(6)(4) + (2)(-2) + (-3)(4)]$

$\underline{\underline{F\cos\theta = 1.143}}$

5.4 Find the projection of $-5\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$ on line parallel to $2\hat{\imath} - 3\hat{\jmath} + \hat{k}$.

$\vec{A} = -5\hat{\imath} + 6\hat{\jmath} - 2\hat{k} \quad \vec{B} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$
$\vec{A}\cdot\vec{B} = AB\cos\theta = (-5)(2) + (6)(-3) + (-2)(1) = -30$
$B = \sqrt{2^2+(-3)^2+1^2} = \sqrt{14}$

$A\cos\theta = -30/\sqrt{14} = \underline{\underline{-8.02}}$

5.5 Find a) resultant $\vec{F}$, b) sum of projections along axis with direction numbers (4, 5, 2).

a) $\vec{F} = \vec{A} + \vec{B} + \vec{C}$
$= (8\hat{\imath} + 10\hat{\jmath} - 6\hat{k}) + (4\hat{\imath} + 3\hat{\jmath} + 10\hat{k}) + (-6\hat{\imath} - 4\hat{\jmath} + 6\hat{k})$
$= (8+4-6)\hat{\imath} + (10+3-4)\hat{\jmath} + (-6+10+6)\hat{k}$

$\underline{\underline{\vec{F} = 6\hat{\imath} + 9\hat{\jmath} + 10\hat{k}}}$

b) $\hat{n} = \frac{4\hat{\imath} + 5\hat{\jmath} + 2\hat{k}}{\sqrt{4^2+5^2+2^2}} = \frac{1}{\sqrt{45}}(4\hat{\imath} + 5\hat{\jmath} + 2\hat{k})$

$\hat{n}\cdot\vec{F} = F\cos\theta = \frac{1}{\sqrt{45}}[(4)(6) + (5)(9) + (2)(10)]$

$\underline{\underline{F\cos\theta = 13.27}}$

5.6 Find a) θ between axes A:(1,2,3) and B:(6,−1,4), b) projection on $\vec{B}$ of 10N force along $\vec{A}$, c) projection on $\vec{A}$ of 10N force along $\vec{B}$.

a) $\vec{A} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \qquad \vec{B} = 6\hat{\imath} - \hat{\jmath} + 4\hat{k}$
$\vec{A}\cdot\vec{B} = AB\cos\theta$
$A = \sqrt{1^2+2^2+3^2} = \sqrt{14} \quad B = \sqrt{6^2+(-1)^2+4^2} = \sqrt{53}$
$\vec{A}\cdot\vec{B} = (1)(6) + (2)(-1) + (3)(4) = \sqrt{14}\sqrt{53}\cos\theta$

$\underline{\underline{\theta = 54.03°}}$

b) $A = 10$ N, $\vec{A}$ on $\vec{B} = A\cos\theta = 10\cos 54.03°$
$= \underline{\underline{5.874 \text{ N}}}$

c) $B = 10$ N, $\vec{B}$ on $\vec{A} = B\cos\theta = 10\cos 54.03°$
$= \underline{\underline{5.874 \text{ N}}}$

5.7 a) Find n so vectors are perpendicular, b) Find n so $\cos\theta = 4/9$.

a) $\vec{A} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k} \quad \vec{B} = 2\hat{\imath} + \hat{\jmath} + n\hat{k}$
For the two vectors to be perpendicular, $\vec{A}\cdot\vec{B} = 0$.

$\vec{A}\cdot\vec{B} = (1)(2) + (-2)(1) + (3)(n) = 0$

$\underline{\underline{n = 0}}$

b) $\vec{A}\cdot\vec{B} = AB\cos\theta = AB(4/9)$
$A = \sqrt{1^2+(-2)^2+3^2} = \sqrt{14}$
$B = \sqrt{2^2+1^2+n^2} = \sqrt{5+n^2}$
$\vec{A}\cdot\vec{B} = (1)(2) + (-2)(1) + (3)(n) = \sqrt{14}\sqrt{5+n^2}\,(4/9)$
$3n = \sqrt{14}\sqrt{5+n^2}\,(4/9)$
$9n^2 = (14)(5+n^2)(16/81)$
$n^2 = 2.218$

$\underline{\underline{n = \pm 1.489}}$

5.8 a) Find $\vec{A}\cdot\vec{B}$, $\vec{A}\cdot\vec{C}$, $(\vec{B}-\vec{C})\cdot\vec{D}$, $(\vec{A}-\vec{B})\cdot\vec{C}$.

$\vec{A}\cdot\vec{B} = (1)(4)+(3)(2)+(-1)(0) = \underline{\underline{10}}$

$\vec{A}\cdot\vec{C} = (1)(8)+(3)(0)+(-1)(-4) = \underline{\underline{12}}$

$(\vec{B}-\vec{C})\cdot\vec{D} = (4-8)(-3)+(2-0)(2)+(0+4)(2) = \underline{\underline{24}}$

$(\vec{A}-\vec{B})\cdot\vec{C} = (1-4)(8)+(3-2)(0)+(-1-0)(-4) = \underline{\underline{-20}}$

b) Find angles between $\vec{A}$ and $\vec{B}$, $\vec{B}$ and $\vec{C}$, $(\vec{A}-\vec{B})$ and $(\vec{C}-\vec{D})$.

$\vec{F}_1\cdot\vec{F}_2 = F_1F_2\cos\theta$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$A = \sqrt{1^2+3^2+(-1)^2} = \sqrt{11} \quad B = \sqrt{4^2+2^2} = \sqrt{20}$

$10 = \sqrt{11}\sqrt{20}\cos\theta$

$\underline{\underline{\theta = 47.6^\circ}}$

$\vec{B}\cdot\vec{C} = BC\cos\theta$

$B = \sqrt{20} \quad C = \sqrt{8^2+(-4)^2} = \sqrt{80}$

$\vec{B}\cdot\vec{C} = (4)(8)+(2)(0)+(0)(-4) = 32$

$32 = \sqrt{20}\sqrt{80}\cos\theta$

$\underline{\underline{\theta = 36.9^\circ}}$

$(\vec{A}-\vec{B})\cdot(\vec{C}-\vec{D}) = |\vec{A}-\vec{B}||\vec{C}-\vec{D}|\cos\theta$

$|\vec{A}-\vec{B}| = \sqrt{(1-4)^2+(3-2)^2+(-1-0)^2} = \sqrt{11}$

$|\vec{C}-\vec{D}| = \sqrt{(8+3)^2+(0-2)^2+(-4-2)^2} = \sqrt{161}$

$(\vec{A}-\vec{B})\cdot(\vec{C}-\vec{D}) = (1-4)(8+3)+(3-2)(0-2)+(-1-0)(-4-2) = -29$

$-29 = \sqrt{11}\sqrt{161}\cos\theta$

$\underline{\underline{\theta = 133.6^\circ}}$

c) Find projection of $(\vec{B}-\vec{D})$ along $\vec{A}$.

$(\vec{B}-\vec{D})$ along $\vec{A}$ is the dot product of $(\vec{B}-\vec{D})$ and unit vector of $\vec{A}$ $(\hat{a})$.

$$\hat{a} = \frac{\vec{A}}{A} = \frac{\hat{\imath}+3\hat{\jmath}-\hat{k}}{\sqrt{1^2+3^2+(-1)^2}} = \frac{1}{\sqrt{11}}\hat{\imath}+\frac{3}{\sqrt{11}}\hat{\jmath}-\frac{1}{\sqrt{11}}\hat{k}$$

$$\hat{a}\cdot(\vec{B}-\vec{D}) = \left(\tfrac{1}{\sqrt{11}}\right)(4+3)+\left(\tfrac{3}{\sqrt{11}}\right)(2-2)+\left(-\tfrac{1}{\sqrt{11}}\right)(0-2)$$

$\therefore \underline{\underline{\hat{a}\cdot(\vec{B}-\vec{D}) = 2.71}}$

5.9 a) Find θ between A: (1, 2, 3) and B(-1, 5, -2).

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$A = \sqrt{1^2+2^2+3^2} = \sqrt{14} \qquad B = \sqrt{(-1)^2+5^2+(-2)^2} = \sqrt{30}$

$\vec{A}\cdot\vec{B} = (1)(-1)+(2)(5)+(3)(-2) = 3$

$3 = \sqrt{14}\sqrt{30}\cos\theta$

$\underline{\underline{\theta = 81.58^\circ}}$

b) Find projection of $\vec{A}$ along $\vec{B}$.

$\vec{A}$ along $\vec{B} = A\cos\theta =$

$\sqrt{14}\cos 81.58^\circ = \underline{\underline{0.548}}$

5.10 Find projection of $\vec{F}$ along $\left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$

$\vec{F} = 100\,\hat{f} = \frac{200}{\sqrt{14}}\hat{\imath} - \frac{300}{\sqrt{14}}\hat{\jmath} + \frac{100}{\sqrt{14}}\hat{k}$

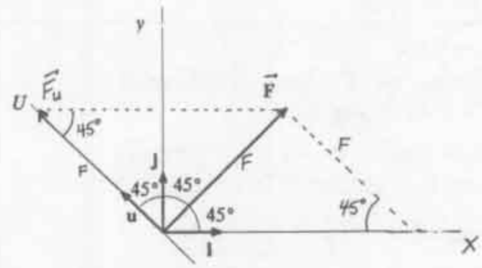
$\hat{\ell} = \frac{1}{\sqrt{6}}\hat{\imath} + \frac{2}{\sqrt{6}}\hat{\jmath} - \frac{1}{\sqrt{6}}\hat{k}$

$\vec{F}$ along $\hat{\ell} = \vec{F}\cdot\vec{\ell}$

$= \left(\frac{200}{\sqrt{14}}\right)\left(\frac{1}{\sqrt{6}}\right)+\left(\frac{-300}{\sqrt{14}}\right)\left(\frac{2}{\sqrt{6}}\right)+\left(\frac{100}{\sqrt{14}}\right)\left(-\frac{1}{\sqrt{6}}\right)$

$= \underline{\underline{-54.55\,N}}$

5.11 a.) Resolve $\vec{F}$ into its (x, y) components



$\underline{\underline{\vec{F} = F\cos 45^\circ\,\hat{\imath} + F\sin 45^\circ\,\hat{\jmath}}}$

b.) Resolve $\vec{F}$ into its components // to (X, U) axes

$X = F^2 + F^2 = \sqrt{2}\,F$

$U = F$

$\underline{\underline{\vec{F} = \sqrt{2}\,F\hat{\imath} + F\hat{u}}}$

5.12 a.) Find projection of $\vec{F} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ along $4\hat{\imath} + 3\hat{k}$

$$\vec{F}\text{ along } 4\hat{\imath}+3\hat{k} = \frac{\vec{F}\cdot(4\hat{\imath}+3\hat{k})}{|4\hat{\imath}+3\hat{k}|}$$

$$= \frac{(2)(4)+(-3)(0)+(6)(3)}{\sqrt{4^2+3^2}} = \underline{\underline{5.2}}$$

b.) Find component of $\vec{F}$ parallel to $4\hat{\imath}+3\hat{k}$.

component of $\vec{F}$ parallel to $4\hat{\imath}+3\hat{k}$ =

$(\vec{F}$ along $4\hat{\imath}+3\hat{k})$(unit vector of $4\hat{\imath}+3\hat{k}$)

$$= 5.2\left(\frac{4\hat{\imath}+3\hat{k}}{\sqrt{4^2+3^2}}\right) = \underline{\underline{4.16\,\hat{\imath} + 3.12\,\hat{k}}}$$

5.13 a.) Show that $\hat{a} = \cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$ and $\hat{b} = \cos\beta\,\hat{\imath} + \cos\beta\,\hat{\jmath}$

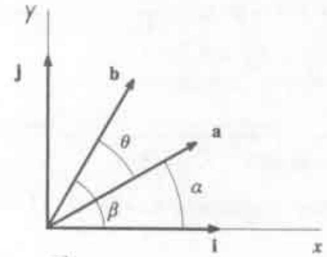
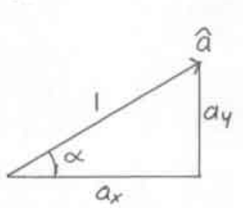
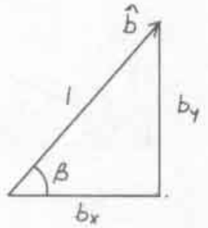


Figure (a.)

Use geometry and the definition of a unit vector:





$\hat{a} = a_x\hat{\imath} + a_y\hat{\jmath}$

$\cos\alpha = a_x/1$

$\sin\alpha = a_y/1$

$a_x = \cos\alpha \quad a_y = \sin\alpha$

$\therefore \hat{a} = \cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$

$\hat{b} = b_x\hat{\imath} + b_y\hat{\jmath}$

$\cos\beta = b_x/1$

$\sin\beta = b_y/1$

$b_x = \cos\beta \quad b_y = \sin\beta$

$\therefore \hat{b} = \cos\beta\,\hat{\imath} + \sin\beta\,\hat{\jmath}$

b.) Form $\hat{a}\cdot\hat{b}$ and show that $\cos(\beta-\alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha$.

$\hat{a}\cdot\hat{b} = ab\cos\theta = a_x b_x + a_y b_y$

$a = b = 1 \quad \cos\theta = \cos\beta\cos\alpha + \sin\beta\sin\alpha$

$\theta = \beta - \alpha$

$\therefore \cos(\beta-\alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha$

5.14 Find unit vector perpendicular to $\vec{r}_1 = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ and $\vec{r}_2 = 3\hat{\imath} - \hat{\jmath} + \hat{k}$

Let $\hat{v} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

For $\hat{v}$ to be perpendicular to $\vec{r}_1$ and $\vec{r}_2$,

$\vec{r}_1\cdot\hat{v} = 0$ and $\vec{r}_2\cdot\hat{v} = 0$.

$\vec{r}_1\cdot\hat{v} = (1)(a) + (4)(b) + (2)(c) = 0$

$\vec{r}_2\cdot\hat{v} = (3)(a) + (-1)(b) + (1)(c) = 0$

$v = 1 = \sqrt{a^2 + b^2 + c^2}$

Solve the three equations simultaneously for a, b, c;

$\hat{v} = 0.397\,\hat{\imath} + 0.330\,\hat{\jmath} - 0.857\,\hat{k}$ -or-

$\hat{v} = -0.397\,\hat{\imath} - 0.330\,\hat{\jmath} + 0.857\,\hat{k}$

5.15 a.) Show that the sum of squares of the numbers in any row or column is 1.

The following vectors are unit vectors along the designated axes.

$x : \hat{x} = l_1\hat{\imath} + l_2\hat{\jmath} + l_3\hat{k}$

$y : \hat{y} = m_1\hat{\imath} + m_2\hat{\jmath} + m_3\hat{k}$

$z : \hat{z} = n_1\hat{\imath} + n_2\hat{\jmath} + n_3\hat{k}$

$X : \hat{X} = l_1\hat{\imath} + m_1\hat{\jmath} + n_1\hat{k}$

$Y : \hat{Y} = l_2\hat{\imath} + m_2\hat{\jmath} + n_2\hat{k}$

$Z : \hat{Z} = l_3\hat{\imath} + m_3\hat{\jmath} + n_3\hat{k}$

$x: \hat{x}\cdot\hat{x} = (l_1)(l_1) + (l_2)(l_2) + (l_3)(l_3)$
$= l_1^2 + l_2^2 + l_3^2 = 1$ By definition of a unit vector.

$y: \hat{y}\cdot\hat{y} = (m_1)(m_1) + (m_2)(m_2) + (m_3)(m_3)$
$= m_1^2 + m_2^2 + m_3^2 = 1$

$z: \hat{z}\cdot\hat{z} = (n_1)(n_1) + (n_2)(n_2) + (n_3)(n_3)$
$= n_1^2 + n_2^2 + n_3^2 = 1$

$X: \hat{X}\cdot\hat{X} = (l_1)(l_1) + (m_1)(m_1) + (n_1)(n_1)$
$= l_1^2 + m_1^2 + n_1^2 = 1$

$Y: \hat{Y}\cdot\hat{Y} = (l_2)(l_2) + (m_2)(m_2) + (n_2)(n_2)$
$= l_2^2 + m_2^2 + n_2^2 = 1$

$Z: \hat{Z}\cdot\hat{Z} = (l_3)(l_3) + (m_3)(m_3) + (n_3)(n_3)$
$= l_3^2 + m_3^2 + n_3^2 = 1$

b.) Show that the product of numbers of any row or column with corresponding numbers in any other row or column = 0.

$\hat{x}\cdot\hat{y} = (l_1)(l_2) + (m_1)(m_2) + (n_1)(n_2)$
$= l_1 l_2 + m_1 m_2 + n_1 n_2 = \cos\theta$

$\hat{x}$ and $\hat{y}$ are orthoganal $(\theta = 90°)$.

$\therefore \hat{x}\cdot\hat{y} = \cos 90° = 0$.

The same opplies for the rest of the columns and rows.

5.16 Challenge Problem

GIVEN A line perpendicular to a given plane forms equal angles with the x, y, and z axes. A vector of magnitude 30 units lies in the plane and forms a 60° angle with the z axis.

FIND Two different vectors that fulfill these conditions.

SOLUTION

For the line perpendicular to the plane and with equal angles $\alpha = \beta = \gamma$ with respect to axes (x, y, z):

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 3\cos^2\alpha = 1$

5.16 cont.

$\therefore \cos^2\alpha = \pm\frac{1}{\sqrt{3}}$. Hence, a unit vector along the line is: $\hat{n} = \frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$

Let the required vector be:

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$

Then, given that $|\vec{v}| = 30$ and $\vec{v}$ forms $60°$ angle with axis z, $v_z = 30\cos 60° = 15$.

Now, since $\vec{v}$ is perpendicular to $\hat{n}$,

$$\vec{v}\cdot\hat{n} = \frac{1}{\sqrt{3}}(v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k})\cdot(\hat{\imath} + \hat{\jmath} + \hat{k}) = 0$$

$$\therefore v_x + v_y + v_z = v_x + v_y + 15 = 0 \qquad (A)$$

Also, $v_x^2 + v_y^2 + 15^2 = 30^2 \qquad (B)$

Eliminating v_y from Eqs (a) and (b) yields:

$$v_x^2 + 15v_x - 225 = 0$$

-or- $v_x = -7.50 \pm 16.77$

$\therefore$ $v_x = 9.27$, or $v_x = -24.27$

For $v_x = 9.27$, $v_y = -v_x - 15 = -24.27$, $v_z = 15$

Hence, a vector that meets the requirements is: $\underline{\underline{\vec{a} = 9.27\hat{\imath} - 24.27\hat{\jmath} + 15\hat{k}}}$

For $v_x = -24.27$, $v_y = -v_x - 15 = 9.27$, $v_z = 15$

Hence, a vector that meets the requirements is: $\underline{\underline{\vec{b} = -24.27\hat{\imath} + 9.27\hat{\jmath} + 15\hat{k}}}$

5.17 Challenge Problem

GIVEN Any vector $\vec{F}$ and a unit vector $\hat{n}$.

a.) Let $\vec{F} = \vec{F}_1 + (\vec{F}\cdot\hat{n})\hat{n}$

Show that $(\vec{F}\cdot\hat{n})\hat{n}$ is the component of $\vec{F}$ in the $\hat{n}$ direction and $\vec{F}_1$ is the component of $\vec{F}$ perpendicular to $\hat{n}$.

$\vec{F}$ along $\hat{n} = \hat{n}\cdot\vec{F} = nF\cos\theta$

Since $\hat{n}$ is a unit vector, $n = 1$ and

$\vec{F}$ along $\hat{n} = (F\cos\theta)\hat{n} \rightarrow$ The projection of $\vec{F}$ onto $\hat{n}$.

If $\vec{F}_1$ is the component of $\vec{F}$ perpendicular to $\hat{n}$, then $\vec{F}_1\cdot\hat{n} = 0$.

Find dot product of $\vec{F}$ and $\hat{n}$.

$\vec{F}\cdot\hat{n} = \vec{F}_1\cdot\hat{n} + (\vec{F}\cdot\hat{n})\hat{n}\cdot\hat{n}, \qquad (\hat{n}\cdot\hat{n} = 1)$

$\vec{F}\cdot\hat{n} - \vec{F}\cdot\hat{n} = 0 = \vec{F}_1\cdot\hat{n} \rightarrow \underline{\underline{\vec{F}_1 \text{ is } \perp \text{ to } \hat{n}}}.$

b.) Let $\vec{F} = 5\hat{\imath} + 8\hat{\jmath} + 2\hat{k}$

Find $\vec{F}_1$ that is perpendicular to line with direction cosines $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

$$\vec{F}_1 = \vec{F} - (\vec{F}\cdot\hat{n})\hat{n}$$

$$= 5\hat{\imath} + 8\hat{\jmath} + 2\hat{k} - \left[(5)(\tfrac{1}{\sqrt{3}}) + (8)(\tfrac{1}{\sqrt{3}}) + (2)(\tfrac{1}{\sqrt{3}})\right]\hat{n}$$

$$= 5\hat{\imath} + 8\hat{\jmath} + 2\hat{k} - 8.66\left(\tfrac{1}{\sqrt{3}}\hat{\imath} + \tfrac{1}{\sqrt{3}}\hat{\jmath} + \tfrac{1}{\sqrt{3}}\hat{k}\right)$$

$\vec{F}_1 = \underline{\underline{0.00015\hat{\imath} + 3\hat{\jmath} - 3\hat{k}}}$

5.18 Challenge Problem

GIVEN Oblique coordinates (x, y, z) · Fig (a).

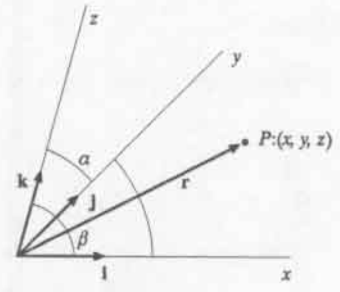


Figure (a)

a.) Show that $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

$\vec{r}$ is a vector from origin $(0,0,0)$ to $P(x, y, z)$.

$\vec{r} = (x-0)\hat{\imath} + (y-0)\hat{\jmath} + (z-0)\hat{k}$

$\therefore \underline{\underline{\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}}}$

b.) Derive formula for r in terms of (x, y, z) and (α, β, γ).

$$\hat{r}\cdot\hat{r} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k})\cdot(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$$

$$= x^2\hat{\imath}\cdot\hat{\imath} + xy\hat{\imath}\cdot\hat{\jmath} + xz\hat{\imath}\cdot\hat{k}$$

$$+ yx\hat{\jmath}\cdot\hat{\imath} + y^2\hat{\jmath}\cdot\hat{\jmath} + yz\hat{\jmath}\cdot\hat{k}$$

$$+ zx\hat{k}\cdot\hat{\imath} + zy\hat{k}\cdot\hat{\jmath} + z^2\hat{k}\cdot\hat{k}$$

$\hat{\jmath}\cdot\hat{k} = \cos\alpha \quad \hat{k}\cdot\hat{\imath} = \cos\beta \quad \hat{\imath}\cdot\hat{\jmath} = \cos\gamma$

$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$

$$r^2 = x^2 + xy\cos\gamma + xz\cos\beta$$

$$+ xy\cos\gamma + y^2 + yz\cos\alpha$$

$$+ xz\cos\beta + zy\cos\alpha + z^2$$

$\therefore \underline{\underline{r = (x^2 + y^2 + z^2 + 2xy\cos\gamma + 2xz\cos\beta + 2yz\cos\alpha)^{1/2}}}$

5.19 Challenge Problem

GIVEN Points A, B, C.

See Fig (a.)

5.19 cont.

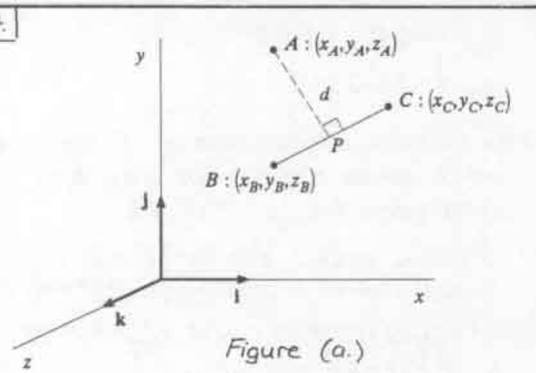


Figure (a.)

a.) Express $\vec{BA}$ in terms of coordinates of B & A.

$$\vec{BA} = (x_A - x_B)\hat{\imath} + (y_A - y_B)\hat{\jmath} + (z_A - z_B)\hat{k}$$

b.) Find projection BP of $\vec{BA}$ on $\vec{BC}$.

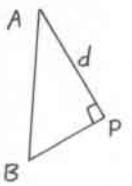
$$\vec{BC} = (x_C - x_B)\hat{\imath} + (y_C - y_B)\hat{\jmath} + (z_C - z_B)\hat{k}$$

$$BP = \frac{\vec{BA}\cdot\vec{BC}}{BC}$$

$$BP = \frac{(x_A-x_B)(x_C-x_B)+(y_A-y_B)(y_C-y_B)+(z_A-z_B)(z_C-z_B)}{\sqrt{(x_C-x_B)^2+(y_C-y_B)^2+(z_C-z_B)^2}}$$

c.) Find d in terms of BA and BP

Minimum distance, d, is perpendicular to $\vec{BC}$ (and BP). Use Pythagorean Theorem to find d.



$$BA^2 = d^2 + BP^2$$

$$d = \sqrt{BA^2 - BP^2}$$

d.) Find d [in] for A: (30, 10, -10), B: (20, 30, 0), C: (-10, 20, 40)

$$\vec{BA} = (30-20)\hat{\imath} + (10-30)\hat{\jmath} + (-10-0)\hat{k} = 10\hat{\imath} - 20\hat{\jmath} - 10\hat{k}$$

$$\vec{BC} = (-10-20)\hat{\imath} + (20-30)\hat{\jmath} + (40-0)\hat{k} = -30\hat{\imath} - 10\hat{\jmath} + 40\hat{k}$$

$$BC = \sqrt{(-30)^2 + (-10)^2 + 40^2} = \sqrt{2600} = 10\sqrt{26}$$

$$BP = \frac{\vec{BA}\cdot\vec{BC}}{BC} = \frac{(10)(-30)+(-20)(-10)+(-10)(40)}{10\sqrt{26}}$$

$$= -50/\sqrt{26}$$

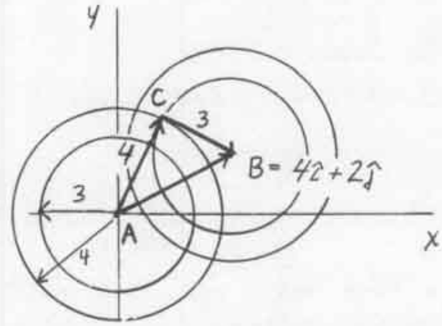
$$BA = \sqrt{10^2 + (-20)^2 + (-10)^2} = \sqrt{600} = 10\sqrt{6}$$

$$d = \sqrt{BA^2 - BP^2} = \sqrt{(10\sqrt{6})^2 - (-50/\sqrt{26})^2}$$

$$d = 22.45 \text{ in}$$

5.20 Design Problem

Find two forces equivalent to $\vec{B} = 4\hat{\imath} + 2\hat{\jmath}$ with magnitudes 3 and 4 units.



If coordinates of B are (4, 2), then A is (0, 0) and C is (x, y).

$r = 4$ circle : $x^2 + y^2 = 16$

$r = 3$ circle : $(x-4)^2 + (y-2)^2 = 9$

$$x^2 + y^2 - 16 = (x-4)^2 + (y-2)^2 - 9$$

$$x^2 + y^2 - 16 = x^2 - 8x + 16 + y^2 - 4y + 4 - 9$$

$$-27 = -8x - 4y$$

$$y = -2x + 27/4$$

Substitute into $r = 4$ circle equation:

$$x^2 + (-2x + 27/4)^2 = 16$$

$$x^2 + 4x^2 - 27x + 729/16 = 16$$

$$5x^2 - 27x + 473/16 = 0$$

$$x = 1.526, \quad 3.874$$

Plug these values into $y = -2x + 27/4$ to obtain the corresponding y values.

$$y = -2(1.526) + 27/4 = 3.698$$

$$y = -2(3.874) + 27/4 = -0.998$$

C is the intersection of any $r = 3$ circle with an $r = 4$ circle. There are 4 possible solutions where $\vec{AC} + \vec{CB} = \vec{B} = 4\hat{\imath} + 2\hat{\jmath}$.

From the solution above:

$$\vec{AC_1} = 1.526\hat{\imath} + 3.698\hat{\jmath}$$

$$\vec{CB_1} = (4 - 1.526)\hat{\imath} + (2 - 3.698)\hat{\jmath}$$

$$\vec{CB_1} = 2.474\hat{\imath} - 1.698\hat{\jmath}$$

$$\vec{AC_2} = 3.874\hat{\imath} - 0.998\hat{\jmath}$$

$$\vec{CB_2} = (4 - 3.874)\hat{\imath} + (2 - (-0.998))\hat{\jmath}$$

$$\vec{CB_2} = 0.126\hat{\imath} + 2.998\hat{\jmath}$$

5.20 cont.

The other two possible solutions are:

$\vec{AC}_3 = 2.474\hat{\imath} - 1.698\hat{\jmath}$

$\vec{BC}_3 = 1.526\hat{\imath} + 3.698\hat{\jmath}$

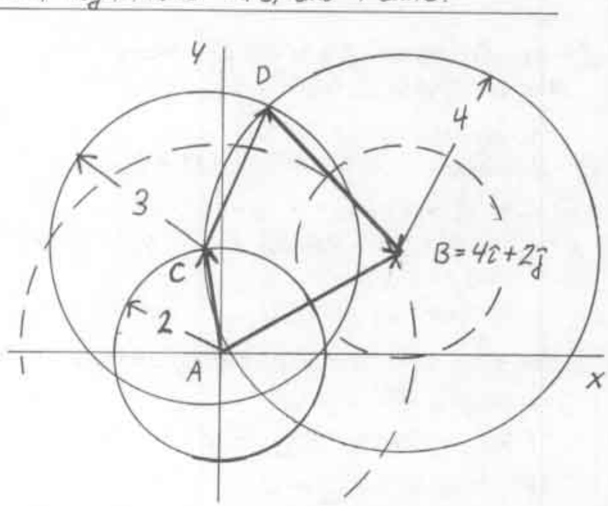
$\vec{AC}_4 = 0.126\hat{\imath} + 2.998\hat{\jmath}$

$\vec{BC}_4 = 3.874\hat{\imath} - 0.998\hat{\jmath}$

For a total of 4 solutions.

5.21 Design Problem

Find three forces equivalent to $\vec{B} = 4\hat{\imath} + 2\hat{\jmath}$ with magnitudes 2, 3, and 4 units.



$\vec{AC} + \vec{CD} + \vec{DB} = \vec{B}$

If coordinates of B are (4,2), then A is (0,0), C is (x_c, y_c), and D is (x_D, y_D).

Consider intersection of 2-radius circle and 4-radius circle.

2-radius circle: $x^2 + y^2 = 4$

4-radius circle: $(x-4)^2 + (y-2)^2 = 16$

$x^2 + y^2 - 4 = (x-4)^2 + (y-2)^2 - 16$

$x^2 + y^2 - 4 = x^2 - 8x + 16 + y^2 - 4y + 4 - 16$

$4y = -8x + 8$

$y = -2x + 2$

Substitute into 2-radius circle equation:

$x^2 + (-2x+2)^2 = 4$

$x^2 + 4x^2 - 8x + 4 = 4$

$5x^2 - 8x = 0$

$x = 0,\ 1.6$

Plug these values into $y = -2x + 2$ to obtain corresponding y values. (These are the coordinates of $C(x_c, y_c)$).

$y_{c_1} = -2(0) + 2 = 2$

$y_{c_2} = -2(1.6) + 2 = -1.2$

Now consider intersection of 3-radius circle and 4-radius circle. For simplicity, use coordinates $(x_c, y_c) = (0, 2)$.

3-radius circle: $x^2 + (y-2)^2 = 9$

4-radius circle: $(x-4)^2 + (y-2)^2 = 16$

$x^2 + (y-2)^2 - 9 = (x-4)^2 + (y-2)^2 - 16$

$x^2 - 9 = x^2 - 8x + 16 - 16$

$8x - 9 = 0$

$x = 9/8 = 1.125 \quad (x_D)$

Plug into 3-radius circle equation:

$(9/8)^2 + (y-2)^2 = 9$

$y^2 - 4y + 4 + 81/64 = 9$

$y^2 - 4y - 239/64 = 0$

$y = 4.781,\ -0.781 \quad (y_D)$

There are 4 solutions for the case of two circles of radii 2 and 4 about points A and B.

From the equations above, the solutions using C_1 (0,2) is:

D_1 (1.125, 4.781)

$\vec{AC}_1 = 2\hat{\jmath}$

$\vec{C_1D_1} = (1.125 - 0)\hat{\imath} + (4.781 - 2)\hat{\jmath}$
$= 1.125\hat{\imath} + 2.781\hat{\jmath}$

$\vec{D_1B} = (4 - 1.125)\hat{\imath} + (2 - 4.781)\hat{\jmath}$
$= 2.875\hat{\imath} - 2.781\hat{\jmath}$

D_2 (1.125, −0.781)

$\vec{AC}_1 = 2\hat{\jmath}$

$\vec{C_1D_2} = 1.125\hat{\imath} - 2.781\hat{\jmath}$

$\vec{D_2B} = 2.875\hat{\imath} + 2.781\hat{\jmath}$

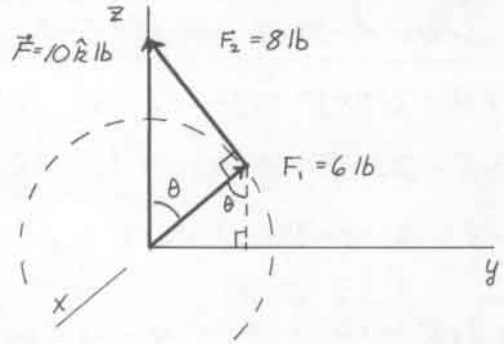
⇒ A total of 2 solutions for C_1.

There are another 2 solutions using C_2 (1.6, −1.2).

There are 4 solutions for the case of two circles of radii 3 and 4 about points A and B; and 4 more solutions for radii 2 and 4. This gives a total of 12 vector sets.

5.22 Design Problem

Replace $\vec{F} = 10\hat{k}$ lb with $F_1 = 6$ lb and $F_2 = 8$ lb.



For simplicity, $\vec{F}$, $\vec{F_1}$, and $\vec{F_2}$ lie in the y-z plane.

Use geometry to find θ:

$$\theta = \sin^{-1}(8/10) = 53.13°$$

$$\vec{F_1} = 0\hat{\imath} + 6\sin 53.13°\,\hat{\jmath} + 6\cos 53.13°\,\hat{k}$$
$$= 4.8\hat{\jmath} + 3.6\hat{k} \text{ lb}$$

$$\vec{F_2} = 0\hat{\imath} + (0-4.8)\hat{\jmath} + (10-3.6)\hat{k}$$
$$= -4.8\hat{\jmath} + 6.4\hat{k} \text{ lb}$$

One possible solution:

$$\vec{F_1} = 4.8\hat{\jmath} + 3.6\hat{k} \text{ lb}, \quad \vec{F_2} = -4.8\hat{\jmath} + 6.4\hat{k} \text{ lb}$$

5.23 Design Problem

Replace $\vec{F} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ kN with $F_1 = 2$ kN and $F_2 = 5$ kN

$$\vec{F} = \vec{F_1} + \vec{F_2}$$

$$F_1 = \sqrt{(F_1x^2) + (F_1y^2) + (F_1z^2)}$$

$$F_2 = \sqrt{F_2x^2 + F_2y^2 + F_2z^2}$$

By examination of $\vec{F}$ components, we see that:

$$\sqrt{3^2 + 4^2} = 5 = F_2 \quad \text{and} \quad \sqrt{2^2} = 2 = F_1$$

So, one possible solution is:

$$\vec{F_1} = 2\hat{k} \text{ kN} \quad \vec{F_2} = 3\hat{\imath} + 4\hat{\jmath} \text{ kN}$$

5.24 Find $\hat{n}$ perpendicular to $\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} - \hat{\jmath} + \hat{k}$

The vector product of two vectors is perpendicular to both vectors.

Let $\vec{A} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ and $\vec{B} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$.

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -1 & 1 \end{vmatrix} = 6\hat{\imath} + 5\hat{\jmath} - 13\hat{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{6^2 + 5^2 + (-13)^2} = 15.17$$

$$\hat{n} = \frac{1}{15.17}(6\hat{\imath} + 5\hat{\jmath} - 13\hat{k})$$

$$\hat{n} = 0.396\hat{\imath} + 0.330\hat{\jmath} - 0.857\hat{k}$$

5.25 Show that if $\vec{q}$ is taken from A to C in Example 5.6, you get the same result for d.

$\vec{q}$ = is from A (50, -75, 25) to C (100, 50, 25).

$$\vec{q} = (100-50)\hat{\imath} + (50+75)\hat{\jmath} + (25-25)\hat{k}$$

$$\vec{q} = 50\hat{\imath} + 125\hat{\jmath}$$

$$d = \hat{e} \cdot \vec{q}, \quad \hat{e} = -0.4441\hat{\imath} + 0.5552\hat{\jmath} - 0.7032\hat{k}$$

$$d = \hat{e} \cdot \vec{q} = (-0.4441)(50) + (0.5552)(125) + (0.7032)(0)$$

$$d = 47.20 \text{ m}$$

5.26 Show that the line passing through (3, 1, -2) and (6, 8, 7) is parallel to the line passing through (-4, 0, -2) and (20, 56, 70).

Let $\vec{A}$ be along first line and $\vec{B}$ be along the second line.

$$\vec{A} = (6-3)\hat{\imath} + (8-1)\hat{\jmath} + (7-(-2))\hat{k}$$
$$= 3\hat{\imath} + 7\hat{\jmath} + 9\hat{k}$$

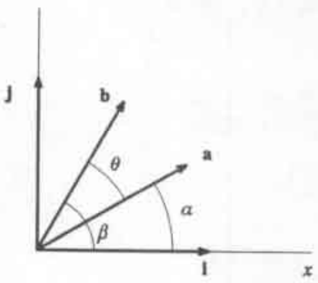
$$\vec{B} = (20-(-4))\hat{\imath} + (56-0)\hat{\jmath} + (70-(-2))\hat{k}$$
$$= 24\hat{\imath} + 56\hat{\jmath} + 72\hat{k}$$

If $\vec{A}$ and $\vec{B}$ are parallel, then $\vec{A} \times \vec{B} = 0$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 7 & 9 \\ 24 & 56 & 72 \end{vmatrix} = 0$$

$\therefore$ $\vec{A}$ and $\vec{B}$ are parallel.

5.27 Find $\vec a \times \vec b$ and show that it yields the trig identity
$\sin(\beta-\alpha) = \sin\beta\cos\alpha - \cos\beta\sin\alpha$
$\vec a$ and $\vec b$ are unit vectors.



$\hat a = \cos\alpha\,\hat\imath + \sin\alpha\,\hat\jmath$
$\hat b = \cos\beta\,\hat\imath + \sin\beta\,\hat\jmath$

$$\hat c = \hat a \times \hat b = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \end{vmatrix}$$

$$= (\sin\beta\cos\alpha - \cos\beta\sin\alpha)\,\hat k$$

By definition; $c = ab\sin\theta$

By figure, $\theta = \beta - \alpha \quad \therefore c = ab\sin(\beta-\alpha)$

But; $a = b = 1 \quad \therefore c = \sin(\beta - \alpha)$

So, $\underline{\underline{\sin(\beta-\alpha) = \sin\beta\cos\alpha - \cos\beta\sin\alpha}}$

5.28 Find a.) $\vec A \times \vec B$, b.) $\vec C \times \vec D$, c.) $(\vec D + \vec B) \times (\vec A + \vec C)$, d.) $\hat\jmath \times \vec B$, where $\vec A = 4\hat\imath + \hat\jmath - \hat k$, $\vec B = 3\hat\imath + 6\hat\jmath$, $\vec C = 3\hat\jmath + 2\hat k$, and $\vec D = -2\hat\imath - 4\hat\jmath - 8\hat k$.

a.)
$$\vec A \times \vec B = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 4 & 1 & -1 \\ 3 & 6 & 0 \end{vmatrix} = \underline{\underline{6\hat\imath - 3\hat\jmath + 21\hat k}}$$

b.)
$$\vec C \times \vec D = \begin{matrix} \hat\imath & \hat\jmath & \hat k \\ 0 & 3 & 2 \\ -2 & -4 & -8 \end{matrix} = \underline{\underline{-16\hat\imath - 4\hat\jmath + 6\hat k}}$$

c.) $\vec D + \vec B = (-2\hat\imath - 4\hat\jmath - 8\hat k) + (3\hat\imath + 6\hat\jmath) = \hat\imath + 2\hat\jmath - 8\hat k$
$\vec A + \vec C = (4\hat\imath + \hat\jmath - \hat k) + (3\hat\jmath + 2\hat k) = 4\hat\imath + 4\hat\jmath + \hat k$

$$(\vec D + \vec B) \times (\vec A + \vec C) = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 1 & 2 & -8 \\ 4 & 4 & 1 \end{vmatrix} = \underline{\underline{34\hat\imath - 33\hat\jmath - 4\hat k}}$$

d)
$$\hat\jmath \times \vec B = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 0 & 1 & 0 \\ 3 & 6 & 0 \end{vmatrix} = \underline{\underline{-3\hat k}}$$

5.29 Find a.) $\vec A + \vec B$, b) $\vec A - \vec B$, c) $\vec A \cdot \vec B$, d.) $\vec C = \vec A \times \vec B$, e) $\hat c$, f) $(\vec A + \vec B)\cdot(\vec A - \vec B)$ g) $(\vec A + \vec B) \times (\vec A - \vec B)$, h) $(\vec A - \vec B) \times (\vec A + \vec B)$ where $\vec A = 3\hat\imath + \hat\jmath$ and $\vec B = \hat\jmath + 2\hat k$

a) $\vec A + \vec B = (3\hat\imath + \hat\jmath) + (\hat\jmath + 2\hat k) = \underline{\underline{3\hat\imath + 2\hat\jmath + 2\hat k}}$

b) $\vec A - \vec B = (3\hat\imath + \hat\jmath) - (\hat\jmath + 2\hat k) = \underline{\underline{3\hat\imath - 2\hat k}}$

c) $\vec A \cdot \vec B = (3)(0) + (1)(1) + (0)(2) = \underline{\underline{1}}$

d)
$$\vec C = \vec A \times \vec B = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = \underline{\underline{2\hat\imath - 6\hat\jmath + 3\hat k}}$$

e) $$\hat c = \frac{\vec C}{|\vec C|} = \frac{2\hat\imath - 6\hat\jmath + 3\hat k}{\sqrt{2^2 + (-6)^2 + 3^2}} = \underline{\underline{\tfrac{2}{7}\hat\imath - \tfrac{6}{7}\hat\jmath + \tfrac{3}{7}\hat k}}$$

f) $(\vec A + \vec B)\cdot(\vec A - \vec B) = (3)(3) + (2)(0) + (2)(-2)$
$= \underline{\underline{5}}$

g)
$$(\vec A + \vec B) \times (\vec A - \vec B) = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 3 & 2 & 2 \\ 3 & 0 & -2 \end{vmatrix} = \underline{\underline{-4\hat\imath + 12\hat\jmath - 6\hat k}}$$

h)
$$(\vec A - \vec B) \times (\vec A + \vec B) = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 3 & 0 & -2 \\ 3 & 2 & 2 \end{vmatrix} = \underline{\underline{4\hat\imath - 12\hat\jmath + 6\hat k}}$$

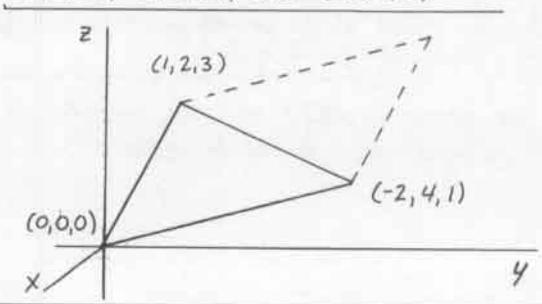
5.30 Find area of parallelogram with the two sides $\vec A = 14\hat\imath - 6\hat\jmath$ and $\vec B = 4\hat\imath + 10\hat\jmath$

Area of parallelogram equals the magnitude of the vector product of its sides.

$$\vec A \times \vec B = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 14 & -6 & 0 \\ 4 & 10 & 0 \end{vmatrix} = 164\hat k$$

$|\vec A \times \vec B| = \underline{\underline{164}}$ (area of parallelogram)

5.31 Find area of triangle with vertices $(0,0,0)$, $(1,2,3)$, and $(-2,4,1)$



5.31 cont.

Area of Triangle = 1/2 Area of Parallelogram

Area of Parallelogram is the magnitude of the vector product of the two sides.

Let $\vec{A} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{B} = -2\hat{\imath} + 4\hat{\jmath} + \hat{k}$

Area of Triangle = $\frac{1}{2} |\vec{A} \times \vec{B}|$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ -2 & 4 & 1 \end{vmatrix} = -10\hat{\imath} - 7\hat{\jmath} + 8\hat{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{(-10)^2 + (-7)^2 + 8^2} = 14.59 \text{ ft}^2$$

Area of Triangle = $\frac{1}{2}(14.59) = \underline{\underline{7.30 \text{ ft}^2}}$

5.32 Find a.) $\vec{A} \times \vec{B}$, b) $\vec{B} \times \vec{A}$, c) $\vec{A} \times (\vec{B} \times \vec{C})$, d) $\hat{\imath} \times \vec{B}$, e) $\vec{C} \times \hat{k}$

where $\vec{A} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$, $\vec{B} = 6\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{C} = -4\hat{\imath} + \hat{\jmath}$

a)
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 4 \\ 6 & 2 & -1 \end{vmatrix} = \underline{\underline{-7\hat{\imath} + 26\hat{\jmath} + 10\hat{k}}}$$

b) $\vec{B} \times \vec{A} = -(\vec{A} \times \vec{B}) = \underline{\underline{7\hat{\imath} - 26\hat{\jmath} - 10\hat{k}}}$

c)
$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} \times \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 6 & 2 & -1 \\ -4 & 1 & 0 \end{vmatrix} = \vec{A} \times (\hat{\imath} + 4\hat{\jmath} + 14\hat{k})$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 4 \\ 1 & 4 & 14 \end{vmatrix} = \underline{\underline{-30\hat{\imath} - 24\hat{\jmath} + 9\hat{k}}}$$

d)
$$\hat{\imath} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 0 \\ 6 & 2 & -1 \end{vmatrix} = \underline{\underline{\hat{\jmath} + 2\hat{k}}}$$

e)
$$\vec{C} \times \hat{k} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \underline{\underline{\hat{\imath} + 4\hat{\jmath}}}$$

5.33 a) Evaluate $\vec{c} = \vec{a} \times \vec{b}$ by expansion.

b) Check with Eq. 5.9,

c) Show that $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b}$ using scalar products

where $\vec{a} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$

and $\vec{b} = -2\hat{\imath} + 6\hat{\jmath} - 4\hat{k}$

a) $\vec{c} = \vec{a} \times \vec{b} = (3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}) \times (-2\hat{\imath} + 6\hat{\jmath} - 4\hat{k})$

$$\begin{aligned} &(3\hat{\imath}) \times (-2\hat{\imath} + 6\hat{\jmath} - 4\hat{k}) = 0 + 18\hat{k} + 12\hat{\jmath} \\ &(5\hat{\jmath}) \times (-2\hat{\imath} + 6\hat{\jmath} - 4\hat{k}) = 10\hat{k} + 0 - 20\hat{\imath} \\ &\underline{+\ (-2\hat{k}) \times (-2\hat{\imath} + 6\hat{\jmath} - 4\hat{k}) = +4\hat{\jmath} + 12\hat{\imath} + 0} \end{aligned}$$

$\therefore \underline{\underline{\vec{c} = \vec{a} \times \vec{b} = -8\hat{\imath} + 16\hat{\jmath} + 28\hat{k}}}$

b)
$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 5 & -2 \\ -2 & 6 & -4 \end{vmatrix} = \underline{\underline{-8\hat{\imath} + 16\hat{\jmath} + 28\hat{k}}}$$

c) When scalar product = 0, then the vectors are perpendicular.

$\vec{a} \cdot \vec{c} = (3)(-8) + (5)(16) + (-2)(28) = 0$

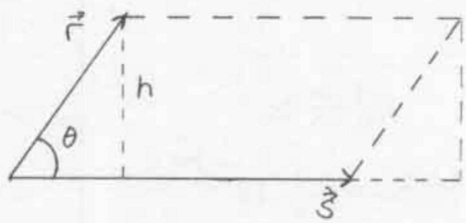
$\vec{b} \cdot \vec{c} = (-2)(-8) + (6)(16) + (-4)(28) = 0$

$\therefore \vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b}$

5.34 a) Prove $|\vec{r} \times \vec{s}|$ = area of parallelogram

b) Find area of parallelogram with sides $\vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{s} = -3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

a)



Area of parallelogram = $h|\vec{s}|$

$h = |\vec{r}| \sin\theta$; $0 \le \theta \le 180°$

Area = $rs \sin\theta$

By definition, $|\vec{r} \times \vec{s}| = rs \sin\theta$

$\therefore$ Area = $|\vec{r} \times \vec{s}|$

b)
$$\vec{r} \times \vec{s} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix} = \underline{\underline{8\hat{\imath} - 10\hat{\jmath} + 4\hat{k} \quad [m]}}$$

$$|\vec{r} \times \vec{s}| = \sqrt{8^2 + (-10)^2 + 4^2} = \underline{\underline{13.42 \text{ m}^2}}$$

5.35 Show that $\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$ is equivalent to $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$ where $\vec{a}, \vec{b}, \vec{c}$ are from sides of a triangle with interior angles α, β, γ (Fig. a)

By Fig. a,

$\vec{a} + \vec{b} = \vec{c}$

But,

$(\vec{a}+\vec{b}) \times (\vec{a}+\vec{b})$
$= (\vec{a}+\vec{b}) \times \vec{c} = 0$

$\therefore \vec{a} \times \vec{c} = \vec{c} \times \vec{b}$

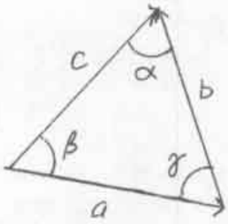


Figure (a)

By theorem 5.2,

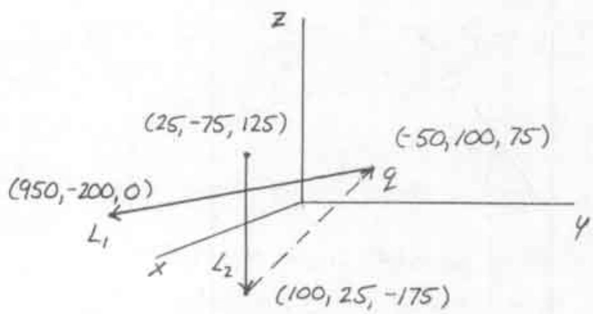
$|\vec{a} \times \vec{c}| = ac\sin\beta$ and $|\vec{c} \times \vec{b}| = bc\sin\alpha$

$\therefore ac\sin\beta = bc\sin\alpha$

or $a\sin\beta = b\sin\alpha$

$$\therefore \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

5.36 Find minimum distance between L_1 and L_2.



$\vec{L_1} = (950-(-50))\hat{\imath} + (-200-100)\hat{\jmath} + (0-75)\hat{k}$
$= 1000\hat{\imath} - 300\hat{\jmath} - 75\hat{k}$

$\vec{L_2} = (100-25)\hat{\imath} + (25-(-75))\hat{\jmath} + (-175-125)\hat{k}$
$= 75\hat{\imath} + 100\hat{\jmath} - 300\hat{k}$

Minimum distance is along a vector $\vec{R} \perp$ to $\vec{L_1}$ and $\vec{L_2}$.

$$\vec{R} = \vec{L_1} \times \vec{L_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1000 & 300 & -75 \\ 75 & 100 & -300 \end{vmatrix}$$

$= 97{,}500\hat{\imath} + 294{,}375\hat{\jmath} + 122{,}500\hat{k}$

Projection of $\vec{q}$ on the unit vector of $\vec{R}$ gives the minimum distance between $\vec{L_1}$ and $\vec{L_2}$. (Min dist $= \hat{e}_R \cdot q$)

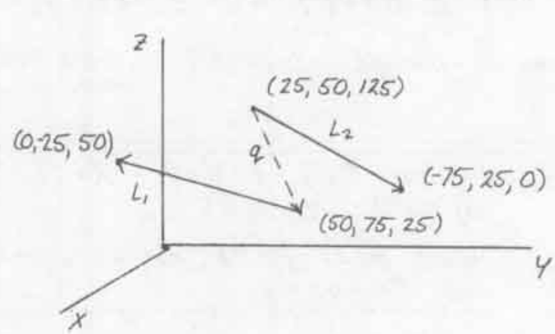
$\vec{q} = (-50-100)\hat{\imath} + (100-25)\hat{\jmath} + (75-(-175))\hat{k}$
$= -150\hat{\imath} + 75\hat{\jmath} + 250\hat{k}$

$$\hat{e}_R = \frac{\vec{R}}{|R|} = \frac{97{,}500\hat{\imath} + 294{,}500\hat{\jmath} + 122{,}500\hat{k}}{\sqrt{97{,}500^2 + 294{,}500^2 + 122{,}500^2}}$$

$= 0.292\hat{\imath} + 0.883\hat{\jmath} + 0.367\hat{k}$

$\hat{e}_R \cdot \vec{q} = \underline{\underline{114.2 \text{ ft}}}$

5.37 Find minimum distance between L_1 and L_2



$\vec{L_1} = (0-50)\hat{\imath} + (-25-75)\hat{\jmath} + (50-25)\hat{k}$
$= -50\hat{\imath} - 100\hat{\jmath} + 25\hat{k}$

$\vec{L_2} = (-75-25)\hat{\imath} + (25-50)\hat{\jmath} + (0-125)\hat{k}$
$= -100\hat{\imath} - 25\hat{\jmath} - 125\hat{k}$

Minimum distance is along a vector $\vec{R} \perp$ to L_1 and L_2.

$$\vec{R} = \vec{L_1} \times \vec{L_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -50 & -100 & 25 \\ -100 & -25 & -125 \end{vmatrix}$$

$= 13{,}125\hat{\imath} - 8750\hat{\jmath} - 8750\hat{k}$

Projection of $\vec{q}$ on the unit vector of $\vec{R}$ gives the minimum distance between $\vec{L_1}$ and $\vec{L_2}$. (Min. dist. $= \hat{e}_R \cdot \vec{q}$)

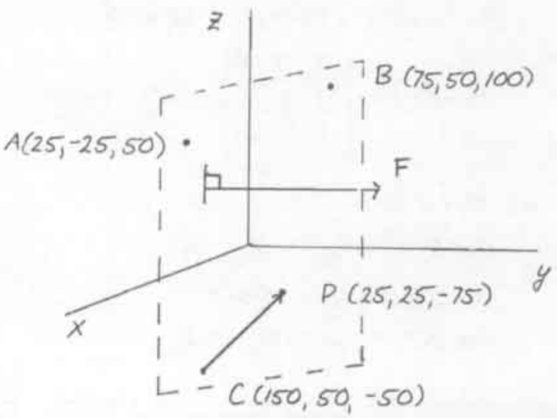
$\vec{q} = (50-25)\hat{\imath} + (75-50)\hat{\jmath} + (25-125)\hat{k}$
$= 25\hat{\imath} + 25\hat{\jmath} - 100\hat{k}$

$$\hat{e}_R = \frac{\vec{R}}{|R|} = \frac{13{,}125\hat{\imath} - 8750\hat{\jmath} - 8750\hat{k}}{\sqrt{13{,}125^2 + (-8750)^2 + (-8750)^2}}$$

$= 0.728\hat{\imath} - 0.485\hat{\jmath} - 0.485\hat{k}$

$\hat{e}_R \cdot \vec{q} = \underline{\underline{54.57 \text{ m}}}$

5.38 Find minimum distance from the plane that contains A, B, C to point P.



Minimum distance between plane ABC and P is a line $\perp$ to plane ABC ($\vec{F}$) that passes through P.

$\vec{F} = \vec{AB} \times \vec{BC}$ = vector $\perp$ to plane ABC

$$\vec{AB} = (75-25)\hat{\imath} + (50-(-25))\hat{\jmath} + (100-50)\hat{k}$$
$$= 50\hat{\imath} + 75\hat{\jmath} + 50\hat{k}$$

$$\vec{BC} = (150-75)\hat{\imath} + (50-50)\hat{\jmath} + (-50-100)\hat{k}$$
$$= 75\hat{\imath} - 150\hat{k}$$

$$\vec{F} = \vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 50 & 75 & 50 \\ 75 & 0 & -150 \end{vmatrix}$$

$$= -11{,}250\hat{\imath} + 11{,}250\hat{\jmath} - 5625\hat{k}$$

$$\hat{e}_F = \frac{\vec{F}}{|\vec{F}|} = \frac{-11{,}250\hat{\imath} + 11{,}250\hat{\jmath} - 5625\hat{k}}{\sqrt{(-11{,}250)^2 + 11{,}250^2 + (-5625)^2}}$$

$$= -0.667\hat{\imath} + 0.667\hat{\jmath} - 0.333\hat{k}$$

$$\vec{CP} = (25-150)\hat{\imath} + (25-50)\hat{\jmath} + (-75-(-50))\hat{k}$$
$$= -125\hat{\imath} - 25\hat{\jmath} - 25\hat{k}$$

Project $\vec{CP}$ onto $\hat{e}_F = \vec{CP} \cdot \hat{e}_F$

$\vec{CP} \cdot \hat{e}_F = \underline{\underline{75 \text{ in}}}$

5.39 Find the equation of a plane passing through the points $A: (1, 0, 2)$, $B: (2.3, 0)$, and $C: (-1, -2, -5)$, where $P: (x, y, z)$ is a general point in the plane. Verify that A, B, C lie in the plane.

The normal to the plane may be calculated as $\vec{N} = \vec{AB} \times \vec{BC}$. Then, since the vector $\vec{AP}$ lies in the plane, $\vec{N} \cdot \vec{AP} = 0$ is the equation of the plane. (Note also that $\vec{N} \cdot \vec{BP} = 0$ or $\vec{N} \cdot \vec{CP} = 0$ gives the equation of the plane as well.)

By the given data:

$$\vec{AB} = (2-1)\hat{\imath} + (3-0)\hat{\jmath} + (0-2)\hat{k} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$$
$$\vec{BC} = (-1-2)\hat{\imath} + (-2-3)\hat{\jmath} + (5-0)\hat{k} = -3\hat{\imath} - 5\hat{\jmath} + 5\hat{k}$$
$$\vec{AP} = (x-1)\hat{\imath} + (y-0)\hat{\jmath} + (z-2)\hat{k} = (x-1)\hat{\imath} + y\hat{\jmath} + (z-2)\hat{k}$$

Therefore,

$$\vec{N} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -2 \\ -3 & -5 & 5 \end{vmatrix} = 5\hat{\imath} + \hat{\jmath} + 4\hat{k}$$

and:

$$\vec{N} \cdot \vec{AP} = 5(x-1) + y + 4(z-2) = 5x + y + 4z - 13 = 0$$

Hence, the equation of the plane is

$$\underline{\underline{5x + y + 4z = 13}}$$

* Note that the coefficients of (x, y, z) are the $\hat{\imath}, \hat{\jmath}, \hat{k}$ direction numbers of $\vec{N}$, the normal to the plane.

<u>Verify A, B, C lie in plane</u>

$A: (1, 0, 2)$; $5(1) + 0 + 4(2) = 13$ ✓
$B: (2, 3, 0)$; $5(2) + 3 + 4(0) = 13$ ✓
$C: (-1, -2, 5)$; $5(-1) - 2 + 4(5) = 13$ ✓

$\therefore$ <u>A, B, C all lie in plane</u>

5.40 a.) Find the equation of a plane Q $ax + by + cz + d = 0$ through points $A: (2, -3, 4)$, $B: (1, 2, 3)$, and perpendicular to the plane R: $2x + 3y - 2z = 0$.
b) Verify that A and B lie in the plane and that Q is perpendicular to R.

a.) If A and B lie in plane Q, the vector $\vec{AB}$ lies in Q. Also, the normal $\vec{N}$ to plane R has direction numbers $(2, 3, -2)$; see the solution of Problem 5.39.

Therefore, $\vec{N} = 2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$, and since Q must be perpendicular to R, $\vec{N}$ is parallel to Q. The normal $\vec{M}$ of Q is therefore,

$$\vec{M} = \vec{AB} \times \vec{N},$$

where $\vec{AB} = (1-2)\hat{\imath} + [2-(-3)]\hat{\jmath} + (3-4)\hat{k}$
$$= -\hat{\imath} + 5\hat{\jmath} - \hat{k}$$

5.40 cont.

$$\therefore \vec{M} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 5 & -1 \\ 2 & 3 & -2 \end{vmatrix} = -7\hat{\imath} - 4\hat{\jmath} - 13\hat{k}$$

So, the equation for Q is

$$-7x - 4y - 13z + d = 0$$

or $7x + 4y + 13z = d$

We may use the coordinates of A (or B) to find d. Thus,

$$7(2) + 4(-3) + 13(4) = 54 = d$$

$$\therefore \quad \underline{\underline{Q: 7x + 4y + 13z - 54 = 0}}$$

b.) A: (2,−3,4): $7(2) + 4(-3) + 13(4) - 54 = 0$
B: (1, 2, 3): $7(1) + 4(2) + 3(13) - 54 = 0$

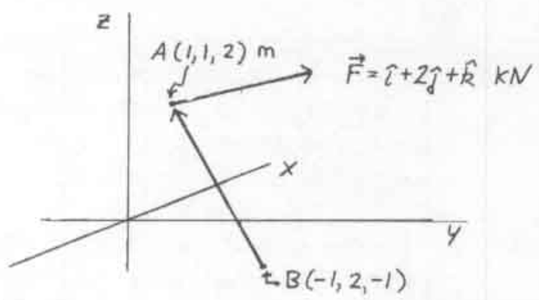
For Q to be perpendicular to R, $\vec{N}\cdot\vec{M} = 0$

$$\vec{N}\cdot\vec{M} = (2\hat{\imath} + 3\hat{\jmath} - 2\hat{k})\cdot(-7\hat{\imath} - 4\hat{\jmath} - 13\hat{k})$$

$$\therefore \vec{N}\cdot\vec{M} = -14 - 12 + 26 = 0$$

⇒ Therefore $\underline{\underline{Q \perp R}}$.

5.41 Find the moment of $\vec{F} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ [kN] about point B: (−1, 2, −1) [m]. $\vec{F}$ passes through A: (1, 1, 2) [m]; See figure:



$$\vec{M}_B = \vec{r} \times \vec{F}$$

$$\vec{r} = (1-(-1))\hat{\imath} + (1-2)\hat{\jmath} + (2-(-1))\hat{k}$$

$$= 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$$

$$\vec{M}_B = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 2 & 1 \end{vmatrix} = \underline{\underline{-7\hat{\imath} + \hat{\jmath} + 5\hat{k} \quad kN\cdot m}}$$

5.42 a.) Find the moment $\vec{M}_o$ about point (0,0,0) of Force $\vec{F} = \hat{\imath} + 3\hat{k}$ [lb]. $\vec{F}$ passes through point A: (−1, 0, 2) [ft]

b.) Find direction cosines for $\vec{M}_o$

a.) $\vec{M}_o = \vec{r} \times \vec{F}$ ($\vec{r}$ is from (0,0,0) to A (−1,0,2))

$$\vec{r} = (-1-0)\hat{\imath} + (0-0)\hat{\jmath} + (2-0)\hat{k} = -\hat{\imath} + 2\hat{k}$$

$$\vec{M}_o = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 0 & 2 \\ 1 & 0 & 3 \end{vmatrix} = \underline{\underline{5\hat{\jmath} \quad lb\cdot ft}}$$

b.) $M_o = 5$ lb·ft

$$\cos\theta_x = M_x/M_o = 0/5 = 0$$

$$\cos\theta_y = M_y/M_o = 5/5 = 1$$

$$\cos\theta_z = M_z/M_o = 0/5 = 0$$

5.43 a.) Find $\vec{M}_o$ about (0,0,0) due to a force $|F| = 50$ N, whose line of action is from A: (1, 2, −3) [m] to B: (2, 0, 1) [m].

b) Find $\vec{M}$ about (0, 1, 2)

a) $\vec{F}$ lies along a line from A (1, 2, −3) to B (2, 0, 1).

Unit vector from A to B given by

$$\vec{r}_{AB} = (2-1)\hat{\imath} + (0-2)\hat{\jmath} + (1-(-3))\hat{k}$$

$$= \hat{\imath} - 2\hat{\jmath} + 4\hat{k}$$

$$\hat{e}_{AB} = \frac{\hat{\imath} - 2\hat{\jmath} + 4\hat{k}}{\sqrt{1^2 + (-2)^2 + 4^2}}$$

$$= 0.2181\,\hat{\imath} - 0.4364\,\hat{\jmath} + 0.8729\,\hat{k}$$

$$\vec{F} = 50\,\hat{e}_{AB} = 10.91\,\hat{\imath} - 21.82\,\hat{\jmath} + 43.64\,\hat{k}$$

$\vec{M}_o = \vec{r} \times \vec{F}$ ($\vec{r}$ is from (0,0,0) to A: (1, 2, −3))

$$\vec{r} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

$$\vec{M}_o = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -3 \\ 10.91 & -21.82 & 43.64 \end{vmatrix}$$

$$\underline{\underline{= 21.8\,\hat{\imath} - 76.4\,\hat{\jmath} - 43.6\,\hat{k}}}$$

b.) $\vec{M}_{(0,1,2)} = \vec{r} \times \vec{F}$ ($\vec{r}$ is from (0,1,2) to A)

$$\vec{r} = (1-0)\hat{\imath} + (2-1)\hat{\jmath} + (-3-2)\hat{k} = \hat{\imath} + \hat{\jmath} - 5\hat{k}$$

$$\vec{M}_{(0,1,2)} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -5 \\ 10.91 & -21.82 & 43.64 \end{vmatrix}$$

$$\underline{\underline{= -65.5\,\hat{\imath} - 98.2\,\hat{\jmath} - 32.7\,\hat{k}}}$$

5.44 Find the sum of the moments of vectors $\vec{F}$, $\vec{G}$, and $\vec{H}$ about (4,5,6) where

$\vec{F} = 4\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ at (2, 1, -4)
$\vec{G} = -4\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ at (-2, 3, 1)
$\vec{H} = 3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ at (3, 2, -1)
P: (4, 5, 6)

Position vectors from P to the points of action:

$\vec{r}_F = (2-4)\hat{\imath} + (1-5)\hat{\jmath} + (-4-6)\hat{k} = -2\hat{\imath} - 4\hat{\jmath} + 10\hat{k}$

$\vec{r}_G = (-2-4)\hat{\imath} + (3-5)\hat{\jmath} + (1-6)\hat{k} = -6\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

$\vec{r}_H = (3-4)\hat{\imath} + (2-5)\hat{\jmath} + (-1-6)\hat{k} = -\hat{\imath} - 3\hat{\jmath} - 7\hat{k}$

$$\vec{M}_F = \vec{r}_F \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -4 & -10 \\ 4 & -3 & 6 \end{vmatrix} = -54\hat{\imath} - 28\hat{\jmath} + 22\hat{k}$$

$$\vec{M}_G = \vec{r}_G \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -6 & -2 & -5 \\ -4 & 2 & 5 \end{vmatrix} = 50\hat{\jmath} - 20\hat{k}$$

$$\vec{M}_H = \vec{r}_H \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -3 & -7 \\ 3 & -4 & -5 \end{vmatrix} = -13\hat{\imath} - 26\hat{\jmath} + 13\hat{k}$$

$$\vec{M} = \vec{M}_F + \vec{M}_G + \vec{M}_H = \underline{\underline{-67\hat{\imath} - 4\hat{\jmath} + 15\hat{k}}}$$

5.45 a.) Find $\vec{M}_E$, b) Find $\vec{M}_D$, c) Find θ_z of $\vec{P}$.

$\vec{P}$ acts along AF and P = 170 N

$\vec{r}_P = 12\hat{\imath} + 8\hat{\jmath} + 9\hat{k}$ [m]

$$\hat{e}_P = \frac{12\hat{\imath} + 8\hat{\jmath} + 9\hat{k}}{\sqrt{12^2 + 8^2 + 9^2}} = 0.706\hat{\imath} + 0.471\hat{\jmath} + 0.529\hat{k}$$

$\vec{P} = 170\,\hat{e}_P = 120\hat{\imath} + 80\hat{\jmath} + 90\hat{k}$ [N]

a) $\vec{r}_{EF} = 12\hat{\imath}$ (From E to F)

$$\vec{M}_E = \vec{r}_{EF} \times \vec{P} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 12 & 0 & 0 \\ 120 & 80 & 90 \end{vmatrix} = \underline{\underline{-1080\hat{\jmath} + 960\hat{k} \ \text{N·m}}}$$

b) $\vec{r}_{DF} = 9\hat{k}$ (From D to F)

$$\vec{M}_D = \vec{r}_{DF} \times \vec{P} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 9 \\ 120 & 80 & 90 \end{vmatrix} = \underline{\underline{-720\hat{\imath} + 1080\hat{\jmath} \ \text{N·m}}}$$

c.) $\cos\theta_z = P_z/P = 90/170 = 9/17$

$\theta_z = \cos^{-1}(9/17) = \underline{\underline{58.0°}}$

5.46 a.) Express $\vec{F}$ in terms of $\hat{\imath}, \hat{\jmath}, \hat{k}$
b) Find $\vec{M}_A$
$\vec{F}$ acts along the diagonal and F = 3000 lb.

a.) $\vec{r}_F = -4\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ [m]

$$\hat{e}_F = \frac{-4\hat{\imath} + 4\hat{\jmath} - 2\hat{k}}{\sqrt{(-4)^2 + 4^2 + (-2)^2}} = -\tfrac{2}{3}\hat{\imath} + \tfrac{2}{3}\hat{\jmath} - \tfrac{1}{3}\hat{k}$$

$\vec{F} = 3000\,\hat{e}_F = \underline{\underline{-2000\hat{\imath} + 2000\hat{\jmath} - 1000\hat{k} \ [\text{lb}]}}$

b.) $\vec{r}_A = -4\hat{\imath}$ (From A to y-axis)

$$\vec{M}_A = \vec{r}_A \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4 & 0 & 0 \\ -2000 & 2000 & -1000 \end{vmatrix}$$

$$= \underline{\underline{-4000\hat{\jmath} - 8000\hat{k} \quad \text{lb·in}}}$$

5.47 a.) Find $\vec{M}_A$, b) Find $\vec{M}_C$
$\vec{F}$ acts along the diagonal and F = 2 kN

$\vec{r}_F = -200\hat{\imath} + 600\hat{\jmath} - 300\hat{k}$ [mm]

$$\hat{e}_F = \frac{-200\hat{\imath} + 600\hat{\jmath} - 300\hat{k}}{\sqrt{(-200)^2 + (600)^2 + (-300)^2}}$$

$= -0.286\hat{\imath} + 0.857\hat{\jmath} - 0.429\hat{k}$

$\vec{F} = 2\hat{e}_F = -0.571\hat{\imath} + 1.714\hat{\jmath} - 0.857\hat{k}$ kN

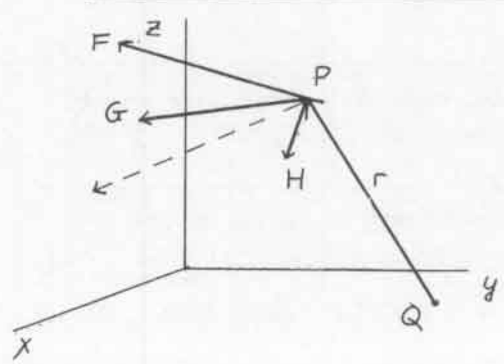
a) $\vec{r}_A = -600\hat{\jmath}$ [mm] (From A to x, z plane)

$$\vec{M}_A = \vec{r}_A \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -600 & 0 \\ -0.571 & 1.714 & -0.857 \end{vmatrix}$$

$$= \underline{\underline{514\hat{\imath} - 343\hat{k} \quad \text{N·m}}}$$

b) $\vec{M}_C = 0$ by inspection
Line of action of $\vec{F}$ acts through C.

5.48 Show that $\Sigma\vec{M}$ ($\vec{F}$, $\vec{G}$, $\vec{H}$ about Q) equals the moment of the resultant, $\vec{R}$, about Q.



5.48 cont.

Sum of the moments about Q

$= \vec{r}\times\vec{F} + \vec{r}\times\vec{G} + \vec{r}\times\vec{H}$ -or-

$Q = \vec{r}\times(\vec{F}+\vec{G}+\vec{H})$

$\therefore \Sigma M_Q = \vec{r}\times(\vec{F}+\vec{G}+\vec{H}) = \vec{r}\times\vec{R}$

where $\vec{R} = \vec{F}+\vec{G}+\vec{H}$ is the resultant

5.49 a.) Find $M_{(5,6,8)}$ of each force

b.) Find magnitude and direction cosines of $\vec{M}$.

$\vec{F_1} = 3\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ at $(1,2,4)$

$\vec{F_2} = -2\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ at $(-3,2,1)$

$\vec{F_3} = 6\hat{\imath} - 8\hat{\jmath} - 5\hat{k}$ at $(4,2,-4)$

$\vec{r_1} = -4\hat{\imath} - 4\hat{\jmath} - 12\hat{k}$

$\vec{r_2} = -8\hat{\imath} - 4\hat{\imath} - 7\hat{k}$

$\vec{r_3} = -\hat{\imath} - 4\hat{\jmath} - 12\hat{k}$

a.)

$$\vec{M_1} = \vec{r_1}\times\vec{F_1} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4 & -4 & -12 \\ 3 & -3 & 5 \end{vmatrix} = \underline{\underline{-56\hat{\imath} - 16\hat{\jmath} + 24\hat{k}}}$$

$$\vec{M_2} = \vec{r_2}\times\vec{F_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -8 & -4 & -7 \\ -2 & 2 & 6 \end{vmatrix} = \underline{\underline{-10\hat{\imath} + 62\hat{\jmath} - 24\hat{k}}}$$

$$\vec{M_3} = \vec{r_3}\times\vec{F_3} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -4 & -12 \\ 6 & -8 & -5 \end{vmatrix} = \underline{\underline{-76\hat{\imath} - 77\hat{\jmath} + 32\hat{k}}}$$

b.) $\vec{M} = \vec{M_1} + \vec{M_2} + \vec{M_3} = -142\hat{\imath} - 31\hat{\jmath} + 32\hat{k}$

$M = \sqrt{(-142)^2 + (-31)^2 + (32)^2} = \underline{\underline{148.8}}$

$\cos\theta_x = M_x/M = -142/148.8 = \underline{\underline{-0.954}}$

$\cos\theta_y = M_y/M = -31/148.8 = \underline{\underline{-0.208}}$

$\cos\theta_z = M_z/M = 32/148.8 = \underline{\underline{0.215}}$

5.50 Show that it is impossible to place a force at $(5,10,-20)$ so that $\Sigma M_0 = 0$.

$\vec{A} = 8\hat{\imath} + 11\hat{\jmath} + 15\hat{k}$ at $(2,4,-1)$

$\vec{B} = -6\hat{\imath} + 9\hat{\jmath} + 10\hat{k}$ at $(4,-2,2)$

$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$ at $(5,10,-20)$

$\vec{r_A} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$

$\vec{r_B} = 4\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$\vec{r_F} = 5\hat{\imath} + 10\hat{\jmath} - 20\hat{k}$

$$\vec{M_A} = \vec{r_A}\times\vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 4 & -1 \\ 8 & 11 & 15 \end{vmatrix} = 71\hat{\imath} - 38\hat{\jmath} - 10\hat{k}$$

$$\vec{M_B} = \vec{r_B}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & 2 \\ -6 & 9 & 10 \end{vmatrix} = -38\hat{\imath} - 52\hat{\jmath} + 24\hat{k}$$

$$\vec{M_F} = \vec{r_F}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & 10 & -20 \\ F_x & F_y & F_z \end{vmatrix} = (10F_z + 20F_y)\hat{\imath} + (-20F_x - 5F_z)\hat{\jmath} + (5F_y - 10F_x)\hat{k}$$

x-projection: $10F_z + 20F_y + 71 - 38 = 0$

y-projection: $-20F_x - 5F_z - 38 - 52 = 0$

z-projection: $5F_y - 10F_x - 10 + 24 = 0$

$F_y = \dfrac{-10F_z - 33}{20} = -0.5F_z - 1.65$

$F_z = \dfrac{20F_x + 90}{-5} = -4F_x - 18$

$14 - 10F_x + 5(-0.5(-4F_x - 18) - 1.65) = 0$

$\underline{\underline{50.75 \neq 0 \quad \therefore \text{It is not possible}}}$

5.51 Find moments about the (x,y,z) axes.

$\vec{F} = 3\hat{\imath} + 4\hat{\jmath} - \hat{k}$ at $(2,4,6)$ [N]

$\vec{r} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$ [m]

$\vec{M} = \vec{F}\cdot(\hat{n}\times\vec{r})$

$$M_x = \vec{F}\cdot(\hat{\imath}\times\vec{r}) = \begin{vmatrix} 3 & 4 & -1 \\ 1 & 0 & 0 \\ 2 & 4 & 6 \end{vmatrix} = \underline{\underline{-28\ N\cdot m}}$$

$$M_y = \vec{F}\cdot(\hat{\jmath}\times\vec{r}) = \begin{vmatrix} 3 & 4 & -1 \\ 0 & 1 & 0 \\ 2 & 4 & 6 \end{vmatrix} = \underline{\underline{20\ N\cdot m}}$$

$$M_z = \vec{F}\cdot(\hat{k}\times\vec{r}) = \begin{vmatrix} 3 & 4 & -1 \\ 0 & 0 & 1 \\ 2 & 4 & 6 \end{vmatrix} = \underline{\underline{-4\ N\cdot m}}$$

5.52 Find ΣM_o.

$\vec{F} = \hat{\imath} - 4\hat{\jmath} + 8\hat{k}$ at $(0,1,4)$ [kN]

$\vec{G} = 6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ at $(2,2,2)$ [kN]

$\vec{r}_F = \hat{\jmath} + 4\hat{k}$ [m]

$\vec{r}_G = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ [m]

$\Sigma M_o = \vec{r}_F \times \vec{F} + \vec{r}_G \times \vec{G}$

$$\vec{M}_F = \vec{r}_F \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 4 \\ 1 & -4 & 8 \end{vmatrix} = 24\hat{\imath} + 4\hat{\jmath} - \hat{k}$$

$$\vec{M}_G = \vec{r}_G \times \vec{G} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 2 & 2 \\ 6 & -3 & 2 \end{vmatrix} = 10\hat{\imath} + 8\hat{\jmath} - 18\hat{k}$$

$M_x = 24 + 10 = 34$ kN·m

$M_y = 4 + 8 = 12$ kN·m

$M_z = -1 - 18 = \underline{-19 \text{ kN·m}}$

5.53 Find moment of $\vec{F}$ about the axis through $(0,0,0)$ with direction cosines $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$.

$\vec{F} = 20\hat{\imath} + 60\hat{\jmath} + 90\hat{k}$ [N] at $(0,40,0)$ [mm]

$\vec{M} = \vec{F} \cdot (\hat{n} \times \vec{r})$

$\hat{n} = 1/\sqrt{3}\,\hat{\imath} + 1/\sqrt{3}\,\hat{\jmath} + 1/\sqrt{3}\,\hat{k}$

$\vec{r} = 40\hat{\jmath}$

$$M = \vec{F} \cdot (\hat{n} \times \vec{r}) = \begin{vmatrix} 20 & 60 & 90 \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & 40 & 0 \end{vmatrix} = \underline{1.617 \text{ N·m}}$$

5.54 Find M about axis through $(0,0,0)$ with direction cosines $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$.

$\vec{F} = 90\hat{\imath} + 60\hat{\jmath} + 20\hat{k}$ at $(0,0,40)$ [lb]

$\vec{M} = \vec{F} \cdot (\hat{n} \times \vec{r})$

$\hat{n} = 1/\sqrt{3}\,\hat{\imath} + 1/\sqrt{3}\,\hat{\jmath} + 1/\sqrt{3}\,\hat{k}$

$\vec{r} = 40\hat{k}$ [in]

$$M = \vec{F} \cdot (\hat{n} \times \vec{r}) = \begin{vmatrix} 90 & 60 & 20 \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & 0 & 40 \end{vmatrix} = \underline{692.8 \text{ lb·in}}$$

5.55 a.) Find M_F about line AB; See Fig. P5.46.
b.) Find M_F about line AC

$F = 3000$ lb

a.) $M_F = \vec{F} \cdot (\hat{n}_{AB} \times \vec{r})$

$\vec{r}$ is from a point on $\overline{AB}$ to the line of action of $\vec{F}$ at right rear of the block.

$\vec{F} = 3000\,\hat{n}_F$

$$\hat{n}_F = \frac{-4\hat{\imath} + 4\hat{\jmath} - 2\hat{k}}{\sqrt{(-4)^2 + 4^2 + (-2)^2}} = -2/3\,\hat{\imath} + 2/3\,\hat{\jmath} - 1/3\,\hat{k}$$

$\vec{F} = -2000\hat{\imath} + 2000\hat{\jmath} - 1000\hat{k}$ lb

$\hat{n}_{AB} = -2/\sqrt{5}\,\hat{\imath} + 1/\sqrt{5}\,\hat{k}$

$\vec{r} = -2\hat{k}$ [in] from point B on $\overline{AB}$ to $\vec{F}$

$$M_F = \vec{F} \cdot (\hat{n}_{AB} \times \vec{r}) = \begin{vmatrix} -2000 & 2000 & -1000 \\ -2/\sqrt{5} & 0 & 1/\sqrt{5} \\ 0 & 0 & -2 \end{vmatrix}$$

$= \underline{-3578 \text{ lb-in}}$

b.) The line of action of $\vec{F}$ intersects line $\overline{AC}$. There fore, the moment of $\vec{F}$ about $\overline{AC}$ is zero by inspection.

NOTE: In part a, we could have taken $\vec{r} = -4\hat{\imath}$ from point A on line $\overline{AB}$ to line of action of $\vec{F}$; or, we could have taken $\vec{r}$ from <u>any</u> point on $\overline{AB}$ to <u>any</u> point on the line of action of $\vec{F}$.

5.56 Use second and third equations of Eq. 5.19 to obtain moment in Ex. 5.9.

Second Eqn: $M_L = \vec{r} \cdot (\vec{F} \times \hat{n})$

Third Eqn: $M_L = \hat{n} \cdot (\vec{r} \times \vec{F})$

$$\vec{r} \cdot (\vec{F} \times \hat{n}) = \begin{vmatrix} 3.25 & 1.5 & 0.5 \\ 1.00 & 0.5 & 2.5 \\ 6/19 & -6/19 & 17/19 \end{vmatrix} = \underline{3.625 \text{ kN·m}}$$

$$\hat{n} \cdot (\vec{r} \times \vec{F}) = \begin{vmatrix} 6/19 & -6/19 & 17/19 \\ 3.25 & 1.5 & 0.5 \\ 1.00 & 0.5 & 2.5 \end{vmatrix} = \underline{3.625 \text{ kN·m}}$$

5.57 Find moments of $\vec{F}$ about the (x, y, z) axes

$\vec{F} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ at $(1,2,3)$ [lb]

$\vec{M} = \vec{F} \cdot (\hat{n} \times \vec{r})$

$\vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ [ft] from $(0,0,0)$ to $(1,2,3)$

$$\vec{M}_x = \vec{F} \cdot (\hat{\imath} \times \vec{r}) = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 0 & 0 \\ 1 & 2 & 3 \end{vmatrix} = -2 \text{ lb·ft}$$

$$\vec{M}_y = \vec{F} \cdot (\hat{\jmath} \times \vec{r}) = \begin{vmatrix} 3 & 4 & 5 \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 4 \text{ lb-ft}$$

$$\vec{M}_z = \vec{F} \cdot (\hat{k} \times \vec{r}) = \begin{vmatrix} 3 & 4 & 5 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = -2 \text{ lb-ft}$$

5.58 Find M_F about the line through $(-4,-7,-2)$ in the direction of $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

$\vec{F} = 10\hat{\imath} + 20\hat{\jmath} + 5\hat{k}$ [kN] at $(3,0,-4)$ [m]

$\vec{V} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ at $(-4,-7,-2)$

$\vec{r} = (3-(-4))\hat{\imath} + (0-(-7))\hat{\jmath} + (-4-(-2))\hat{k}$

$= 7\hat{\imath} + 7\hat{\jmath} - 2\hat{k}$ [m]

$$\hat{n} = \frac{\vec{V}}{V} = \frac{3\hat{\imath} + \hat{\jmath} - 2\hat{k}}{\sqrt{3^2 + 1^2 + (-2)^2}} = \tfrac{3}{\sqrt{14}}\hat{\imath} + \tfrac{1}{\sqrt{14}}\hat{\jmath} - \tfrac{2}{\sqrt{14}}\hat{k}$$

$$M = \vec{F} \cdot (\hat{n} \times \vec{r}) = \begin{vmatrix} 10 & 20 & 5 \\ 3/\sqrt{14} & 1/\sqrt{14} & -2/\sqrt{14} \\ 7 & 7 & -2 \end{vmatrix} = 8.018 \text{ kN·m}$$

5.59 Prove that the moments about (x, y, z) axes are equal to the projections of $\vec{r} \times \vec{F}$ on these axes.

$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$ at (x, y, z)

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ at $(0,0,0)$

$$\vec{R} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = \begin{cases} R_x = yF_z - zF_y \\ R_y = zF_x - xF_z \\ R_z = xF_y - yF_z \end{cases}$$

$$M_x = \vec{F} \cdot (\hat{\imath} \times \vec{r}) = \begin{vmatrix} F_x & F_y & F_z \\ 1 & 0 & 0 \\ x & y & z \end{vmatrix} = yF_z - zF_y = R_x$$

$$M_y = \vec{F} \cdot (\hat{\jmath} \times \vec{r}) = \begin{vmatrix} F_x & F_y & F_z \\ 0 & 1 & 0 \\ x & y & z \end{vmatrix} = zF_x - xF_z = R_y$$

$$M_z = \vec{F} \cdot (\hat{k} \times \vec{r}) = \begin{vmatrix} F_x & F_y & F_z \\ 0 & 0 & 1 \\ x & y & z \end{vmatrix} = xF_y - yF_x = R_z$$

5.60 Use second and third equations of Eq. 5.19 to obtain moment in Ex 5.10.

$\vec{F} = 50\hat{\imath} + 100\hat{\jmath} - 30\hat{k}$ $\quad \hat{n} = 0.8\hat{\imath} + 0.6\hat{k}$

$\vec{r} = 0\hat{\imath} + .75\hat{\jmath} + 0\hat{k}$

$$\hat{n} \cdot (\vec{r} \times \vec{F}) = \begin{vmatrix} 0.8 & 0 & 0.6 \\ 0 & 0.75 & 0 \\ 50 & 100 & -30 \end{vmatrix} = -40.5 \text{ kN·m}$$

$$\vec{r} \cdot (\vec{F} \times \hat{n}) = \begin{vmatrix} 0 & 0.75 & 0 \\ 50 & 100 & -30 \\ 0.8 & 0 & 0.6 \end{vmatrix} = -40.5 \text{ kN·m}$$

5.61 Find M_F about the line through $(-4,-7,-2)$ in the direction of $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\vec{F} = 10\hat{\imath} + 20\hat{\jmath} + 5\hat{k}$ [lb] at $(3,0,-4)$ [ft]

$\vec{V} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ at $(-4,-7,-2)$

$\vec{r} = (3-(-4))\hat{\imath} + (0-(-7))\hat{\jmath} + (-4-(-2))\hat{k}$

$= 7\hat{\imath} + 7\hat{\jmath} - 2\hat{k}$ [ft]

$$\hat{n} = \frac{\vec{V}}{V} = \frac{3\hat{\imath} + \hat{\jmath} - 2\hat{k}}{\sqrt{3^2 + 1^2 + (-2)^2}} = \tfrac{3}{\sqrt{14}}\hat{\imath} + \tfrac{1}{\sqrt{14}}\hat{\jmath} - \tfrac{2}{\sqrt{14}}\hat{k}$$

$$M = \vec{F} \cdot (\hat{n} \times \vec{r}) = \begin{vmatrix} 10 & 20 & 5 \\ 3/\sqrt{14} & 1/\sqrt{14} & -2/\sqrt{14} \\ 7 & 7 & -2 \end{vmatrix} = 8.018 \text{ lb-ft}$$

5.62 a.) Find M of $\vec{F}$ about line AB

b.) Find M of $\vec{F}$ about line AC

Refer to Fig P5.47

$\vec{F} = 2\hat{n}_F$ kN

$$\hat{n}_F = \frac{-0.2\hat{\imath} + 0.6\hat{\jmath} - 0.3\hat{k}}{\sqrt{(-0.2)^2 + 0.6^2 + (-0.3)^2}} = -\tfrac{2}{7}\hat{\imath} + \tfrac{6}{7}\hat{\jmath} - \tfrac{3}{7}\hat{k}$$

a.) $M = \vec{F} \cdot (\hat{n}_{AB} \times \vec{r})$

$$\hat{n}_{AB} = \frac{-0.2\hat{\imath} - 0.6\hat{\jmath}}{\sqrt{(-0.2)^2 + (-0.6)^2}} = -0.316\hat{\imath} - 0.949\hat{\jmath}$$

$\vec{r} = -0.6\hat{\jmath}$ [m]; ($\vec{r}$ is from A to $(0.2, 0, 0.3)$)

5.62 cont

$$M = \vec{F}\cdot(\hat{n}_{AB}\times\vec{r}) = \begin{vmatrix} -4/7 & 12/7 & -6/7 \\ -.316 & -.949 & 0 \\ 0 & -0.6 & 0 \end{vmatrix} = \underline{\underline{-0.163 \text{ KN·m}}}$$

b.) $M = \vec{F}\cdot(\hat{n}_{AC}\cdot\vec{r})$ (Can use the same $\vec{r}$ because A lies on AB and AC.)

$$\hat{n}_{AC} = \frac{-0.2\hat{\imath} - 0.3\hat{k}}{\sqrt{(-0.2)^2+(-0.3)^2}} = -0.555\hat{\imath} - 0.832\hat{k}$$

$$M = \vec{F}\cdot(\hat{n}_{AC}\times\vec{r}) = \begin{vmatrix} -4/7 & 12/7 & -6/7 \\ -0.555 & 0 & -0.832 \\ 0 & -0.6 & 0 \end{vmatrix} = \underline{\underline{0}}$$

This could have been expected because the line of action of $\vec{F}$ passes through point C.

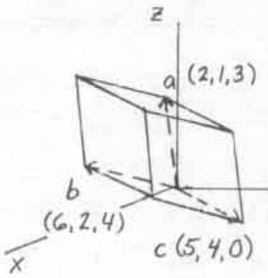
5.63 Find the volume of the parallelepiped using the scalar triple product.

Volume $= \vec{a}\cdot(\vec{b}\times\vec{c})$

$\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ [ft]

$\vec{b} = 6\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ [ft]

$\vec{c} = 5\hat{\imath} + 4\hat{\jmath}$ [ft]



$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 2 & 1 & 3 \\ 6 & 2 & 4 \\ 5 & 4 & 0 \end{vmatrix}$$

$\underline{\underline{\text{Volume} = 30 \text{ ft}^3}}$

5.64 a.) Prove that vectors (1, -3, 2), (-2, 4, 3), and (0, -4, 14) are parallel to the same plane.
b.) Find $\hat{n}$ (unit normal vector)

a.) Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ represent the vectors respectively. If $\vec{a}\cdot(\vec{b}\times\vec{c}) = 0$, then they are parallel.

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 1 & -3 & 2 \\ -2 & 4 & 3 \\ 0 & -4 & 14 \end{vmatrix} = 0$$

∴ The vectors are parallel to the same plane.

b.) $\hat{n}$ is perpendicular to all three vectors. So, $\hat{n}$ is the unit vector of the vector product of any pair of the vectors.

$$\vec{N} = \vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -3 & 2 \\ -2 & 4 & 3 \end{vmatrix} = -17\hat{\imath} - 7\hat{\jmath} - 2\hat{k}$$

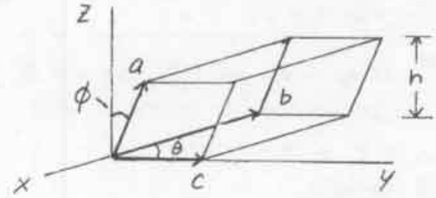
$$\hat{n} = \frac{-17\hat{\imath} - 7\hat{\jmath} - 2\hat{k}}{\sqrt{(-17)^2+(-7)^2+(-2)^2}} = \underline{\underline{-0.919\hat{\imath} - 0.379\hat{\jmath} - 0.108\hat{k}}}$$

or: $\underline{\underline{0.919\hat{\imath} + 0.379\hat{\jmath} + 0.108\hat{k}}}$

(Same unit vector with opposite sense.)

5.65 a.) Show that $|\vec{a}\cdot(\vec{b}\times\vec{c})|$ = volume of parallelepiped.
b.) Show that $\vec{a}\cdot(\vec{b}\times\vec{c}) = 0$ if, and only if $\vec{a}$, $\vec{b}$, $\vec{c}$ are all parallel to the same plane.

a.)



$|\vec{b}\times\vec{c}| = bc\sin\theta$ = area of the base

$h = a\cos\phi$

Volume = base × height $= (bc\sin\theta)(a\cos\phi)$

$\vec{a}\cdot(\vec{b}\times\vec{c}) = abc\sin\theta\cos\phi$ = Volume

b.) If $\vec{a}\cdot(\vec{b}\times\vec{c}) = 0 = abc\sin\theta\cos\phi$, then either $\sin\theta = 0$ or $\cos\phi = 0$.

If $\sin\theta = 0$, then $\theta = 0°$ or $180°$, and $\vec{b}$ and $\vec{c}$ are parallel.

If $\cos\phi = 0$, then $\phi = 90°$, and $\vec{a}$ is perpendicular to the same plane as $\vec{b}$ and $\vec{c}$.

5.66 Find moment of couple

$\vec{F}_1$ at origin $= -\vec{F}_2$

$\vec{F}_2 = 3\hat{\imath} + 4\hat{\jmath} - \hat{k}$ [KN] at (2, 4, 6) [m]

$\vec{M} = \vec{r}_{1,2}\times\vec{F}_2$ ($\vec{r}_{1,2}$ is from (0,0,0) to (2,4,6) that is from $\vec{F}_1$ to $\vec{F}_2$)

$$\vec{M} = \vec{r}\times\vec{F}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 4 & 6 \\ 3 & 4 & -1 \end{vmatrix}$$

$$= \underline{\underline{-28\hat{\imath} + 20\hat{\jmath} - 4\hat{k} \quad \text{kN·m}}}$$

5.67 Find (x, y, z) projections of the resultant couple $\vec{M}$, given $\vec{F}$ at (0,0,0) and $-\vec{F}$ at (9, 8.4, -3) [m]

$\vec{F} = 800\,(0.36\,\hat{\imath} - 0.48\,\hat{\jmath} + 0.8\,\hat{k})$ N at (0,0,0)

$-\vec{F}$ at (9, 8.4, -3) [m]

$\vec{M} = \vec{r} \times \vec{F}$ ($\vec{r}$ is from (9, 8.4, -3) to (0,0,0))

$\therefore \vec{r} = -9\hat{\imath} - 8.4\hat{\jmath} + 3\hat{k}$ [m]

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -9 & -8.4 & 3 \\ 800(0.36) & 800(-0.48) & 800(0.8) \end{vmatrix}$$

$$= -4224\,\hat{\imath} + 6624\,\hat{\jmath} + 5875\,\hat{k} \text{ N·m}$$

$M_x = -4224$ N·m, $M_y = 6624$ N·m, $M_z = 5875$ N·m

5.68 Find the torque turning the steering wheel. See Fig. P5.68

$\vec{M} = \vec{r} \times \vec{F}$ ($\vec{r}$ is from point B to point A, perpendicular to the x-axis)

$\vec{F} = 50\cos 18°\,\hat{\imath} + 50\sin 18°\,\hat{k}$

$\vec{r} = 2(150\sin 60°)\hat{\jmath}$

$$\vec{M} = (\vec{r} \times \vec{F}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 0 \\ \cos 18° & 0 & \sin 18° \end{vmatrix} (300\sin 60°)(50)$$

$$= 4.01\,\hat{\imath} - 12.35\,\hat{k} \text{ kN·m}$$

Torque on the z-axis = -12.35 kN·m

5.69 a.) Find the resultant couple
b.) Find magnitude and direction angles of the resultant couple

a.) $\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3$

$\vec{M} = (3+4-2)\hat{\imath} + (-1+5+1)\hat{\jmath} + (2+0+4)\hat{k}$

$\vec{M} = 5\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$ N·m

b.) $M = \sqrt{5^2+5^2+6^2} = 9.27$ N·m

$\theta_x = \cos^{-1}(5/9.27) = 57.4°$

$\theta_y = \cos^{-1}(5/9.27) = 57.4°$

$\theta_z = \cos^{-1}(6/9.27) = 49.7°$

5.70 a.) Find a so that $\vec{A} \perp \vec{B}$.
b.) Find resultant couple

a.) $\vec{A} = 3\hat{\imath} + 4\hat{\jmath} - \hat{k}$ $\quad \vec{B} = 8\hat{\imath} - 6\hat{\jmath} + a\hat{k}$

If $\vec{A} \perp \vec{B}$, then $\vec{A} \cdot \vec{B} = 0$

$\vec{A} \cdot \vec{B} = (3)(8) + (4)(-6) + (-1)(a) = 0 \quad a = 0$

b.) $\vec{M} = \vec{A} + \vec{B} = (3+8)\hat{\imath} + (4-6)\hat{\jmath} + (-1+0)\hat{k}$
$= 11\hat{\imath} - 2\hat{\jmath} - \hat{k}$

5.71 Find resultant couple

$\vec{M}_1 = 6\hat{\imath} + 5\hat{\jmath} - 8\hat{k}$ [kN·m]

$\vec{M}_2 = \hat{\imath} - 3\hat{\jmath} + \hat{k}$ [kN·m]

$\vec{M}_3 = -4\hat{\imath} + 2\hat{\jmath} - 6\hat{k}$ [kN·m]

$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3$
$= (6+1-4)\hat{\imath} + (5-3+2)\hat{\jmath} + (-8+1-6)\hat{k}$
$= 3\hat{\imath} + 4\hat{\jmath} - 13\hat{k}$ [kN·m]

5.72 Find resultant couple

$\vec{M} = \sum \vec{r} \times \vec{F}$

$\vec{F}_1 = 20\hat{\imath}$, $\quad \vec{r}_1 = 2\hat{\imath} - 2\hat{\jmath}$

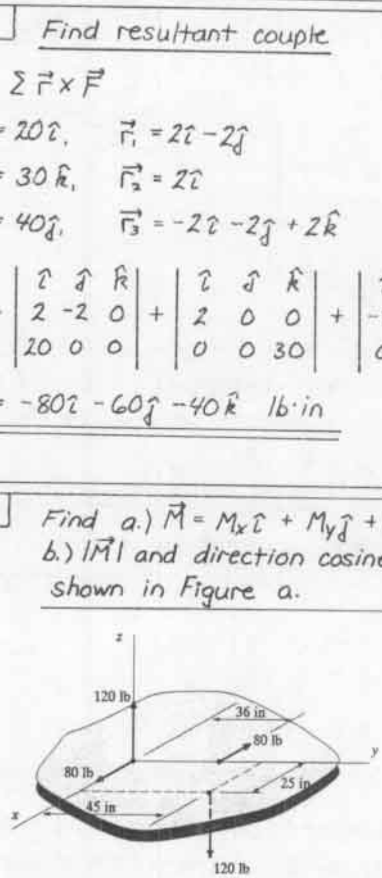
$\vec{F}_2 = 30\hat{k}$, $\quad \vec{r}_2 = 2\hat{\imath}$

$\vec{F}_3 = 40\hat{\jmath}$, $\quad \vec{r}_3 = -2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$$\vec{M} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -2 & 0 \\ 20 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 0 \\ 0 & 0 & 30 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -2 & 2 \\ 0 & 40 & 0 \end{vmatrix}$$

$\vec{M} = -80\hat{\imath} - 60\hat{\jmath} - 40\hat{k}$ lb·in

5.73 Find a.) $\vec{M} = M_x\hat{\imath} + M_y\hat{\jmath} + M_z\hat{k}$
b.) $|\vec{M}|$ and direction cosines for forces shown in Figure a.



a) $\vec{M} = \vec{M}_{120} + \vec{M}_{80}$

5.73 cont.

$\vec{M}_{120} = -120(45)\hat{\imath} + 120(25)\hat{\jmath} = -5400\hat{\imath} + 3000\hat{\jmath}$

$\vec{M}_{80} = 80(36)\hat{k} = 2880\hat{k}$

$\underline{\vec{M} = -5400\hat{\imath} + 3000\hat{\jmath} + 2880\hat{k}}$

b.) $M = \sqrt{(-5400)^2 + 3000^2 + 2880^2} = \underline{\underline{6816 \text{ lb-in}}}$

$\cos\alpha = -5400/6816 = -0.792$

$\cos\beta = 3000/6816 = 0.440$

$\cos\gamma = 2880/6816 = \underline{\underline{0.423}}$

5.74 Find $\vec{M} = M_x\hat{\imath} + M_y\hat{\jmath} + M_z\hat{k}$, due to

$\vec{F} = 7\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ at $(3,1,4)$; $-\vec{F}$ at $(6,1,7)$

$\vec{G} = -8\hat{\imath} + 3\hat{\jmath} + 10\hat{k}$ at $(6,8,2)$; $-\vec{G}$ at $(8,-4,-10)$

$\vec{H} = -10\hat{\imath} + 4\hat{\jmath} - 6\hat{k}$ at $(1,-4,-3)$; $-\vec{H}$ at $(7,0,1)$ [kN]

The position vectors are

From $-\vec{F}$ to $\vec{F}$ $\quad \vec{r}_F = -3\hat{\imath} + 0\hat{\jmath} + 11\hat{k}$ [m]

From $-\vec{G}$ to $\vec{G}$ $\quad \vec{r}_G = -2\hat{\imath} + 12\hat{\jmath} + 12\hat{k}$

From $-\vec{H}$ to $\vec{H}$ $\quad \vec{r}_H = -6\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$

$\therefore \vec{M} = \vec{M}_F + \vec{M}_G + \vec{M}_H$

$$\vec{M}_F = \vec{r}_F \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3 & 0 & 11 \\ 7 & -2 & 4 \end{vmatrix} = 22\hat{\imath} + 89\hat{\jmath} + 6\hat{k}$$

$$\vec{M}_G = \vec{r}_G \times \vec{G} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 12 & 12 \\ -8 & 3 & 10 \end{vmatrix} = 84\hat{\imath} - 76\hat{\jmath} + 90\hat{k}$$

$$\vec{M}_H = \vec{r}_H \times \vec{H} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -6 & -4 & -4 \\ -10 & 4 & -6 \end{vmatrix} = 40\hat{\imath} + 4\hat{\jmath} - 64\hat{k}$$

$$\vec{M} = (22+84+40)\hat{\imath} + (89-76+4)\hat{\jmath} + (6+90-64)\hat{k}$$
$$= \underline{\underline{146\hat{\imath} + 17\hat{\jmath} + 32\hat{k}}} \quad [\text{kN}\cdot\text{m}]$$

5.75 Find $\vec{M}$ expressed in magnitude and direction angles.

$\vec{M} = \vec{M}_1 + \vec{M}_2$

$\vec{M}_1 = 10\hat{\jmath} \qquad \vec{M}_2 = 8\hat{k}$

$\vec{M} = 10\hat{\jmath} + 8\hat{k}$

$M = \sqrt{10^2 + 8^2} = \underline{\underline{12.806 \text{ kN}\cdot\text{m}}}$

$\alpha = \cos^{-1}(0/12.806) = 90°$

$\beta = \cos^{-1}(10/12.806) = 38.66°$

$\gamma = \cos^{-1}(8/12.806) = \underline{\underline{51.34°}}$

5.76 Find the required compensating couple $\vec{M}_o$, when $\vec{F}$ is moved from $(1,1,-2)$ to $(0,0,0)$.

$\vec{M}_o = \vec{r} \times \vec{F}$, where $\vec{r}$ is from $(0,0,0)$ to $(1,1,-2)$.

$\vec{F} = 6\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$

$\vec{r} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$

$$\vec{M}_o = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -2 \\ 6 & 5 & -3 \end{vmatrix} = \underline{\underline{7\hat{\imath} - 9\hat{\jmath} - \hat{k} \text{ kN}\cdot\text{m}}}$$

5.77 Find $\vec{F}$ and $\vec{M}_o$ relative to origin O (Figure a.)

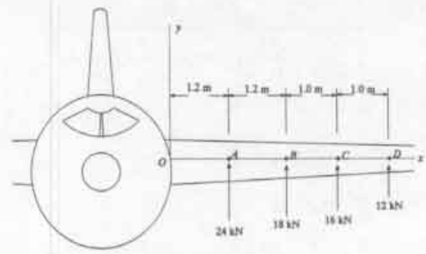


Figure a.

$\vec{F} = \sum F_y = 24 + 18 + 16 + 12 = \underline{\underline{70 \text{ kN}}}$

$\vec{M}_o = 24(1.2) + 18(2.4) + 16(3.4) + 12(4.4)$
$= \underline{\underline{179.2 \text{ kN}\cdot\text{m}}}$

5.78 Replace $\vec{F}_1$, $\vec{F}_2$, and $\vec{M}$ with a force through the origin and M_o

$\vec{F}_1 = 3\hat{\imath} + 4\hat{\jmath}$ at $(1,2,3)$

$\vec{F}_2 = \hat{\imath} + \hat{\jmath} + \hat{k}$ at $(-1,1,4)$

$\vec{M} = 40\hat{\imath} + 20\hat{k}$

$\vec{r}_1 = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \qquad \vec{r}_2 = -\hat{\imath} + \hat{\jmath} + 4\hat{k}$

$\vec{F} = \vec{F}_1 + \vec{F}_2 = \underline{\underline{4\hat{\imath} + 5\hat{\jmath} + \hat{k} \text{ kN}}}$

5.78 cont.

$\vec{M}_o = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{M}$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix} + 40\hat{\imath} + 20\hat{k}$$

$\vec{M}_o = 25\hat{\imath} + 14\hat{\jmath} + 16\hat{k}$ kN·m

5.79 Replace the given force system with $\vec{F}$ at $(5, 9, -35)$ and $\vec{M}$.

$\vec{F}_1 = 600\hat{\imath} - 350\hat{\jmath} + 90\hat{k}$ at $(3, 8, 21)$

$\vec{F}_2 = -1000\hat{\imath} + 260\hat{\jmath} - 720\hat{k}$ at $(7, -9, -40)$

$\vec{F}_3 = 300\hat{\imath} - 950\hat{\jmath} + 850\hat{k}$ at $(60, -3, 25)$

$\vec{M}_1 = 4000\hat{\imath} - 2800\hat{\jmath} + 10{,}000\hat{k}$ at $(1, 14, 28)$

$\vec{M}_2 = -1600\hat{\imath} - 24{,}000\hat{\jmath} - 8000\hat{k}$ at $(-4, -6, -80)$

$\therefore \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$= (600 - 1000 + 300)\hat{\imath} + (-350 + 260 - 950)\hat{\jmath} + (90 - 720 + 850)\hat{k}$

$\vec{F} = -100\hat{\imath} - 1040\hat{\jmath} + 220\hat{k}$ lb

Position vectors from $(5, 9, -35)$ to the given point of action.

$\vec{r}_1 = -2\hat{\imath} - \hat{\jmath} + 56\hat{k}$

$\vec{r}_2 = 2\hat{\imath} - 18\hat{\jmath} - 5\hat{k}$

$\vec{r}_3 = 55\hat{\imath} - 12\hat{\jmath} + 60\hat{k}$

$\vec{M} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 + \vec{M}_1 + \vec{M}_2$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -1 & 56 \\ 600 & -350 & 90 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -18 & -5 \\ -1000 & 260 & -720 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 55 & -12 & 60 \\ 300 & -950 & 850 \end{vmatrix}$$

$+ (4000\hat{\imath} - 2800\hat{\jmath} + 10{,}000\hat{k})$

$+ (-1600\hat{\imath} - 24{,}000\hat{\jmath} - 8000\hat{k})$

$\vec{M} = 82{,}970\hat{\imath} - 15{,}330\hat{\jmath} - 62{,}830\hat{k}$ lb·in

5.80 Replace the given forces with $\vec{F}$ at $(5, 9, -35)$ and $\vec{M}$.

$\vec{F}_1 = 600\hat{\imath} - 350\hat{\jmath} + 90\hat{k}$ at $(3, 8, 21)$

$\vec{F}_2 = -1000\hat{\imath} + 260\hat{\jmath} - 720\hat{k}$ at $(7, -9, -40)$

$\vec{F}_3 = 300\hat{\imath} - 950\hat{\jmath} + 850\hat{k}$ at $(60, -3, 25)$

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = -100\hat{\imath} - 1040\hat{\jmath} + 220\hat{k}$ lb

Position vectors from $(5, 9, -35)$ to the given point of action.

$\vec{r}_1 = -2\hat{\imath} - \hat{\jmath} + 56\hat{k}$

$\vec{r}_2 = 2\hat{\imath} - 18\hat{\jmath} - 5\hat{k}$

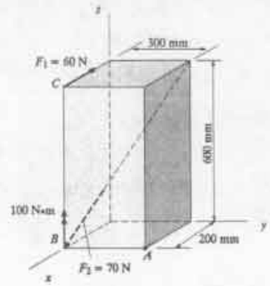
$\vec{r}_3 = 55\hat{\imath} - 12\hat{\jmath} + 60\hat{k}$

$\vec{M} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -1 & 56 \\ 600 & -350 & 90 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -18 & -5 \\ -1000 & 260 & -720 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 55 & -12 & 60 \\ 300 & -950 & 850 \end{vmatrix}$$

$\vec{M} = 80{,}570\hat{\imath} + 11{,}470\hat{\jmath} - 64{,}830\hat{k}$ lb-ft

5.81 Replace the given force system with $\vec{F}$ at point A and $\vec{M}$ (See Fig. a)



$\vec{F}_1 = -60\hat{\imath}$ N

$\vec{F}_2 = 70\left(\dfrac{0.2\hat{\imath} - 0.3\hat{\jmath} - 0.6\hat{k}}{\sqrt{0.2^2 + (-0.3)^2 + (-0.6)^2}}\right) = 20\hat{\imath} - 30\hat{\jmath} - 60\hat{k}$

$\vec{F} = \vec{F}_1 + \vec{F}_2 = (-60\hat{\imath}) + (20\hat{\imath} - 30\hat{\jmath} - 60\hat{k})$

$= -40\hat{\imath} - 30\hat{\jmath} - 60\hat{k}$ N

Position vectors

$\vec{r}_1 = -0.3\hat{\jmath} + 0.6\hat{k}$ (from A to C)

$\vec{r}_2 = -0.3\hat{\jmath}$ (from A to B)

$\vec{M} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{M}_B$

$\vec{M}_B = 100\hat{k}$ N·m

$$\vec{M} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -0.3 & 0.6 \\ -60 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -0.3 & 0 \\ 20 & -30 & -60 \end{vmatrix} + 100\hat{k}$$

$\vec{M} = 18\hat{\imath} - 36\hat{\jmath} + 88\hat{k}$ N·m

5.82 Replace the given force system with $\vec{F}$ at O and $\vec{M}$ (See Fig a.)

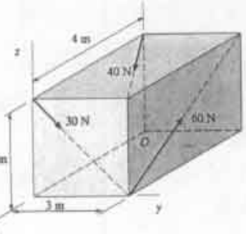


Figure a.

$$\vec{F_{30}} = 30\left(\frac{3\hat{j}-3\hat{k}}{\sqrt{3^2+(-3)^2}}\right) = 21.21\hat{j} - 21.21\hat{k}$$

$$\vec{F_{40}} - 40\left(\frac{4\hat{i}+3\hat{j}}{\sqrt{4^2+3^2}}\right) = 32\hat{i} + 24\hat{j}$$

$$\vec{F_{60}} = 60\left(\frac{-4\hat{i}+3\hat{k}}{\sqrt{(-4)^2+3^2}}\right) = -48\hat{i} + 36\hat{k}$$

$$\vec{F} = \vec{F_{30}} + \vec{F_{40}} + \vec{F_{60}} = \underline{\underline{-16\hat{i} + 45.21\hat{j} + 14.79\hat{k}\ N}}$$

Position Vectors:

$\vec{r_{30}} = 4\hat{i} + 3\hat{k}$ From O to (0,0,3)

$\vec{r_{40}} = 3\hat{k}$ From O to (-4,0,3)

$\vec{r_{60}} = 4\hat{i} + 3\hat{j}$ From O to (0,3,0)

$$\vec{M} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2} + \vec{r_3}\times\vec{F_3}$$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 3 \\ 0 & 21.21 & -21.21 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 3 \\ 32 & 24 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ -48 & 0 & 36 \end{vmatrix}$$

$$\underline{\underline{\vec{M} = -27.63\hat{i} + 36.84\hat{j} + 228.84\hat{k}\ \ N\cdot m}}$$

5.83 Find a.) $\vec{M_A}$, b.) $\vec{M_B}$, c.) $\vec{M_{BA}}$

$$\vec{F} = 10\sqrt{45}\left(\frac{6\hat{j}-3\hat{k}}{\sqrt{6^2+(-3)^2}}\right) = 60\hat{j} - 30\hat{k}$$

$$-\vec{F} = -60\hat{j} + 30\hat{k}$$

a.) Position vectors:

$\vec{r_F} = -2\hat{i} - 3\hat{k}$ $\quad \vec{r_{-F}} = -3\hat{k}$

$$\vec{M_A} = \vec{r_F}\times\vec{F} + \vec{r_{-F}}\times -\vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -3 \\ 0 & 60 & -30 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -3 \\ 0 & -60 & 30 \end{vmatrix}$$

$$\underline{\underline{\vec{M_A} = -60\hat{j} - 120\hat{k}\ \ N\cdot m}}$$

ALTERNATIVELY, Since $\Sigma F = 0$, the moment is the same for all points. Thus,

$\vec{M} = \vec{r}\times\vec{F}$, where $\vec{r}$ is position vector from $-\vec{F}$ to $\vec{F}$ or $\vec{r} = -2\hat{i}$.

$$\therefore \vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 0 \\ 0 & 60 & -30 \end{vmatrix} = \underline{\underline{-60\hat{j} - 120\hat{k}\ \ N\cdot m}}$$

b.) $\vec{F}$ acts through B, so it causes no moment.

$$\vec{r_{-F}} = 2\hat{i}$$

$$\vec{M_B} = \vec{r_{-F}}\times -\vec{F} = (2\hat{i})\times(-60\hat{j} + 30\hat{k})$$

$$= \underline{\underline{-60\hat{j} - 120\hat{k}\ \ N\cdot m}}$$

ALTERNATIVELY, (See alternative sol'n to part a.)

c.) $\vec{M_{BA}} = \hat{n}_{BA}\cdot(\vec{r_{-F}}\times -\vec{F})$

$$\hat{n}_{BA} = \left(\frac{2\hat{i}+6\hat{j}}{\sqrt{2^2+6^2}}\right) = 0.3162\hat{i} + 0.9487\hat{j}$$

$$M_{BA} = (0.3162)(0) + (0.9487)(-60) + (0)(-120)$$

$$= \underline{\underline{-56.92\ \ N\cdot m}}$$

ALTERNATIVELY,

$$M_{BA} = \vec{M_A}\cdot\hat{n}_{BA}$$

$$= (-60\hat{j} - 120\hat{k})\cdot(0.3162\hat{i} + 0.9487\hat{j})$$

$$= -60(0.9487) = \underline{\underline{-56.92\ \ N\cdot m}}$$

5.84 a.) Replace the given force system with $\vec{F}$ at (8,0,-2) and $\vec{M_1}$

b.) Find M_2 about line (1,2,2) through the origin.

$\vec{F_1} = 4\hat{i} - 3\hat{j} + 4\hat{k}$ at (0,1,4)

$\vec{F_2} = -\hat{i} + 4\hat{j} - 6\hat{k}$ at (-1,2,0)

$\vec{M} = 8\hat{i} - 5\hat{j} + 18\hat{k}$

a.) $\vec{F} = \vec{F_1} + \vec{F_2} = \underline{3\hat{i} + \hat{j} - 2\hat{k}\ \ \text{kip}}$

Position vectors from (8,0,-2) to the given point of action:

$\vec{r_1} = -8\hat{i} + \hat{j} + 6\hat{k}$

$\vec{r_2} = -9\hat{i} + 2\hat{j} + 2\hat{k}$

$$\vec{M_1} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2} + \vec{M}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & 1 & 6 \\ 4 & -3 & 4 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -9 & 2 & 2 \\ -1 & 4 & -6 \end{vmatrix} + 8\hat{i} - 5\hat{j} + 18\hat{k}$$

$$\underline{\vec{M_1} = 10\hat{i} - 5\hat{j} + 4\hat{k}\ \ \text{kip-ft}}$$

5.84 cont.

b.) $M_z = \hat{n}\cdot(\vec{r}\times\vec{F}) + n\cdot\vec{M}_1$

$\hat{n} = \left(\dfrac{\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{1^2+2^2+2^2}}\right) = \frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

$\vec{r} = 8\hat{i} - 2\hat{k}$ (From origin to (8,0,-2))

$$M_z = \begin{vmatrix} 1/3 & 2/3 & 2/3 \\ 8 & 0 & -2 \\ 3 & 1 & -2 \end{vmatrix} + (1/3)(10) + (2/3)(-5) + (2/3)(4)$$

$\underline{\underline{M_z = 15.33 \text{ kip-ft}}}$

ALTERNATIVELY, With the initial forces $\vec{F_1}$, $\vec{F_2}$ and moment $\vec{M}$, we have

$\vec{r}_1 = \hat{j} + 4\hat{k}$ from (0,0,0) to (0,1,4)

$\vec{r}_2 = -\hat{i} + 2\hat{j}$ from (0,0,0) to (-1,2,0)

Hence,

$$\vec{r}_1\times\vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 4 \\ 4 & -3 & 4 \end{vmatrix} = 16\hat{i} + 16\hat{j} - 4\hat{k}$$

$\hat{n}\cdot\vec{r}_1\times\vec{F}_1 = \frac{1}{3}(\hat{i}+2\hat{j}+2\hat{k})\cdot(16\hat{i}+16\hat{j}-4\hat{k}) = 40/3$

$$\vec{r}_2\times\vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 4 & -6 \end{vmatrix} = -12\hat{i} - 6\hat{j} - 2\hat{k}$$

$\hat{n}\cdot\vec{r}_2\times\vec{F}_2 = \frac{1}{3}(\hat{i}+2\hat{j}+2\hat{k})\cdot(-12\hat{i}-6\hat{j}-2\hat{k}) = -28/3$

$\hat{n}\cdot\vec{M} = \frac{1}{3}(\hat{i}+2\hat{j}+2\hat{k})\cdot(8\hat{i}-5\hat{j}+18\hat{k}) = 34/3$

Therefore

$\underline{\underline{M_z = 40/3 - 28/3 + 34/3 = 46/3 = 15.33 \text{ kip}\cdot\text{ft}}}$

5.85 Replace the given force system with $\vec{P}$ at the origin and $\vec{M}$. (See Fig. a.)

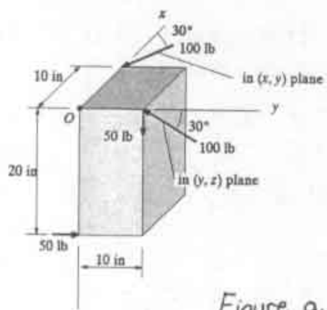


Figure a.

$\vec{A} = 50\hat{j}$ $\quad \vec{B} = 50\hat{k}$

$\vec{D} = -100\cos 30°\,\hat{i} - 100\sin 30°\,\hat{j}$

$\vec{P} = \vec{A}+\vec{B}+\vec{C}+\vec{D} = \underline{\underline{-86.6\hat{i} - 86.6\hat{j} \text{ lb}}}$

$\vec{M} = \vec{M}_A + \vec{M}_B + \vec{M}_C + \vec{M}_D$

$\vec{r}_A = 20\hat{k}$ $\quad \vec{r}_B = \vec{r}_C = 10\hat{j}$ $\quad \vec{r}_D = 10\hat{i}$

$\vec{M}_A = \vec{r}_A\times\vec{A} = (20\hat{k})\times(50\hat{j}) = -1000\hat{i}$

$\vec{M}_B = \vec{r}_B\times\vec{B} = (10\hat{j})\times(50\hat{k}) = 500\hat{i}$

$\vec{M}_C = \vec{r}_C\times\vec{C} = (10\hat{j})\times(-100\cos 30°\hat{j} - 100\sin 30°\hat{k})$
$= -500\hat{i}$

$\vec{M}_D = \vec{r}_D\times\vec{D} = (10\hat{i})\times(-100\cos 30°\hat{i} - 100\sin 30°\hat{k})$
$= -500\hat{k}$

$\vec{M} = -1000\hat{i} + 500\hat{i} - 500\hat{i} - 500\hat{k}$
$\underline{\underline{= -1000\hat{i} - 500\hat{k} \text{ lb-in}}}$

5.86 Replace the given force system with $\vec{P}$ at the origin and $\vec{M}$. (See Fig. a)

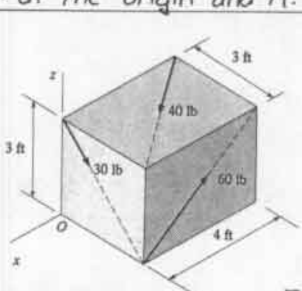


Figure a.

$\vec{F}_{30} = 30\left(\dfrac{3\hat{j} - 3\hat{k}}{\sqrt{3^2+(-3)^2}}\right) = 21.21\hat{j} - 21.21\hat{k}$

$\vec{F}_{40} = 40\left(\dfrac{4\hat{i} + 3\hat{j}}{\sqrt{4^2+3^2}}\right) = 32\hat{i} + 24\hat{j}$

$\vec{F}_{60} = 60\left(\dfrac{-4\hat{i} + 3\hat{k}}{\sqrt{(-4)^2+3^2}}\right) = -48\hat{i} + 36\hat{k}$

$\vec{P} = \vec{F}_{30} + \vec{F}_{40} + \vec{F}_{60} = \underline{\underline{-16\hat{i} + 45.21\hat{j} + 14.79\hat{k} \text{ lb}}}$

$\vec{M} = \vec{M}_{30} + \vec{M}_{40} + \vec{M}_{60}$

$\vec{r}_{30} = 3\hat{j}$ $\quad \vec{r}_{40} = 3\hat{j} + 3\hat{k}$ $\quad \vec{r}_{60} = 3\hat{j}$

$\vec{M}_{30} = \vec{r}_{30}\times\vec{F}_{30} = (3\hat{j})\times(21.21\hat{j} - 21.21\hat{k})$
$= -63.63\hat{i}$

$\vec{M}_{40} = \vec{r}_{40}\times\vec{F}_{40} = (3\hat{j}+3\hat{k})\times(32\hat{i}+24\hat{j})$
$= -72\hat{i} + 96\hat{j} - 96\hat{k}$

$\vec{M}_{50} = \vec{r}_{50}\times\vec{F}_{50} = (3\hat{j})\times(-48\hat{i} + 36\hat{k})$
$= 108\hat{i} + 144\hat{k}$

$\underline{\underline{\vec{M} = -27.63\hat{i} + 96\hat{j} + 48\hat{k}}}$

5.87 a) Replace the given force system with $\vec{F}$ at O and $\vec{M}$.

b.) Find magnitudes and direction cosines of $\vec{F}$ and $\vec{M}$. (See Fig a.)

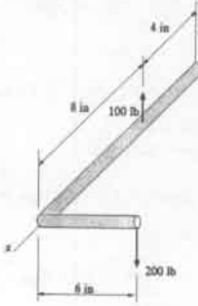


Figure a.

a.) $\vec{F} = \vec{F}_{100} + \vec{F}_{200}$

$\vec{F}_{100} = 100\,\hat{k} \qquad \vec{F}_{200} = -200\,\hat{k}$

$\underline{\vec{F} = -100\,\hat{k} \text{ lb}}$

$\vec{M} = \vec{r}_{100} \times \vec{F}_{100} + \vec{r}_{200} \times \vec{F}_{200}$

$\vec{r}_{100} = 4\hat{\imath} \qquad \vec{r}_{200} = 12\hat{\imath} + 6\hat{\jmath}$

$\vec{M} = (4\hat{\imath}) \times (100\,\hat{k}) + (12\hat{\imath} + 6\hat{\jmath}) \times (-200\,\hat{k})$

$= (-400\hat{\jmath}) + (-1200\,\hat{\imath} + 2400\hat{\jmath})$

$= \underline{-1200\,\hat{\imath} + 2000\hat{\jmath} \quad \text{lb-in}}$

b.) $F = 100\text{ lb} \qquad M = \sqrt{(-1200)^2 + 2000^2} = 2{,}332 \text{ lb}$

$\cos\alpha = 0/100 = 0 \qquad \cos\alpha = -1200/2332 = -0.515$

$\cos\beta = 0/100 = 0 \qquad \cos\beta = 2000/2332 = 0.858$

$\cos\gamma = -100/100 = -1 \quad \cos\gamma = 0/2332 = 0$

$\underline{F = 100 \text{ lb};\ (0,0,-1) \quad M = 2332 \text{ lb};\ (-0.515, 0.858, 0)}$

5.88 Replace the given force system with $\vec{F}$ at (0,0,20) and $\vec{M}$.

$\vec{F}_1 = 500\hat{\imath} - 340\hat{\jmath} - 650\,\hat{k}$ at (9,7,23)

$\vec{F}_2 = -360\hat{\imath} + 550\hat{\jmath} + 720\,\hat{k}$ at (30,8,-22)

$\vec{F}_3 = 860\hat{\imath} - 25\hat{\jmath} - 150\,\hat{k}$ at (-60,45,10)

$\vec{M}_1 = 10{,}500\,\hat{\imath} - 6400\hat{\jmath} + 13{,}600\,\hat{k}$

$\vec{M}_2 = 16{,}000\,\hat{\imath} + 9500\,\hat{\jmath}$

$\underline{\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 1000\hat{\imath} + 185\hat{\jmath} - 80\,\hat{k} \ \ N}$

Position vectors from point (0,0,20) to the given point of action:

$\vec{r}_{F1} = 9\hat{\imath} + 7\hat{\jmath} + 3\,\hat{k}$

$\vec{r}_{F2} = 30\hat{\imath} + 8\hat{\jmath} - 42\,\hat{k}$

$\vec{r}_{F3} = -60\hat{\imath} + 45\hat{\jmath} - 10\,\hat{k}$

$$\vec{M}_{F1} = \vec{r}_{F1} \times \vec{F}_1 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 9 & 7 & 3 \\ 500 & -340 & -650 \end{vmatrix}$$

$$= -3530\hat{\imath} + 7350\hat{\jmath} - 6560\,\hat{k}$$

$$\vec{M}_{F2} = \vec{r}_{F2} \times \vec{F}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 30 & 8 & -42 \\ -360 & 550 & 720 \end{vmatrix}$$

$$= 28{,}860\,\hat{\imath} - 6480\hat{\jmath} + 19{,}380\,\hat{k}$$

$$\vec{M}_{F3} = \vec{r}_{F3} \times \vec{F}_3 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -60 & 45 & -10 \\ 860 & -25 & -150 \end{vmatrix}$$

$$= -7000\hat{\imath} - 17{,}600\hat{\jmath} - 37{,}200\,\hat{k}$$

$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_{F1} + \vec{M}_{F2} + \vec{M}_{F3}$

$= 44{,}830\,\hat{\imath} - 13{,}630\hat{\jmath} - 10{,}780\,\hat{k} \ \ N\cdot cm$

$= \underline{448.3\,\hat{\imath} - 136.3\hat{\jmath} - 107.8\,\hat{k} \ \ N\cdot m}$

5.89 a.) Find resultant $\vec{F}$ and $\vec{M}$ at A.

b.) Find magnitude and direction angles of $\vec{M}$ (See Fig a.)

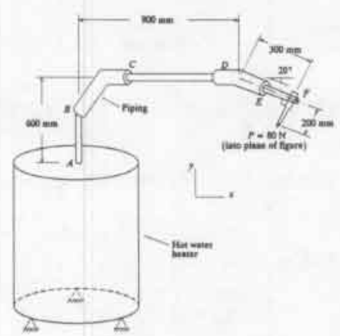


Figure a.

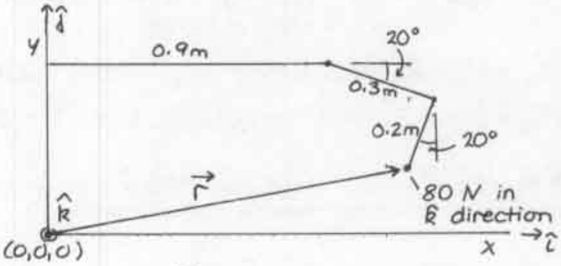


Figure b

a.) By Fig b,

$\underline{\vec{F} = -80\,\hat{k} \ \ N}$

5.89 cont.

$\vec{r} = (0.9 + 0.3\cos 20° - 0.2\sin 20°)\hat{\imath}$
$\quad + (0.6 - 0.3\sin 20° - 0.2\cos 20°)\hat{\jmath}$
$\quad = 1.1135\hat{\imath} + 0.3095\hat{\jmath}$

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1.1135 & 0.3095 & 0 \\ 0 & 0 & -80 \end{vmatrix}$$

$$\vec{M} \;\; 24.76\hat{\imath} + 89.08\hat{\jmath} \;\; \text{N·m}$$

b.) $M = \sqrt{(-24.76)^2 + 89.08^2} = \underline{92.46 \text{ N·m}}$

$\alpha = \cos^{-1}(-24.76/92.46) = 105.5°$

$\beta = \cos^{-1}(89.08/92.46) = 15.54°$

$\gamma = \cos^{-1}(0/92.46) = \underline{90°}$

5.90

Replace the given force system with $\vec{F}$ at O and $\vec{M}$ (See Fig a)

$\vec{F}_{30,1} = 30\hat{\imath}$ at (0,0,0)

$\vec{F}_{30,2} = -30\hat{\imath}$ at (5,4,3)

$\vec{F}_{40} = 40\hat{\jmath}$ at (5,0,3)

$\vec{F}_{20} = 20\hat{k}$ at (5,4,0)

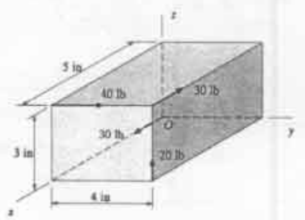


Figure a.

$\vec{F} = \Sigma \vec{F}_i = 30\hat{\imath} - 30\hat{\imath} + 40\hat{\jmath} + 20\hat{k} = \underline{40\hat{\jmath} + 20\hat{k} \text{ lb}}$

Position vectors from (0,0,0) to the given point of action:

$\vec{r}_{30,2} = 5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$

$\vec{r}_{40} = 5\hat{\imath} + 3\hat{k} \qquad \vec{r}_{20} = 5\hat{\imath} + 4\hat{\jmath}$

$\vec{M}_{30} = \vec{r}_{30,2} \times \vec{F}_{30,2} = (5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}) \times (-30\hat{\imath})$
$\qquad = -90\hat{\jmath} + 120\hat{k}$

$\vec{M}_{40} = \vec{r}_{40} \times \vec{F}_{40} = (5\hat{\imath} + 3\hat{k}) \times (40\hat{\jmath}) = -120\hat{\imath} + 200\hat{k}$

$\vec{M}_{20} = \vec{r}_{20} \times \vec{F}_{20} = (5\hat{\imath} + 4\hat{\jmath}) \times (20\hat{k}) = 80\hat{\imath} - 100\hat{\jmath}$

$$\Sigma\vec{M} = \vec{M} = -40\hat{\imath} - 190\hat{\jmath} + 320\hat{k} \;\; \text{lb-in}$$

5.91

Replace the given force system with $\vec{F}$ at the origin and $\vec{M}_o$.

$\vec{F}_1 = 3\hat{\imath} + 4\hat{\jmath} - \hat{k}$ at (-1, 4, 0)

$\vec{F}_2 = -2\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$ at (0, 8, 6)

$\vec{M} = 4\hat{\imath} + 6\hat{k}$

$\vec{F} = \vec{F}_1 + \vec{F}_2 = (3-2)\hat{\imath} + (4+5)\hat{\jmath} + (-1+5)\hat{k}$
$\qquad = \underline{\hat{\imath} + 9\hat{\jmath} + 4\hat{k} \text{ lb}}$

Position vectors from (0,0,0) to the given point of action:

$\vec{r}_1 = -\hat{\imath} + 4\hat{\jmath} \qquad \vec{r}_2 = 8\hat{\jmath} + 6\hat{k}$

$\vec{M}_o = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{M}$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 4 & 0 \\ 3 & 4 & -1 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 8 & 6 \\ -2 & 5 & 5 \end{vmatrix} + 4\hat{\imath} + 6\hat{k}$$

$$\vec{M}_o = 10\hat{\imath} - 13\hat{\jmath} + 6\hat{k} \;\; \text{lb-ft}$$

5.92

Replace the given force system with a force at A and a couple. (See Fig a.).

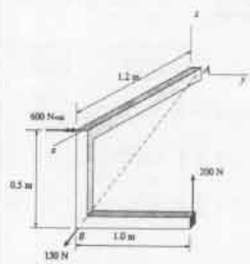


Figure a.

$\vec{F}_{130} = 130\,(1.2/1.3\,\hat{\imath} - 0.5/1.3\,\hat{k})$
$\qquad = 120\hat{\imath} - 50\hat{k}$ at (1.2, 0, -0.5)

$\vec{F}_{200} = 200\hat{k}$ at (1.2, 1, -0.5)

$\vec{M}_{600} = 600\hat{\jmath}$

$\vec{F} = \vec{F}_{130} + \vec{F}_{200} = \underline{120\hat{\imath} + 150\hat{k} \text{ N}}$

The line of action of $\vec{F}_{130}$ passes through A, therefore it does not cause a moment about A.

$\vec{M}_{200} = \vec{r}_{200} \times \vec{F}_{200} = (1.2\hat{\imath} + \hat{\jmath} - 0.5\hat{k}) \times (200\hat{k})$
$\qquad = 200\hat{\imath} - 240\hat{\jmath}$

$\vec{M}_A = \vec{M}_{200} + \vec{M}_{600} = \underline{200\hat{\imath} + 360\hat{\jmath} \text{ N·m}}$

5.93 Replace the given force system with $\vec{F}$ at O and $\vec{M}$ (See Fig a.)

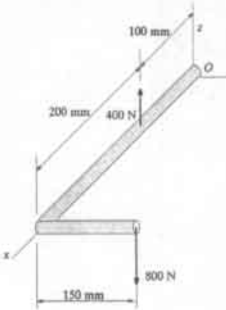


Figure a.

$\vec{F}_{400} = 400\,\hat{k}$ at $(0.1, 0, 0)$ [m]

$\vec{F}_{800} = -800\,\hat{k}$ at $(0.3, 0.15, 0)$ [m]

$\vec{F} = \vec{F}_{400} + \vec{F}_{800} = \underline{\underline{-400\,\hat{k}\ N}}$

$\vec{r}_{400} = 0.1\,\hat{\imath}$

$\vec{r}_{800} = 0.3\,\hat{\imath} + 0.15\,\hat{\jmath}$

$$\vec{M} = \vec{r}_{400} \times \vec{F}_{400} + \vec{r}_{800} \times \vec{F}_{800}$$
$$= (0.1)\hat{\imath} \times (400)\hat{k} + (0.3\,\hat{\imath} + 0.15\,\hat{\jmath}) \times (-800\,\hat{k})$$
$$= \underline{\underline{-120\,\hat{\imath} + 200\,\hat{\jmath}\quad N\cdot m}}$$

5.94 Replace the given force system with a force at A and a couple (See Fig a.)

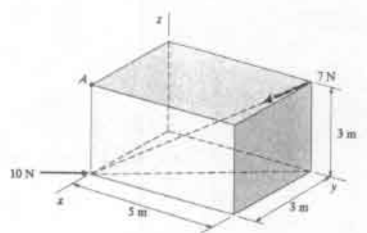


Figure a.

$$\vec{F}_{10} = 10\left(\frac{-3\hat{\imath} + 5\hat{\jmath}}{\sqrt{(-3)^2 + 5^2}}\right) = -5.145\,\hat{\imath} + 8.575\,\hat{\jmath}$$

$$\vec{F}_7 = 7\left(\frac{3\hat{\imath} - 5\hat{\jmath} - 3\hat{k}}{\sqrt{3^2 + (-5)^2 + (-3)^2}}\right) = 3.202\,\hat{\imath} - 5.337\,\hat{\jmath} - 3.202\,\hat{k}$$

$$\vec{F} = \vec{F}_7 + \vec{F}_{10}$$
$$= (3.202 - 5.145)\hat{\imath} + (-5.337 + 8.575)\hat{\jmath} - 3.202\,\hat{k}$$
$$= \underline{\underline{-1.943\,\hat{\imath} + 3.238\,\hat{\jmath} - 3.202\,\hat{k}\ N}}$$

The same position vector $(-3\hat{k})$ can be used for both forces because they both act through $(3,0,0)$.

$$\vec{M} = \vec{r} \times \vec{F}_7 + \vec{r} \times \vec{F}_{10} = \vec{r} \times (\vec{F}_7 + \vec{F}_{10}) = \vec{r} \times \vec{F}$$
$$= (-3\hat{k}) \times (-1.943\,\hat{\imath} + 3.238\,\hat{\jmath} - 3.202\,\hat{k})$$
$$= \underline{\underline{9.72\,\hat{\imath} + 5.83\,\hat{\jmath}\quad N\cdot m}}$$

5.95 a.) Replace the given force system with a force at O and a couple
b.) Find moment about A
c.) Find moment about line from point A to E (see Fig a)

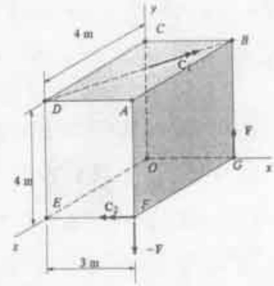


Figure a.

a.)
$\vec{F} = 100\,\hat{\jmath}$ at $(3,0,0)$

$-\vec{F} = -100\,\hat{\jmath}$ at $(3,0,4)$

$\vec{C}_1 = 500\left(\tfrac{3}{5}\hat{\imath} - \tfrac{4}{5}\ \right)$
$= 300\,\hat{\imath} - 400\,\hat{k}$

$\vec{C}_2 = -200\,\hat{\imath}$

$\vec{F}_o = \vec{F} + (-\vec{F}) = \underline{\underline{0}}$

$\vec{M}_o = \vec{C}_1 + \vec{C}_2 + \vec{r}_F \times \vec{F}_1$ where $\vec{r}_F = -4\,\hat{k}$ (from $-\vec{F}$ to F)

$$\vec{M}_o = (300\,\hat{\imath} - 400\,\hat{k}) + (-200\,\hat{\imath}) + (-4\hat{k}) \times (100\,\hat{\jmath})$$
$$= \underline{\underline{500\,\hat{\imath} - 400\,\hat{k}\quad N\cdot m}}$$

b.) In this problem, all of the forces are couples and the equivalent systems about any point are equal.
$\therefore\ \vec{M}_A = 500\,\hat{\imath} - 400\,\hat{k}\ \ N\cdot m$

c.) Project the system along $\overline{AE}$.
$\vec{M}$ along $\overline{AE} = \hat{n}_{AE} \cdot \vec{M}$

$$\hat{n}_{AE} = \frac{\overrightarrow{AE}}{AE} = \left(\frac{-3\hat{\imath} - 4\hat{\jmath}}{\sqrt{(-3)^2 + (-4)^2}}\right) = -\tfrac{3}{5}\hat{\imath} - \tfrac{4}{5}\hat{\jmath}$$

$$\hat{n}_{AE} \cdot \vec{M} = (-\tfrac{3}{5})(500) + (-\tfrac{4}{5})(0) + (0)(-400)$$
$$= \underline{\underline{-300\ N\cdot m}}$$

5.96 a.) Find the dynamically equivalent force and couple at O
b.) Find M_x, M_z (see Fig a.)

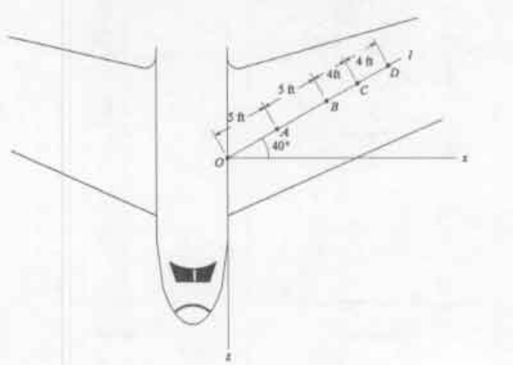


Figure a.

5.96 cont.

$\vec{A} = 6000\hat{j}$ lb at 5 ft along ℓ

$\vec{B} = -4500\hat{j}$ lb at 10 ft along ℓ

$\vec{C} = 4000\hat{j}$ lb at 14 ft along ℓ

$\vec{D} = 3000\hat{j}$ lb at 18 ft along ℓ

a.) $\vec{F} = \vec{A} + \vec{B} + \vec{C} + \vec{D} = \underline{\underline{8500\hat{j} \text{ lb}}}$

$\hat{n}_\ell = \cos 40° \hat{i} - \sin 40° \hat{k} = 0.766\hat{i} - 0.643\hat{k}$

$\vec{r}_A = 5\hat{n}_\ell = 3.830\hat{i} - 3.214\hat{k}$

$\vec{r}_B = 10\hat{n}_\ell = 7.66\hat{i} - 6.428\hat{k}$

$\vec{r}_C = 14\hat{n}_\ell = 10.725\hat{i} - 8.999\hat{k}$

$\vec{r}_D = 18\hat{n}_\ell = 13.789\hat{i} - 11.570\hat{k}$

$\vec{M} = \vec{M}_A + \vec{M}_B + \vec{M}_C + \vec{M}_D$

$\vec{M}_A = \vec{r}_A \times \vec{A} = (3.830\hat{i} - 3.214\hat{k}) \times (6000\hat{j})$
$= 19{,}284\hat{i} + 22{,}980\hat{k}$

$\vec{M}_B = \vec{r}_B \times \vec{B} = (7.660\hat{i} - 6.428\hat{k}) \times (-4500\hat{j})$
$= -28{,}926\hat{i} - 34{,}470\hat{k}$

$\vec{M}_C = \vec{r}_C \times \vec{C} = (10.725\hat{i} - 8.999\hat{k}) \times (4000\hat{j})$
$= 35{,}996\hat{i} + 42{,}900\hat{k}$

$\vec{M}_D = \vec{r}_D \times \vec{D} = (13.789\hat{i} - 11.570\hat{k}) \times (3000\hat{j})$
$= 34{,}710\hat{i} + 41{,}367\hat{k}$

$\underline{\underline{\vec{M} = 61{,}064\hat{i} + 72{,}777\hat{k} \text{ lb-ft}}}$

b.) $\underline{\underline{M_x = 61{,}064 \text{ lb-ft} \qquad M_z = 72{,}777 \text{ lb-ft}}}$

5.97 a.) Replace given forces with a force at O and a couple;

b.) Find M_{CD};

c.) Find M_{DE};

d.) Find F and W (See Fig a.)

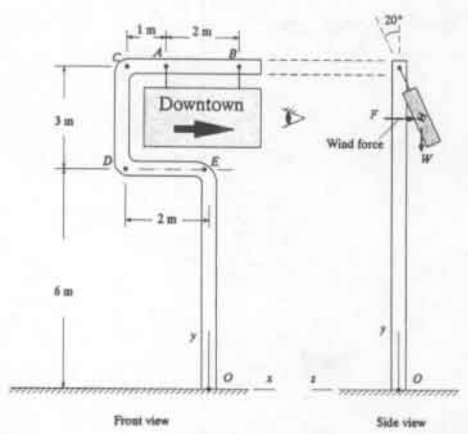


Figure a.

a.) $\vec{F}_A = \vec{F}_B = -2\cos 20°\hat{j} - 2\sin 20°\hat{k}$ [kN]

$\underline{\underline{\vec{F}_O = \vec{F}_A + \vec{F}_B = -3.759\hat{j} - 1.368\hat{k} \text{ [kN]}}}$

$\vec{r}_A = -\hat{i} + 9\hat{j} \qquad \vec{r}_B = \hat{i} + 9\hat{j}$

$\vec{M}_O = \vec{r}_A \times \vec{F}_A + \vec{r}_B \times \vec{F}_B = (\vec{r}_A + \vec{r}_B) \times \vec{F}_A$

$\vec{M}_O = (18\hat{j}) \times (-1.879\hat{j} - 0.6840\hat{k})$
$= \underline{\underline{-12.312\hat{i} \text{ [kN·m]}}}$

b.) $\vec{r} = 2\hat{i} - 6\hat{j}$ (Vector from point D to point O.)

$\hat{n} = \hat{j}$ (Unit vector along line CD.)

$M_{CD} = \hat{n} \cdot \vec{M}_O + \hat{n} \cdot (\vec{r} \times \vec{F}_O)$
$= \hat{j} \cdot (-12.312\hat{i}) + \hat{j} \cdot [(2\hat{i} - 6\hat{j}) \times (-3.759\hat{j} - 1.368k)]$

$\underline{\underline{M_{CD} = 2.74 \text{ KN·m}}}$

c.) $\vec{r} = -6\hat{j}$ (Vector from point E to point O)

$\hat{n} = \hat{i}$ (Unit vector along line DE)

$M_{DE} = \hat{n} \cdot M_O + \hat{n} \cdot (\vec{r} \times \vec{F}_O)$
$= \hat{i} \cdot (-12.312\hat{i}) + \hat{i} \cdot [(-6\hat{j}) \times (-3.759\hat{j} - 1.368\hat{k})]$

$\underline{\underline{M_{DE} = -4.10 \text{ KNm}}}$

d.)

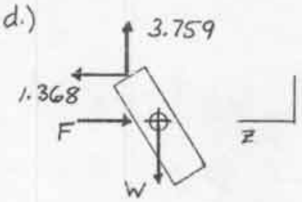


Figure b

By Fig b.)

$\Sigma F_z = 0;$

$1.368 - F = 0$

$\underline{\underline{F = 1.368 \text{ KN}}}$

$\Sigma F_y = 0;$

$3.759 - W = 0$

$\underline{\underline{W = 3.759 \text{ kN}}}$

5.98 Find magnitude of resultant $\vec{R}$ and direction cosines of resultant axis of 4 forces that act at origin of (x, y, z)

$\vec{F}_1 = 4\left(\frac{3\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{3^2 + 2^2 + 1^2}}\right) = \frac{12}{\sqrt{14}}\hat{i} + \frac{8}{\sqrt{14}}\hat{j} + \frac{4}{\sqrt{14}}\hat{k}$ [kN]

$\vec{F}_2 = 6\left(\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{1^2 + 1^2 + 1^2}}\right) = \frac{6}{\sqrt{3}}\hat{i} + \frac{6}{\sqrt{3}}\hat{j} + \frac{6}{\sqrt{3}}\hat{k}$ [kN]

$\vec{F}_3 = 3\left(\frac{3\hat{i} - 2\hat{j} - \hat{k}}{\sqrt{3^2 + (-2)^2 + (-1)^2}}\right) = \frac{9}{\sqrt{14}}\hat{i} - \frac{6}{\sqrt{14}}\hat{j} - \frac{3}{\sqrt{14}}\hat{k}$ [kN]

$\vec{F}_4 = 10\left(\frac{\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2}}\right) = \frac{10}{\sqrt{5}}\hat{j} + \frac{20}{\sqrt{5}}\hat{k}$ [kN]

$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 9.077\hat{i} + 8.471\hat{j} + 12.676\hat{k}$ [kN]

$R = \sqrt{9.077^2 + 8.471^2 + 12.676^2} = \underline{17.74 \text{ kN}}$

5.98 cont.

$\cos\alpha = 9.077/17.74 = \underline{\underline{0.512}}$

$\cos\beta = 8.471/17.74 = \underline{\underline{0.477}}$

$\cos\gamma = 12.676/17.74 = \underline{\underline{0.714}}$

$\vec{R} = 17.74$ kN : (0.512, 0.477, 0.714 m)

5.99 Find $\vec{R}$, $\vec{M}$, and point of intersection of resultant axis with (x, z) plane of following 4 forces.

$\vec{F} = -50\hat{j}$ at (1, 4, 2)
$\vec{G} = 40\hat{j}$ at (3, 1, −5)
$\vec{H} = 25\hat{j}$ at (−10, 0, 6)
$\vec{R} = \vec{F} + \vec{G} + \vec{H} = \underline{\underline{15\hat{j}\ \text{lb}}}$

Position vectors from point (a, 0, c) to the point of action.

$\vec{r}_F = (1-a)\hat{i} + 4\hat{j} + (2-c)\hat{k}$
$\vec{r}_G = (3-a)\hat{i} + \hat{j} + (-5-c)\hat{k}$
$\vec{r}_H = (-10-a)\hat{i} + (6-c)\hat{k}$

$\vec{M} = \vec{r}_F \times \vec{F} + \vec{r}_G \times \vec{G} + \vec{r}_H \times \vec{H}$
$= [(1-a)\hat{i} + 4\hat{j} + (2-c)\hat{k}] \times (-50\hat{j})$
$[(3-a)\hat{i} + \hat{j} + (-5-c)\hat{k}] \times (40\hat{j})$
$+ [(-10-a)\hat{i} + (6-c)\hat{k}] \times (25\hat{j})$

$\vec{M} = (150 + 15c)\hat{i} + (-180 - 15a)\hat{k}$
$\vec{R} \times \vec{M} = 0 = (15\hat{j}) \times [(150 + 15c)\hat{i} + (-180 - 15a)\hat{k}]$
$\hat{i}:\ 15(-180 - 15a) = 0 \longrightarrow a = -12$
$\hat{k}:\ -15(150 + 15c) = 0 \longrightarrow c = -10$

Intersection of (x, z) plane: $\underline{\underline{(-12, 0, -10)}}$
$\underline{\underline{\vec{M} = 0}}$

5.100 a.) Replace existing force system with $\vec{R}$ and $\vec{M}$ parallel to $\vec{R}$;
b.) Find resultant axis by its intersection (a, b, 0) with (x, y) plane.

$\vec{F} = \hat{i} - \hat{j}$ at (1, 1, 1) [kN]
$\vec{G} = 2\hat{j} + \hat{k}$ at (−1, 0, 4) [kN]
$\vec{H} = -3\hat{i} + \hat{j} + \hat{k}$ at (3, 4, 0) [kN]
$\vec{C} = 10\hat{i} - 5\hat{j} + 4\hat{k}$ [kN·m]

a.) $\vec{R} = \vec{F} + \vec{G} + \vec{H} = \underline{\underline{-2\hat{i} + 2\hat{j} + 2\hat{k}\ \text{kN}}}$

$\vec{R}$ acts at point (a, b, 0)
Position vectors from (a, b, 0) to point of action:

$\vec{r}_F = (1-a)\hat{i} + (1-b)\hat{j} + \hat{k}$
$\vec{r}_G = (-1-a)\hat{i} + (-b)\hat{j} + 4\hat{k}$
$\vec{r}_H = (3-a)\hat{i} + (4-b)\hat{j}$

$\vec{M} = \vec{r}_F \times \vec{F} + \vec{r}_G \times \vec{G} + \vec{r}_H \times \vec{H} + \vec{C}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1-a & 1-b & 1 \\ 1 & -1 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (-1-a) & -b & 4 \\ 0 & 2 & 1 \end{vmatrix}$$

$$+ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-a & 4-b & 0 \\ -3 & 1 & 1 \end{vmatrix} + \vec{C}$$

$\underline{\underline{\vec{M} = (7-2b)\hat{i} + (-6+2a)\hat{j} + (15-2a-2b)\hat{k}\ \ \text{kN·m}}}$

b) If $\vec{M}$ is parallel to $\vec{R}$, $\vec{R} \times \vec{M} = 0$.

$$\vec{R} \times \vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 2 & 2 \\ 7-2b & -6+2a & 15-2a-2b \end{vmatrix} = 0$$

$\hat{i}:\ -8a - 4b + 42 = 0$
$\hat{j}:\ -4a - 8b + 44 = 0$
$\hat{k}:\ -4a + 4b - 2 = 0$

$a = 3.333 \qquad b = 3.833$

$\underline{\underline{\vec{M} = -0.666\hat{i} + 0.666\hat{j} + 0.666\hat{k}\ \ \text{kN·m}}}$

Resultant axis has the same direction cosines as $\vec{R}$ and intersects the (x, y) plane at (3.333, 3.833, 0).

$R = \sqrt{(-2)^2 + 2^2 + 2^2} = \sqrt{12}$
$\alpha = \cos^{-1}\left(-2/\sqrt{12}\right) = 125.8°$
$\beta = \gamma = \cos^{-1}\left(2/\sqrt{12}\right) = 54.7°$

The resultant axis passes through (3.333, 3.833, 0) with $\alpha = 125.8°$, $\underline{\underline{\beta = \gamma = 54.7°}}$

5.101 a.) Replace the given force system with $\vec{R}$ at $(a, b, 0)$ and $\vec{M}$
b.) Write condition of parallelism of $\vec{R}$ and $\vec{M}$
c.) Solve for a and b with 2 of the scalar equations;
d.) Show that the 3rd equation is satisfied.

$\vec{F} = 3\hat{i} - 2\hat{j} + \hat{k}$ at $(1, 4, 0)$

$\vec{M}_o = 20\hat{i} - 10\hat{j} + 30\hat{k}$

a.) $\vec{R} = \vec{F}$ now acting at $(a, b, 0)$.

$M = \vec{r} \times \vec{R} + \vec{M}_o$, where $\vec{r} = (1-a)\hat{i} + (4-b)\hat{j}$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1-a & 4-b & 0 \\ 3 & -2 & 1 \end{vmatrix} + (20\hat{i} - 10\hat{j} + 30\hat{k})$$

$$\vec{M} = (24-b)\hat{i} + (-11+a)\hat{j} + (2a+3b+16)\hat{k} \quad \text{kN·m}$$

b.) $\vec{R} \times \vec{M} = 0$

$$\vec{R} \times \vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 24-b & a-11 & 2a+3b+16 \end{vmatrix} = 0$$

$$(-5a-6b-21)\hat{i} + (-6a-10b-24)\hat{j} + (3a-2b+15)\hat{k} = 0$$

c.) $-5a - 6b - 21 = 0$

$-6a - 10b - 24 = 0$

$a = -4.714, \quad b = 0.429$

$\therefore \vec{R}$ acts at the point $(-4.714, 0.429, 0)$

d.) $3a - 2b + 15 = 0$

$3(-4.714) - 2(0.429) + 15 = 0$

$0 = 0$

5.102 a) Replace the given force system with $\vec{F}$ at $(a, b, 0)$ and $\vec{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$;
b.) Write condition of parallelism $(\vec{F} \times \vec{M} = 0)$;
c.) Solve for a and b using to scalar equations
d.) Show that the third equation is satisfied.

$\vec{F} = -10\hat{i} + 5\hat{j} - 8\hat{k}$ kip at $(3, 2, -5)$ ft

$\vec{M}_o = 120\hat{i} + 60\hat{j} - 80\hat{k}$ (kip·ft)

a.) $\vec{F}_o = \vec{F}$

$\vec{M} = \vec{M}_o + \vec{r} \times \vec{F}$, where $\vec{r}$ is from $(a, b, 0)$ to $(3, 2, -5)$

$\vec{r} = (3-a)\hat{i} + (2-b)\hat{j} - 5\hat{k}$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-a & 2-b & -5 \\ -10 & 5 & -8 \end{vmatrix} + 120\hat{i} + 60\hat{j} - 80\hat{k}$$

$$\vec{M} = (129+8b)\hat{i} + (134-8a)\hat{j} + (-45-5a-10b)\hat{k} \quad \text{(kip-ft)}$$

b.) If $\vec{F}$ and $\vec{M}$ are parallel, then $\vec{F} \times \vec{M} = 0$

$$\vec{F} \times \vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -10 & 5 & -8 \\ 129+8b & 134-8a & -45-5a-10b \end{vmatrix} = 0$$

$$= (847 - 89a - 50b)\hat{i} + (-1482 - 50a - 164b)\hat{j} + (-1985 + 80a - 40b)\hat{k} = 0 \qquad (a)$$

c.) Scalar equations from Eq (a):

$$\begin{aligned} 89a + 50b &= 847 \\ 50a + 164b &= -1482 \\ 80a - 40b &= 1985 \end{aligned} \qquad (b)$$

By first two of Eqs (b),

$a = 17.610$ ft, $\quad b = -14.405$ ft

d.) By third of Eqs (b),

$80a - 40b = 1985$

$80(17.610) - 40(-14.405) = 1985$

5.103 a.) Replace the given force system with $\vec{F}$ at $(a, 0, c)$ and $\vec{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$;
b.) Write the condition of parallelism $(\vec{F} \times \vec{M} = 0)$
c.) Solve for a and c using two of the scalar equations
d.) Show the third equation is satisfied.

$\vec{F} = 10\hat{i} - 5\hat{j} + 8\hat{k}$ (lb) at $(3, 2, -5)$ (in)

$\vec{M} = 120\hat{i} + 60\hat{j} - 80\hat{k}$ (lb·in)

a.) $\vec{F}_o = \vec{F}$

$\vec{M} = \vec{M}_o + \vec{r} \times \vec{F}$, where $\vec{r}$ is from $(a, 0, c)$ to $(3, 2, -5)$

$\vec{r} = (3-a)\hat{i} + 2\hat{j} + (-5-c)\hat{k}$

5.103 cont.

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-a & 2 & -5-c \\ 10 & -5 & 8 \end{vmatrix} + 120\hat{i} + 60\hat{j} - 80\hat{k}$$

$$= (111-5c)\hat{i} + (-14+8a-10c)\hat{j} + (-115+5a)\hat{k} \ (\text{lb·in})$$

b.) If $\vec{F}$ and $\vec{M}$ are parallel, then $\vec{F}\times\vec{M}=0$.

$$\vec{F}\times\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & -5 & 8 \\ 111-5c & -14+8a-10c & -115+5a \end{vmatrix} = 0$$

$$= (-89a+80c+687)\hat{i} + (-50a-40c+2038)\hat{j} + (80a-125c+415)\hat{k} = 0 \qquad (a)$$

c.) Scalar equations from Eq (a):

$$-89a + 80c = -687$$
$$-50a - 40c = -2038 \qquad (b)$$
$$80a - 125c = -415$$

By first two Eqs (b).

$a = 25.201$ in, $\quad c = 19.449$ in

d.) By third of Eqs (b),

$$80a - 125c = -415$$
$$80(25.201) - 125(19.449) = -415$$

5.104 Solve Example 5.17 for $\vec{F} = 3\hat{i} - 2\hat{j} + \hat{k}$ (lb) and $\vec{M}_o = 20\hat{i} - 10\hat{j} + 30\hat{k}$ (lb-ft)

a.) $\vec{F}_o = \vec{F}$

$\vec{M} = \vec{M}_o + \vec{r}\times\vec{F}$ where $\vec{r}$ is from $(a,b,0)$ to $(3,2,-5)$.

$\vec{r} = (3-a)\hat{i} + (2-b)\hat{j} - 5\hat{k}$ (ft)

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-a & 2-b & -5 \\ 3 & -2 & 1 \end{vmatrix} + 20\hat{i} - 10\hat{j} + 30\hat{k}$$

$$= (12-b)\hat{i} + (-28+a)\hat{j} + (18+2a+3b)\hat{k} \quad (\text{lb·ft})$$

b.) If $\vec{F}$ and $\vec{M}$ are parallel, than $\vec{F}\times\vec{M}=0$

$$\vec{F}\times\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 12-b & -28+a & 18+2a+3b \end{vmatrix} = 0$$

$$= (-8-5a-6b)\hat{i} + (-42-6a-10b)\hat{j} + (-60+3a-2b)\hat{k} = 0 \qquad (a)$$

The resultant axis is along $\vec{F}$ at $(a,b,0)$. The direction cosines are the same as those for $\vec{F}$.

$$F\sqrt{3^2+(-2)^2+1^2} = \sqrt{14}$$

$$\cos\alpha = 3/\sqrt{14} = 0.8018$$

$$\cos\beta = -2/\sqrt{14} = -0.5345$$

$$\cos\gamma = 1/\sqrt{14} = 0.2673$$

Direction cosines: $(0.8018, -0.5345, 0.2673)$

Check: $0.8018^2 + (-0.5345)^2 + (0.2673)^2 = 1.0$

c.) Scalar Equations from Eqs (a):

$$5a + 6b = -8$$
$$6a + 10b = -42 \qquad (b)$$
$$3a - 2b = 60$$

By first two of Eqs (b),

$a = 12.286$ ft, $\quad b = -11.571$ ft

d.) By the third of Eqs (b),

$$3a - 2b = 60$$
$$3(12.286) - 2(-11.571) = 60$$

5.105 Solve Problem 5.103 for $\vec{F} = 3\hat{i} - 2\hat{j} + \hat{k}$ (kN) at $(1,2,-3)$ (m) and $\vec{M}_o = 20\hat{i} - 10\hat{j} + 30\hat{k}$ (kN·m) at point $(a,0,c)$.

a.) $\vec{F}_o = \vec{F}$

$\vec{M} = \vec{M}_o + \vec{r}\times\vec{F}$, where $\vec{r}$ is from $(a,0,c)$ to $(1,2,-3)$.

$\vec{r} = (1-a)\hat{i} + 2\hat{j} + (-3-c)\hat{k}$ (m)

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1-a & 2 & -3-c \\ 3 & -2 & 1 \end{vmatrix} + 20\hat{i} - 10\hat{j} + 30\hat{k}$$

$$= (16-2c)\hat{i} + (-20+a-3c)\hat{j} + (22+2a)\hat{k} \ (\text{kN·m})$$

b.) If $\vec{F}$ and $\vec{M}$ are parallel, then $\vec{F}\times\vec{M}=0$.

$$\vec{F}\times\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 16-2c & -20+a-3c & 22+2a \end{vmatrix} = 0$$

$$= (-24-5a+3c)\hat{i} + (-50-6a-2c)\hat{j} + (-28+3a-13c)\hat{k} = 0 \qquad (a)$$

5.105 cont.

c.) Scalar equations by Eqs (a):

$$\begin{aligned} 5a - 3c &= -24 \\ 6a + 2c &= -50 \qquad (b) \\ 3a - 13c &= 28 \end{aligned}$$

By first two of Eqs (b),

$\underline{\underline{a = -7.071 \text{ m}, \quad c = -3.786 \text{ m}}}$

d.) By third of Eqs (b),

$$3a - 13c = 28$$

$$3(-7.071) - 13(-3.786) = 28$$

5.106 Find the wrench of the force system shown in Fig. a.

Let

$\vec{A} = 100\,\hat{k}$ (lb) at $(4,0,0)$ (in)

$\vec{B} = -200\,\hat{k}$ (lb) at $(12,6,0)$ (in)

$\underline{\underline{\vec{F} = \vec{A} + \vec{B} = -100\,\hat{k} \text{ lb}}}$

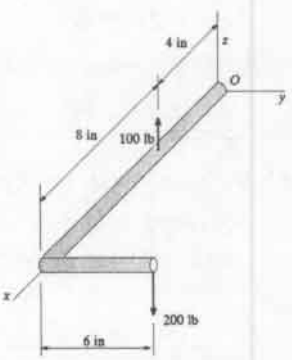


Figure a.

Position vectors from $(a,b,0)$ to the points of action of $\vec{A}$ and $\vec{B}$:

$$\vec{r}_A = (4-a)\hat{\imath} + (-b)\hat{\jmath} = (4-a)\hat{\imath} - b\hat{\jmath}$$

$$\vec{r}_B = (12-a)\hat{\imath} + (6-b)\hat{\jmath}$$

$$\vec{M} = \vec{r}_A \times \vec{A} + \vec{r}_B \times \vec{B}$$

$$\vec{M} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4-a & -b & 0 \\ 0 & 0 & 100 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 12-a & 6-b & 0 \\ 0 & 0 & -200 \end{vmatrix}$$

$$= (-1200 + 100b)\hat{\imath} + (2000 - 100a)\hat{\jmath} \quad (\text{lb}\cdot\text{in})$$

If $\vec{F}$ and $\vec{M}$ are parallel, then $\vec{F} \times \vec{M} = 0$.

$$\vec{F} \times \vec{M} = (-100\hat{k}) \times [(-1200 + 100b)\hat{\imath} + (2000 - 100a)\hat{\jmath}] = 0$$

$$= (200{,}000 - 10{,}000a)\hat{\imath} + (120{,}000 - 10{,}000b)\hat{\jmath} = 0$$

Scalar equations are:

$$10{,}000\,a = 200{,}000$$

$$10{,}000\,b = 120{,}000$$

$$a = 20, \qquad b = 12$$

Hence, $\vec{M} = 0$.

The resultant axis is parallel to $\vec{F}$ at $(20, 12, 0)$.

$$\vec{n} = \vec{F}/F = \frac{-100\,\hat{k}}{100} = -\hat{k}$$

Wrench:

Consists of: $\vec{F} = -100\,\hat{k}$ lb, $\vec{M} = 0$

Direction: $\vec{n} = -\hat{k}$

At point: $(20, 12, 0)$

5.107 Find the wrench of the force system in Problem 5.80 at $(0, b, c)$.

$\vec{F}_1 = 600\hat{\imath} - 350\hat{\jmath} + 90\hat{k}$ (lb) at $(3, 8, 21)$ (ft)

$\vec{F}_2 = -1000\hat{\imath} + 260\hat{\jmath} - 720\hat{k}$ (lb) at $(7, -9, -40)$ (ft)

$\vec{F}_3 = 300\hat{\imath} - 950\hat{\jmath} + 850\hat{k}$ (lb) at $(60, -3, 25)$ (ft)

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = -100\hat{\imath} - 1040\hat{\jmath} + 220\hat{k}$ (lb)

Position vectors from $(0, b, c)$ to the points of action of forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ are

$\vec{r}_1 = 3\hat{\imath} + (8-b)\hat{\jmath} + (21-c)\hat{k}$ (ft)

$\vec{r}_2 = 7\hat{\imath} + (-9-b)\hat{\jmath} + (-40-c)\hat{k}$ (ft)

$\vec{r}_3 = 60\hat{\imath} + (-3-b)\hat{\jmath} + (25-c)\hat{k}$ (ft)

$$\vec{M} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3$$

$$\vec{M} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 8-b & 21-c \\ 600 & -350 & 90 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 7 & -9-b & -40-c \\ -1000 & 260 & -720 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 60 & -3-b & 25-c \\ 300 & -950 & 850 \end{vmatrix}$$

$$\vec{M} = (46{,}150 - 220b - 1040c)\hat{\imath} + (13{,}870 + 100c)\hat{\jmath} + (-69{,}130 - 100b)\hat{k} \quad (\text{lb}\cdot\text{ft})$$

If $\vec{F}$ and $\vec{M}$ are parallel, then $\vec{F} \times \vec{M} = 0$.

$$\vec{F} \times \vec{M} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -100 & -1040 & 220 \\ (46{,}150 - 220b - 1040c) & (13{,}870 + 100c) & (-69{,}130 - 100b) \end{vmatrix}$$

$$= (68{,}843{,}800 + 104{,}000b - 22{,}000c)\hat{\imath} + (3{,}240{,}000 - 58{,}400b - 228{,}800c)\hat{\jmath} + (46{,}609{,}000 - 228{,}800b - 1{,}091{,}600c)\hat{k} = 0$$

5.107 cont.

Scalar equations:

$$1040\,b - 220\,c = -688{,}438$$
$$584\,b + 2288\,c = 32{,}400 \qquad (a)$$
$$2288\,b + 10{,}916\,c = 466{,}090$$

By first two of Eqs (a),

$$b = -625.20 \text{ ft}, \quad c = 173.74 \text{ ft}$$

By third of Eqs (a),

$$2{,}288\,(-625.20) + 10{,}916\,(173.74) = 466{,}090$$

The resultant axis is parallel to $\vec{F}$ at $(0, -625.20, 173.74)$.

$$\vec{n} = \vec{F}/F = \frac{-100\hat{\imath} - 1040\hat{\jmath} + 220\hat{k}}{\sqrt{(-100)^2 + (-1040)^2 + 220^2}}$$

$$\vec{n} = -0.09366\hat{\imath} - 0.97405\hat{\jmath} + 0.20605\hat{k}$$

check: $(0.09366)^2 + (-0.97405)^2 + (0.20605)^2 = 1$

Wrench:

Consists of: $\vec{F} = -100\hat{\imath} - 1040\hat{\jmath} + 220\hat{k}$ lb

$\vec{M} = 3004\hat{\imath} + 31{,}244\hat{\jmath} - 6610\hat{k}$ ft·lb

Direction: $\vec{n} = -0.094\hat{\imath} - 0.974\hat{\jmath} - 0.206\hat{k}$

At point: $(0, -625.20, 173.74)$ ft

5.108 Find the wrench of the force system of Problem 5.88 at point $(a, 0, c)$.

$\vec{F}_1 = 500\hat{\imath} - 340\hat{\jmath} - 650\hat{k}$ [N] at $(9, 7, 23)$ [cm]

$\vec{F}_2 = -360\hat{\imath} + 550\hat{\jmath} + 720\hat{k}$ [N] at $(30, 8, -22)$ [cm]

$\vec{F}_3 = 860\hat{\imath} - 25\hat{\jmath} - 150\hat{k}$ [N] at $(-60, 45, 10)$ [cm]

$\vec{M}_1 = 10{,}500\hat{\imath} - 6400\hat{\jmath} + 13{,}600\hat{k}$ [N·cm]

$\vec{M}_2 = 16{,}000\hat{\imath} + 9500\hat{\jmath}$ [N·cm]

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 1000\hat{\imath} + 185\hat{\jmath} - 80\hat{k}$ [N]

Position vectors from $(a, 0, c)$ to points of action of $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$:

$\vec{r}_1 = (9-a)\hat{\imath} + 7\hat{\jmath} + (23-c)\hat{k}$ [cm]

$\vec{r}_2 = (30-a)\hat{\imath} + 8\hat{\jmath} + (-22-c)\hat{k}$ [cm]

$\vec{r}_3 = (-60-a)\hat{\imath} + 45\hat{\jmath} + (10-c)\hat{k}$ [cm]

$$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3$$

$$\vec{r}_1 \times \vec{F}_1 = (3270 - 340c)\hat{\imath} + (17{,}350 - 650a - 500c)\hat{\jmath} + (-6560 - 340a)\hat{k}$$

$$\vec{r}_2 \times \vec{F}_2 = (17{,}860 + 550c)\hat{\imath} + (-13{,}680 + 720a + 360c)\hat{\jmath} + (19{,}380 - 550a)\hat{k}$$

$$\vec{r}_3 \times \vec{F}_3 = (-6500 - 25c)\hat{\imath} + (-400 - 150a - 860c)\hat{\jmath} + (-37{,}200 + 25a)\hat{k}$$

If $\vec{F}$ and $\vec{M}$ are parallel, then $\vec{F} \times \vec{M} = 0$.

$$\vec{F} \times \vec{M} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1000 & 185 & -80 \\ 41130 + 185c & 6370 - 80a - 1000c & -10780 - 185a \end{vmatrix} = 0$$

Scalar equations for $\vec{F} \times \vec{M} = 0$:

$$\hat{\imath}: 1{,}484{,}700 + 40{,}625a + 80{,}000c = 0$$
$$\hat{\jmath}: 7{,}489{,}600 + 185{,}000a - 14{,}800c = 0 \qquad (a)$$
$$\hat{k}: 1{,}239{,}050 + 80{,}000a + 1{,}034{,}225c = 0$$

By the first two of Eqs (a);

$$a = -40.3306, \quad c = 1.9216$$

By third of Eqs (a);

$$0 = 0$$

The resultant axis is along $\vec{F}$ through $(-40.3306, 0, 1.9216)$.

$$\hat{n} = \vec{F}/F = \frac{1000\hat{\imath} + 185\hat{\jmath} - 80\hat{k}}{\sqrt{1000^2 + 185^2 + (-80)^2}}$$

$$\hat{n} = 0.980\hat{\imath} + 0.181\hat{\jmath} - 0.078\hat{k}$$

$$\vec{M} = 41{,}485\hat{\imath} + 7675\hat{\jmath} - 3319\hat{k} \text{ N·cm}$$

Wrench:

Consists of: $\vec{F} = 1000\hat{\imath} + 185\hat{\jmath} - 80\hat{k}$ N

$\vec{M} = 41{,}485\hat{\imath} + 7675\hat{\jmath} - 3319\hat{k}$ N·cm

Direction: $\hat{n} = 0.980\hat{\imath} + 0.181\hat{\jmath} - 0.078\hat{k}$

At point: $(-40.33, 0, 1.92)$ cm

5.109 Find the wrench of the force system shown in Fig a. Forces in lb, length in inches.

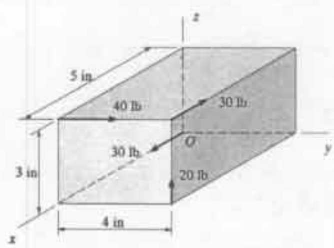


Figure a.

5.109 cont.

$\vec{F}_1 = 40\hat{j}$ at $(5,0,3)$

$\vec{F}_2 = -30\hat{i}$ at $(5,4,3)$

$\vec{F}_3 = 30\hat{k}$ at $(0,0,0)$

$\vec{F}_4 = 20\hat{k}$ at $(5,4,0)$

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 40\hat{j} + 20\hat{k}$ [lb]

Find the wrench at $(a,b,0)$.

Position vectors from $(a,b,0)$ to points of action of $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$

$\vec{r}_1 = (5-a)\hat{i} - b\hat{j} + 3\hat{k}$ [in]

$\vec{r}_2 = (5-a)\hat{i} + (4-b)\hat{j} + 3\hat{k}$ [in]

$\vec{r}_3 = -a\hat{i} - b\hat{j}$ [in]

$\vec{r}_4 = (5-a)\hat{i} + (4-b)\hat{j}$ [in]

$$\vec{M} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 + \vec{r}_4 \times \vec{F}_4$$

$$\vec{M} = [(5-a)\hat{i} - b\hat{j} + 3\hat{k}] \times (40\hat{j})$$
$$[(5-a)\hat{i} + (4-b)\hat{j} + 3\hat{k}] \times (-30\hat{i})$$
$$(-a\hat{i} - b\hat{j}) \times (30\hat{i})$$
$$+ [(5-a)\hat{i} + (4-b)\hat{j}] \times (20\hat{k})$$

$$= (-20b-40)\hat{i} + (20a-190)\hat{j} + (-40a+320)\hat{k} \quad \text{lb-in}$$

If $\vec{F}$ and $\vec{M}$ are parallel, then $\vec{F} \times \vec{M} = 0$.

$$\vec{F} \times \vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 40 & 20 \\ -20b-40 & 20a-190 & -40a+320 \end{vmatrix} = 0$$

Scalar equations of $\vec{F} \times \vec{M} = 0$

$\hat{i}$: $-2000a + 16{,}600 = 0$

$\hat{j}$: $400b + 800 = 0$ (a)

$\hat{k}$: $800b + 1600 = 0$

By first two of Eqs (a):

$a = 8.3$ in, $b = -2$ in (b)

By last of Eqs (a): $0 = 0$

With Eqs. b, $\vec{M} = -24\hat{j} - 12\hat{k}$

The resultant axis is parallel to $\vec{F}$ at $(8.3, -2, 0)$.

$$\therefore \hat{n} = \vec{F}/F = \frac{40\hat{j} + 120\hat{k}}{\sqrt{40^2 + 20^2}} = 0.894\hat{j} + 0.447\hat{k}$$

Wrench:

Consists of: $\vec{F} = 40\hat{j} + 20\hat{k}$ lb

$\vec{M} = -24\hat{j} - 12\hat{k}$ lb·in

Direction: $\hat{n} = 0.894\hat{j} + 0.447\hat{k}$

At point: $(8.3, -2, 0)$ in

5.110 Find the wrench of the force system shown in Fig a.

$\vec{F}_1 = -60\hat{i}$ [N]

at $(0.2, 0, 0.6)$ [m]

$$\vec{F}_2 = 70\left(\frac{0.2\hat{i} - 0.3\hat{j} - 0.6\hat{k}}{\sqrt{.2^2 + (-.3)^2 + (-.6)^2}}\right)$$

$= 20\hat{i} - 30\hat{j} - 60\hat{k}$ [N]

at $(0.2, 0, 0)$ [m]

$\vec{M}_1 = 100\hat{k}$ [N·m]

$\vec{F} = \vec{F}_1 + \vec{F}_2$

$= -40\hat{i} - 30\hat{j} - 60\hat{k}$ N

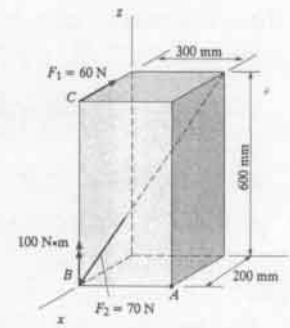


Figure a.

Find the wrench at $(a,b,0)$.

Position vectors from $(a,b,0)$ to the points of action of forces $\vec{F}_1, \vec{F}_2$:

$\vec{r}_1 = (0.2-a)\hat{i} - b\hat{j} + 0.6\hat{k}$ [m]

$\vec{r}_2 = (0.2-a)\hat{i} - b\hat{j} + 0\hat{k}$ [m]

$$\therefore \vec{M} = \vec{M}_1 + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.2-a & -b & 0.6 \\ -60 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.2-a & -b & 0 \\ 20 & -30 & -60 \end{vmatrix} + 100\hat{k}$$

$$= (60b)\hat{i} + (-24-60a)\hat{j} + (94+30a-40b)\hat{k} \quad \text{N·m}$$

If $\vec{F}$ and $\vec{M}$ are parallel, then $\vec{F} \times \vec{M} = 0$.

$$\vec{F} \times \vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -40 & -30 & -60 \\ 60b & -24-60a & 94+30a-40b \end{vmatrix} = 0$$

Scalar Equations for $\vec{F} \times \vec{M} = 0$:

$\hat{i}$: $-4260 - 4500a + 1200b = 0$

$\hat{j}$: $3760 + 1200a - 5200b = 0$ (a)

$\hat{k}$: $960 + 2400a + 1800b = 0$

By first two of Eqs (a):

$a = -0.8033$ m, $b = 0.5378$ m (b)

5.110 cont.

By Eqs (b),

$$\vec{M}=32.267\hat{i}+24.200\hat{j}+48.389\hat{k} \quad [N\cdot m]$$

The resultant axis along $\vec{F}$ at (-0.8033, 0.5378, 0).

$$\hat{n}=\vec{F}/F=\frac{-40\hat{i}-30\hat{j}-60\hat{k}}{\sqrt{(-40)^2+(-30)^2+(-60)^2}}$$

$$=-0.512\hat{i}-0.384\hat{j}-0.768\hat{k}$$

Wrench:

Consists of: $\vec{F}=-40\hat{i}-30\hat{j}-60\hat{k}$ [N]

$\vec{M}=32.27\hat{i}+24.20\hat{j}+48.39\hat{k}$ [N·m]

Direction: $\hat{n}=-0.512\hat{i}-0.384\hat{j}-0.768\hat{k}$

At point: (-0.803, 0.538, 0) [m]

5.111 Find the wrench of the force system shown in Fig a.

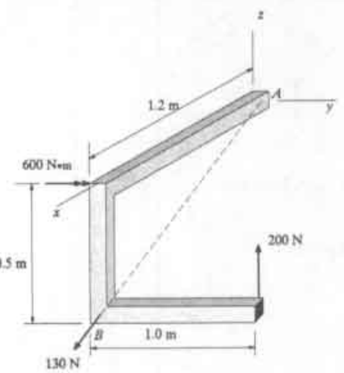


Figure a

$\vec{F}_1=200\hat{k}$ [N] at (1.2, 1, -0.5) [m]

$\vec{F}_2=130\left(\frac{1.2}{1.3}\hat{i}-\frac{0.5}{1.3}\hat{k}\right)=120\hat{i}-50\hat{k}$ [N]

at (1.2, 0, -0.5) [m]

$\vec{M}_1=600\hat{j}$ [N·m]

$\vec{F}=\vec{F}_1+\vec{F}_2=120\hat{i}+150\hat{k}$ [N]

Find the wrench at (a, b, 0).

Position vectors from (a, b, 0) to points of action of forces $\vec{F}_1$, $\vec{F}_2$;

$\vec{r}_1=(1.2-a)\hat{i}+(1-b)\hat{j}-0.5\hat{k}$ [m]

$\vec{r}_2=(1.2-a)\hat{i}-b\hat{j}-0.5\hat{k}$ [m]

$$\vec{M}=\vec{M}_1+\vec{r}_1\times\vec{F}_1+\vec{r}_2\times\vec{F}_2$$

$$\vec{F}\times\vec{M}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 120 & 0 & 150\\ 200-150b & 150a+360 & 120b\end{vmatrix}=0$$

Scalar equations for $\vec{F}\times\vec{M}=0$:

$\hat{i}$: $-54{,}000-22{,}500a=0$

$\hat{j}$: $30{,}000-36900b=0$ (a)

$\hat{k}$: $43{,}200+18{,}000a=0$

By last two of Eqs (a),

$a=-2.400$ m, $b=0.8130$ m (b)

By Eqs (b):

$\vec{M}=78.050\hat{i}+97.560\hat{k}$ N·m

The resultant axis along $\vec{F}$ at (-2.4, 0.813, 0).

$$\therefore \hat{n}=\vec{F}/F=\frac{120\hat{i}+150\hat{k}}{\sqrt{120^2+150^2}}=0.6247\hat{i}+7809\hat{k}$$

Wrench:

Consists of: $\vec{F}=120\hat{i}+150\hat{k}$

$\vec{M}=78.05\hat{i}+97.56\hat{k}$ [N·m]

Direction: $\hat{n}=0.625\hat{i}+0.781\hat{k}$

At point: (-2.4, 0.813, 0) [m]

5.112 Find the wrench of the force system shown in Fig a.

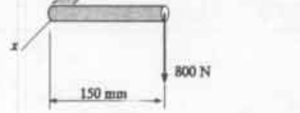


Figure a.

$\vec{F}_1=400\hat{k}$ at (0.1, 0, 0) [m]

$\vec{F}_2=-800\hat{k}$ at (0.3, 0.15, 0) [m]

$\vec{F}=\vec{F}_1+\vec{F}_2=-400\hat{k}$ [N]

Find the wrench at (a, b, 0).

5.112 cont.

Position vectors from $(a,b,0)$ to the points of action of $\vec{F_1}, \vec{F_2}$:

$\vec{r_1} = (0.1-a)\hat{\imath} - b\hat{\jmath}$ [m]

$\vec{r_2} = (0.3-a)\hat{\imath} + (0.15-b)\hat{\jmath}$ [m]

$\vec{M} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2}$

$$\vec{M} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0.1-a & -b & 0 \\ 0 & 0 & 400 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0.3-a & 0.15-b & 0 \\ 0 & 0 & -800 \end{vmatrix}$$

$= (-120 + 400b)\hat{\imath} + (200-400a)\hat{\jmath}$ N·m

If $\vec{F}$ and $\vec{M}$ are parallel, then $\vec{F}\times\vec{M} = 0$

$$\vec{F}\times\vec{M} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & -400 \\ -120+400b & 200-400a & 0 \end{vmatrix} = 0$$

Scalar equations for $\vec{F}\times\vec{M} = 0$:

$\hat{\imath}$: $80{,}000 - 160{,}000\,a = 0$

$\hat{\jmath}$: $48{,}000 - 160{,}000\,b = 0$ (a)

By Eqs. (a), $a = 0.5$ m, $b = 0.3$ m (b)

By Eqs (b), $\vec{M} = 0$

The resultant axis is along $\vec{F}$ at $(0.5, 0.3, 0)$.

$\therefore \hat{n} = \vec{F}/F = \dfrac{-400\,k}{400} = -\hat{k}$

Wrench:

Consists of: $\vec{F} = -400\,k$ [N]

$\vec{M} = 0$ [N·m]

Direction: $\hat{n} = -\hat{k}$

At point: $(0.5, 0.3, 0)$ [m]

5.113

Draw Free-body diagram of rod OAB (see Fig a)

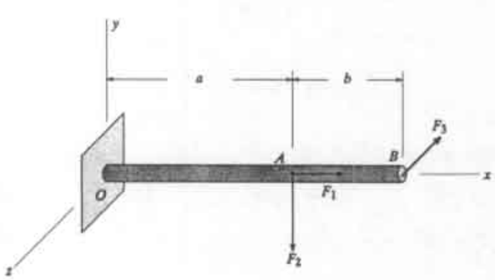


Figure a.

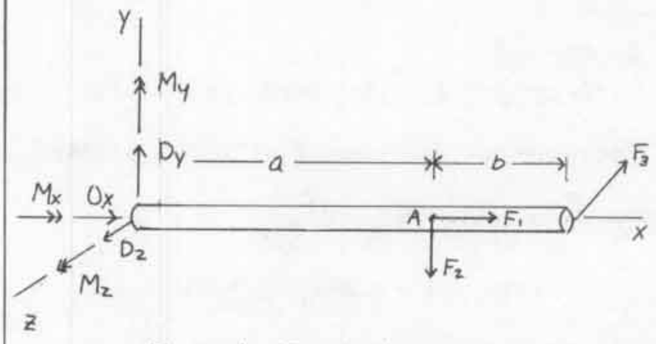


Figure b. Free-body Diagram of rod OAB

5.114

Draw Free-body Diagram of Spool S. (See Fig a.).

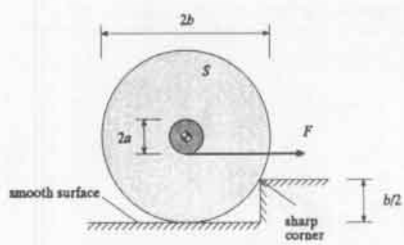


Figure a.

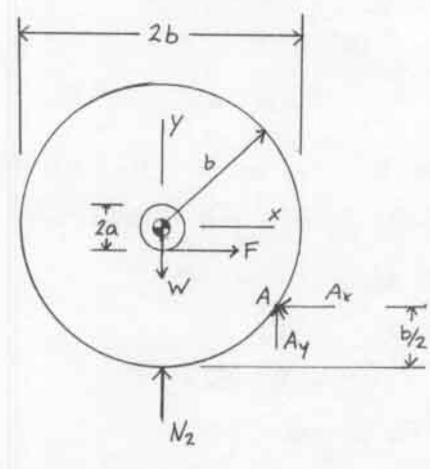


Figure b. Free-body Diagram of Spool S

5.115

Draw Free-Body Diagrams of Bar OA and Weight W (Fig. a).

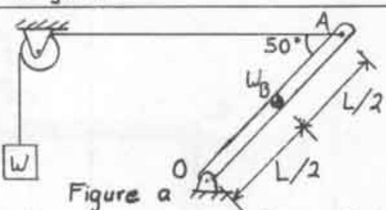


Figure a

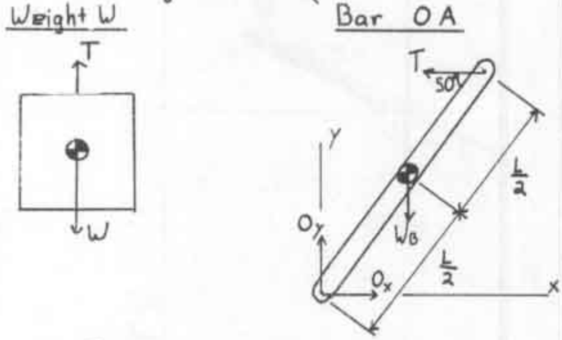


Figure b. Free-Body Diagrams

5.116

Draw Free-Body Diagrams of the sign (side view) (Fig. a.)

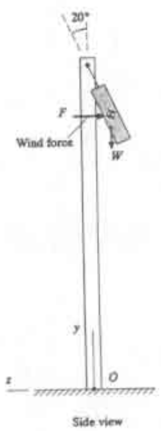


Figure a

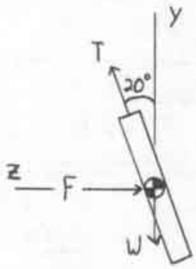
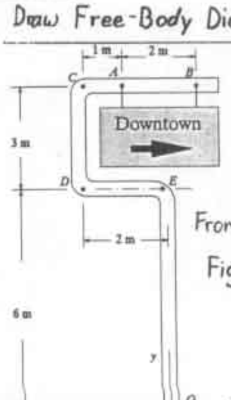


Figure b. Free-body diagram

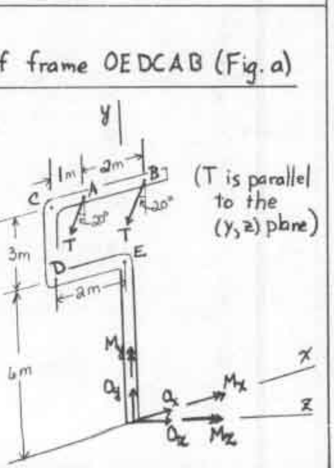
5.117

Draw Free-Body Diagram of frame OEDCAB (Fig. a)



Front view

Figure a



5.118

Draw Free-Body Diagram of the bar fixed at point A (Fig. a).

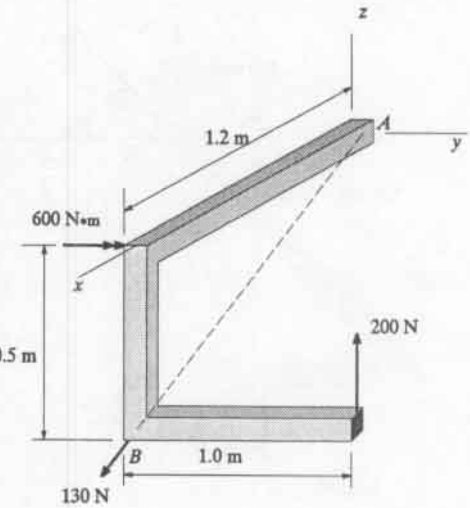


Figure a

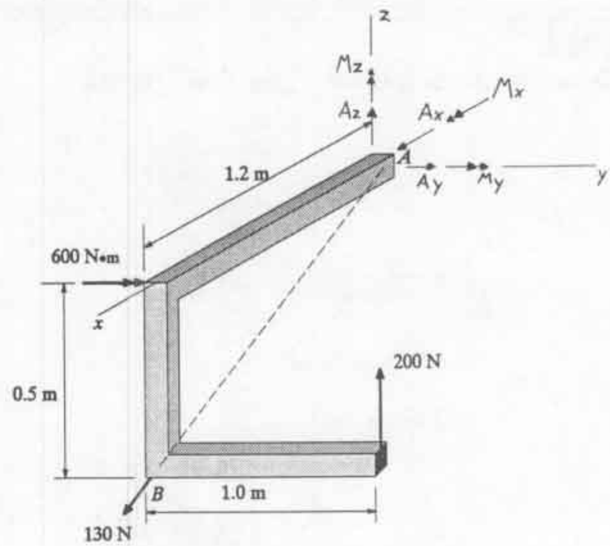


Figure b. Free-body diagram

5.119

Draw Free-Body Diagram of L-shaped bar fixed at O (Fig. a)

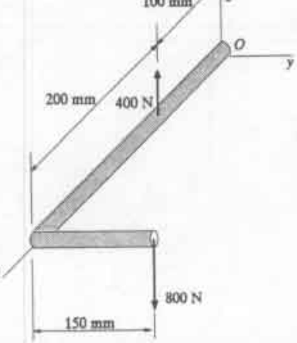


Figure a

5.119 Cont.

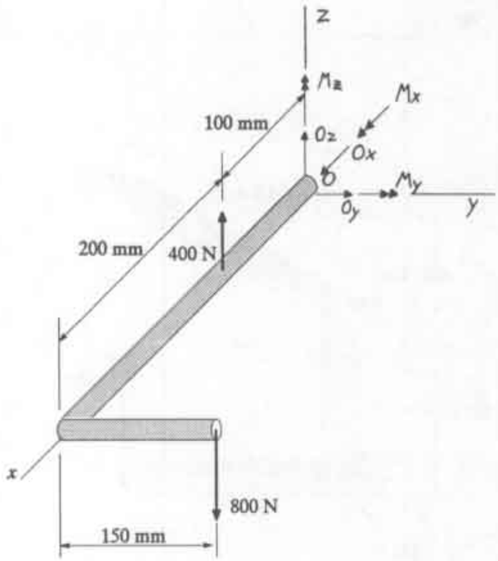


Figure b. Free-body diagram

5.120

Draw Free-Body Diagram of Bar OAB (Fig. a)

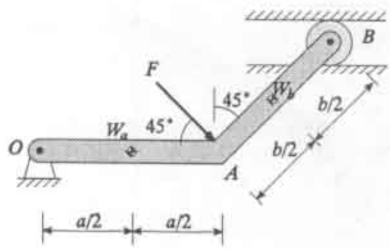


Figure a

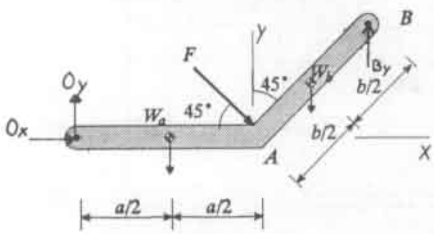


Figure b. Free-body diagram

5.121

Draw Free-Body Diagram of bar ABCDO fixed at A (Fig. a).

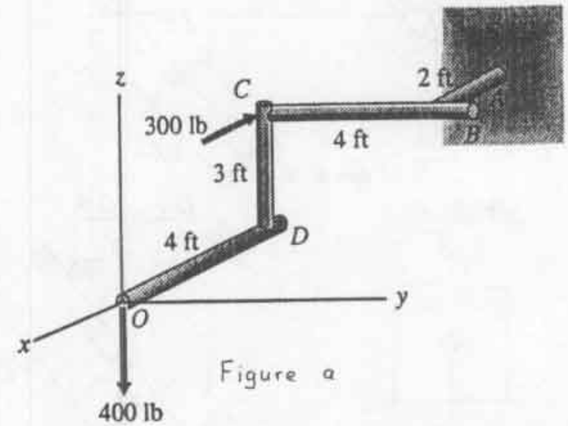


Figure a

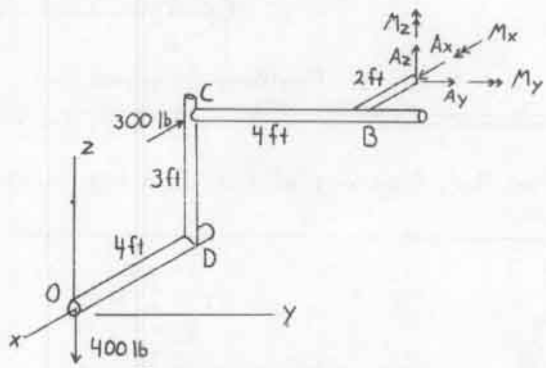


Figure b. Free-body diagram

5.122

Draw Free-Body Diagram of member ABC (Fig. a)

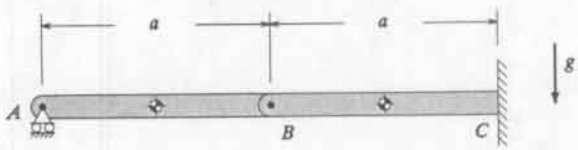


Figure a

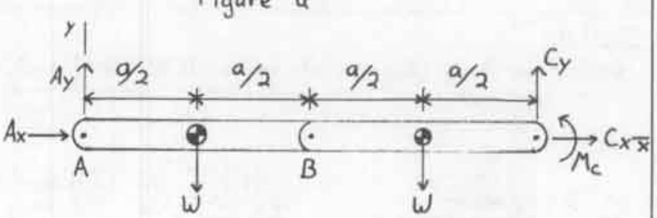


Figure b. Free-body diagram

5.123

Draw Free-Body Diagrams of Bar AB and Bar BC separately (Fig. a)

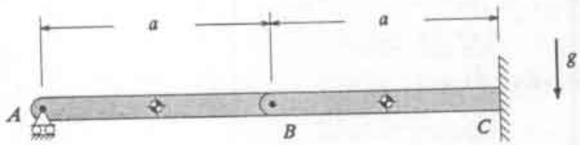


Figure a

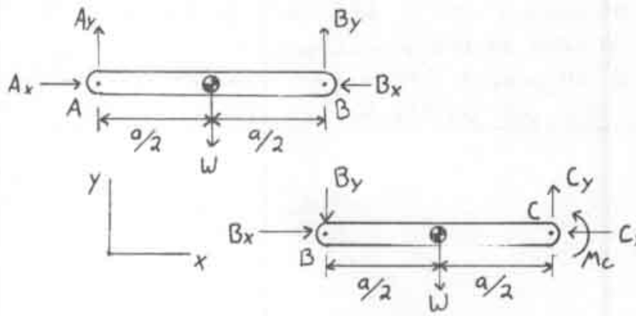


Figure b. Free-body diagrams

5.124

Draw Free-Body Diagram of car hood OABC (Fig. a)

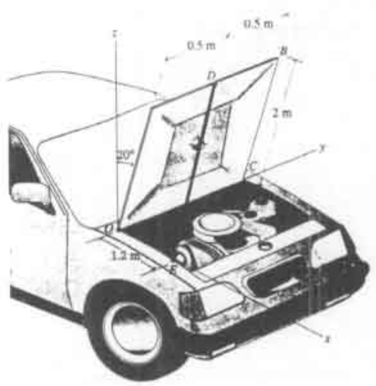


Figure a.

Note: -Rod DE is a two-force member.
-Supports at O and C do not exert moments.
-Support at C cannot exert force in y-direction.

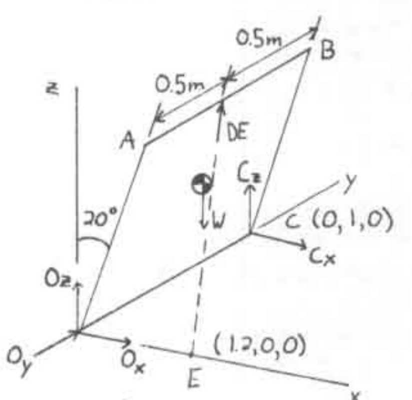


Figure b. Free-body diagram

5.125

Draw Free-Body Diagram of barn door (Fig. a)

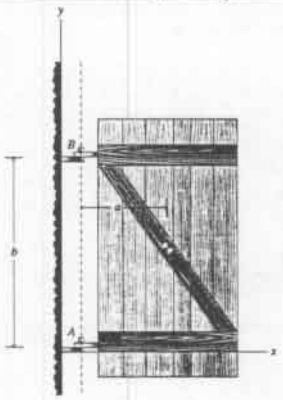


Figure a

Note: Hinges at A and B are not capable of resisting moments

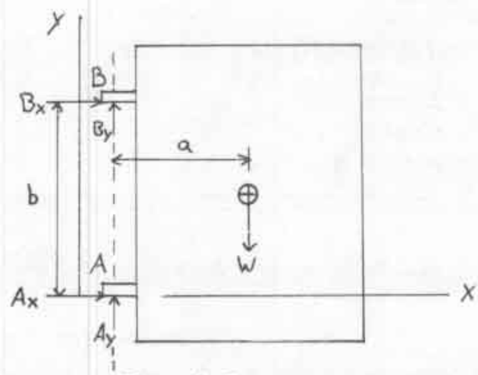


Figure b. Free-body diagram

5.126

Find the reactions on barn door at A and B (Fig.a)

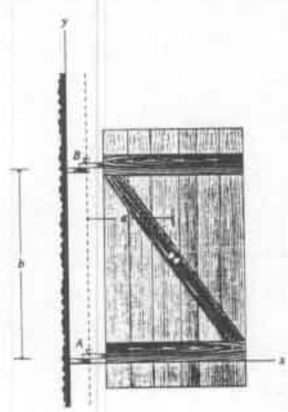


Figure a

Note: Hinge at B supports no vertical load.
Hinges at A and B cannot resist moments
$a = 1.0$ m
$b = 2.0$ m
$W = 900$ N

5.126 Cont.

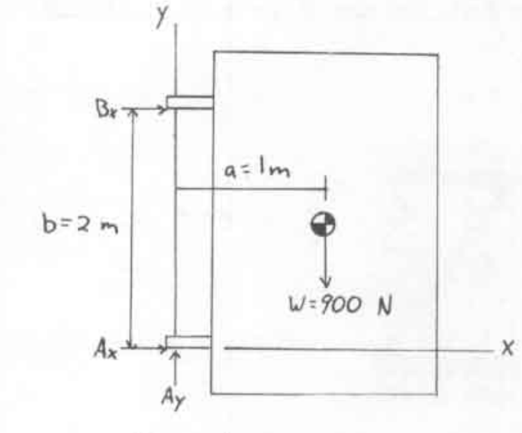


Figure b. Free-body diagram

solution by Fig. b,

$\Sigma F_y = A_y - 900 = 0$

$\underline{\underline{A_y = 900\ N}}$

$\Sigma M_A = -B_x(2) - 900(1) = 0$

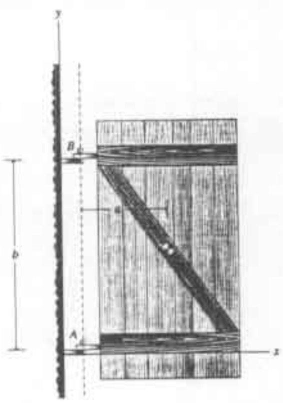
$\underline{\underline{B_x = -450\ N}}$

$\Sigma F_x = A_x + B_x = 0$

$\underline{\underline{A_x = 450\ N}}$

5.127

Find the reactions on the barn door at A and B (Fig. a)



Note: Hinge at A supports no vertical load. Hinges A and B cannot resist moments.
$a = 2.5$ ft
$b = 6$ ft
$W = 160$ lb

Figure a

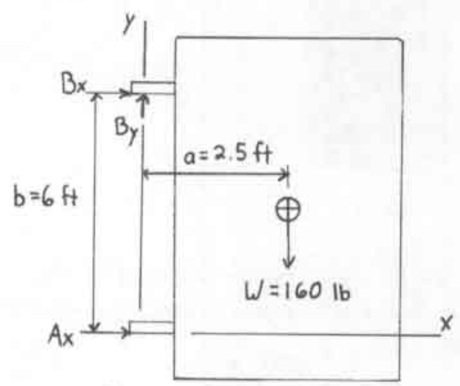


Figure b. Free-body diagram

Solution by Fig. b,

$\Sigma F_y = B_y - 160 = 0$

$\underline{\underline{B_y = 160\ lb}}$

$\Sigma M_B = A_x(6) - 160(2.5) = 0$

$\underline{\underline{A_x = 66.67\ lb}}$

$\Sigma F_x = A_x + B_x = 0$

$\underline{\underline{B_x = -66.67\ lb}}$

5.128

a) Replace $\vec{F} = -40\hat{i} + 60\hat{j} + 30\hat{k}$ in Fig. a with a force at B and a couple

b) Determine the force and moment exerted by the wall on the bar at A.

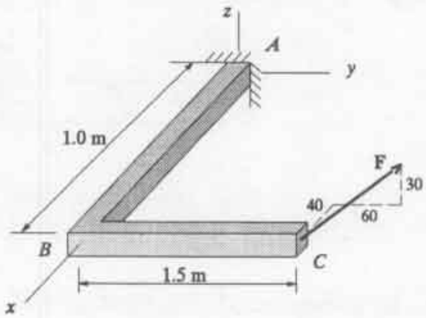


Figure a

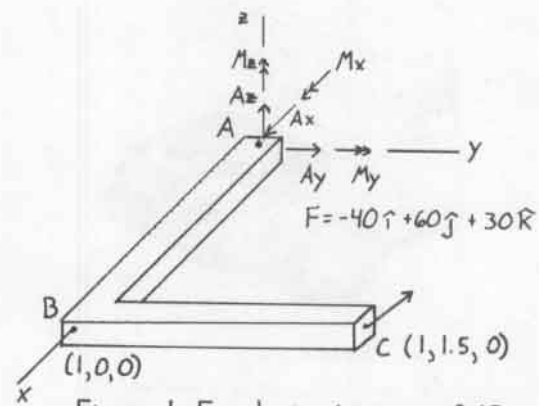


Figure b. Free-body diagram of ABC

Solutions by Fig. b;

a) $\underline{\underline{\vec{R} = \vec{F} = -40\hat{i} + 60\hat{j} + 30\hat{k}\ N}}$

$\vec{M} = \vec{r} \times \vec{F}$, where $\vec{r}$ is from B to C

$\vec{r} = 1.5\hat{j}$

$\vec{M} = (1.5\hat{j}) \times (-40\hat{i} + 60\hat{j} + 30\hat{k})$

$\underline{\underline{\vec{M} = 45\hat{i} + 60\hat{k}\ N\cdot m}}$

b) The reactions at A may be determined by considering the equilibrium of Fig. b.

Thus,

$\Sigma \vec{F} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} + \vec{F} = 0$

$\Sigma M_A = M_x\hat{i} + M_y\hat{j} + M_z\hat{k} + \vec{r} \times \vec{F} = 0$

where $\vec{r} = \hat{i} + 1.5\hat{j}$; $\vec{F} = -40\hat{i} + 60\hat{j} + 30\hat{k}$

5.128 Cont.

Hence, $\vec{r}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1.5 & 0 \\ 40 & 60 & 30 \end{vmatrix} = 45\hat{\imath} - 30\hat{\jmath} + 120\hat{k}$

The solution is,

$\underline{\underline{A_x = 40\ N\ ,\ A_y = -60\ N\ ,\ A_z = -30\ N}}$

$\underline{\underline{M_x = -45\ N\cdot m,\ M_y = 30\ N\cdot m,\ M_z = -120\ N\cdot m}}$

Alternatively, with the results of part a, Consider the free-body diagram of bar AB (Fig. C).

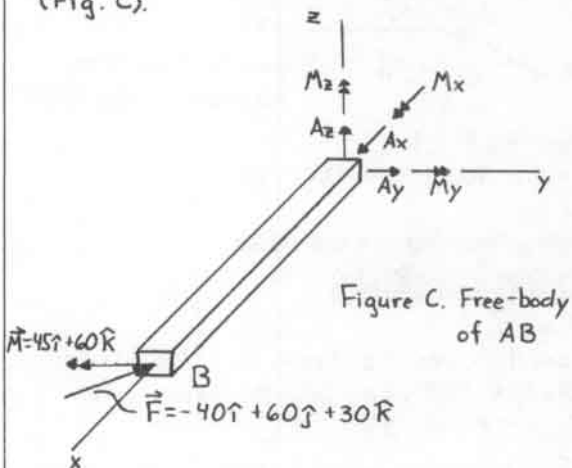


Figure C. Free-body diagram of AB

By Fig. C,

$\Sigma\vec{F} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} - 40\hat{\imath} + 60\hat{\jmath} + 30\hat{k} = 0$

or $\underline{\underline{A_x = 40\ N\ ,\ A_y = -60\ N\ ,\ A_z = -30\ N}}$

$\Sigma M_A = M_x\hat{\imath} + M_y\hat{\jmath} + M_z\hat{k} + 45\hat{\imath} + 60\hat{k} + \vec{r}\times\vec{F} = 0$

where $\vec{r} = 1\hat{\imath}$ m

Hence, $\vec{r}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 0 \\ -40 & 60 & 30 \end{vmatrix} = -30\hat{\jmath} + 60\hat{k}$

Therefore,

$\underline{\underline{M_x = -45\ N\cdot m\ ,\ M_y = 30\ N\cdot m,\ M_z = -120\ N\cdot m}}$

5.129

Find $F_x, F_y, F_z, M_x, M_y,$ and M_z exerted on bar ABC by the clamp at A (Fig. a).

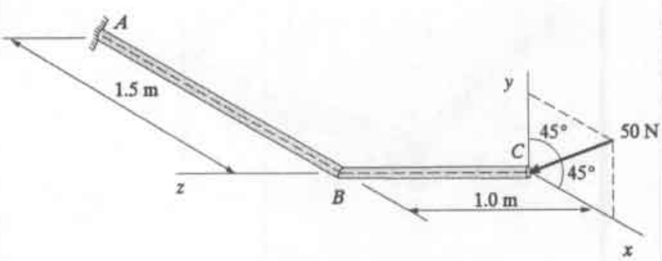


Figure a

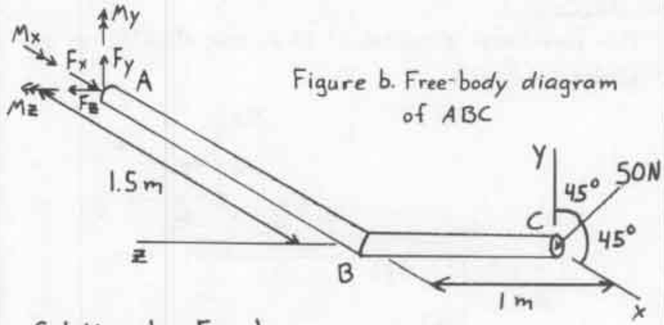


Figure b. Free-body diagram of ABC

Solution by Fig. b,

$\vec{C} = 50(-\cos 45°\hat{\imath} - \sin 45°\hat{\jmath}) = -35.35\hat{\imath} - 35.35\hat{\jmath}$

For equilibrium,

$\Sigma F = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k} + \vec{C} = 0$

Therefore,

$\underline{\underline{F_x = 35.35\ N\ ,\ F_y = 35.35\ N,\ F_z = 0}}$

Also, for equilibrium,

$\Sigma M_A = M_x\hat{\imath} + M_y\hat{\jmath} + M_z\hat{k} + \vec{r}\times\vec{C} = 0$

where $\vec{r}$ is from A to C, or

$\vec{r} = 1.5\hat{\imath} - 1\hat{k}$

Therefore, $\vec{r}\times\vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1.5 & 0 & -1 \\ -35.35 & -35.35 & 0 \end{vmatrix}$

$\vec{r}\times\vec{C} = -35.35\hat{\imath} + 35.35\hat{\jmath} - 53.03\hat{k}\ \ (N\cdot m)$

Hence,

$\underline{\underline{M_x = 35.35\ N\cdot m\ ,\ M_y = -35.35\ N\cdot m,\ M_z = 53.03\ N\cdot m}}$

5.130

Find $\vec{F_A}$ and $\vec{M_A}$, the force and moment that the wall at A exerts on the bar (Fig. a)

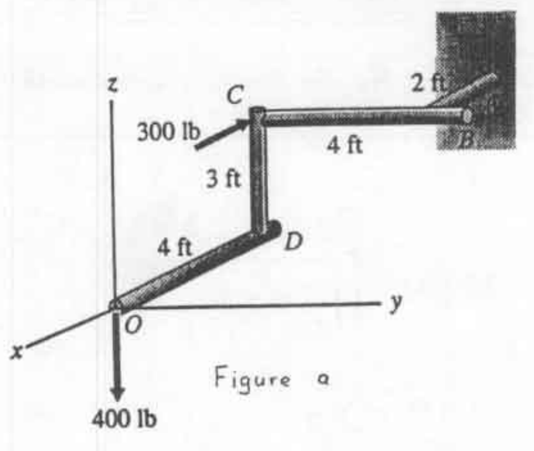


Figure a

5.130 Cont.

The free-body diagram of the bar ABCDO is shown in Fig. b.

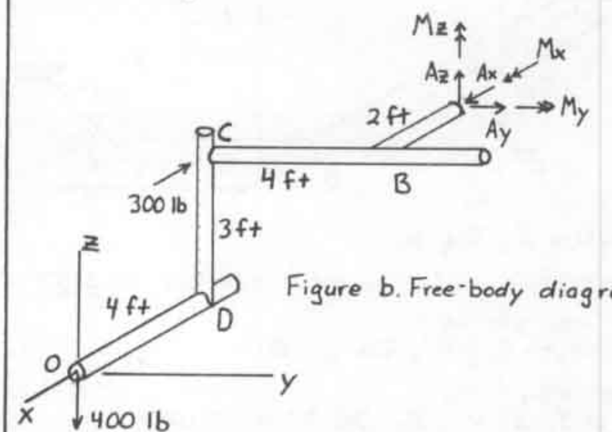


Figure b. Free-body diagram

Solution by Fig. b,

$\Sigma F_x = A_x - 300 = 0 \rightarrow A_x = 300$ lb

$\Sigma F_y = A_y = 0$

$\Sigma F_z = A_z - 400 = 0 \rightarrow A_z = 400$ lb

$\therefore \vec{F}_A = 300\,\hat{\imath} + 400\,\hat{k}$ (lb)

The position vectors from A to points of action of 400 and 300 lb forces are,

$\vec{r}_{400} = 6\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ $\quad \vec{r}_{300} = 2\hat{\imath} - 4\hat{\jmath}$

so, $\vec{M}_{400} = (6\hat{\imath} - 4\hat{\jmath} - 3\hat{k}) \times (-400\,\hat{k}) = 1600\,\hat{\imath} + 2400\,\hat{\jmath}$

$\vec{M}_{300} = (2\hat{\imath} - 4\hat{\jmath}) \times (-300\,\hat{\imath}) = -1200\,\hat{k}$

By Fig. b,

$\Sigma \vec{M} = \vec{M}_A + \vec{M}_{400} + \vec{M}_{300} = 0$

$\therefore \vec{M}_A = -1600\,\hat{\imath} - 2400\,\hat{\jmath} + 1200\,\hat{k}$ (lb·ft)

5.131

Find $\vec{F}_B$ and $\vec{M}_B$, the forces exerted on the bar at B by the wall (Fig. a).

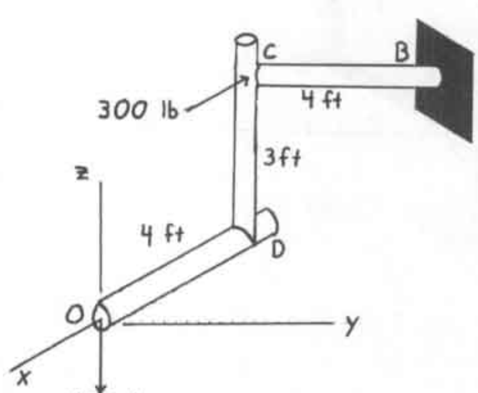


Figure a

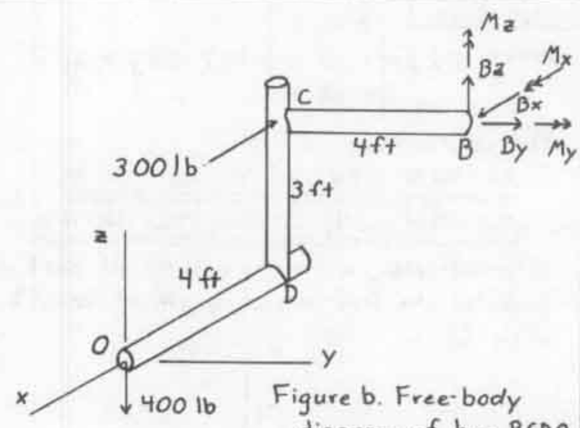


Figure b. Free-body diagram of bar BCDO.

Solution by Fig. b,

$\Sigma F_x = B_x - 300 = 0 \rightarrow B_x = 300$

$\Sigma F_y = B_y = 0$

$\Sigma F_z = B_z - 400 = 0 \rightarrow B_z = 400$

$\vec{F}_B = 300\,\hat{\imath} + 400\,\hat{k}$ (lb)

By Fig. b,

the position vectors from B to points of action of 400 and 300 lb forces are

$\vec{r}_{400} = 4\,\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ $\quad \vec{r}_{300} = -4\,\hat{\jmath}$

$\therefore \vec{M}_{400} = (4\hat{\imath} - 4\hat{\jmath} - 3\hat{k}) \times (-400\,\hat{k}) = 1600\,\hat{\imath} + 1600\,\hat{\jmath}$

$\vec{M}_{300} = (-4\hat{\jmath}) \times (-300\,\hat{\imath}) = -1200\,\hat{k}$

By Fig. b, equilibrium of moments yields,

$\Sigma \vec{M} = \vec{M}_B + \vec{M}_{400} + \vec{M}_{300} = 0$

or, $\vec{M}_B = -1600\,\hat{\imath} - 1600\,\hat{\jmath} + 1200\,\hat{k}$ (lb·ft)

5.132

Find tension T in cable CF (Fig. a).

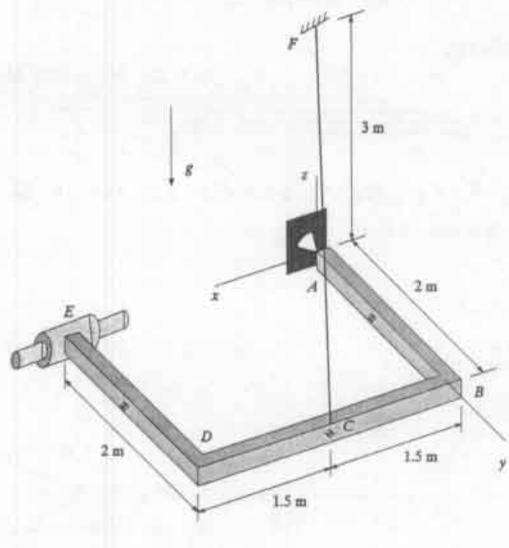


Figure a

5.132 Cont.

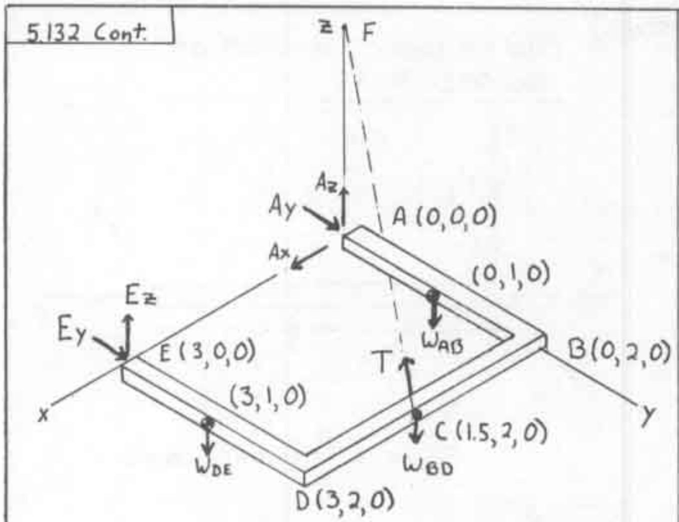


Figure b. Free-body diagram

Distribute the weight (140 KN) among the three bars proportional to their lengths.

By Fig. b,

$$W_{AB} = 140\left(\frac{L_{AB}}{L_{AB}+L_{BD}+L_{DE}}\right) = 140\left(\frac{2}{2+3+2}\right) = 40 \text{ kN}$$

$$W_{BD} = 140\left(\frac{3}{7}\right) = 60 \text{ kN}$$

$$W_{DE} = 140\left(\frac{2}{7}\right) = 40 \text{ kN}$$

$$\vec{T} = T\left(\frac{-1.5\hat{i} - 2\hat{j} + 3\hat{k}}{\sqrt{(-1.5)^2+(-2)^2+(3)^2}}\right) = (-0.3841\hat{i} - 0.5121\hat{j} + 0.7682\hat{k})T$$

By Fig. b the six equilibrium equations (in six unknowns) are

$\Sigma F_x = 0 = A_x - 0.3841\,T$

$\Sigma F_y = 0 = A_y + E_y - 0.5121T$

$\Sigma F_z = 0 = A_z + E_z + 0.7682T - 140$

$\Sigma M_x = 0 = F_z(y) - F_y(z)$
$= [(-40)(1) + (-60)(2) + (-40)(1) + (0.7682\,T)(2)] - [(-0.5121\,T)(0)]$

$\Sigma M_y = 0 = F_x(z) - F_z(x)$
$= [A_x(0)] - [(0.7682\,T)(1.5) + (E_z)(3) + (-60)(1.5) + (-40)(3)]$

$\Sigma M_z = 0 = F_y(x) - F_x(y)$
$= [(-0.5121\,T)(1.5) + (E_y)(3)] - [(-0.3841\,T)(2)]$

By computer software (simultaneous equation solver),

$A_x = 50$ kN	$E_y = 0$ kN
$A_y = 66.67$ kN	$E_z = -20$ kN
$A_z = 60$ kN	$T = 130.2$ kN

Alternatively, since only T is required, by examination, for moments summed about the x-axis, T is the only variable. Therefore,

$$0.7682\,T(2) = 40(1) + 60(2) + 40(1) = 200$$

or,

$$\underline{\underline{T = 130.2 \text{ kN}}}$$

5.133 Given Figure a:

a.) Write the six equilibrium equations

b.) Solve for G_x, G_y, G_z in terms of F_x, F_y, F_z, and W and show that one of the remaining equations is redundant.

c.) Explain the fact that one of the equations is redundant.

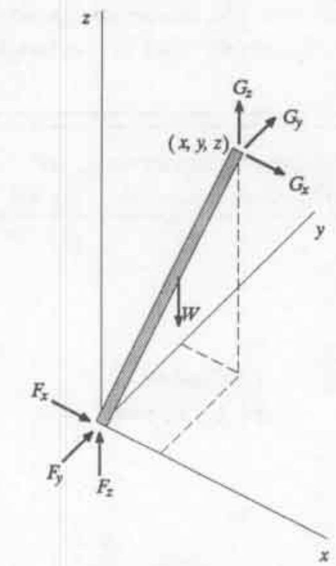


Figure a.

a.) By Figure a,

W acts at $(x/2,\ y/2,\ z/2)$.

The force equilibrium equations are

$$\Sigma F_x = F_x + G_x = 0 \qquad (1)$$

$$\Sigma F_y = F_y + G_y = 0 \qquad (2)$$

$$\Sigma F_z = F_z + G_z - W = 0 \qquad (3)$$

The moment equilibrium equations about the x, y, z axes are:

$$\Sigma M_x = -z\,G_y + y\,G_z - \tfrac{y}{2}W = 0 \qquad (4)$$

$$\Sigma M_y = z\,G_x - x\,G_z + \tfrac{x}{2}W = 0 \qquad (5)$$

$$\Sigma M_z = -y\,G_x + x\,G_y = 0 \qquad (6)$$

b.) By eqs. (1), (2), and (3),

$G_x = -F_x$

$G_y = -F_y$

$G_z = W - F_z$

Rearrange (4): $W = (2/y)(y\,G_z - z\,G_y)$

$$W = 2G_z - \frac{2z}{y}G_y \qquad (7)$$

5.133 cont.

Rearrange (5): $W = (2/x)(xG_z - zG_x)$

$$W = 2G_z - 2\tfrac{z}{x}G_x \qquad (8)$$

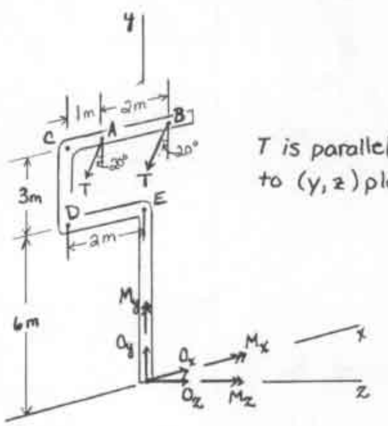
By eqs. (7) and (8);

$$2G_z - 2\tfrac{z}{y}G_y = 2G_z - 2\tfrac{z}{x}G_x$$

$$\therefore xG_y - yG_x = 0 \qquad (9)$$

Equation (9) is the same as equation (6). ∴ Equation (6) is redundant.

5.134 Find the support reactions at O of the frame shown in Figure P5.97



T is parallel to (y, z) plane

Figure a. FBD of Prob 5.117

From Problem 5.97, $T = 2\,kN$.
The free-body diagram of the frame of Fig. P5.97 is shown in Fig a.

By Fig a., Sum the forces and moments about O.

$\Sigma F_x = O_x = 0$

$\Sigma F_y = O_y - 2(2\cos 20°) = 0;\quad O_y = 3.759\ kN$

$\Sigma F_z = O_z - 2(2\sin 20°) = 0;\quad O_z = 1.368\ kN$

$\Sigma M_x = M_x - 2(2\sin 20°)(9) = 0;\quad M_x = 12.31\ kN{\cdot}m$

$\Sigma M_y = M_y = 0;$

$\Sigma M_z = M_z = 0;$

$$\therefore\ \vec{O} = 3.759\,\hat{j} + 1.368\,\hat{k}\ \ kN$$

$$\vec{M}_o = 12.31\,\hat{\imath}\ \ kN{\cdot}m$$

5.135 Find the support reactions of rod OAB (Fig a.)

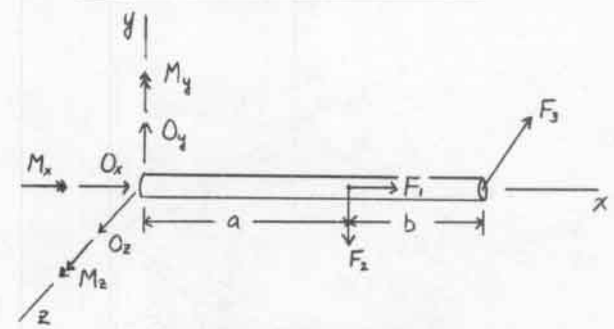


Figure a. Free body diagram

By Fig. a:

$\Sigma F_x = O_x + F_1 = 0;\quad O_x = -F_1$

$\Sigma F_y = O_y - F_2 = 0;\quad O_y = F_2$

$\Sigma F_z = O_z - F_3 = 0;\quad O_z = F_3$

$\Sigma M_x = M_x = 0$

$\Sigma M_y = M_y + F_3(a+b) = 0;\quad M_y = -F_3(a+b)$

$\Sigma M_z = M_z - F_2(a) = 0;\quad M_z = +F_2 a$

$$\therefore\ \vec{O} = -F_1\hat{\imath} + F_2\hat{j} + F_3\hat{k}$$

$$\vec{M}_o = -F_3(a+b)\hat{j} + F_2 a\hat{k}$$

5.136 Find all the forces that act on the spool when it is on the verge of rolling over the corner.

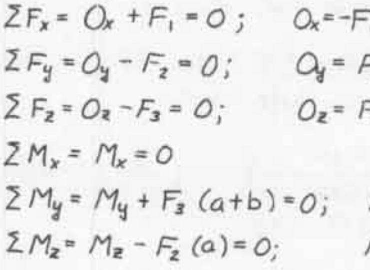


Figure a.

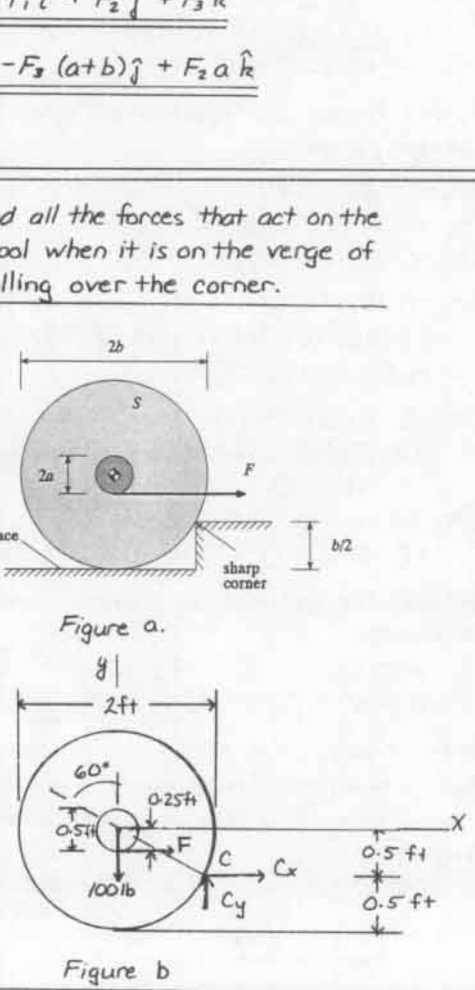


Figure b

5.136 cont.

By the free-body diagram of the spool (Fig b),

$\Sigma F_x = C_x + F = 0;$ $\quad C_x = -F$ lb

$\Sigma F_y = C_y - 100 = 0;$ $\quad C_y = 100$ lb

$\Sigma M_c = -F(0.5 - 0.25) + 100(\sin 60°) = 0$

$\vec{F} = 346.4\,\hat{i}$ lbs, $\quad \vec{C} = -346.4\,\hat{i} + 100\,\hat{j}$ lb

5.137

a.) Find all forces acting on the bar OA in Fig a. and show them in a diagram.

b.) Find W required for equilibrium.

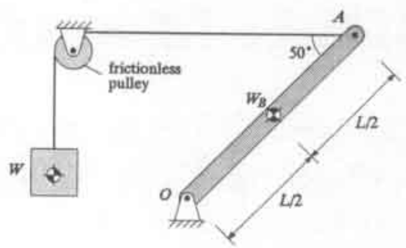


Figure a.

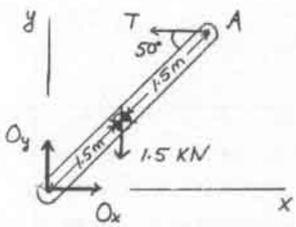


Figure b. Free body diagram

a.) By the free-body diagram of OA (Fig b.),

$\Sigma F_x = O_x - T = 0;$ $\quad O_x = T$

$\Sigma F_y = O_y - 1.5 = 0;$ $\quad O_y = 1.5$ kN

$\Sigma M_o = T(3\sin 50°) - 1.5(1.5\cos 50°) = 0;$

$T = 0.63$ kN

$\therefore O_x = 0.63$ kN, $\quad O_y = 1.5$ kN, $\quad T = 0.63$ kN

b.) By the free-body diagram of W (Fig c),

$\Sigma F_y = T - W = 0;$

$W = T$

$\therefore W = 0.63$ kN

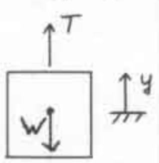


Figure c

5.138

Find the support reactions of bar OAB (weight = 0.5 kN/m), for a = 1m, b = 1.2m, and F = 2 kN.

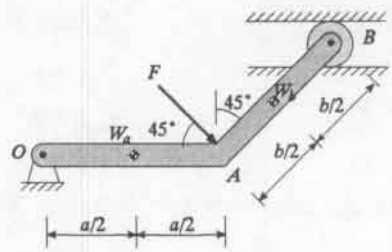


Figure a.

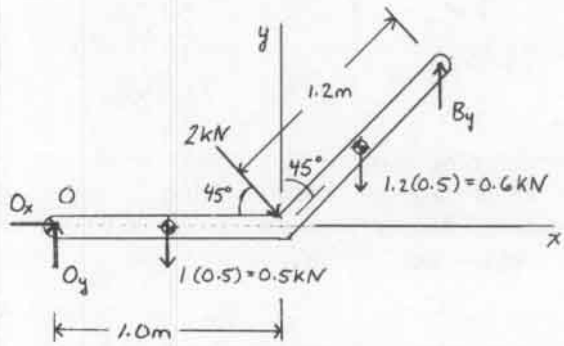


Figure b.

By the free-body diagram of bar OAB (Fig b.),

$\Sigma F_x = O_x + 2\cos 45° = 0;$ $\quad O_x = -1.414$ kN (1)

$\Sigma F_y = O_y + B_y - 0.5 - 0.6 - 2\sin 45° = 0$ $\quad$ (2)

$\Sigma M_o = -0.5(0.5) - 2\sin 45°(1)$

$-0.6(1 + 0.6\cos 45°) + B_y(1 + 1.2\cos 45°) = 0;$

$B_y = 1.363$ kN (3)

By Eqs (2) and (3); $\quad O_y = 1.152$ kN

$\therefore \vec{O} = -1.414\,\hat{i} + 1.152\,\hat{j}$ kN, $\quad \vec{B} = 1.363\,\hat{j}$ kN

5.139

Find the force systems that act on each bar in terms of a and W. (Each bar, AB and BC, in Fig a has weight W.)

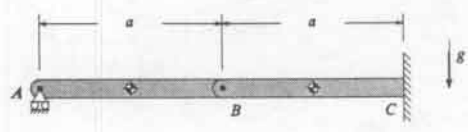


Figure a.

5.139 cont.

By the free-body diagram of bar AB (Fig. b);

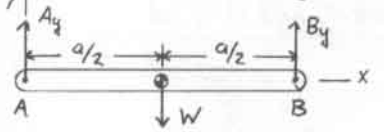


Figure b.

$\Sigma F_x = A_y + B_y - W = 0$

$\Sigma M_A = B_y(a) - W(a/2) = 0$

$\therefore\ A_y = W/2$

$B_y = W/2$

By the free-body diagram for bar BC (Fig c.);

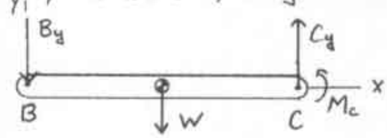


Figure c

$\Sigma F_y = -B_y + C_y - W = 0$

$\Sigma M_c = M_c + W(a/2) + B_y(a) = 0$

$\therefore\ C_y = 3/2\, W$

$M_c = Wa$

Note the reactions in the x-direction are zero because there are no horizontal forces that act on either bar AB or BC.

5.140

Find the force systems that act on bars AB and BC (Fig a). Each bar weighs 40 lb and has length $a = 4$ ft.

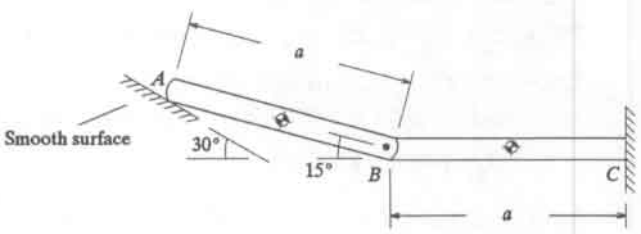


Figure a

The free-body diagrams of bars AB and BC are shown in Figs. b and c, respectively.

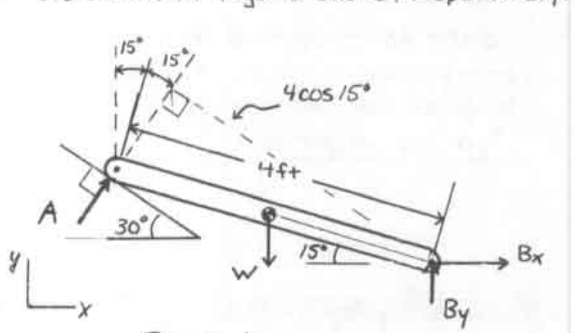


Figure b.

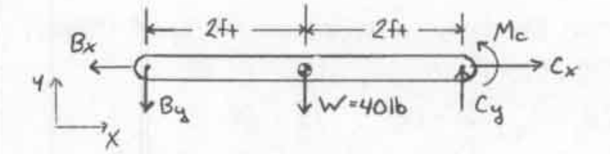


Figure c.

By Fig.b,

$$\left.\begin{aligned} &\Sigma F_x = B_x + A\cos 60° = 0 \\ &\Sigma F_y = A\sin 60° + B_y - 40 = 0 \\ &\Sigma M_B = (40)(2\cos 15°) - A(4\cos 15°) = 0 \end{aligned}\right\} \text{(a)}$$

The solution of Eqs (a) is

$A = 20.00$ lb, $B_x = -10.00$ lb, $B_y = 22.68$ lb

By Fig.c,

$\Sigma F_x = C_x - B_x = 0$; $\quad C_x = B_x = -10.00$ lb

$\Sigma F_y = C_y - B_y - 40 = 0$; $\quad C_y = 40 + B_y = 62.68$ lb

$\Sigma M_c = M_c + (40)(2) + B_y(4) = 0$;

$M_c = -170.72$ lb·ft

Hence,

$C_x = -10.00$ lb, $C_y = 62.68$ lb, $M_c = -170.72$ lbft

5.141

Find tensions in cable AB and AC and the reactions at O for the structural system shown in Fig. a. The boom weighs 3.6 kN.

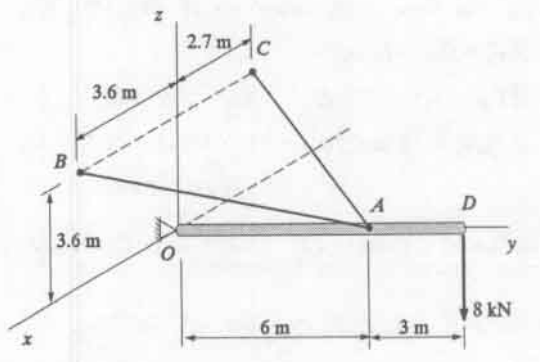


Figure a.

5.141 cont.

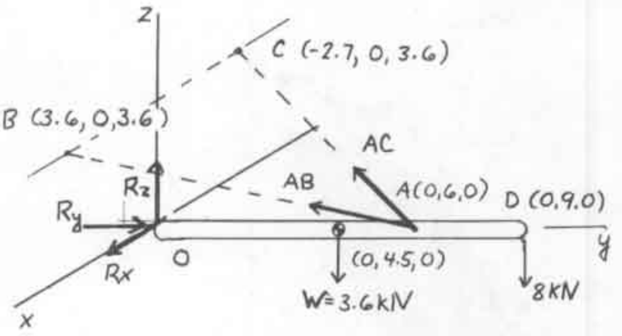


Figure b.

By Fig a., the unit vectors directed from A to B and from A to C are, respectively,

$$n_{AB} = \frac{3.6\hat{i} - 6\hat{j} + 3.6\hat{k}}{\sqrt{3.6^2 + (-6)^2 + 3.6^2}}$$

$$= 0.457\hat{i} - 0.762\hat{j} + 0.457\hat{k}$$

$$n_{AC} = \frac{-2.7\hat{i} - 6\hat{j} + 3.6\hat{k}}{\sqrt{(-2.7)^2 + (-6)^2 + 3.6^2}}$$

$$= -0.36\hat{i} - 0.8\hat{j} + 0.48\hat{k}$$

$$\vec{AB} = AB\,\vec{n}_{AB}, \qquad \vec{AC} = AC\,\vec{n}_{AC}$$

By Fig b., the free-body diagram of the boom, we have

$$\Sigma F_x = R_x + 0.457AB - 0.36AC = 0$$

$$\Sigma F_y = R_y - 0.762\,AB - 0.8\,AC = 0 \qquad (a)$$

$$\Sigma F_z = R_z - 8 - 3.6 + 0.457AB + 0.48AC = 0$$

$$\Sigma M_A = \vec{r}_o \times \vec{R} + \vec{r}_8 \times (-8\hat{k}) + \vec{r}_w \times (-3.6\hat{k})$$

$$\vec{r}_o = -6\hat{j} \quad \vec{r}_8 = 3\hat{j} \quad \vec{r}_w = -1.5\hat{j}$$

$$\Sigma M_A = (-6\hat{j}) \times (R_x\hat{i} + R_y\hat{j} + R_z\hat{k}) + (3\hat{j}) \times (-8\hat{k}) + (-1.5\hat{j}) \times (-3.6\hat{k}) = 0$$

$$= -6R_z\hat{i} + 6R_x\hat{k} - 24\hat{i} + 5.4\hat{i} = 0 \qquad (b)$$

By Eq. (b),

$$\hat{i} \quad -6R_z - 18.6\hat{i} = 0 \quad \rightarrow \quad \underline{R_z = -3.1 \text{ kN}}$$

$$\hat{j}: \quad 6R_x = 0 \quad \rightarrow \quad \underline{R_x = 0 \text{ kN}}$$

Then, by Eq (a),

$$\underline{AB = 13.8 \text{ kN}, \quad AC = 17.5 \text{ kN}, \quad R_y = 24.5 \text{ kN}}$$

5.142 Given Fig. a., find the forces on the links BD and BE.

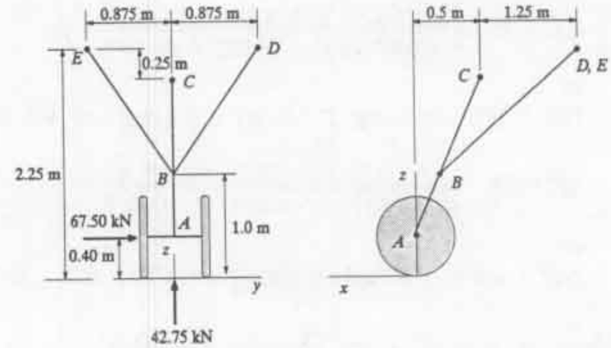


Figure a.

By Fig. a., relative to axis x, y, z, the coordinates of A, B, C, D, and E are

A (0, 0, 0.4) D (-1.75, 0.875, 2.25)
C (-0.5, 0, 2) E (-1.75, -0.875, 2.25)

By geometry, the coordinates of B are (See Fig. b. where now X, Y, Z axes are taken with origin at A.)

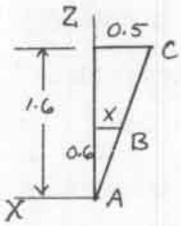


Figure b.

$$\frac{X}{0.5} = \frac{0.6}{1.6}$$

$$X = 0.1875$$

$$B = (-0.1875, 0, 1)$$

Hence, with respect to X.Y Z axes,

A (0,0,0)
B (-0.1875, 0, 0.6)
C (-0.5, 0, 1.6)
D (-1.75, 0.875, 1.85)
E (-1.75, -0.875, 1.85)

The free-body diagram of link AC is shown in Fig. c.

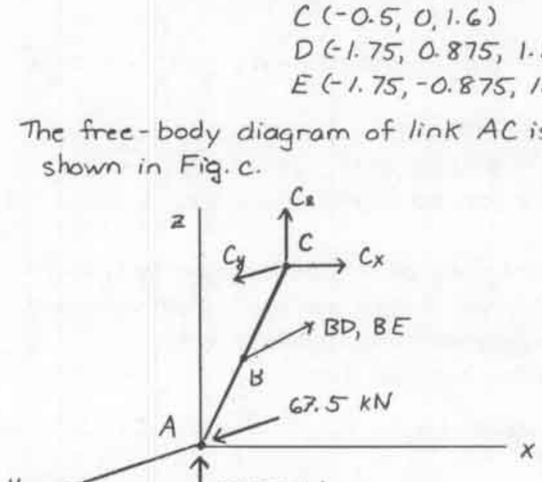


Figure c.

5.142 cont.

By Fig. c., resolving forces into vector components, we find;

$$\vec{BD} = BD\left(\frac{-1.563\hat{i} + 0.875\hat{j} + 1.25\hat{k}}{\sqrt{(-1.563)^2 + 0.875^2 + 1.25^2}}\right)$$

or

$$\vec{BD} = BD(-0.7156\,\hat{i} + 0.4006\,\hat{j} + 0.5724\,\hat{k}) \quad (a)$$

$$\vec{BE} = BE \quad \frac{-1.563\hat{i} - 0.875\hat{j} + 1.25\hat{k}}{\sqrt{(-1.563)^2 + (-0.875)^2 + 1.25^2}}$$

or

$$\vec{BE} = BE(-0.7156\hat{i} - 0.4006\hat{j} + 0.5724\hat{k}) \quad (b)$$

Also, by Fig. c., with Eqs. (a) and (b),

$$\left.\begin{aligned} \Sigma F_x &= C_x - 0.7156\ BD - 0.7156\ BE = 0 \\ \Sigma F_y &= C_y + 0.4006BD - 0.4006\,BE + 67.5 = 0 \\ \Sigma F_z &= C_z + 0.5724\,BD + 0.5724\,BE + 42.75 = 0 \end{aligned}\right\} (c)$$

$$\vec{M}_A = \vec{r}_{AB} \times \vec{BD} + \vec{r}_{AB} \times \vec{BE} + \vec{r}_{AC} \times \vec{C} = 0$$

where $\vec{r}_{AB} = -0.1875\,\hat{i} + 0.6\hat{k}$,

$\vec{r}_{AC} = -0.5\,\hat{i} + 1.6\,\hat{k}$

$$\therefore\ \vec{r}_{AB} \times \vec{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.1875 & 0 & 0.6 \\ -0.7156\,BD & 0.4006\,BD & 0.5724\,BD \end{vmatrix}$$

$$= -0.2404\,BD\hat{i} - 0.3221\,BD\hat{j} - 0.0751\,BD\,\hat{k}$$

$$\vec{r}_{AB} \times \vec{BE} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.1875 & 0 & 0.6 \\ -0.7156\,BE & -0.4006\,BE & 0.5724\,BE \end{vmatrix}$$

$$= 0.2404\,BE\,\hat{i} - 0.3221\,BE\,\hat{j} + 0.0751\,BE\,\hat{k}$$

$$\vec{r}_{AC} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.5 & 0 & 1.6 \\ C_x & C_y & C_z \end{vmatrix}$$

$$= -1.6\,C_y\,\hat{i} + (1.6\,C_x + 0.5\,C_z)\,\hat{j} - 0.5\,C_y\,\hat{k}$$

$$\begin{aligned} \vec{M}_A &= (-0.2404\ BD + 0.2404\ BE - 1.6\ C_y)\hat{i} \\ &+ (-0.3221\,BD - 0.3221\,BE + 1.6\,C_x + 0.5\,C_z)\hat{j} \\ &+ (-0.0751\,BD + 0.0751\,BE - 0.5\,C_y)\,\hat{k} = 0 \end{aligned} \quad (d)$$

Converting Eq. (d) into scalar equations, with Eq. (c), we obtain six equilibrium equations. Solving them simultaneously with computer aide, we find:

$C_x = 28.5$ kN, $C_y = 40.5$ kN, $C_z = -65.5$ kN

and

$BD = -114.9$ kN (compression)

$BE = 154.7$ kN (tension)

5.143

Refer to Problem 5.124. See Fig. a. below.

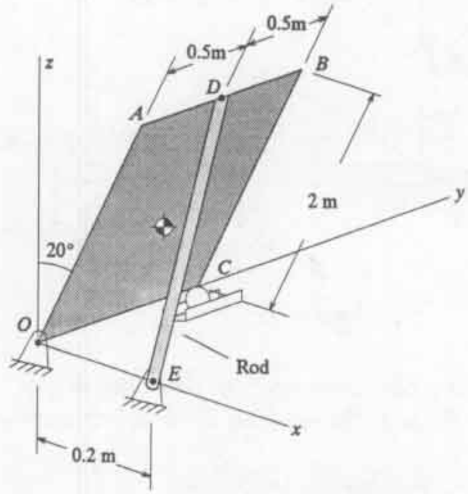


Figure a.

The rod DE is shortened and its upper end D pinned to the side OA of the hood, 0.3 m from point O. The center of gravity of the hood is located centrally in OABC.

Find the support reactions at O and C and the force in rod DE.

The free-body diagram of the hood is shown in Fig. (b).

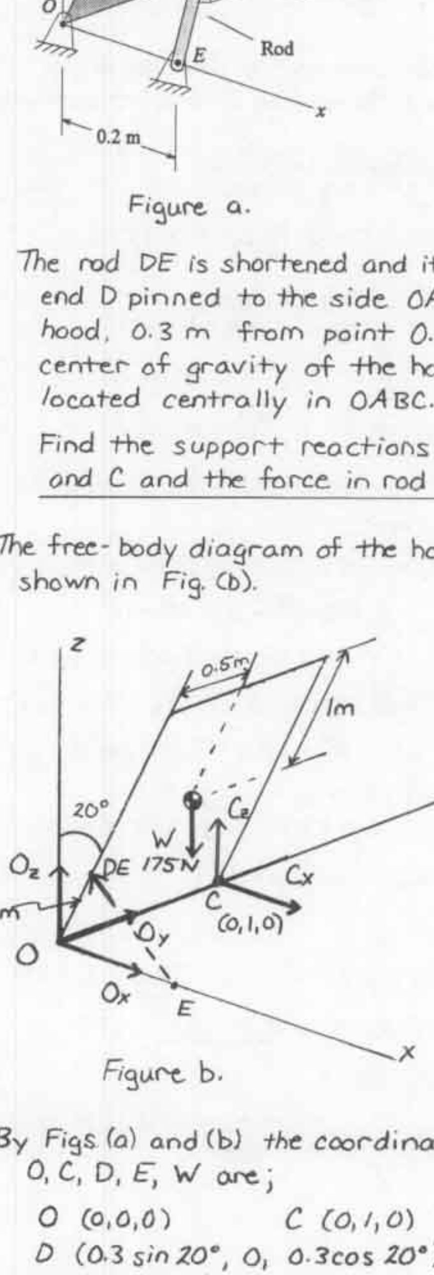


Figure b.

By Figs (a) and (b) the coordinates of O, C, D, E, W are;

O (0,0,0) $\qquad$ C (0,1,0)

D (0.3 sin 20°, 0, 0.3cos 20°)

E (0.2, 0, 0)

W (sin 20°, 0.5, cos 20°)

5.143 cont.

Therefore, the unit vector directed from E to D is:

$$\vec{n}_{DE} = \frac{(0.3\sin 20° - 0.2)\hat{i} + 0.3\cos 20°\hat{k}}{\sqrt{(0.3\sin 20° - 0.2)^2 + (0.3\cos 20°)^2}}$$

$$\vec{n}_{DE} = -0.3265\,\hat{i} + 0.9452\,\hat{k}$$

Hence

$$\vec{DE} = DE\,\vec{n}_{DE}$$

By Fig (b),

$$\Sigma F_x = O_x + C_x - 0.3265\,DE = 0$$

$$\Sigma F_y = O_y = 0 \qquad (a)$$

$$\Sigma F_z = O_z + C_z + 0.9452\,DE - 175 = 0$$

Also,

$$\vec{M}_o = \vec{r}_{DO} \times \vec{DE} + \vec{r}_{WO} \times \vec{W} + \vec{r}_{CO} \times \vec{C}$$

where by Fig (b),

$$\vec{r}_{DO} = 0.3\,(\sin 20°)\,\hat{i} + 0.3\,(\cos 20°)\,\hat{k}$$

$$\vec{r}_{WO} = (1)(\sin 20°)\,\hat{i} + 0.5\,\hat{j} + (1)(\cos 20°)\,\hat{k}$$

$$\vec{r}_{CO} = (1)\,\hat{j}$$

Therefore,

$$\vec{r}_{DO} \times \vec{DE} = (0.3\sin 20°\,\hat{i} + 0.3\cos 20°\,\hat{k}) \times (-0.3265\,\hat{i} + 0.9452\,\hat{k})\,DE = -0.1890\,DE\,\hat{j}$$

$$\vec{r}_{WO} \times \vec{W} = (\sin 20°\,\hat{i} + 0.5\,\hat{j} + \cos 20°\,\hat{k}) \times (-175\,\hat{k}) = -87.5\,\hat{i} + 59.85\,\hat{j}$$

$$\vec{r}_c \times \vec{C} = (\hat{j}) \times (C_x\,\hat{i} + C_z\,\hat{k}) = C_z\,\hat{i} - C_x\,\hat{k}$$

$$\therefore \vec{M}_o = (C_z - 87.5)\,\hat{i} + (59.85 - 0.1890\,DE)\,\hat{j} + (-C_x)\,\hat{k} = 0 \qquad (b)$$

Converting Eqn (b) into scalar equations, with Eqs. (a), we have six equilibrium equations. Solving them simultaneosly, we obtain the solution:

$C_x = 0$ N	$O_x = 103.4$ N
$C_z = 87.5$ N	$O_y = 0$ N
$DE = 316.7$ N	$O_z = -211.8$ N

Therefore

$$\vec{C} = 87.5\,\hat{k}\ \text{N}, \qquad \vec{O} = 103.4\,\hat{i} - 211.8\,\hat{k}\ \text{N}$$

$$\vec{DE} = -103.4\,\hat{i} + 299.3\,\hat{k}\ \text{N}$$

Alternatively, by Eq (b),

$$C_z - 87.5 = 0; \qquad C_z = 87.5\ \text{N}$$

$$59.85 - 0.1890\,DE = 0; \quad DE = 316.7\,\text{N} \qquad (c)$$

$$C_x = 0$$

Then with Eqs (a) and (c), we obtain

$$O_x = (0.3265)(316.7) = 103.4\ \text{N}$$

$$O_y = 0$$

$$O_z = 175 - (0.9452)(316.7) - 87.5 = -211.8\ \text{N}$$

as above.

5.144

Refer to roblem 5.143. The rod DE is attached as in Problem 5.143 (Fig b, Problem 5.143 solution). But now the supports at O and C are moved to $y_o = 0.2$ m and $y_c = 0.8$ m, respectively.

Find the support reactions at O and C and the force in rod DE

The free-body diagram of the hood is shown in Fig (a).

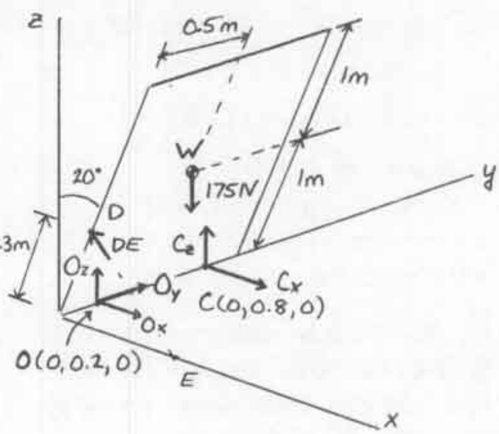


Figure a.

By Fig. a, we obtain the coordinates

O: (0, 0.2, 0)

C: (0, 0.8, 0)

D: (0.3 sin 20°, 0, 0.3 cos 20°)

E: (0.2, 0, 0)

W: (sin 20°, 0.5, cos 20°)

5.144 cont.

The unit vector directed from E to D is

$$\vec{n}_{DE} = \frac{(0.3\sin 20° - 0.2)\hat{\imath} + 0.3\cos 20°\hat{k}}{\sqrt{(0.3\sin 20° - 0.2)^2 + (0.3\cos 20°)^2}}$$

$$\vec{n}_{DE} = -0.3265\,\hat{\imath} + 0.9452\,\hat{k}$$

Hence, $\overrightarrow{DE} = DE\,\vec{n}_{DE}$

By Fig a.

$$\begin{aligned} \Sigma F_x &= O_x + C_x - 0.3265\,DE = 0 \\ \Sigma F_y &= O_y = 0 \\ \Sigma F_z &= O_z + C_z + 0.9452\,DE - 175 = 0 \end{aligned} \qquad (a)$$

Also,

$$\vec{M}_o = \vec{r}_{Do} \times \overrightarrow{DE} + \vec{r}_{wo} \times \vec{W} + \vec{r}_{co} \times \vec{C} = 0$$

Where, by Fig a.;

$$\vec{r}_{Do} = 0.3\sin 20°\,\hat{\imath} - 0.2\,\hat{\jmath} + 0.3\cos 20°\,\hat{k}$$

$$\vec{r}_{wo} = \sin 20°\,\hat{\imath} + 0.3\,\hat{\jmath} + \cos 20°\,\hat{k}$$

$$\vec{r}_{co} = 0.6\,\hat{\jmath}$$

$$\therefore \vec{r}_{Do} \times \overrightarrow{DE} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0.3\sin 20° & -0.2 & 0.3\cos 20° \\ -0.3265\,DE & 0 & 0.9452\,DE \end{vmatrix}$$

$$= -0.1890\,DE\,\hat{\imath} - 0.1890\,DE\,\hat{\jmath} - 0.0653\,DE\,\hat{k}$$

$$\vec{r}_{wo} \times \vec{W} = (\sin 20°\,\hat{\imath} + 0.3\,\hat{\jmath} + \cos 20°\,\hat{k}) \times (-175\,\hat{k})$$

$$= -52.5\,\hat{\imath} + 59.85\,\hat{\jmath}$$

$$\vec{r}_{co} \times \vec{C} = (0.6\,\hat{\jmath}) \times (C_x\hat{\imath} + C_z\hat{k})$$

$$= 0.6\,C_z\,\hat{\imath} - 0.6\,C_x\,\hat{k}$$

$$\begin{aligned} \vec{M}_o = {} & (-0.189\,DE - 52.5 + 0.6\,C_z)\hat{\imath} \\ & + (-0.189\,DE + 59.85)\hat{\jmath} \\ & + (-0.0653\,DE - 0.6\,C_x)\hat{k} = 0 \end{aligned} \qquad (b)$$

Converting Eq (b) into scalar equations, with Eqs (a), we have six equilibrium equations. Solving them simultaneously, we obtain the solution.

$C_x = -34.5$ N $\qquad O_x = 137.9$ N

$C_z = 187.3$ N $\qquad O_y = 0$ N

$DE = 316.7$ N $\qquad O_z = -311.6$ N

Therefore,

$$\vec{C} = -34.5\,\hat{\imath} + 187.3\,\hat{k}\ \text{N}$$

$$\vec{O} = 137.9\,\hat{\imath} - 311.6\,\hat{k}\ \text{N}$$

$$\overrightarrow{DE} = -103.4\,\hat{\imath} + 299.3\,\hat{k}\ \text{N}$$

5.145

Given Figure E 5.21 a (simulated here as Fig a.),

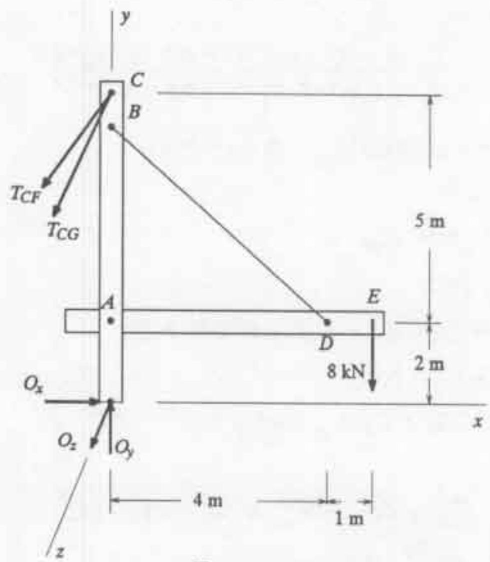


Figure a.

a.) Find tensions in DB, CF, and CG.
b.) Find the forces at A that act on ADE.
c.) Find support reaction at O.

From Example 5.21, the lengths of Cables CF and CG are each 9.219 m. To determine the location of point F, note by Fig. a that (after F is moved):

$$x_F^{\,2} + 7^2 + z_F^{\,2} = 9.219^2$$

$$\tan 45° = \frac{z_F}{x_F} \qquad x_F = z_F = -4.242\ \text{m}$$

And the coordinates of F, G, and C are

$$\begin{aligned} &F: (-4.242,\ 0,\ -4.242) \\ &G: (-5.196,\ 0,\ 3.000) \\ &C: (0, 7, 0) \end{aligned} \qquad (a)$$

a.) With Eqs (a), we have

$$\vec{T}_{CF} = T_{CF}\left(\frac{-4.242\,\hat{\imath} - 7\,\hat{\jmath} - 4.242\,\hat{k}}{9.219}\right)$$

$$= T_{CF}\,(-0.4601\,\hat{\imath} - 0.7593\,\hat{\jmath} - 0.4601\,\hat{k}) \qquad (b)$$

$$\vec{T}_{CG} = T_{CG}\left(\frac{-5.196\,\hat{\imath} - 7\,\hat{\jmath} + 3.000\,\hat{k}}{9.219}\right)$$

$$= T_{CG}\,(-0.5636\,\hat{\imath} - 0.7593\,\hat{\jmath} + 0.3254\,\hat{k}) \qquad (c)$$

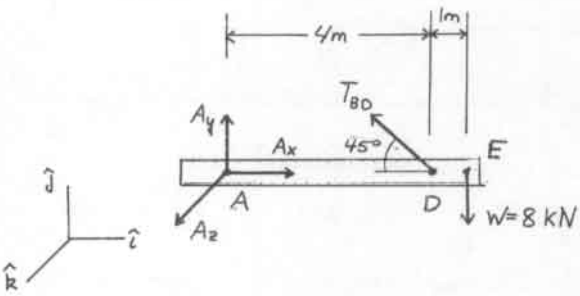
5.145 cont.

By Fig a,

$$\vec{T}_{DB} = T_{DB}\left(\frac{-4\hat{\imath}+4\hat{\jmath}}{\sqrt{(-4)^2+4^2}}\right)$$

$$= T_{DB}\,(-0.7071\,\hat{\imath} + 0.7071\,\hat{\jmath}) \qquad (d)$$

From the free-body diagram of Bar AD (See Fig b.) and Eq (d),



$$\Sigma M_A = \vec{r}_{AD} \times \vec{T}_{DB} + \vec{r}_{AE} \times \vec{W} = 0$$

$$= (4\hat{\imath}) \times (-0.7071\,T_{DB}\,\hat{\imath} + 0.7071\,T_{DB}\,\hat{\jmath})$$
$$+ (5\hat{\imath}) \times (-8\hat{\jmath}) = 0$$

$$= (2.8284\,T_{DB} - 40)\,\hat{k} = 0$$

$$\therefore \underline{\underline{T_{DB} = 14.14\text{ kN}}} \qquad (e)$$

Consider the free-body diagram of the whole crane (Fig c.);

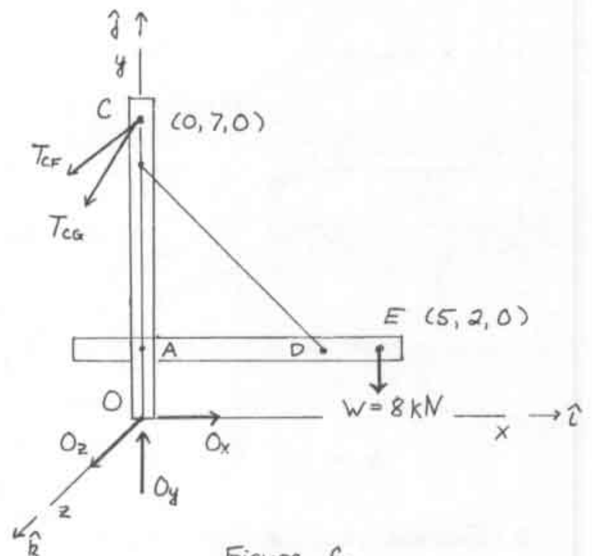


Figure c

By Fig. c,

$$\Sigma M_o = \vec{r}_{oc} \times (\vec{T}_{CF} + \vec{T}_{CG}) + \vec{r}_{oE} \times \vec{W} = 0 \qquad (f)$$

where

$$\vec{r}_{oc} = 7\hat{\jmath}, \quad \vec{r}_{DE} = 5\hat{\imath} + 2\hat{\jmath}$$

and

$$\vec{W} = -8\hat{\jmath}$$

By Eqs. (b), (c), and (f), we have

$$(2.2778\,T_{CG} - 3.2207\,T_{CF})\,\hat{\imath}$$
$$+ (3.9452\,T_{CG} + 3.2207\,T_{CF} - 40)\,\hat{k} = 0$$

or

$$2.2778\,T_{CG} - 3.2207\,T_{CF} = 0$$
$$3.9452\,T_{CG} + 3.2207\,T_{CF} = 40$$

The solution of these equations is

$$\underline{\underline{T_{CF} = 4.55\text{ kN}, \quad T_{CG} = 6.43\text{ kN}}} \qquad (g)$$

b.) To determine the forces at A that act on Bar ADE, by Fig b, we have

$$\Sigma F_x = A_x - 0.7071\,T_{BD} = 0$$
$$\Sigma F_y = A_y + 0.7071\,T_{BD} - 8 = 0 \qquad (h)$$
$$\Sigma F_z = A_z = 0$$

By Eqs. (e) and (h),

$$\underline{\underline{A_x = 10.0\text{ kN}, \quad A_y = 2\text{ kN}, \quad A_z = 0}}$$

c.) To determine the support reactions at O, by Fig. c, we find with Eqs. (b) and (c),

$$\Sigma F_x = O_x - 0.4601\,T_{CF} - 0.5636\,T_{CG} = 0$$
$$\Sigma F_y = O_y - 0.7593\,T_{CF} - 0.7593\,T_{CG} - 8 = 0$$
$$\Sigma F_z = O_z - 0.4601\,T_{CF} + 0.3254\,T_{CG} = 0$$

With Eq (g),

$$\underline{\underline{O_x = 5.72\text{ kN}, \quad O_y = 16.34\text{ kN}, \quad O_z = 0}}$$

5.146

Given Figure a. The point E of cable BE may be varied horizontally a distance d from the z axis.

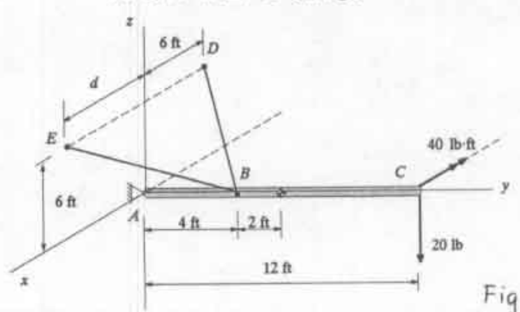


Figure a.

5.146 cont.

a.) Plot force BE in cable BE for $0 \le d \le 6$ ft.
b.) What restrictions must be placed on d, if $BE_{max} = 100$ lb?

The free-body diagram of the flagpole is shown in Fig. b;

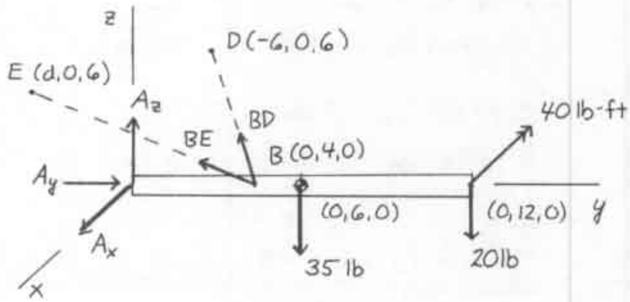


Figure b.

a.) By Fig. b,

$$\vec{BD} = BD\left(\frac{-6\hat{i} - 4\hat{j} + 6\hat{k}}{\sqrt{(-6)^2 + (-4)^2 + 6^2}}\right)$$

$$= BD\,(-0.6396\,\hat{i} - 0.4264\,\hat{j} + 0.6396\,\hat{k})$$

$$\vec{BE} = BE\left(\frac{d\hat{i} - 4\hat{j} + 6\hat{k}}{\sqrt{d^2 + (-4)^2 + 6^2}}\right) = BE\left(\frac{d\hat{i} - 4\hat{j} + 6\hat{k}}{\sqrt{d^2 + 52}}\right)$$

Since the support reactions are not required, the moment equations about the x, y, z axis need only be considered as follows:

$$\Sigma M_x = (0.6396\,BD)(4) + \left(\frac{6BE}{\sqrt{d^2+52}}\right)(4) - 35(6) - 20(12) - 40 = 0$$

$$= 2.5584\,BD + \frac{24\,BE}{\sqrt{d^2+52}} - 490 = 0$$

$$\Sigma M_y = 0$$

$$\Sigma M_z = (0.6396\,BD)(4) - \left(\frac{d\,BE}{\sqrt{d^2+52}}\right)(4) = 0$$

$$= 2.5584\,BD - \left(\frac{4d\,BE}{\sqrt{d^2+52}}\right) = 0$$

Solving for BE and BD in terms of d, we find:

$$BE = \frac{122.5\sqrt{d^2+52}}{d+6}$$

$$BD = \frac{191.5\,d}{d+6}$$

The plot of BE for d=0 to 6ft is shown in Fig. c.

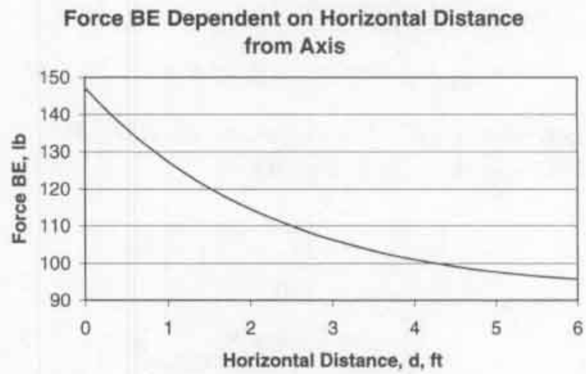


Figure c.

b.) If BE maximum is 100 lb, then:

$$100 \le \frac{122.5\sqrt{d^2+52}}{(d+6)}$$

$$\therefore \underline{\underline{d \ge 4.26 \text{ ft}}}$$

5.147

Given Figure a. in which the mechanism supports a weight W in various positions. Pins A, B, C, D are part of member ABCD.

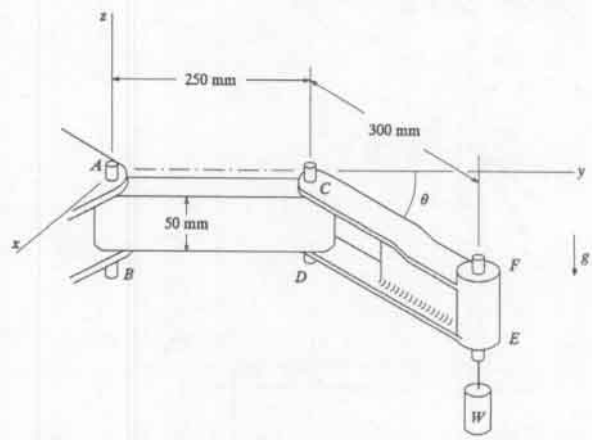


Figure a.

a.) Express reactions at A and B in terms of W and θ. The pin reactions are tangent to the surfaces of ABCD and there are no vertical forces at A and D.

b.) Let W = 200 N and plot reactions at A and B for $0 \le \theta \le 150°$.

5.147 cont.

The free-body diagrams of members ABCD and CDEF are shown in Figs. b. and c. respectively.

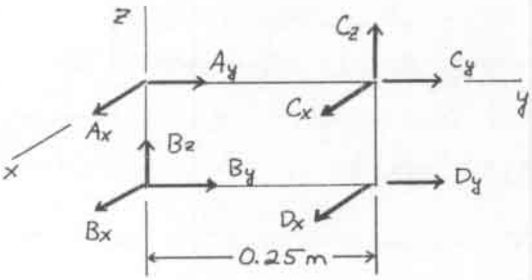


Figure b.

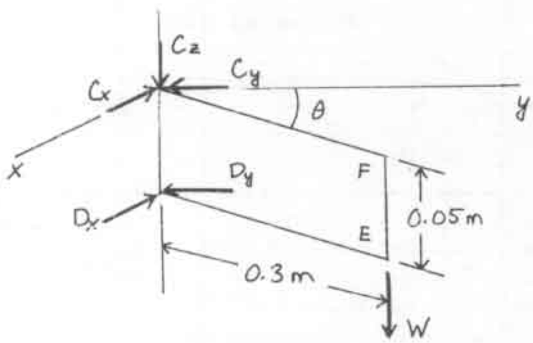


Figure c.

a.) By Fig. c,

$$\Sigma F_x = -C_x - D_x = 0; \quad C_x = -D_x$$

$$\Sigma F_y = -C_y - D_y = 0; \quad C_y = -D_y \qquad (a)$$

$$\Sigma F_z = -C_z - W = 0; \quad C_z = -W$$

$$\Sigma M_{cx} = -W(0.3\cos\theta) - D_y(0.05) = 0;$$

$$D_y = -6W\cos\theta \qquad (b)$$

$$\Sigma M_{cy} = W(0.3\sin\theta) + D_x(0.05) = 0;$$

$$D_x = -6W\sin\theta$$

Hence by Eqs (a) and (b),

$$C_x = 6W\sin\theta, \quad C_y = 6W\cos\theta, \quad C_z = -W$$

By Fig. b,

$$\Sigma F_x = A_x + B_x + 6W\sin\theta - 6W\sin\theta = 0;$$

$$A_x = -B_x$$

$$\Sigma F_y = A_y + B_y + 6W\cos\theta - 6W\cos\theta = 0; \qquad (c)$$

$$A_y = -B_y$$

$$\Sigma F_z = B_z - W = 0; \quad B_z = W$$

$$\Sigma M_x = -6W\cos\theta(0.05) - W(0.25) + B_y(0.05) = 0$$

$$B_y = 6W\cos\theta + 5W$$

$$B_y = -A_y = 6W\cos\theta + 5W \qquad (d)$$

$$\Sigma M_y = -B_x(0.5) + 6W\sin\theta(0.05) = 0$$

$$B_x = 6W\sin\theta$$

$$B_x = -A_x = 6W\sin\theta \qquad (e)$$

Hence, by Eqs. (c), (d), and (e),

$$A_x = -6W\sin\theta \qquad B_x = 6W\sin\theta$$

$$A_y = -6W\cos\theta - 5W \qquad B_y = 6W\cos\theta + 5W$$

$$B_z = W$$

b.) For $W = 200$ N

$$A_x = -1200\sin\theta$$
$$A_y = -1200\cos\theta - 1000$$
$$B_x = 1200\sin\theta$$
$$B_y = 1200\cos\theta + 1000$$
$$B_z = 200$$

The plots of the reactions for $0 \le \theta \le 150°$ are given in Fig. d.

The plot of A and B for $0 \le \theta \le 150°$ is given in Fig. e.

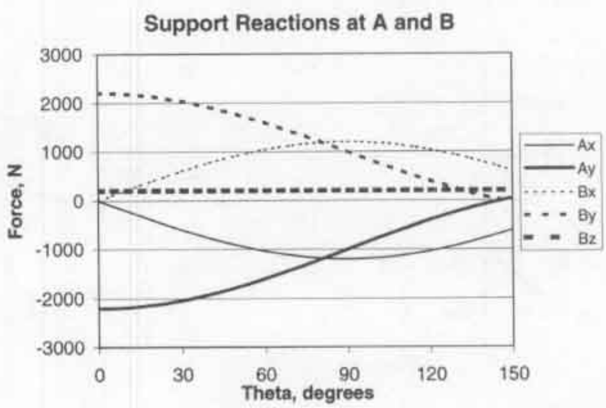


Figure d.

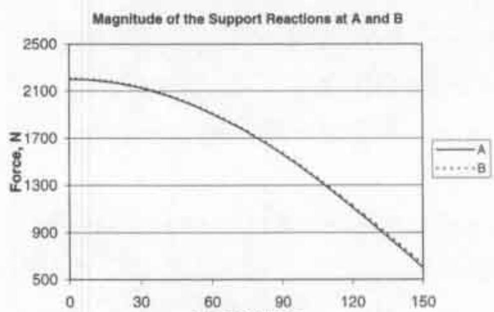


Figure e.

5.148 Given Figure a; forces P lie in a vertical plane.

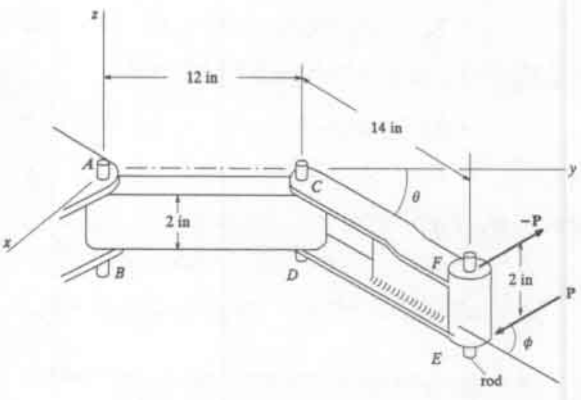


Figure a.

a.) Find (x, y, z) projections of reactions at A and B in terms of P and ϕ.

b.) Plot (x, y, z) projections of the reactions at A and B for P = 200 lb and $0 \le \phi \le 90°$.

c.) Plot magnitudes of A and B for P = 200 lb and $0 \le \phi \le 90°$.

The free-body diagram of the mechanism is given in Fig. b.

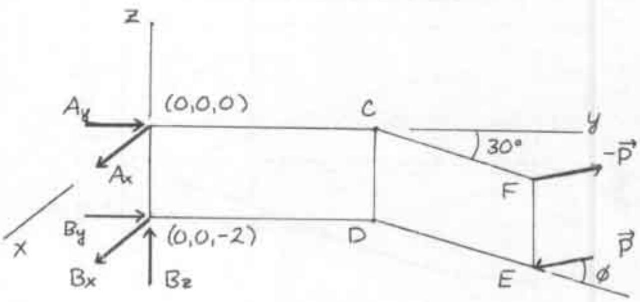


Figure b

By Fig. b,

$$\vec{P} = P\sin(30° - \phi)\hat{\imath} + P\cos(30° - \phi)\hat{\jmath}$$

$$-\vec{P} = -P\sin(30° - \phi)\hat{\imath} - P\cos(30° - \phi)\hat{\jmath} \qquad (a)$$

Combining $\vec{P}$ and $-\vec{P}$ into a couple, we have;

$$\vec{M}_F = \vec{r} \times (-\vec{P}) = (-2\hat{k}) \times (-P\sin(30° - \phi)\hat{\imath} - P\cos(30° - \phi)\hat{\jmath})$$

$$\vec{M}_F = -2P\cos(30 - \phi)\hat{\imath} + 2P\sin(30 - \phi)\hat{\jmath} \qquad (b)$$

$$\Sigma F_x = A_x + B_x = 0 \qquad A_x = -B_x$$

$$\Sigma F_y = A_y + B_y = 0 \qquad A_y = -B_y$$

$$\Sigma F_z = B_z = 0$$

$$\Sigma M_x = B_y(2) - 2P\cos(30° - \phi) = 0$$

$$\Sigma M_y = -B_x(2) + 2P\sin(30° - \phi) = 0$$

$$A_x = -P\sin(30° - \phi) \qquad B_x = P\sin(30° - \phi)$$

$$A_y = -P\cos(30° - \phi) \qquad B_y = P\cos(30° - \phi)$$

$$B_z = 0$$

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b.) The plots of A_x, A_y, B_x, B_y, B_z, for $\phi = 0$ to $90°$ and P = 200 lb are given in Fig. c.

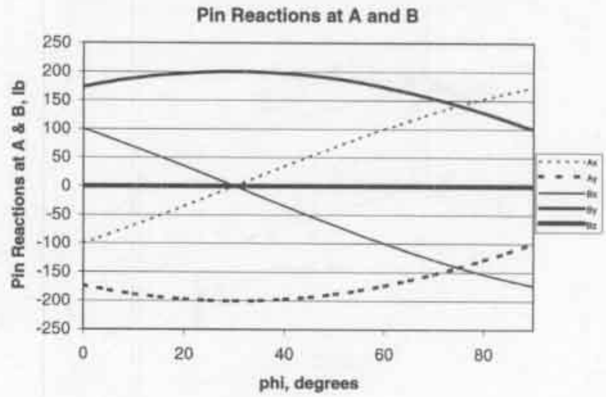


Figure c.

c.) $A = \sqrt{A_x^2 + A_y^2}$

$$= [(-200\sin(30° - \phi))^2 + (-200\cos(30° - \phi))^2]^{1/2}$$

$$= (40{,}000(\sin^2(30° - \phi) + \cos^2(30° - \phi)))^{1/2}$$

$$= 200 \text{ lb} = P$$

$B = \sqrt{B_x^2 + B_y^2 + B_z^2}$

$$= [(200\sin(30° - \phi))^2 + (200\cos(30° - \phi))^2]^{1/2}$$

$$= (40{,}000(\sin^2(30° - \phi) + \cos^2(30° - \phi)))^{1/2}$$

$$= 200 \text{ lb} = P$$

Therefore, A and B are independent of ϕ and have magnitude P = 200 lb. Therefore, the plot is a straight line (Fig. d).

This result could have been expected because the (x, y, z) projections of A and B are equal but opposite in sense. Therefore, their magnitudes are the same.

Also, $[P\sin(30° - \phi)]^2 + [P\cos(30° - \phi)]^2 = P^2 = A^2$

Therefore, A = P and since A = B, then B = P.

5.148 cont.

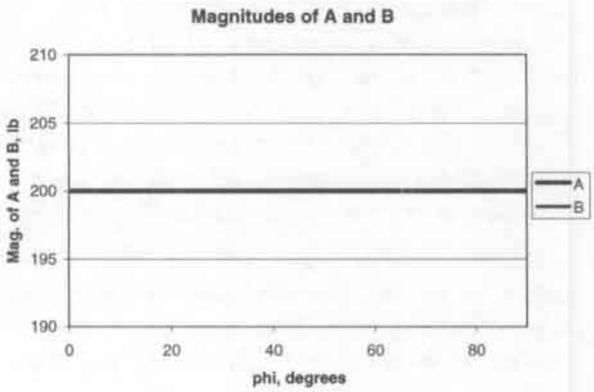


Figure d.

5.149

Given: In a preliminary design of a large uniform sign that weighs 1.6 kN, it is originally suggested that the sign be supported by two cables and a ball and socket joint. (Fig. a)

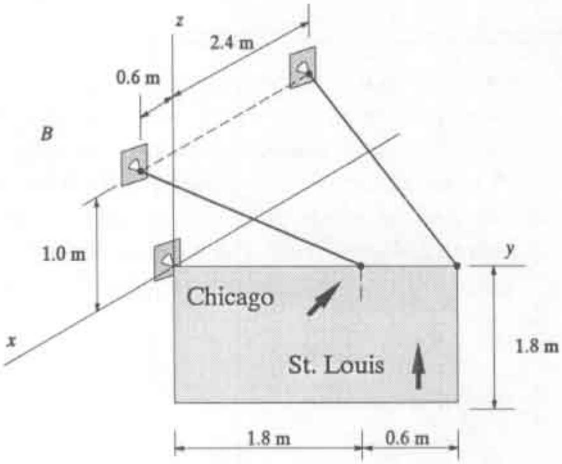


Figure a.

a.) Suggest two alternative placements of the cables to reduce the cable tensions.

b.) Write the advantages and disadvantages of your suggestion compared to the original.

a.) The free-body diagram of the sign is shown in Fig. b.

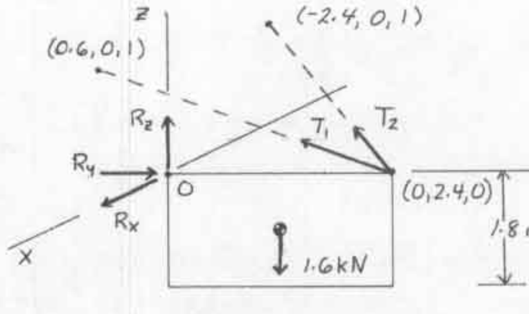


Figure b.

By Fig. b;

$$\vec{T}_1 = T_1\left(\frac{0.6\hat{\imath} - 1.8\hat{\jmath} + \hat{k}}{\sqrt{0.6^2 + (-1.8)^2 + 1^2}}\right)$$

$$= T_1\,(0.2798\,\hat{\imath} - 0.8394\,\hat{\jmath} + 0.4663\,\hat{k})$$

$$\vec{T}_2 = T_2\left(\frac{-2.4\hat{\imath} - 2.4\hat{\jmath} + \hat{k}}{\sqrt{(-2.4) + (-2.4)^2 + 1^2}}\right) \qquad (a)$$

$$= T_2\,(-0.6783\,\hat{\imath} - 0.6783\,\hat{\jmath} + 0.2826\,\hat{k})$$

$$\vec{W} = -1.6\,\hat{k}$$

$$\vec{r}_{T_1} = 1.8\hat{\jmath} \qquad \vec{r}_{T_2} = 2.4\hat{\jmath} \qquad \vec{r}_W = 1.2\hat{\jmath} - 0.9\hat{k}$$

Therefore,

$$\vec{M}_o = \vec{r}_{T_1} \times \vec{T}_1 + \vec{r}_{T_2} \times \vec{T}_2 + \vec{r}_W \times \vec{W} = 0$$

$$T_1\ 0.8393\,\hat{\imath} - 0.5036\,\hat{k}) + T_2(0.6782\,\hat{\imath} + 1.6279\,\hat{k})$$
$$-1.92\,\hat{\imath} = 0$$

So,
$$0.8393\,T_1 + 0.6782\,T_2 - 1.92 = 0$$
$$-0.5036\,T_1 + 1.6279\,T_2 = 0 \qquad (b)$$

The solution of Eqs (b) is

$$T_1 = 1.83\ \text{kN}, \qquad T_2 = 0.566\ \text{kN} \qquad (c)$$

Also, by Fig b. and Eqs (a) and (c),

$$\Sigma F_x = R_x + 0.2798\,T_1 - 0.6783\,T_2 = 0$$
$$\Sigma F_y = R_y - 0.8394\,T_1 - 0.6783\,T_2 = 0$$
$$\Sigma F_z = R_z + 0.4663\,T_1 + 0.2826\,T_2 - 1.92 = 0$$

The solution of these equations is:

$$R_x = -0.128\ \text{kN}, \quad R_y = 1.92\ \text{kN}, \quad R_z = 0.907\ \text{kN}$$

Design #1

Attach both cables to the sign at (0, 2.4, 0). Keep the anchor points of the other ends of the cables as in Fig. a.

5.149 cont.

Then,

$$\vec{T}_1 = T_1\left(\frac{0.6\hat{\imath} - 2.4\hat{\jmath} + \hat{k}}{\sqrt{0.6^2 + (-2.4)^2 + 1^2}}\right)$$

$$= T_1(0.2249\hat{\imath} - 0.8994\hat{\jmath} + 0.3748\hat{k}) \quad \text{(d)}$$

$$\vec{T}_2 = T_2(-0.6783\hat{\imath} - 0.6783\hat{\jmath} + 0.2826\hat{k})$$

(as before)

$$\therefore \vec{M}_o = \vec{r}_T \times (\vec{T}_1 + \vec{T}_2) + \vec{r}_w \times \vec{W} = 0 \quad (\vec{r}_T = 2.4\hat{\jmath})$$

$$(0.8995\,T_1 + 0.6782\,T_2)\hat{\imath} + (-0.5398\,T_1 + 1.6279\,T_2)\hat{k} - 1.92\,\hat{\imath} = 0$$

So,
$$0.8995\,T_1 + 0.6782\,T_2 - 1.92 = 0 \quad \text{(e)}$$
$$-0.5398\,T_1 + 1.6279\,T_2 = 0$$

The solution of Eqs (e) is

$$T_1 = 1.708 \text{ kN}, \quad T_2 = 0.566 \text{ kN} \quad \text{(f)}$$

For this configuration, T_1 is reduced by about 122 N without affecting T_2. Also, with Eqs (d) and (f):

$$\Sigma F_x = R_x + 0.2249\,T_1 - 0.6783\,T_2 = 0$$

$$\Sigma F_y = R_y - 0.8994\,T_1 - 0.6783\,T_2 = 0$$

$$\Sigma F_z = R_z + 0.3748\,T_1 + 0.2826\,T_2 - 1.92 = 0$$

The solution of these equations is:

$$R_x = 0, \quad R_y = 1.92 \text{ kN}, \quad R_z = 1.12 \text{ kN}$$

Design #2

Attach both cables to the sign at $(0, 2.4, 0)$. Locate the ends of the cables symmetrically so that the anchor points are $x = \pm 2.4$ m, $y = 0$, $z = 1$ m. Then,

$$\vec{T}_1 = T_1(0.6783\hat{\imath} - 0.6783\hat{\jmath} + 0.2826\hat{k})$$

$$\vec{T}_2 = T_2(-0.6783\hat{\imath} - 0.6783\hat{\jmath} + 0.2826\hat{k}) \quad \text{(g)}$$

$$\vec{r}_T = 2.4\hat{\jmath}$$

$$\vec{M}_o = \vec{r}_T \times (\vec{T}_1 + \vec{T}_2) + \vec{r}_w \times \vec{W} = 0 \quad (T_1 = T_2 = T)$$

or $1.3565\,T\,\hat{\imath} - 1.92\,\hat{\imath} = 0$

Therefore, $T = 1.415$ kN, $T_1 = T_2 = T$ (h)

By Eqs (g) and (h)

$$\Sigma F_x = R_x + 0.6783\,T - 0.6783\,T = 0$$

$$\Sigma F_y = R_y - 0.6783\,T - 0.6783\,T = 0$$

$$\Sigma F_z = R_z + 0.2826\,T + 0.2826\,T - 1.92 = 0$$

The solution of the above equations is,

$$R_x = 0, \quad R_y = 1.92 \text{ kN}, \quad R_z = 1.12 \text{ kN}$$

b.) By attaching both cables to the end of the sign (Design #1), the tension in the longer cable is reduced slightly without altering the tension in the other cable. Also, the out-of-plane reaction $R_x = 0$, $R_y = 1.92$ kN is unchanged and $R_z = 1.12$ kN is increased about 24%, compared to the original configuration.

If the anchor point of the shorter cable is moved farther away from the sign (Design #2), the tension in the cables is evened out. In doing so, the previously shorter cable picks up a significant amount of tension while the tension in the longer cable is reduced. Also, again $R_x = 0$, and $R_y = 1.92$ kN, $R_z = 1.12$ kN as in Design #1.

From the viewpoint of symmetry and the reduction of the out-of-plane reaction ($R_x = 0$), Design #2 is preferable.

5.150

Given: Figure a and Example 5.21, in which tensions in cables CF, CG, and DB and support reactions at A are determined. Note that the tension in cable DB (14.14 kN) is much larger than the tensions in cables CF and CG (5.07 kN).

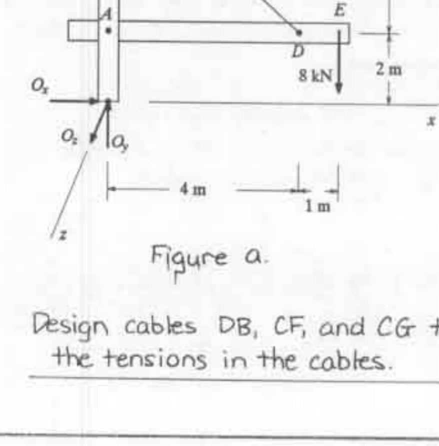


Figure a.

Design cables DB, CF, and CG to equalize the tensions in the cables.

5.150 cont.

Assumptions and Constraints:

- T_{CF} and T_{CG} remain symmetrical. Thus, $T_{CF} = T_{CG}$, $x_F = x_G$, $z_F = -z_G$
- Supports F and G remain in the (x, y) plane. Thus $y_F = y_G = 0$.
- Lengths of CBAO and AE remain unchanged at 7ft and 5ft respectively.
- Angle of cable DB remains at $45°$; $y_B = x_D$.
- Location of end supports fluctuate through a specific range:

  $-7 \le x_F \le 0, \quad -10 \le z_F \le 0$

  $0 \le y_C \le 7, \quad 0 \le x_D \le 5$

The design goal is to make the tensions in the cables equal; that is, to have

$T_{DB} = T_{CF} = T_{CG} = T$.

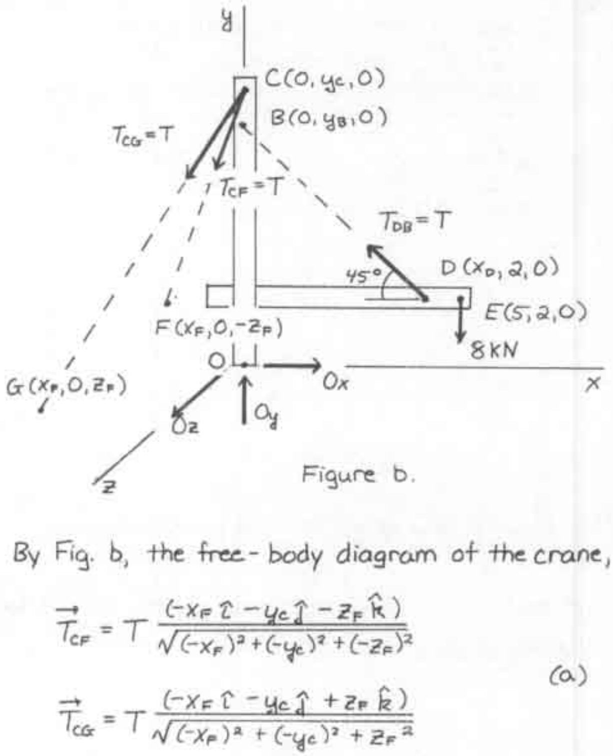


Figure b.

By Fig. b, the free-body diagram of the crane,

$$\vec{T}_{CF} = T\,\frac{(-x_F\,\hat{\imath} - y_C\,\hat{\jmath} - z_F\,\hat{k})}{\sqrt{(-x_F)^2 + (-y_C)^2 + (-z_F)^2}}$$

$$\vec{T}_{CG} = T\,\frac{(-x_F\,\hat{\imath} - y_C\,\hat{\jmath} + z_F\,\hat{k})}{\sqrt{(-x_F)^2 + (-y_C)^2 + z_F^2}} \qquad (a)$$

$$\vec{T}_{DB} = T\left(\frac{-\cos 45°\,\hat{\imath} + \sin 45°\,\hat{\jmath}}{\sqrt{(-\cos 45°)^2 + (\sin 45°)^2}}\right)$$

$$= T/\sqrt{2}\ \hat{\imath} + T/\sqrt{2}\ \hat{\jmath}$$

The free-body diagram of the boom is shown in Fig. c:

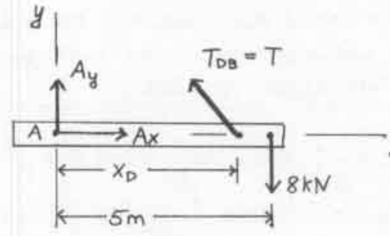


Figure c.

By Fig. c,

$$\Sigma M_A = T x_D(\sin 45°) - 8(5) = 0$$

or $\quad T x_D = 80/\sqrt{2} \qquad (b)$

By Fig. b and Eqs (a),

$$\Sigma M_x = \frac{T(-z_F)}{\sqrt{x_F^2 + y_C^2 + z_F^2}}\,(y_C) + \frac{T(z_F)}{\sqrt{x_F^2 + y_C^2 + z_F^2}}\,(y_C) = 0$$

$$\Sigma M_y = 0$$

$$\Sigma M_z = \frac{2T(-x_F)}{\sqrt{x_F^2 + y_C^2 + z_F^2}}\,(y_C) - 8(5) = 0$$

$$\frac{-T x_F y_C}{\sqrt{x_F^2 + y_C^2 + z_F^2}} = 20 \qquad (c)$$

By Eq (b), $\quad T = 80/x_D\sqrt{2} \qquad (d)$

Therefore, by Eqs. (c) and (d):

$$\frac{-80\,x_F\,y_C}{x_D\sqrt{2(x_F^2 + y_C^2 + z_F^2)}} = 20$$

or

$$\frac{x_F\,y_C}{x_D\sqrt{x_F^2 + y_C^2 + z_F^2}} = -0.3536 \qquad (e)$$

Equation (e) may be examined for the effects of moving the supports.

One solution is:

$x_D = 5\text{m} \quad y_C = 4\text{ m} \quad x_F = -5.3\text{ m}$

$z_F = -10\text{m} \quad T = 11.314\text{ kN} \qquad (f)$

Then, by Fig c, and Eq (e),

$$\Sigma F_x = A_x - \sqrt{2}/2\ T = 0$$

$$\Sigma F_y = A_y - \sqrt{2}/2\ T - 8 = 0$$

or $\quad A_x = (0.7071)(11.314) = 8\text{ kN}$

$\quad\ \ A_y = 8 - (0.7071)(11.314) = 0$

Thus, with the design of Eqs (f), A_x, A_y are reduced from their original values $A_x = 10\text{ kN}$, $A_y = -2\text{ kN}$. Other designs may be selected to satisfy Eq. (e).

5.157 Derive a formula for the angle between two lines with given direction cosines.

$\vec{A} = \cos\alpha_A \hat{\imath} + \cos\beta_A \hat{\jmath} + \cos\gamma_A \hat{k}, \qquad A = 1$

$\vec{B} = \cos\alpha_B \hat{\imath} + \cos\beta_B \hat{\jmath} + \cos\gamma_B \hat{k}, \qquad B = 1$

$\vec{A}\cdot\vec{B} = \cos\alpha_A \cos\alpha_B + \cos\beta_A \cos\beta_B + \cos\gamma_A \cos\gamma_B$

$= AB\cos\theta \quad = \cos\theta$

$\theta = \cos^{-1}\left[\cos\alpha_A\cos\alpha_B + \cos\beta_A\cos\beta_B + \cos\gamma_A\cos\gamma_B\right]$

5.162 Find the angle between $\vec{A}$ and $\vec{B}$.

$\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k} \qquad \vec{B} = 5\hat{\imath} - 2\hat{\jmath} - 4\hat{k}$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{AB}$$

$$\cos\theta = \frac{(2)(5) + (3)(-2) + (1)(-4)}{\sqrt{2^2+3^2+1^2}\ \sqrt{5^2+(-2)^2+(-4)^2}} = 0$$

$\theta = 90°$

5.164 Given: A particle is displaced a distance $AB = \sqrt{201}$ m along the line whose direction cosines are $(4/\sqrt{201},\ -4/\sqrt{201},\ 13/\sqrt{201})$

Find the projections of this displacement on the (x, y, z) axes.

$AB = \sqrt{201}$

$\hat{n}_{AB} = 4/\sqrt{201}\ \hat{\imath} - 4/\sqrt{201}\ \hat{\jmath} + 13/\sqrt{201}\ \hat{k}$

$\vec{AB} = AB\,\hat{n}_{AB} = 4\hat{\imath} - 4\hat{\jmath} + 13\hat{k}$

x- projection = 4

y- projection = −4

z - projection = 13

5.165 Given: The rectangular coordinates of a point P are $(4, -3, 0)$. The straight-line segment PO joins P to the origin.

Find the direction cosines and length L of PO.

The unit vector from O to P is:

$$\vec{n} = \frac{4\hat{\imath} - 3\hat{\jmath}}{\sqrt{4^2+(-3)^2}} = 0.8\hat{\imath} - 0.6\hat{\jmath}$$

Therefore, the direction cosines are

$\cos\alpha = 0.8, \quad \cos\beta = -0.6, \quad \cos\gamma = 0$

and the length L is $\sqrt{4^2+(-3)^2} = 5$

5.169 Show that the magnitude of $\vec{a}\times\vec{b}$ equals the area of the parallelogram with sides $\vec{a}$ and $\vec{b}$.

By definition,

$\vec{a}\times\vec{b} = ab\sin\theta$

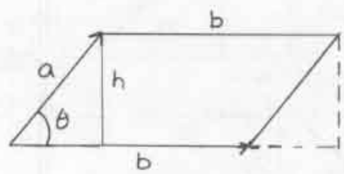


Figure a.

By Fig a, the area of the parallelogram is:

Area = bh, where $h = a\sin\theta$; $0 \le \theta \le 180°$

$\therefore$ Area $= ab\sin\theta = \vec{a}\times\vec{b}$

6.1

For Stability (Eqn. 6.1): $m + r \geq 2j$

For Static Determinancy: $m + r = 2j$

a) $m = 15 \quad r = 4 \quad j = 9$

$m + r = 19 \qquad 2j = 18$

$19 \geq 18 \rightarrow$ Stable

$19 \neq 18 \rightarrow$ Statically Indeterminate

b) $m = 15 \quad r = 3 \quad j = 9$

$m + r = 18 \qquad 2j = 18$

$18 = 18 \rightarrow$ Stable and Statically Determinate

c) Unstable, since the 3 reaction components are concurrent.

d) $m = 15 \quad r = 4 \quad j = 9$

$m + r = 19 \qquad 2j = 18$

$19 > 18 \rightarrow$ Stable

$19 \neq 18 \rightarrow$ Statically Indeterminate

e) $m = 10 \quad r = 4 \quad j = 7$

$m + r = 14 \qquad 2j = 14$

$14 = 14 \rightarrow$ Stable and Statically Determinate

f) $m = 11 \quad r = 4 \quad j = 8$

$m + r = 15 \qquad 2j = 16$

$15 < 16 \rightarrow$ Unstable

g) Unstable, since the 3 reaction components are concurrent.

h) $m = 10 \quad r = 3 \quad j = 6$

$m + r = 13 \qquad 2j = 12$

$13 > 12 \rightarrow$ Stable

$13 \neq 12 \rightarrow$ Statically Indeterminate

i) $m = 12 \quad r = 3 \quad j = 7$

$m + r = 15 \qquad 2j = 14$

$15 > 14 \rightarrow$ Stable

$15 \neq 14 \rightarrow$ Statically Indeterminate

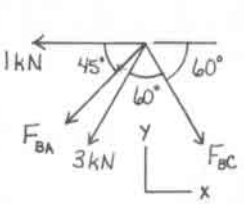
j) $m = 13 \quad r = 3 \quad j = 8$

$m + r = 16 \qquad 2j = 16$

$16 = 16 \rightarrow$ Stable and Statically Determinate

6.2

FBD of Joint B



$\Sigma F_x = -1 - F_{BA}\cos 45° - 3\cos 60° + F_{BC}\cos 60° = 0$

$\Sigma F_y = -F_{BA}\sin 45° - 3\sin 60° - F_{BC}\sin 60° = 0$

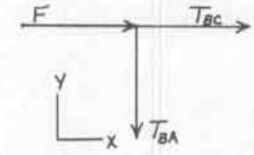
2 Equations and 2 Unknowns

$F_{BA} = -3.59$ kN (Compression)

$F_{BC} = -0.0717$ kN (Compression)

6.3

FBD of Joint B



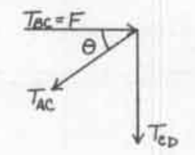
$\Sigma F_x = F + T_{BC} = 0$

$T_{BC} = -F$ (Compression)

$\Sigma F_y = -T_{BA} = 0$

$T_{BA} = 0$

FBD of Joint C



$\Sigma F_x = F - T_{AC}\cos\theta = 0$

$F = T_{AC}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$

$T_{AC} = \frac{F\sqrt{a^2+b^2}}{b}$ (Tension)

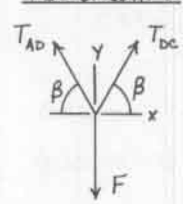
$\Sigma F_y = -T_{CD} - T_{AC}\sin\theta = 0$

$T_{CD} = -T_{AC}\left(\frac{a}{\sqrt{a^2+b^2}}\right)$

$T_{CD} = -\frac{Fa}{b}$ (Compression)

6.4

FBD of Joint D



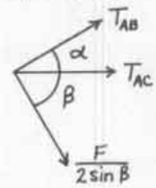
$\Sigma F_x = -T_{AD}\cos\beta + T_{DC}\cos\beta = 0$

$T_{AD} = T_{DC}$

$\Sigma F_y = -F + T_{DC}\sin\beta + T_{AD}\sin\beta = 0$

$T_{DC} = T_{AD} = \frac{F}{2\sin\beta}$ (Tension)

FBD of Joint A



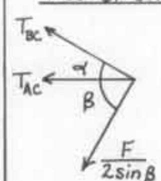
$\Sigma F_y = T_{AB}\sin\alpha - (F/2\sin\beta)\sin\beta = 0$

$T_{AB} = \frac{F}{2\sin\alpha}$ (Tension)

$\Sigma F_x = \frac{F}{2\sin\alpha}\cos\alpha + \frac{F}{2\sin\beta}\cos\beta + T_{AC} = 0$

$T_{AC} = -\frac{F}{2}\left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)$ (Compression)

FBD of Joint C

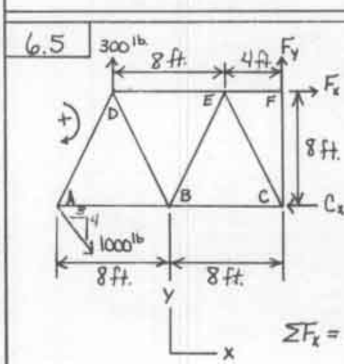


$\Sigma F_y = T_{BC}\sin\alpha - \left(\frac{F}{2\sin\beta}\right)\sin\beta = 0$

$T_{BC} = \frac{F}{2\sin\alpha}$ (Tension)

Also, by symmetry $T_{BC} = T_{AB}$.

6.5



$\Sigma F_y = 300 + F_y - 1000\left(\frac{4}{5}\right) = 0$

$F_y = 500$ lb.

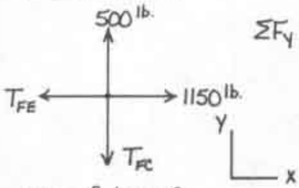
$\Sigma M_F = C_x(8) - 300(12) + 1000\left(\frac{4}{5}\right)(16) + 1000\left(\frac{3}{5}\right)(8) = 0$

$C_x = 1750$ lb.

$\Sigma F_x = 1000\left(\frac{3}{5}\right) + F_x - C_x = 0$

$F_x = 1150$ lb. (continued)

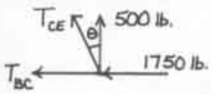
FBD of Joint F



$\Sigma F_y = 500 - T_{FC} = 0$

$T_{FC} = 500$ lb. (Tension)

FBD of Joint C



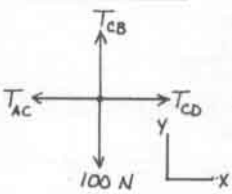
$\Sigma F_y = 500 + T_{CE} \cos\theta = 0$

$T_{CE} \frac{8}{\sqrt{4^2+8^2}} = -500$

$\underline{\underline{T_{CE} = -559 \text{ lb.}}}$ (Compression)

6.6

FBD of Joint C



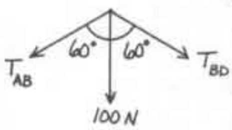
$\Sigma F_x = -T_{AC} + T_{CD} = 0$

$T_{AC} = T_{CD}$

$\Sigma F_y = -100 + T_{CB} = 0$

$\underline{\underline{T_{CB} = 100 \text{ N}}}$ (TENSION)

FBD of Joint B



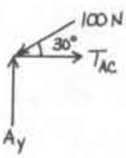
$\Sigma F_x = -T_{AB} \sin 60° + T_{BD} \sin 60° = 0$

$T_{AB} = T_{BD}$

$\Sigma F_y = -100 - T_{AB} \cos 60° - T_{BD} \cos 60° = 0$

$\underline{\underline{T_{AB} = T_{BD} = -100 \text{ N}}}$ (Compression)

FBD of Joint A



$\Sigma F_x = -100 \cos 30° + T_{AC} = 0$

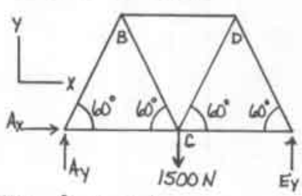
$\underline{\underline{T_{AC} = T_{CD} = 86.6 \text{ N}}}$ (Tension)

6.7

Because all triangles are equilateral, all member lengths are equal.

Due to symmetry: $T_{BC} = T_{DC}$, $T_{AC} = T_{EC}$, $T_{AB} = T_{ED}$, $A_y = E_y$

FBD of entire truss



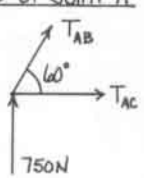
$\Sigma F_y = 2A_y - 1500 = 0$

$A_y = E_y = 750 \text{ N}$

$\Sigma F_x = A_x = 0$

$A_x = 0$

FBD of Joint A



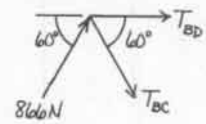
$\Sigma F_y = T_{AB} \sin 60° + 750 = 0$

$\underline{\underline{T_{AB} = T_{ED} = -866 \text{ N}}}$ (Compression)

$\Sigma F_x = T_{AB} \cos 60° + T_{AC} = 0$

$\underline{\underline{T_{AC} = T_{EC} = 433 \text{ N}}}$ (Tension)

FBD of Joint B



$\Sigma F_y = 866 \sin 60° - T_{BC} \sin 60° = 0$

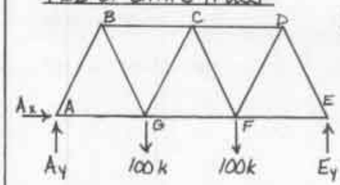
$\underline{\underline{T_{BC} = T_{DC} = 866 \text{ N}}}$ (Tension)

$\Sigma F_x = T_{BD} + 866 \cos 60° + 866 \cos 60° = 0$

$\underline{\underline{T_{BD} = -866 \text{ N}}}$ (Compression)

6.8

FBD of entire truss



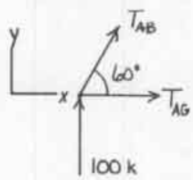
Due to symmetry the reactions at A and E are as follows:

$A_x = 0$

$A_y = E_y = 100k$

Also: $T_{AB} = T_{DE}$, $T_{AG} = T_{FE}$, $T_{BG} = T_{DF}$, $T_{GC} = T_{FC}$, $T_{BC} = T_{CD}$

FBD of Joint A



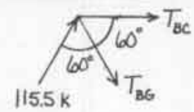
$\Sigma F_y = T_{AB} \sin 60° + 100 = 0$

$\underline{\underline{T_{AB} = T_{DE} = -115.5 \text{ k}}}$ (Compression)

$\Sigma F_x = T_{AB} \cos 60° + T_{AG} = 0$

$\underline{\underline{T_{AG} = T_{FE} = 57.7 \text{ k}}}$ (Tension)

FBD of Joint B



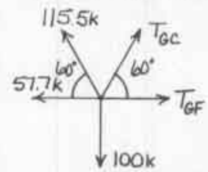
$\Sigma F_y = 115.5 \sin 60° - T_{BG} \sin 60° = 0$

$\underline{\underline{T_{BG} = T_{DF} = 115.5 \text{ k}}}$ (Tension)

$\Sigma F_x = 115.5 \cos 60° + 115.5 \cos 60° + T_{BC} = 0$

$\underline{\underline{T_{BC} = T_{CD} = -115.5 \text{ k}}}$ (Compression)

FBD of Joint G



$\Sigma F_y = 115.5 \sin 60° - 100 + T_{GC} \sin 60° = 0$

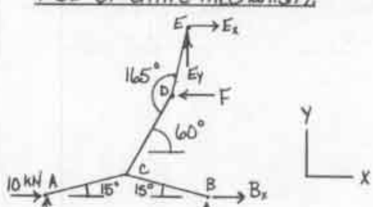
$\underline{\underline{T_{GC} = T_{FC} = 0 \text{ k}}}$

$\Sigma F_x = -57.7 - 115.5 \cos 60° + T_{GC} \cos 60° + T_{GF} = 0$

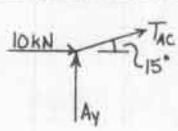
$\underline{\underline{T_{GF} = 115.5 \text{ k}}}$ (Tension)

6.9

FBD of entire mechanism.



FBD of Joint A

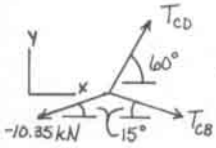


$\Sigma F_x = 10 + T_{AC} \cos 15° = 0$

$T_{AC} = -10.35 \text{ kN}$ (Compression)

(continued)

FBD of Joint C



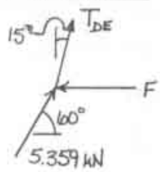
$\Sigma F_x = 10.35\cos 15° + T_{CD}\cos 60° + T_{CB}\cos 15° = 0$

$\Sigma F_y = 10.35\sin 15° + T_{CD}\sin 60° - T_{CB}\sin 15° = 0$

Solve these simultaneous equations

$T_{CD} = -5.359$ kN (Compression)

FBD of Joint D



$\Sigma F_y = 5.359\sin 60° + T_{DE}\cos 15° = 0$

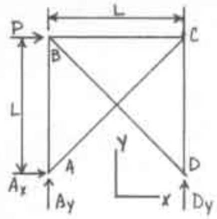
$T_{DE} = -4.80$ kN (Compression)

$\Sigma F_x = T_{DE}\sin 15° - F + 5.359\cos 60° = 0$

$\underline{F = 1.436 \text{ kN}}$

6.10

FBD of entire truss



$\Sigma F_x = P + A_x = 0$

$\underline{A_x = -P}$

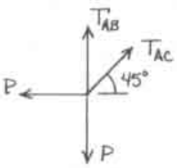
$\circlearrowleft + \ \Sigma M_A = -P(L) + D_y(L) = 0$

$\underline{D_y = P}$

$\Sigma F_y = A_y + D_y = 0$

$\underline{A_y = -P}$

FBD of Joint A



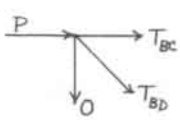
$\Sigma F_x = T_{AC}\cos 45 - P = 0$

$\underline{T_{AC} = \frac{P}{\cos 45°}}$ (Tension)

$\Sigma F_y = T_{AB} - P + T_{AC}\sin 45° = 0$

$\underline{T_{AB} = 0}$

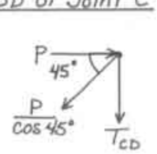
FBD of Joint B



$\underline{T_{BD} = 0}$

$\underline{T_{BC} = -P}$ (Compression)

FBD of Joint C

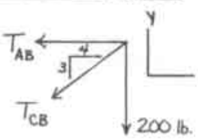


$\Sigma F_y = -T_{CD} - \frac{P}{\cos 45°}\sin 45° = 0$

$\underline{T_{CD} = -P}$ (Compression)

6.11

FBD of Joint B



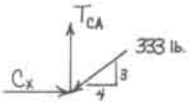
$\Sigma F_y = -200 - T_{CB}\left(\frac{3}{5}\right) = 0$

$\underline{T_{CB} = -333 \text{ lb.}}$ (Compression)

$\Sigma F_x = -T_{AB} - T_{CB}\left(\frac{4}{5}\right) = 0$

$\underline{T_{AB} = 267 \text{ lb. (Tension)}}$

FBD of Joint C

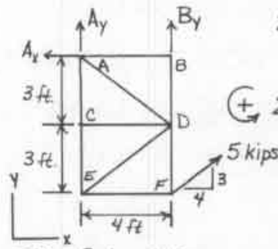


$\Sigma F_y = -333\left(\frac{3}{5}\right) + T_{CA} = 0$

$\underline{T_{CA} = 200 \text{ lb.}}$ (Tension)

6.12

FBD of entire truss



$\Sigma F_x = -A_x + 5\left(\frac{4}{5}\right) = 0$

$\underline{A_x = 4 \text{ kips}}$

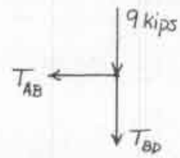
$\circlearrowleft + \ \Sigma M_B = -A_y(4) + 5\left(\frac{4}{5}\right)(6) = 0$

$\underline{A_y = 6 \text{ kips}}$

$\Sigma F_y = 6 + B_y + 5\left(\frac{3}{5}\right) = 0$

$\underline{B_y = -9 \text{ kips}}$

FBD of Joint B

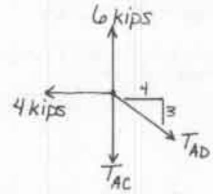


$\Sigma F_x = T_{AB} = 0$

$\Sigma F_y = -9 - T_{BD} = 0$

$\underline{T_{BD} = -9 \text{ kips}}$ (Compression)

FBD of Joint A



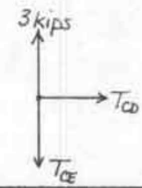
$\Sigma F_x = -4 + T_{AD}\left(\frac{4}{5}\right) = 0$

$\underline{T_{AD} = 5 \text{ kips}}$ (Tension)

$\Sigma F_y = -5\left(\frac{3}{5}\right) + 6 - T_{AC} = 0$

$\underline{T_{AC} = 3 \text{ kips}}$ (Tension)

FBD of Joint C

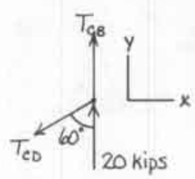


$\Sigma F_x = T_{CD} = 0$

$\underline{T_{CD} = 0}$

6.13

FBD of Joint C



$\Sigma F_x = T_{CD}\sin 60° = 0$

$\underline{T_{CD} = 0}$

$\Sigma F_y = T_{CB} + 20 = 0$

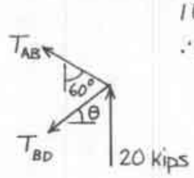
$\underline{T_{CB} = -20 \text{ kips}}$ (Compression)

By geometry, $DC = AB = 6$ ft. Also,

FBD of Joint B

$\left.\begin{array}{l} AB\sin 60° = DB\cos\theta \\ 10 - AB\cos 60° = DB\sin\theta \end{array}\right\} \Rightarrow \tan\theta = \frac{7}{3\sqrt{3}}$

$\therefore \ \theta = 53.4°$



$\Sigma F_x = -T_{AB}\sin 60° - T_{BD}\cos 53.4° = 0$

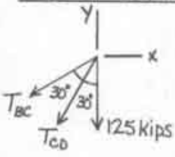
$\Sigma F_y = T_{AB}\cos 60° - T_{BD}\sin 53.4° + 20 = 0$

$\underline{T_{BD} = 17.44 \text{ kips}}$ (Tension)

$\underline{T_{AB} = -12.00 \text{ kips (Compression)}}$

6.14

FBD of Joint C



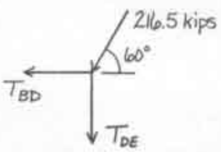
$\Sigma F_x = -T_{BC}\cos 30° - T_{CD}\sin 30° = 0$

$\Sigma F_y = -125 - T_{BC}\sin 30° - T_{CD}\cos 30° = 0$

$\underline{T_{CD} = -217 \text{ kips}}$ (Compression)

$\underline{T_{BC} = 125.0 \text{ kips}}$ (Tension)

FBD of Joint D



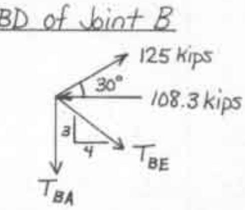
$\Sigma F_x = -216.5\cos 60° - T_{BD} = 0$

$\underline{T_{BD} = -108.3 \text{ kips}}$ (Compression)

$\Sigma F_y = -216.5\sin 60° - T_{DE} = 0$

$\underline{T_{DE} = -187.5 \text{ kips}}$ (Compression)

FBD of Joint B



$\Sigma F_x = 125\cos 30° - 108.3 + T_{BE}\left(\frac{4}{5}\right) = 0$

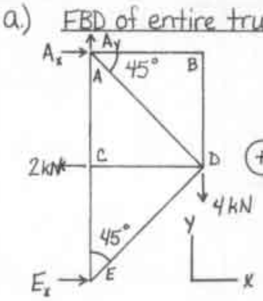
$\underline{T_{BE} = 0}$

$\Sigma F_y = 125\sin 30° - T_{BA} = 0$

$\underline{T_{BA} = 62.5 \text{ kips}}$ (Tension)

6.15

a) FBD of entire truss



$\Sigma F_y = A_y - 4 = 0$

$\underline{A_y = 4 \text{ kN}}$

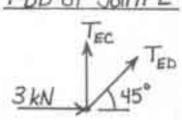
$(+\!\!\circlearrowleft)\ \Sigma M_A = -2(2) - 4(2) + E_x(4) = 0$

$\underline{E_x = 3 \text{ kN}}$

$\Sigma F_x = A_x + 3 - 2 = 0$

$\underline{A_x = -1 \text{ kN}}$

b) FBD of Joint E



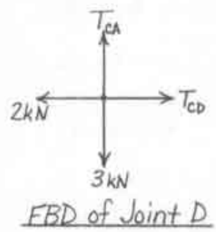
$\Sigma F_x = 3 + T_{ED}\cos 45° = 0$

$\underline{T_{ED} = -4.24 \text{ kN}}$ (Compression)

$\Sigma F_y = T_{ED}\sin 45° + T_{EC} = 0$

$\underline{T_{EC} = 3 \text{ kN}}$ (Tension)

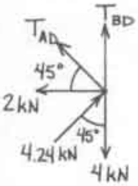
FBD of Joint C



$\underline{T_{CD} = 2 \text{ kN}}$ (Tension)

$\underline{T_{CA} = 3 \text{ kN}}$ (Tension)

FBD of Joint D



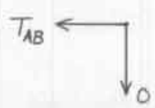
$\Sigma F_x = -T_{AD}\cos 45° - 2 + 4.24\cos 45° = 0$

$\underline{T_{AD} = 1.414 \text{ kN}}$ (Tension)

$\Sigma F_y = T_{BD} - 4 + 4.24\sin 45° + T_{AD}\sin 45° = 0$

$\underline{T_{BD} = 0}$

FBD of Joint B

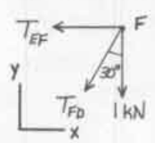


$\Sigma F_x = T_{AB} = 0$

$\underline{T_{AB} = 0}$

6.16

FBD of Joint F



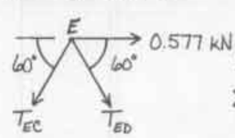
$\Sigma F_y = -1 - T_{FD}\cos 30° = 0$

$T_{FD} = -1.155 \text{ kN}$ (Compression)

$\Sigma F_x = -T_{FD}\sin 30° - T_{EF} = 0$

$T_{EF} = 0.577 \text{ kN}$ (Tension)

FBD of Joint E



$\Sigma F_x = 0.577 + T_{ED}\cos 60° - T_{EC}\cos 60° = 0$

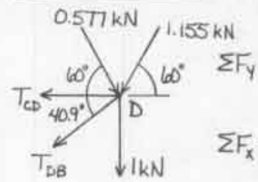
$\Sigma F_y = -T_{EC}\sin 60° - T_{ED}\sin 60° = 0$

$T_{EC} = -T_{ED}$

$\underline{T_{EC} = 0.577 \text{ kN}}$ (Tension)

$T_{ED} = -0.577 \text{ kN}$ (Compression)

FBD of Joint D



$\Sigma F_y = -1.155\sin 60° - 0.577\sin 60° - 1 - T_{DB}\sin 40.9° = 0$

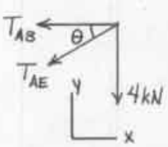
$\underline{T_{DB} = -3.82 \text{ kN}}$ (Compression)

$\Sigma F_x = -1.155\cos 60° + 0.577\cos 60° - T_{CD} - T_{DB}\cos 40.9° = 0$

$\underline{T_{CD} = 2.60 \text{ kN}}$ (Tension)

6.17

FBD of Joint A



$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$

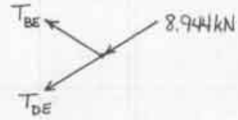
$\Sigma F_y = -4 - T_{AE}\sin 26.6° = 0$

$T_{AE} = -8.944 \text{ kN}$ (Compression)

$\Sigma F_x = -T_{AB} - T_{AE}\cos 26.6° = 0$

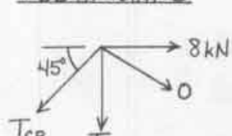
$T_{AB} = 8 \text{ kN}$ (Tension)

FBD of Joint E



By inspection $T_{BE} = 0$

FBD of Joint B



$\Sigma F_x = 8 - T_{CB}\cos 45° = 0$

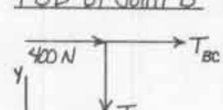
$T_{CB} = 11.31 \text{ kN}$ (Tension)

$\Sigma F_y = -T_{BD} - T_{CB}\sin 45° = 0$

$\underline{T_{BD} = -8 \text{ kN}}$ (Compression)

6.18

FBD of Joint B

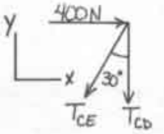


$\underline{T_{AB} = 0}$

$\underline{T_{BC} = -400 \text{ N}}$ (Compression)

(continued)

FBD of Joint C



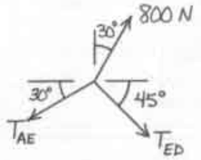
$\Sigma F_x = 400 - T_{CE}\sin 30° = 0$

$\underline{T_{CE} = 800\ N}$ (Tension)

$\Sigma F_y = -T_{CE}\cos 30° - T_{CD} = 0$

$\underline{\underline{T_{CD} = -693\ N}}$ (Compression)

FBD of Joint E



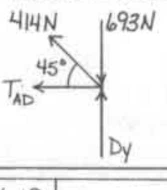
$\Sigma F_x = 800\sin 30° - T_{AE}\cos 30° + T_{ED}\cos 45° = 0$

$\Sigma F_y = 800\cos 30° - T_{AE}\sin 30° - T_{ED}\sin 45° = 0$

$\underline{\underline{T_{AE} = 800\ N}}$ (Tension)

$\underline{\underline{T_{ED} = 414\ N}}$ (Tension)

FBD of Joint D

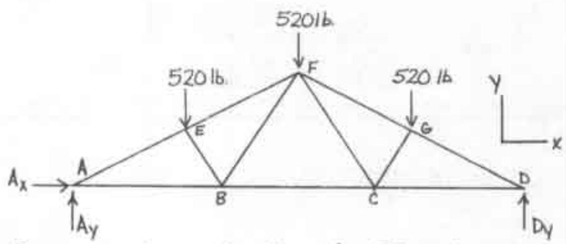


$\Sigma F_x = -T_{AD} - 414\cos 45° = 0$

$\underline{\underline{T_{AD} = -293\ N}}$ (Compression)

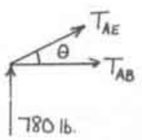
6.19

FBD of entire truss



By symmetry $\underline{A_x = 0}$ $\underline{A_y = 780\ lb}$ $\underline{D_y = 780\ lb}$

FBD of Joint A



$\theta = \tan^{-1}\left(\frac{6}{12}\right)$

$\theta = 26.6°$

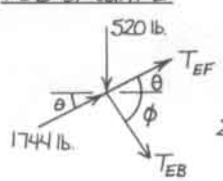
$\Sigma F_y = T_{AE}\sin 26.6° + 780 = 0$

$\underline{\underline{T_{AE} = -1{,}744\ lb.}}$ (Compression)

$\Sigma F_x = T_{AB} + T_{AE}\cos 26.6° = 0$

$\underline{\underline{T_{AB} = 1{,}560\ lb.}}$ (Tension)

FBD of Joint E



$\phi = \tan^{-1}\left(\frac{3}{2}\right)$

$\phi = 56.3°$

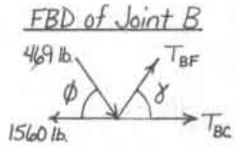
$\Sigma F_x = 1744\cos 26.6° + T_{EF}\cos 26.6° + T_{EB}\cos 56.3° = 0$

$\Sigma F_y = -520 + 1744\sin 26.6° + T_{EF}\sin 26.6° - T_{EB}\sin 56.3° = 0$

$\underline{\underline{T_{EF} = -1{,}453\ lb.}}$ (Compression)

$\underline{\underline{T_{EB} = -469\ lb.}}$ (Compression)

FBD of Joint B



$\gamma = \tan^{-1}\left(\frac{6}{4}\right)$

$\gamma = 56.3°$

$\Sigma F_y = -469\sin 56.3° + T_{BF}\sin 56.3° = 0$

$\underline{\underline{T_{BF} = 469\ lb.}}$ (Tension)

$\Sigma F_x = T_{BC} + 469\cos 56.3° + 469\cos 56.3° - 1560 = 0$

$\underline{\underline{T_{BC} = 1{,}040\ lb.}}$ (Tension)

By symmetry

$\underline{\underline{T_{DG} = -1{,}744\ lb.}}$ (Compression)

$\underline{\underline{T_{DC} = 1{,}560\ lb.}}$ (Tension)

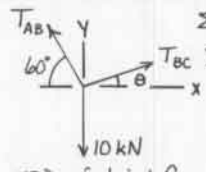
$\underline{\underline{T_{CG} = -469\ lb.}}$ (Compression)

$\underline{\underline{T_{GF} = -1{,}453\ lb.}}$ (Compression)

$\underline{\underline{T_{CF} = 469\ lb.}}$ (Tension)

6.20

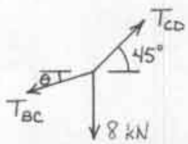
FBD of Joint B



$\Sigma F_x = -T_{AB}\cos 60° + T_{BC}\cos\theta = 0$ (a)

$\Sigma F_y = T_{AB}\sin 60° + T_{BC}\sin\theta - 10 = 0$ (b)

FBD of Joint C



$\Sigma F_x = -T_{BC}\cos\theta + T_{CD}\cos 45° = 0$ (c)

$\Sigma F_y = -T_{BC}\sin\theta + T_{CD}\sin 45° - 8 = 0$ (d)

Add (b) and (d)

$T_{AB}\sin 60° - 18 + T_{CD}\sin 45° = 0$ (e)

Add (a) and (c)

$-T_{AB}\cos 60° + T_{CD}\cos 45° = 0$ (f)

Solve (f) for T_{CD} and sub into (e)

$\underline{\underline{T_{AB} = 13.18\ kN}}$ (Tension)

Sub for T_{AB} in (f)

$\underline{\underline{T_{CD} = 9.32\ kN}}$ (Tension)

Sub for T_{AB} and T_{CD} in (a) and (d)

$T_{BC}\cos\theta = 6.588$ (g)

$T_{BC}\sin\theta = -1.4115$ (h)

Divide (h) by (g)

$\tan\theta = -0.2143$

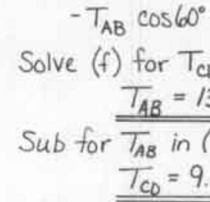
$\underline{\underline{\theta = -12.09°}}$

Sub for θ in (h)

$\underline{\underline{T_{BC} = 6.74\ kN}}$ (Tension)

6.21

FBD of Section (Cut through CD, BD, BE)



$\Sigma F_y = -5\left(\frac{4}{5}\right) + T_{BD}\sin 45° = 0$

$\underline{\underline{T_{BD} = 5.66\ kN}}$ (Tension)

$\circlearrowleft\!+\ \Sigma M_B = 5\left(\frac{4}{5}\right)(4) - T_{CD}(4m) = 0$

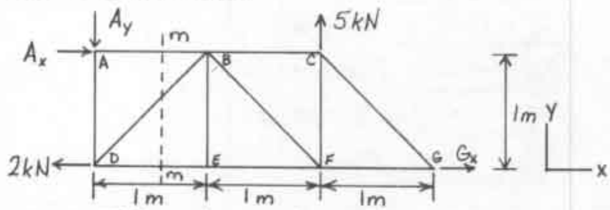
$\underline{\underline{T_{CD} = 4\ kN}}$ (Tension)

$\Sigma F_x = T_{CD} + T_{BE} + T_{BD}\cos 45° + 5\left(\frac{3}{5}\right) = 0$

$\underline{\underline{T_{BE} = -11\ kN}}$ (Compression)

6.22

FBD of entire truss



$\Sigma F_y = -A_y + 5 = 0$

$A_y = 5\ kN$

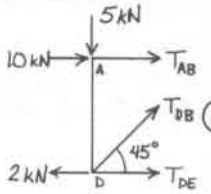
$\circlearrowleft + \ \Sigma M_A = -2(1) + 5(2) + G_x(1) = 0$

$G_x = -8\ kN$

$\Sigma F_x = A_x + G_x - 2 = 0$

$A_x = 10\ kN$

FBD of Section m-m



$\Sigma F_y = -5 + T_{DB} \sin 45^\circ = 0$

$\underline{T_{DB} = 7.07\ kN}$ (Tension)

$\circlearrowleft + \ \Sigma M_D = -10(1) - T_{AB}(1) = 0$

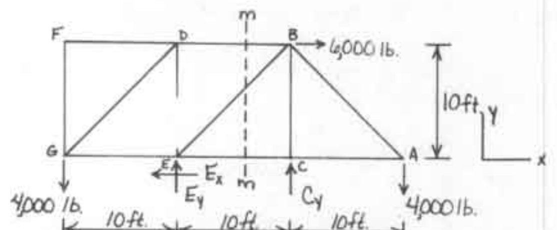
$\underline{T_{AB} = -10\ kN}$ (Compression)

$\Sigma F_x = 10 - 10 - 2 + 7.07 \cos 45^\circ + T_{DE} = 0$

$\underline{T_{DE} = -3\ kN}$ (Compression)

6.23

FBD of entire truss



$\Sigma F_x = -E_x + 6000 = 0$

$E_x = 6{,}000\ lb.$

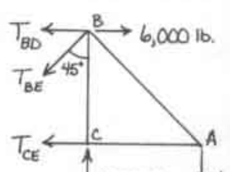
$\circlearrowleft + \ \Sigma M_E = 4000(10) + C_y(10) - 4000(20) - 6000(10) = 0$

$C_y = 10{,}000\ lb.$

$\Sigma F_y = -4000 + 10{,}000 - 4000 + E_y = 0$

$E_y = -2{,}000\ lb.$

FBD of Section m-m



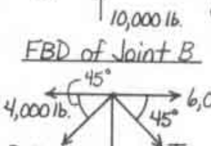
$\Sigma F_y = -4000 + 10{,}000 - T_{BE} \sin 45^\circ = 0$

$\underline{T_{BE} = 8{,}485\ lb.}$ (Tension)

$\circlearrowleft + \ \Sigma M_C = T_{BD}(10) - 6000(10) - 4000(10) + T_{BE} \cos 45^\circ (10)$

$\underline{T_{BD} = 4{,}000\ lb.}$ (Tension)

FBD of Joint B



$\Sigma F_x = -4000 - 8485 \cos 45^\circ + 6000 + T_{AB} \cos 45^\circ = 0$

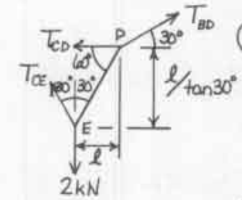
$T_{AB} = 5{,}656\ lb$ (Tension)

$\Sigma F_y = -8485 \sin 45^\circ - T_{BC} - 5656 \sin 45^\circ = 0$

$\underline{T_{BC} = -10{,}000\ lb.}$ (Compression)

6.24

FBD of Section (Cut through BD, CD, CE)



$\circlearrowleft + \ \Sigma M_D = 2(\ell) - T_{CE} \cos 30^\circ (\ell) - \frac{T_{CE} \sin 30^\circ \ell}{\tan 30^\circ} = 0$

$\underline{T_{CE} = 1.155\ kN}$ (Tension)

$\Sigma F_y = T_{BD} \sin 30^\circ + 1.155 \cos 30^\circ - 2 = 0$

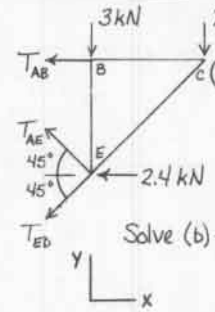
$\underline{T_{BD} = 2\ kN}$ (Tension)

$\Sigma F_x = 2 \cos 30^\circ - T_{CD} - 1.155 \sin 30^\circ = 0$

$\underline{T_{CD} = 1.155\ kN}$ (Tension)

6.25

FBD of Section (Cut through AB, AE, ED)



$\circlearrowleft + \ \Sigma M_E = T_{AB}(1) - 2(1) = 0$

$\underline{T_{AB} = 2\ kN}$ (Tension)

$\Sigma F_x = -T_{AB} - T_{AE} \cos 45^\circ - T_{ED} \cos 45^\circ - 2.4 = 0$ (a)

$\Sigma F_y = -3 - 2 + T_{AE} \sin 45^\circ - T_{ED} \sin 45^\circ = 0$ (b)

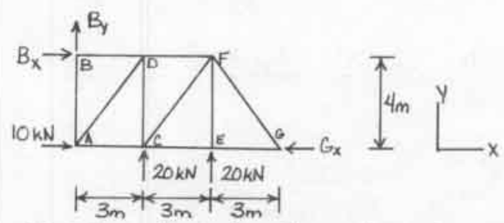
Solve (b) for T_{ED} and sub into (a) to get

$\underline{T_{AE} = 0.424\ kN}$ (Tension)

$\underline{T_{ED} = -6.65\ kN}$ (Compression)

6.26

FBD of entire truss

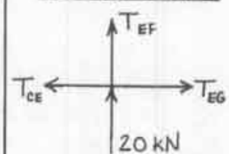


With the method of sections it is only necessary to find G_x in order to find the required member forces.

$\circlearrowleft + \ \Sigma M_B = 10(4) + 20(3) + 20(6) - G_x(4) = 0$

$G_x = 55\ kN$

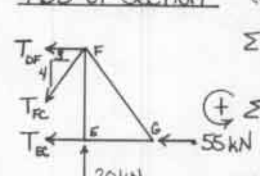
FBD of Joint E



$\Sigma F_y = T_{EF} + 20 = 0$

$\underline{T_{EF} = -20\ kN}$ (Compression)

FBD of Section (Cut through DF, FC, EC)



$\Sigma F_y = 20 - T_{FC}\left(\frac{4}{5}\right) = 0$

$T_{FC} = 25\ kN$ (Tension)

$\circlearrowleft + \ \Sigma M_F = -55(4) - T_{EC}(4) = 0$

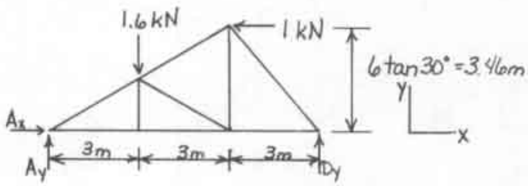
$\underline{T_{EC} = -55\ kN}$ (Compression)

$\Sigma F_x = -T_{EC} - 55 - T_{DF} - 25\left(\frac{3}{5}\right) = 0$

$\underline{T_{DF} = -15\ kN}$ (Compression)

6.27

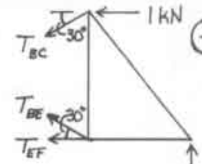
FBD of entire truss



$\circlearrowleft\!+\ \Sigma M_A = D_y(9) + 1(3.46) - 1.6(3) = 0$

$D_y = 0.1484\ kN$

FBD of Section (Cut through BC, BE, EF)



$\circlearrowleft\!+\ \Sigma M_E = 0.1484(3) + 1(3.46) + T_{BC}\cos 30°(3.46) = 0$

$\underline{\underline{T_{BC} = -1.303\ kN}}$ (Compression)

$\Sigma F_y = 0.1484 + 1.303\sin 30° + T_{BE}\sin 30° = 0$

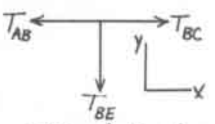
$\underline{\underline{T_{BE} = -1.60\ kN}}$ (Compression)

$\Sigma F_x = -1 + 1.303\cos 30° - T_{EF} + 1.6\cos 30° = 0$

$\underline{\underline{T_{EF} = 1.52\ kN}}$ (Tension)

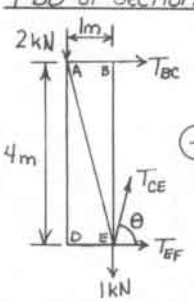
6.28

FBD of Joint B



$\underline{\underline{T_{BE} = 0}}$ (By inspection)

FBD of Section (Cut through BC, CE, EF)



$\tan\theta = 4 \quad \theta = 76.0°$

$\Sigma F_y = -2 - 1 + T_{CE}\sin 76.0° = 0$

$T_{CE} = 3.09\ kN$ (Tension)

$\circlearrowleft\!+\ \Sigma M_E = -T_{BC}(4) + 2(1) = 0$

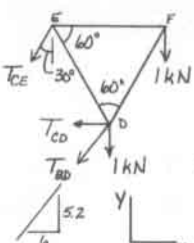
$\underline{\underline{T_{BC} = 0.5\ kN}}$ (Tension)

$\Sigma F_x = T_{BC} + T_{EF} + T_{CE}\cos 76.0° = 0$

$\underline{\underline{T_{EF} = -1.25\ kN}}$ (Compression)

6.29

FBD of Section (Cut through CE, CD, BD)



$\circlearrowleft\!+\ \Sigma M_D = T_{CE}\cos 30°(1.5) + T_{CE}\sin 30°(1.5\tan 60°) - 1(1.5) = 0$

$\underline{\underline{T_{CE} = 0.577\ kN}}$ (Tension)

$\Sigma F_y = -0.577\cos 30° - 2 - T_{BD}\left(\frac{5.2}{\sqrt{63}}\right) = 0$

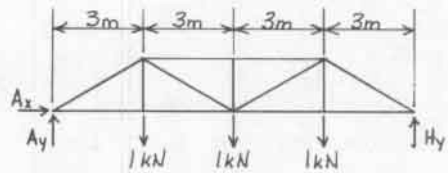
$\underline{\underline{T_{BD} = -3.82\ kN}}$ (Compression)

$\Sigma F_x = -T_{CD} - 0.577\sin 30° + 3.82\left(\frac{6}{\sqrt{63}}\right) = 0$

$\underline{\underline{T_{CD} = 2.60\ kN}}$ (Tension)

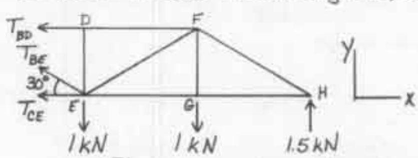
6.30

FBD of entire truss



By symmetry, $A_y = H_y = 1.5\ kN$

FBD of Section (Cut through BD, BE, CE)



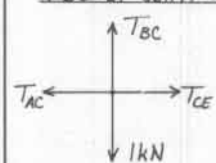
$\circlearrowleft\!+\ \Sigma M_E = T_{BD}(3\tan 30°) - 1(3) + 1.5(6) = 0$

$\underline{\underline{T_{BD} = -3.46\ kN}}$ (Compression)

$\Sigma F_y = -1.0 - 1.0 + 1.5 + T_{BE}\sin 30° = 0$

$\underline{\underline{T_{BE} = 1\ kN}}$ (Tension)

FBD of Joint C

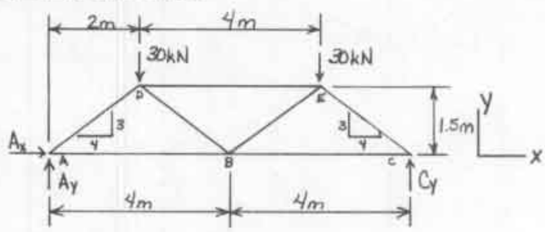


$\Sigma F_y = T_{BC} - 1 = 0$

$\underline{\underline{T_{BC} = 1\ kN}}$ (Tension)

6.31

FBD of entire truss

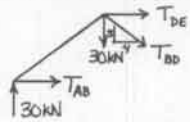


By symmetry

$\underline{\underline{A_x = 0}}$, $\underline{\underline{A_y = 30kN}}$, $\underline{\underline{C_y = 30kN}}$

Also, $T_{AD} = T_{CE}$, $T_{AB} = T_{BC}$, $T_{BD} = T_{BE}$

FBD of Section (Cut through AB, BD, DE)



$\Sigma F_y = 30 - 30 - T_{BD}\left(\frac{3}{5}\right) = 0$

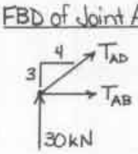
$\underline{\underline{T_{BD} = T_{BE} = 0}}$

$\Sigma F_x = T_{DE} + T_{AB} = 0$

$\circlearrowleft\!+\ \Sigma M_D = -30(2) + T_{AB}(1.5) = 0$

$\underline{\underline{T_{BC} = T_{AB} = 40\ kN}}$ (Tension)

FBD of Joint A:



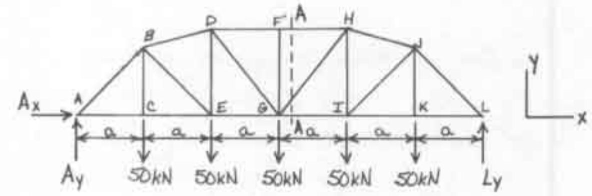
$\Sigma F_y = 30 + T_{AD}\left(\frac{3}{5}\right) = 0$

$\underline{\underline{T_{AD} = T_{CE} = -50\ kN}}$ (Compression)

$\underline{\underline{T_{DE} = -40\ kN}}$ (Compression)

6.32

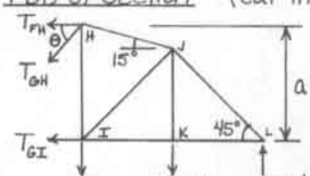
FBD of entire truss



By symmetry

$$L_y = \frac{5(50)}{2} = 125\ kN$$

FBD of Section (Cut through A-A)



$$\theta = \tan^{-1}\left(\frac{a + a \cdot \tan 15°}{a}\right)$$

$$\theta = 51.74°$$

$$\circlearrowleft + \ \Sigma M_H = -T_{GI}(a + a \cdot \tan 15°) - 50(a) + 125(2a) = 0$$

$$\underline{\underline{T_{GI} = 157.7\ kN}}\ (Tension)$$

$$\Sigma F_y = 125 - 50 - 50 - T_{GH} \sin 51.74° = 0$$

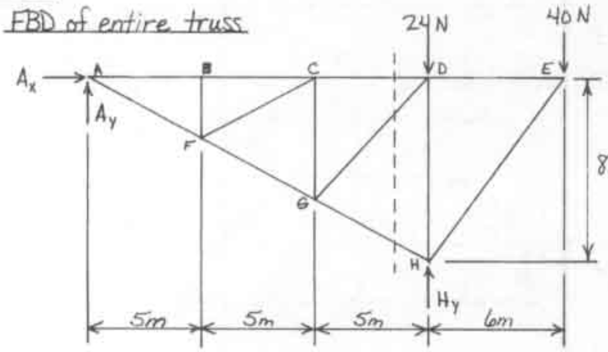
$$\underline{\underline{T_{GH} = 31.8\ kN}}\ (Tension)$$

$$\Sigma F_x = -157.7 - T_{FH} - 31.8 \cos 51.74° = 0$$

$$\underline{\underline{T_{FH} = -177.5\ kN}}\ (Compression)$$

6.33

FBD of entire truss



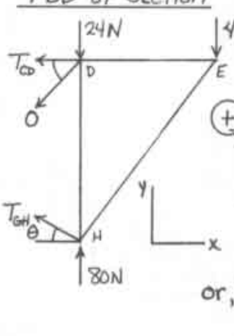
$$\circlearrowleft + \ \Sigma M_A = H_y(15) - 24(15) - 40(21) = 0$$

$$H_y = 80\ N$$

By inspection, BF is a zero-force member. Consequently, CF is zero-force, as are CG and DG.

$$T_{DG} = 0$$

FBD of Section



$$\tan\theta = \frac{8}{15}$$

$$\theta = 28.1°$$

$$\circlearrowleft + \ \Sigma M_D = -T_{GH} \cos 28.1°(8) - 40(6) = 0$$

$$T_{GH} = -34.0\ N\ (Compression)$$

$$\Sigma F_x = -T_{CD} - T_{GH} \cos 28.1° = 0$$

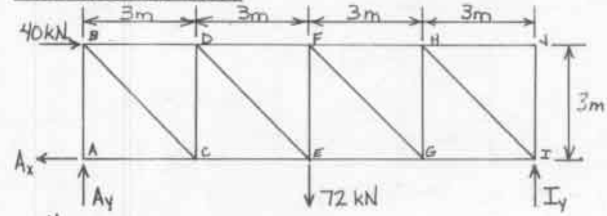
$$\underline{\underline{T_{CD} = 30.0\ N}}\ (Tension)$$

or, $\circlearrowleft + \ \Sigma M_H = 8T_{CD} - 40(6) = 0$

$$\underline{\underline{T_{CD} = 30.0 N\ (Tension)}}$$

6.34

FBD of entire truss



$$\Sigma F_x = 40 - A_x = 0$$

$$A_x = 40\ kN$$

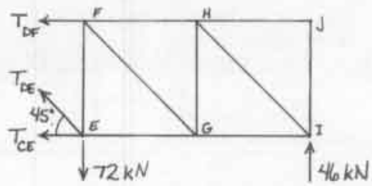
$$\circlearrowleft + \ \Sigma M_A = -40(3) - 72(6) + I_y(12) = 0$$

$$I_y = 46.0\ kN$$

$$\Sigma F_y = A_y + 46.0 - 72 = 0$$

$$A_y = 26.0\ kN$$

FBD of Section (Cut through DF, DE, CE)



$$\Sigma F_y = T_{DE} \sin 45° - 72 + 46 = 0$$

$$T_{DE} = 36.8\ kN\ (Tension)$$

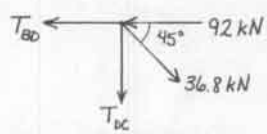
$$\circlearrowleft + \ \Sigma M_E = T_{DF}(3) + 46(6) = 0$$

$$\underline{\underline{T_{DF} = -92.0\ kN}}\ (Compression)$$

$$\Sigma F_x = 92.0 - T_{CE} - 36.8 \cos 45° = 0$$

$$\underline{\underline{T_{CE} = 66.0\ kN}}\ (Tension)$$

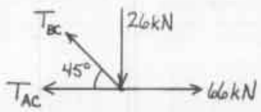
FBD of Joint D



$$\Sigma F_y = -36.8 \sin 45° - T_{DC} = 0$$

$$\underline{\underline{T_{DC} = -26.0\ kN}}\ (Compression)$$

FBD of Joint C

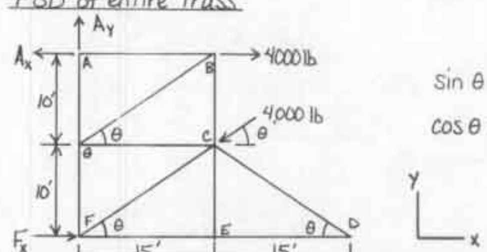


$$\Sigma F_y = -26 + T_{BC} \sin 45° = 0$$

$$\underline{\underline{T_{BC} = 36.8\ kN}}\ (Tension)$$

6.35

FBD of entire truss



$$\sin\theta = \frac{2}{\sqrt{13}} = 0.5547$$

$$\cos\theta = \frac{3}{\sqrt{13}} = 0.8321$$

$$\circlearrowleft + \ \Sigma M_F = A_x(20) - 4000(20) - 3000(15) - 2000(30) = 0$$

$$A_x = 9{,}250\ lb.$$

$$\Sigma F_y = A_y - 3000 - 2000 - 4000 \sin\theta = 0$$

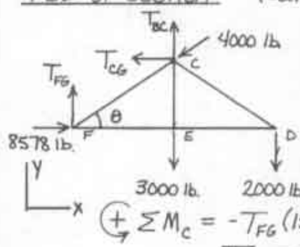
(continued)

$A_y = 7{,}219$ lb

$\Sigma F_x = F_x - 9250 - 4000\cos\theta + 4000 = 0$

$F_x = 8{,}578$ lb.

FBD of Section (Cut through BC, CG, FG)



$\Sigma F_x = -T_{CG} + 8578 - 4000\cos\theta = 0$

$\underline{\underline{T_{CG} = 5{,}250 \text{ lb (Tension)}}}$

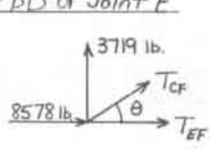
$\circlearrowleft\!\!+\ \Sigma M_C = -T_{FG}(15) + 8578(10) - 2000(15) = 0$

$\underline{\underline{T_{FG} = 3{,}719 \text{ lb. (Tension)}}}$

$\Sigma F_y = T_{FG} + T_{BC} - 4000\sin\theta - 3000 - 2000 = 0$

$\underline{\underline{T_{BC} = 3{,}500 \text{ lb. (Tension)}}}$

FBD of Joint E



$\Sigma F_y = 3719 + T_{CF}\sin\theta = 0$

$\underline{\underline{T_{CF} = -6{,}704 \text{ lb. (Compression)}}}$

$\Sigma F_x = 8578 + T_{CF}\cos\theta + T_{EF} = 0$

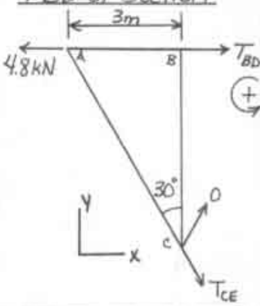
$\underline{\underline{T_{EF} = -3{,}000 \text{ lb. (Compression)}}}$

6.36

By inspection, BC is a zero-force member, $\underline{\underline{T_{BC} = 0}}$

Hence, CD is also zero-force, $\underline{\underline{T_{CD} = 0}}$

FBD of Section (Cut through BD, CD, CE)



$\circlearrowleft\!\!+\ \Sigma M_C = 4.8\left(\frac{3}{\tan 30°}\right) - T_{BD}\left(\frac{3}{\tan 30°}\right) = 0$

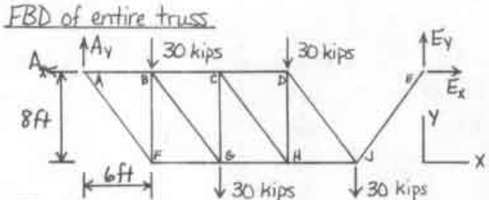
$\underline{\underline{T_{BD} = 4.8 \text{ kN (Tension)}}}$

$\Sigma F_x = T_{CE}(\sin 30°) + T_{BD} - 4.8 = 0$

$\underline{\underline{T_{CE} = 0}}$

6.37

FBD of entire truss



$\circlearrowleft\!\!+\ \Sigma M_A = -30(6) - 30(12) - 30(18) - 30(24) + E_y(30) = 0$

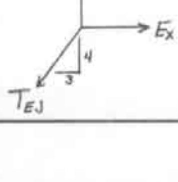
$E_y = 60$ kips

$\Sigma F_y = A_y + E_y - 120 = 0$

$A_y = 60$ kips

$\Sigma F_x = E_x - A_x = 0 \qquad A_x = E_x \quad (1)$

FBD of Joint E



$\Sigma F_y = 60 - T_{EJ}\left(\frac{4}{5}\right) = 0$

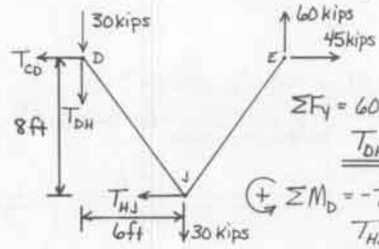
$T_{EJ} = 75$ kips

$\Sigma F_x = E_x - 75\left(\frac{3}{5}\right) = 0$

$E_x = 45$ kips

$A_x = E_x \qquad A_x = 45$ kips

FBD of Section (Cut through CD, DH, HJ)



$\Sigma F_y = 60 - 30 - 30 - T_{DH} = 0$

$\underline{\underline{T_{DH} = 0}}$

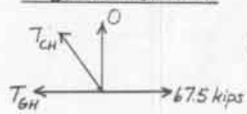
$\circlearrowleft\!\!+\ \Sigma M_D = -T_{HJ}(8) - 30(6) + 60(12) = 0$

$T_{HJ} = 67.5$ kips

$\Sigma F_x = 45 - 67.5 - T_{CD} = 0$

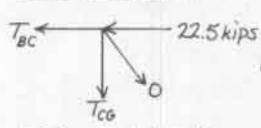
$T_{CD} = -22.5$ kips

FBD of Joint H



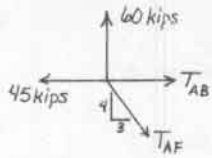
By inspection, $\underline{\underline{T_{CH} = 0}}$

FBD of Joint C



By inspection, $\underline{\underline{T_{CG} = 0}}$

FBD of Joint A



$\Sigma F_y = 60 - T_{AF}\left(\frac{4}{5}\right) = 0$

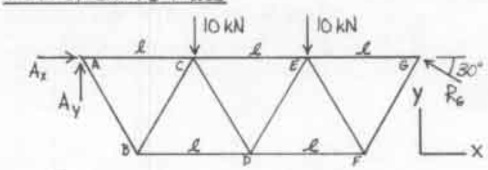
$T_{AF} = 75$ kip

$\Sigma F_x = -45 + T_{AB} + 75\left(\frac{3}{5}\right) = 0$

$\underline{\underline{T_{AB} = 0}}$

6.38

FBD of entire truss



$\circlearrowleft\!\!+\ \Sigma M_G = 10(\ell) + 10(2\ell) - A_y(3\ell) = 0$

$A_y = 10$ kN

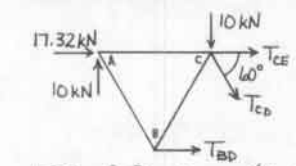
$\Sigma F_y = A_y - 10 - 10 + R_G\sin 30° = 0$

$R_G = 20$ kN

$\Sigma F_x = A_x - R_G\cos 30° = 0$

$A_x = 17.32$ kN

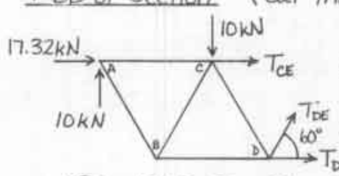
FBD of Section (Cut through CE, CD, BD)



$\Sigma F_y = -10 + 10 - T_{CD}\sin 60° = 0$

$\underline{\underline{T_{CD} = 0}}$

FBD of Section (Cut through CE, DE, DF)



$\Sigma F_y = 10 - 10 + T_{DE}\sin 60° = 0$

$\underline{\underline{T_{DE} = 0}}$

$\circlearrowleft\!\!+\ \Sigma M_C = -10(\ell) + T_{DF}(\ell\sin 60°) = 0$

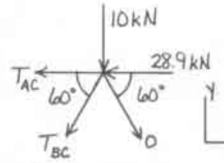
$T_{DF} = 11.55$ kN (Tension)

$\Sigma F_x = 17.32 + T_{CE} + T_{DF} = 0$

$\underline{\underline{T_{CE} = -28.9 \text{ kN (Compression)}}}$

(continued)

FBD of Joint C

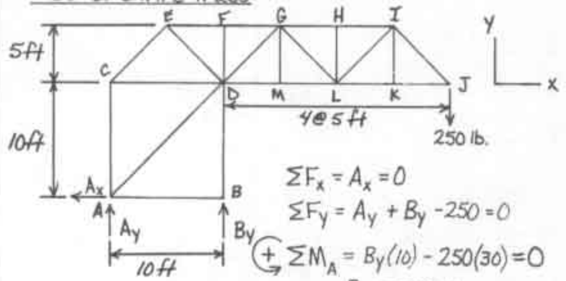


$\Sigma F_y = -10 - T_{BC}\sin 60° = 0$

$\underline{\underline{T_{BC} = -11.55\ kN}}$ (Compression)

6.39

FBD of entire truss



$\Sigma F_x = A_x = 0$

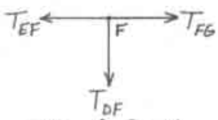
$\Sigma F_y = A_y + B_y - 250 = 0$

$\circlearrowleft\!+\ \Sigma M_A = B_y(10) - 250(30) = 0$

$B_y = 750\ lb$

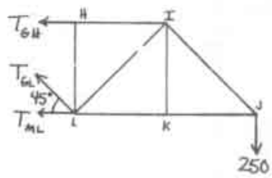
$A_y = -500\ lb$

FBD of Joint F



By inspection, $\underline{\underline{T_{DF} = 0}}$

FBD of Section (Cut through GH, GL, ML)



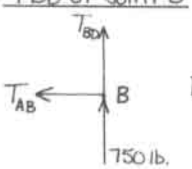
$\Sigma F_y = -250 + T_{GL}\sin 45° = 0$

$\underline{\underline{T_{GL} = 354\ lb.}}$ (Tension)

$\circlearrowleft\!+\ \Sigma M_L = T_{GH}(5) - 250(10) = 0$

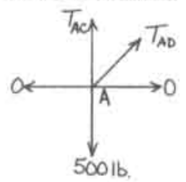
$\underline{\underline{T_{GH} = 500\ lb.}}$ (Tension)

FBD of Joint B



By inspection, $T_{AB} = 0$

FBD of Joint A

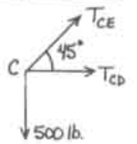


By inspection, $\underline{\underline{T_{AD} = 0}}$

$\Sigma F_y = T_{AC} - 500 = 0$

$T_{AC} = 500\ lb.$ (Tension)

FBD of Joint C



$\Sigma F_y = T_{CE}\sin 45° - 500 = 0$

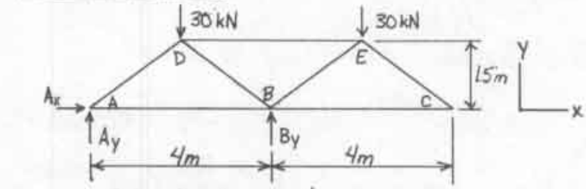
$T_{CE} = 707.1\ lb.$ (Tension)

$\Sigma F_x = T_{CD} + T_{CE}\cos 45° = 0$

$\underline{\underline{T_{CD} = -500\ lb.}}$ (Compression)

6.40

FBD of entire truss



$\Sigma F_x = A_x = 0 \qquad \underline{\underline{A_x = 0}}$

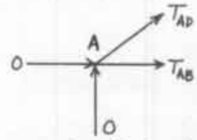
$\circlearrowleft\!+\ \Sigma M_A = -30(2) - 30(6) + B_y(4) = 0$

$\underline{\underline{B_y = 60\ kN}}$

$\Sigma F_y = -60 + A_y + B_y = 0$

$\underline{\underline{A_y = 0}}$

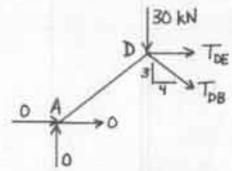
FBD of Joint A



By inspection,

$\underline{\underline{T_{AD} = T_{AB} = 0}}$

FBD of Section (Cut through DE, DB, AB)



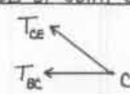
$\Sigma F_y = -30 - T_{DB}\left(\frac{3}{5}\right) = 0$

$\underline{\underline{T_{DB} = -50\ kN}}$ (Compression)

$\Sigma F_x = T_{DB}\left(\frac{4}{5}\right) + T_{DE} = 0$

$\underline{\underline{T_{DE} = 40\ kN}}$ (Tension)

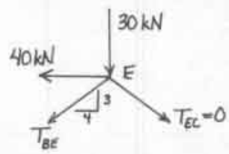
FBD of Joint C



By inspection,

$\underline{\underline{T_{CE} = T_{BC} = 0}}$

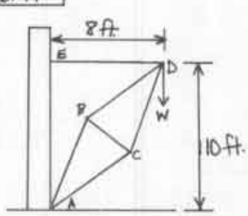
FBD of Joint E



$\Sigma F_y = -30 - T_{BE}\left(\frac{3}{5}\right) = 0$

$\underline{\underline{T_{BE} = -50\ kN}}$ (Compression)

6.41



Requirements:

$W \leq 2000\ lb.$

$500\ lb. < T_{BC} < 900\ lb.$

$AE \leq 10\ ft.$

$DE \leq 8\ ft.$

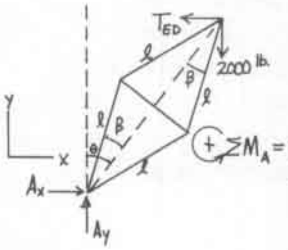
Assumptions: (To simplify the problem, add additional constraints)

$AB = AC = BD = CD = \ell$

$AE = 10\ ft.$
$DE = 8\ ft.$ > To meet the requirements on AE and DE, Point D must remain fixed.

(continued)

FBD of entire truss



$\Sigma F_x = A_x - T_{ED} = 0 \quad A_x = T_{ED}$

$\Sigma F_y = A_y - 2000 = 0 \quad A_y = 2000\text{ lb.}$

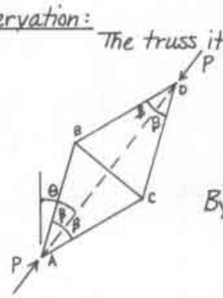
$(+ \Sigma M_A = T_{ED}(2\ell\cos\beta)\cos\theta - 2000(2\ell\cos\beta)\sin\theta = 0$

$T_{ED} = 2000\tan\theta$

$\tan\theta = \frac{8}{10}$ (θ is a constant value due to point D being fixed.)

$\underline{\underline{T_{ED} = 1600\text{ lb.}}}$ (at all times)

Observation:
The truss itself can be viewed as a 2-force member.



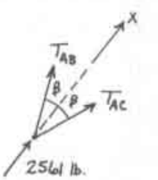
$P = \sqrt{2000^2 + 1600^2}$

$P = 2561\text{ lb.}$

By symmetry

$T_{AB} = T_{AC} = T_{BD} = T_{CD}$

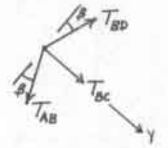
FBD of Joint A



$\Sigma F_x = 2561 + 2T_{AB}\cos\beta = 0$

$T_{AB} = \frac{-1280}{\cos\beta}$ (Compression) (1)

FBD of Joint B



$\Sigma F_y = T_{BC} + 2T_{AB}\sin\beta = 0$

Sub for T_{AB} from (1)

$T_{BC} = 2561\tan\beta$ (Tension) (2)

Range of β: From Eq. (2) $\tan\beta = \frac{T_{BC}}{2561}$

For $T_{BC} = 500\text{ lb}$, $\beta = 11.05°$

For $T_{BC} = 900\text{ lb}$, $\beta = 19.36°$

Member Lengths:

$2\ell\cos\beta = \sqrt{8^2 + 10^2}$ $\qquad \frac{BC}{2} = \ell\sin\beta$

$\ell = \frac{6.403}{\cos\beta}$ $\qquad BC = 12.806\tan\beta$

For $\beta = 11.05°$ $\quad \ell = 6.52'$ $\quad BC = 2.50'$

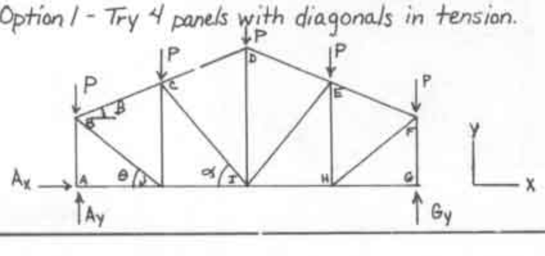
For $\beta = 19.36°$ $\quad \ell = 6.79'$ $\quad BC = 4.50'$

Summary of Design Options

T_{BC}	β	ℓ	BC	T_{AB}
500 lb.	11.05°	6.52'	2.50'	−1304 lb.
900 lb.	19.36°	6.79'	4.50'	−1357 lb.
(Tension)				(Compression)

6.42

Option 1 - Try 4 panels with diagonals in tension.



$\tan\beta = \frac{1}{2.5} \quad \beta = 21.8°$

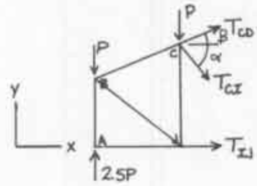
$\tan\theta = \frac{2}{2.5} \quad \theta = 38.7°$

$\tan\alpha = \frac{3}{2.5} \quad \alpha = 50.2°$

$\Sigma F_x = A_x = 0$

From symmetry $A_y = G_y = 2.5P$

FBD of Section (Cut through CD, CI, IJ)



$(+ \Sigma M_C = P(2.5) - 2.5P(2.5) + T_{IJ}(3) = 0$

$T_{IJ} = 1.25P$ (Tension)

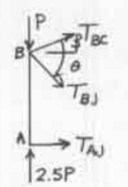
$\Sigma F_x = T_{CD}\cos\beta + T_{CI}\cos\alpha + T_{IJ} = 0$

$\Sigma F_y = .5P + T_{CD}\sin\beta - T_{CI}\sin\alpha = 0$

$T_{CD} = -1.346P$ (Compression)

$T_{CI} = 0$ (Zero-force member)

FBD of Section (Cut through BC, BJ, AJ)



$\Sigma M_B = T_{AJ}(2) = 0$

$T_{AJ} = 0$ (Zero-force member)

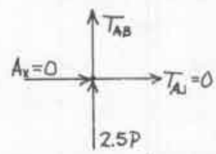
$\Sigma F_x = T_{BC}\cos\beta + T_{BJ}\cos\theta + T_{AJ} = 0$

$\Sigma F_y = 1.5P + T_{BC}\sin\beta - T_{BJ}\sin\theta = 0$

$T_{BJ} = 1.6P$ (Tension)

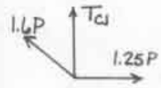
$T_{BC} = -1.346P$ (Compression)

FBD of Joint A



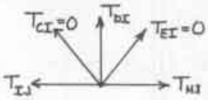
$T_{AB} = -2.5P$ (Compression

FBD of Joint J



$T_{CJ} = -P$ (Compression)

FBD of Joint I

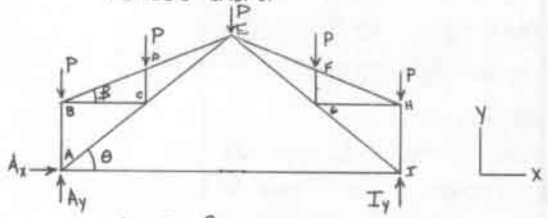


By inspection $T_{DI} = 0$ (Zero-force member)

Member	Length	Unit Cost	Cost
T_{AB}	2m	\$5L²	\$20
T_{FG}	2m	\$5L²	\$20
T_{AJ}	2.5m	\$5L	\$12.50
T_{GH}	2.5m	\$5L	\$12.50
T_{BC}	2.69m	\$5L²	\$36.25
T_{EF}	2.69m	\$5L²	\$36.25
T_{BJ}	3.2m	\$5L	\$16.01
T_{FH}	3.2m	\$5L	\$16.01
T_{CD}	2.69m	\$5L²	\$36.25
T_{DE}	2.69m	\$5L²	\$36.25
T_{CJ}	3m	\$5L²	\$45
T_{EH}	3m	\$5L²	\$45
T_{CI}	3.905m	\$5L	\$19.53
T_{EI}	3.905m	\$5L	\$19.53
T_{IJ}	2.5m	\$5L	\$12.50
T_{HI}	2.5m	\$5L	\$12.50
T_{DI}	4m	\$5L	\$20
		Total Cost	\$416.08

(continued)

Option 2 - Minimize interior members, keep compression members short.



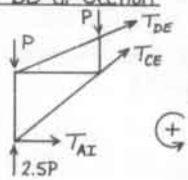
$\tan\beta = \frac{2}{5}$ $\quad \beta = 21.8°$

$\tan\theta = \frac{4}{5}$ $\quad \theta = 38.7°$

$\Sigma F_x = A_x = 0$

By symmetry $\quad A_y = I_y = 2.5P$

<u>FBD of Section</u> (Cut through DE, CE, AI)



$\Sigma F_x = T_{DE}\cos\beta + T_{CE}\cos\theta + T_{AI} = 0$

$\Sigma F_y = T_{DE}\sin\beta + T_{CE}\sin\theta + .5P = 0$

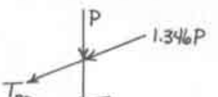
$\circlearrowleft + \; \Sigma M_A = -T_{DE}\cos\beta(3) + T_{DE}\sin\beta(2.5) - P(2.5) = 0$

$T_{DE} = -1.346P$ (Compression)

$T_{CE} = 0$ (Zero-force member)

$T_{AI} = 1.25P$ (Tension)

<u>FBD of Joint D</u>

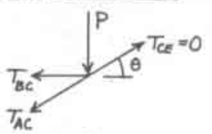


By inspection

$T_{DC} = -P$ (Compression)

$T_{BD} = -1.346P$ (Compression)

<u>FBD of Joint C</u>



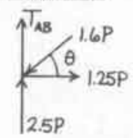
$\Sigma F_y = -P - T_{AC}\sin\theta = 0$

$T_{AC} = -1.6P$ (Compression)

$\Sigma F_x = -T_{BC} - T_{AC}\cos\theta = 0$

$T_{BC} = 1.25P$ (Tension)

<u>FBD of Joint A</u>



$\Sigma F_y = T_{AB} + 2.5P - 1.6P\sin\theta = 0$

$T_{AB} = -3.5P$ (Compression)

Member	Length	Unit Cost	Cost
AB	2m	$\$5L^2$	\$20
HI	2m	$5L^2$	20
AI	10m	$5L$	50
AC	3.2m	$5L^2$	51.25
GI	3.2m	$5L^2$	51.25
BC	2.5m	$5L$	12.50
GH	2.5m	$5L$	12.50
BD	2.69m	$5L^2$	36.25
FH	2.69m	$5L^2$	36.25
DC	1m	$5L^2$	5
FG	1m	$5L^2$	5
DE	2.69m	$5L^2$	36.25
EF	2.69m	$5L^2$	36.25
CE	3.2m	$5L$	16.01
GE	3.2m	$5L$	16.01

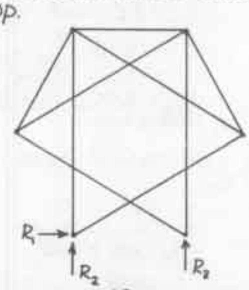
Total Cost \$404.52

6.43

a.) Compound Truss
b.) Compound Truss
c.) Simple Truss
d.) Simple Truss
e.) Compound Truss
f.) Complex Truss
g.) Simple Truss
h.) Compound Truss

6.44

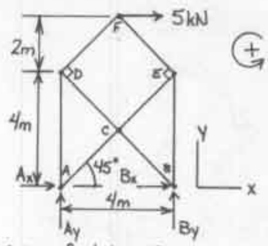
Move the horizontal member between the supports to the top.



There are three different solutions.

6.45

<u>FBD of entire truss</u>



$\circlearrowleft + \; \Sigma M_A = B_y(4m) - 5(6m) = 0$

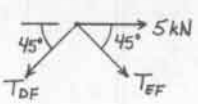
$\underline{B_y = 7.5\,kN}$

$\Sigma F_y = A_y + B_y = 0$

$\underline{A_y = -7.5\,kN}$

$\Sigma F_x = 5 + A_x + B_x = 0$

<u>FBD of Joint F</u>



$\Sigma F_y = -T_{DF}\sin 45° - T_{EF}\sin 45° = 0$

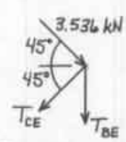
$T_{DF} = -T_{EF}$

$\Sigma F_x = -T_{DF}\cos 45° + T_{EF}\cos 45° + 5 = 0$

$T_{DF} = 3.536\,kN$ (Tension)

$T_{EF} = -3.536\,kN$ (Compression)

<u>FBD of Joint E</u>



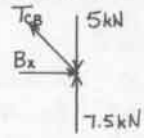
$\Sigma F_x = -T_{CE}\cos 45° + 3.536\cos 45° = 0$

$T_{CE} = 3.536\,kN$ (Tension)

$\Sigma F_y = -3.536\sin 45° - T_{CE}\sin 45° - T_{BE} = 0$

$T_{BE} = -5\,kN$ (Compression)

<u>FBD of Joint B</u>



$\Sigma F_y = 7.5 - 5 + T_{CB}\sin 45° = 0$

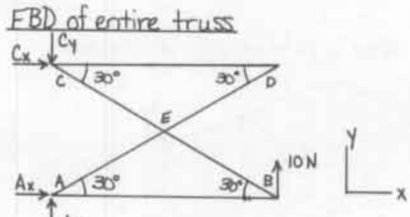
$T_{CB} = -3.536\,kN$ (Compress)

$\Sigma F_x = T_{CB}\cos 45° + B_x = 0$

$\underline{B_x = -2.5\,kN}$ $\quad \underline{A_x = -2.5\,kN}$

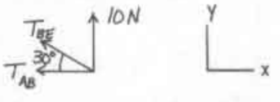
6.46

a.) <u>FBD of entire truss</u>



(continued)

FBD of Joint B



$\Sigma F_y = T_{BE}\sin 30° + 10 = 0$

$\underline{T_{BE} = -20\ N}$ (Compression)

$\Sigma F_x = -T_{BE}\cos 30° - T_{AB} = 0$

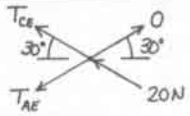
$\underline{T_{AB} = 17.32\ N}$ (Tension)

FBD of Joint D

$\underline{T_{CD} = 0}$

$\underline{T_{DE} = 0}$

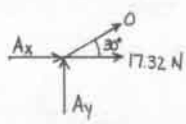
FBD of Joint E



$\underline{T_{AE} = 0}$

$\underline{T_{CE} = -20\ N}$ (Compression)

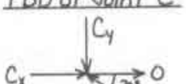
FBD of Joint A



$\underline{A_x = -17.32\ N}$

$\underline{A_y = 0}$

FBD of Joint C



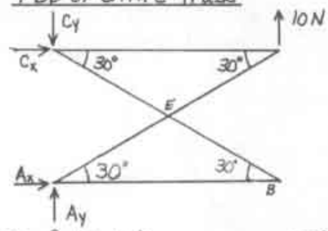
$\Sigma F_x = C_x - 20\cos 30° = 0$

$\underline{C_x = 17.32\ N}$

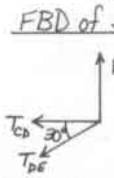
$\Sigma F_y = -C_y + 20\sin 30° = 0$

$\underline{C_y = 10\ N}$

b.) FBD of entire truss



FBD of Joint D



$\Sigma F_y = 10 - T_{DE}\sin 30° = 0$

$\underline{T_{DE} = 20\ N}$ (Tension)

$\Sigma F_x = -T_{CD} - T_{DE}\cos 30° = 0$

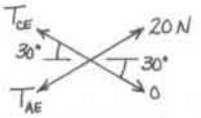
$\underline{T_{CD} = -17.32\ N}$ (Compression)

FBD of Joint B

$\underline{T_{BE} = 0}$

$\underline{T_{AB} = 0}$

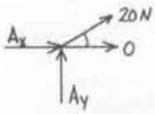
FBD of Joint E



$\underline{T_{CE} = 0}$

$\underline{T_{AE} = 20\ N}$ (Tension)

FBD of Joint A



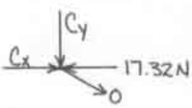
$\Sigma F_y = 20\sin 30° + A_y = 0$

$\underline{A_y = -10\ N}$

$\Sigma F_x = 20\cos 30° + A_x = 0$

$\underline{A_x = -17.32\ N}$

FBD of Joint C

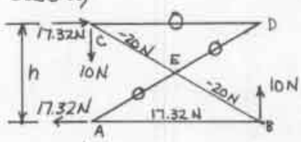


$\underline{C_y = 0}$

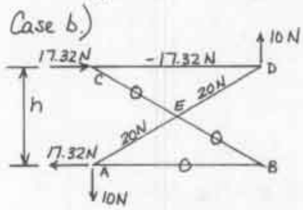
$\underline{C_x = 17.32\ N}$

Summary of Member Forces and Reactions

Case a)



Case b.)



Discussion

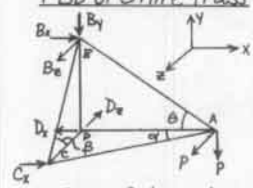
The principle of transmissibility does not prohibit changes in internal (member) forces when the 10 N force is moved along its line of action.

Although the change in member forces causes a change in the individual reaction components, the net reactions do not change. For both cases:

$A_x + C_x = 0 \quad , \quad A_y + C_y = -10\ N, \; A_x h = C_x h = -17.32h$

6.47

FBD of entire truss

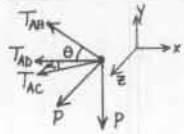


$\tan\theta = \frac{4}{6} \quad \theta = 33.7°$

$\tan\alpha = \frac{2}{6} \quad \alpha = 18.43°$

$\tan\beta = \frac{4}{2} \quad \beta = 63.4°$

FBD of Joint A



$\Sigma F_y = T_{AB}\sin\theta - P = 0$

$\underline{T_{AB} = 1.803P}$ (Tension)

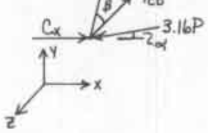
$\Sigma F_z = P + T_{AC}\sin\alpha = 0$

$\underline{T_{AC} = -3.16P}$ (Compression)

$\Sigma F_x = -T_{AB}\cos\theta - T_{AD} - T_{AC}\cos\alpha = 0$

$\underline{T_{AD} = 1.5P}$ (Tension)

FBD of Joint C



$\Sigma F_x = C_x - 3.16P\cos\alpha = 0$

$\underline{C_x = 3P}$

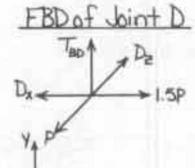
$\Sigma F_y = T_{BC}\sin\beta = 0$

$\underline{T_{BC} = 0}$

$\Sigma F_z = 3.16P\sin\alpha - T_{CD} = 0$

$\underline{T_{CD} = P}$ (Tension)

FBD of Joint D



$\Sigma F_y = \underline{T_{BD} = 0}$

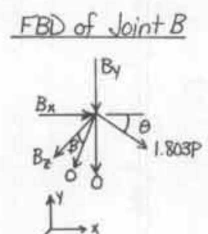
$\Sigma F_x = 1.5P - D_x = 0$

$\underline{D_x = 1.5P}$

$\Sigma F_z = P - D_z = 0$

$\underline{D_z = P}$

FBD of Joint B



$\Sigma F_z = \underline{B_z = 0}$

$\Sigma F_x = B_x + 1.803P\cos\theta = 0$

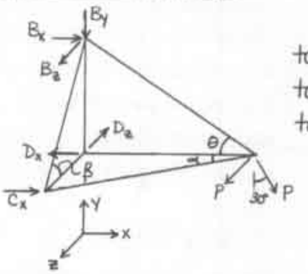
$\underline{B_x = -1.5P}$

$\Sigma F_y = -B_y - 1.803\sin\theta = 0$

$\underline{B_y = -P}$

6.48

FBD of entire truss

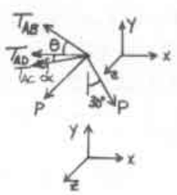


$\tan\theta = \frac{4}{6}$ $\theta = 33.69°$

$\tan\alpha = \frac{2}{6}$ $\alpha = 18.43°$

$\tan\beta = \frac{4}{2}$ $\beta = 63.43°$

FBD of Joint A



$\Sigma F_y = T_{AB}\sin\theta - P\cos 30° = 0$

$\underline{\underline{T_{AB} = 1.56P}}$ (Tension)

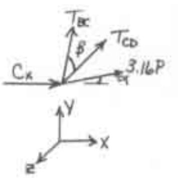
$\Sigma F_z = P + T_{AC}\sin\alpha = 0$

$\underline{\underline{T_{AC} = -3.16P}}$ (Compression)

$\Sigma F_x = P\sin 30° - T_{AB}\cos\theta - T_{AD} - T_{AC}\cos\alpha = 0$

$\underline{\underline{T_{AD} = 2.2P}}$ (Tension)

FBD of Joint C



$\Sigma F_x = C_x - 3.16P\cos\alpha = 0$

$\underline{\underline{C_x = 3P}}$

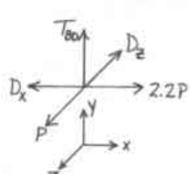
$\Sigma F_y = T_{BC}\sin\beta = 0$

$\underline{\underline{T_{BC} = 0}}$

$\Sigma F_z = 3.16P\sin\alpha - T_{CD} = 0$

$\underline{\underline{T_{CD} = P}}$ (Tension)

FBD of Joint D



$\Sigma F_y = \underline{\underline{T_{BD} = 0}}$

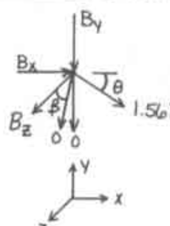
$\Sigma F_x = -D_x + 2.2P = 0$

$\underline{\underline{D_x = 2.2P}}$

$\Sigma F_z = -D_z + P = 0$

$\underline{\underline{D_z = P}}$

FBD of Joint B



$\Sigma F_z = \underline{\underline{B_z = 0}}$

$\Sigma F_y = -B_y - 1.56P\sin\theta = 0$

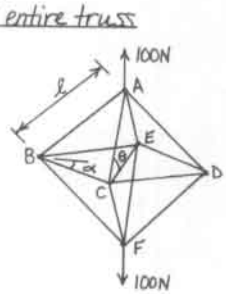
$\underline{\underline{B_y = -.866P}}$

$\Sigma F_x = B_x + 1.56P\cos\theta = 0$

$\underline{\underline{B_x = -1.299P}}$

6.49

FBD of entire truss



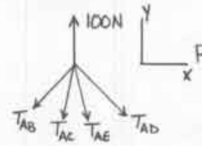
$AB = BC = \ldots = \ell$

$CE = \sqrt{2}\,\ell$

$\cos\theta = \frac{\sqrt{2}\ell}{2\ell}$ $\theta = 45°$

$\sin\alpha = \frac{\sqrt{2}\ell}{2\ell}$ $\alpha = 45°$

FBD of Joint A

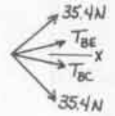


From symmetry, $T_{AB} = T_{AC} = T_{AD} = T_{AE} = T_1$

$\Sigma F_y = -4T\sin 45° + 100 = 0$

$T_1 = \underline{\underline{T_{AB} = 35.4\,N}}$ (Tension)

FBD of Joint B



From symmetry, $T_{BE} = T_{BC} = T_{CD} = T_{DE} = T_2$

$\Sigma F_x = 2(35.4\cos 45°) + 2T_2\cos 45° = 0$

$T_2 = \underline{\underline{T_{BC} = -35.4\,N}}$ (Compression)

By inspection of Joint C:

$\underline{\underline{T_{CE} = 0}}$

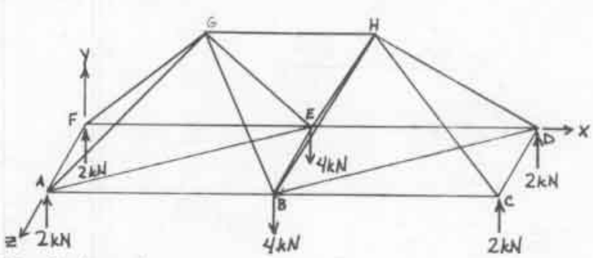
6.50

Although this structure is indeterminate the reactions can be found due to the symmetry of the loading and design of the truss.

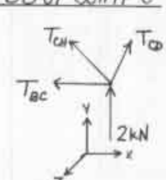
- All reactions are 2kN

Line	Distance Vector	Length	θ_x	θ_y	θ_z
CH	$-2.5\underline{i} + 2\underline{j} - \underline{k}$	3.354	-.7454	.5963	-.2981
DH	$-2.5\underline{i} + 2\underline{j} + \underline{k}$	3.354	-.7454	.5963	.2981
BH	$1.5\underline{i} + 2\underline{j} - \underline{k}$	2.693	.5571	.7428	-.3714
EH	$1.5\underline{i} + 2\underline{j} + \underline{k}$	2.693	.5571	.7428	.3714

FBD of entire truss



FBD of Joint C

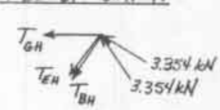


$\Sigma F_y = T_{CH}(.5963) + 2 = 0$

$T_{CH} = -3.354\,kN$ (Compression)

From symmetry $T_{DH} = -3.354\,kN$ (Compression)

FBD of Joint H



From symmetry: $T_{EH} = T_{BH}$

$\Sigma F_y = 2(3.354)(.5963) - T_{EH}(2)(.7428) = 0$

$T_{EH} = 2.693\,kN$ (Tension)

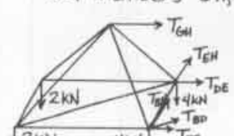
$\Sigma F_x = -T_{GH} - 2T_{EH}(.5571) - 3.354(2)(.7454) = 0$

$\underline{\underline{T_{GH} = -8.00\,kN}}$ (Compression)

Alternate Solution

Use the method of sections.

Cut members GH, EH, BH, BC, BD, ED

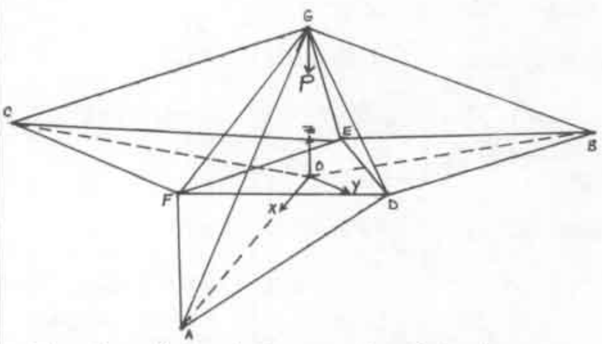


Take moments about Line BE

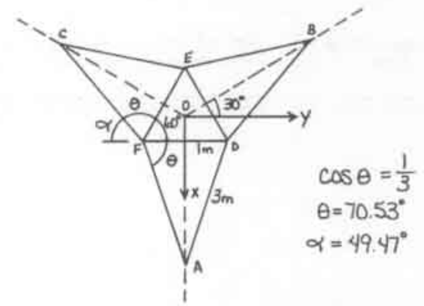
$\Sigma M_{BE} = 2(2kN)(4m) + T_{GH}(2m) = 0$

$\underline{\underline{T_{GH} = -8\,kN}}$ (Compression)

6.51



Top view of the bottom portion of the truss.



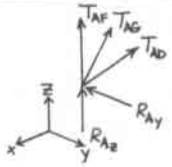
$\cos\theta = \frac{1}{3}$

$\theta = 70.53°$

$\alpha = 49.47°$

Line	Distance Vector	Length	θ_x	θ_y	θ_z
AG	$-3.4058\underline{i} + 0\underline{j} + 3\underline{k}$	4.539 m	-0.7504	0.0	0.6609
AD	$-\sqrt{8}\,\underline{i} + 1\underline{j} + 0\underline{k}$	3	-0.9428	0.3333	0.0
AF	$-\sqrt{8}\,\underline{i} - 1\underline{j} + 0\underline{k}$	3	-0.9428	-0.3333	0.0
DB	————	3	-0.7601	0.6498	0.0
DE	————	2	-0.8660	-0.5	0.0

FBD of Joint A



By symmetry of the loading, reactions at A, B, and C are ⅓P.

$R_{Az} = \frac{1}{3}(10) = 3.333\ kN$

The only member that can carry loading in the z direction is T_{AG}.

$\Sigma F_z = T_{AG}(.6609) + 3.333 = 0$

$\underline{T_{AG} = -5.04\ kN}$ (Compression)

$T_{AF} = T_{AD}$ from symmetry and the geometry of the truss.

$\Sigma F_x = -T_{AG}(0.7504) - 2T_{AD}(0.9428) = 0$

$\underline{T_{AD} = 2.01\ kN}$ (Tension)

FBD of Joint D

From symmetry: $T_{BD} = T_{AD} = 2.01\ kN\ (T)$

By inspection: $T_{DG} = 0$

$\Sigma F_x = -2.007(.7601) + 2.007(.9428) - T_{DE}(.866) = 0$

$T_{DE} = 0.423\ kN$ (Tension)

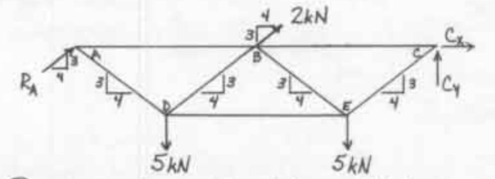
$\Sigma F_y = 2.007(.6498) - 2.007(.3333) - T_{DE}(.5) - T_{DF} = 0$

$\underline{\underline{T_{DF} = 0.423\ kN}}$ (Tension)

(This result could have been obtained by the use of symmetry)

6.58

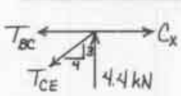
FBD of entire truss



$\circlearrowleft\!+\ \Sigma M_A = C_y(16) - 5(4) - 5(12) + 2(\tfrac{3}{5})(8) = 0$

$C_y = 4.40\ kN$

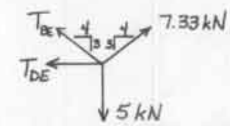
FBD of Joint C



$\Sigma F_y = 4.4 - T_{CE}(\tfrac{3}{5}) = 0$

$T_{CE} = 7.33\ kN$ (Tension)

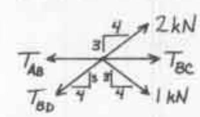
FBD of Joint E



$\Sigma F_y = -5 + 7.333(\tfrac{3}{5}) + T_{BE}(\tfrac{3}{5}) = 0$

$\underline{\underline{T_{BE} = 1\ kN}}$ (Tension)

FBD of Joint B

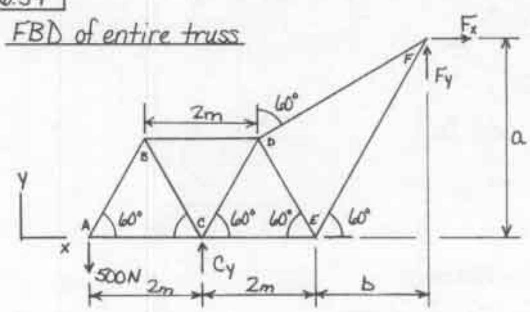


$\Sigma F_y = 2(\tfrac{3}{5}) - 1(\tfrac{3}{5}) - T_{BD}(\tfrac{3}{5}) = 0$

$\underline{\underline{T_{BD} = 1\ kN}}$ (Tension)

6.59

FBD of entire truss



$\cos 60° = \frac{2}{EF}$ $EF = 4m$

$\sin 60° = \frac{a}{4}$ $a = 3.46m$

$\cos 60° = \frac{b}{4}$ $b = 2m$

$\Sigma F_x = F_x = 0$

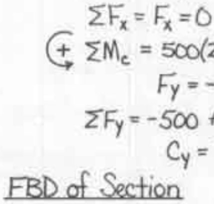
$\circlearrowleft\!+\ \Sigma M_c = 500(2) + F_y(4) = 0$

$F_y = -250\ N$

$\Sigma F_y = -500 + C_y - 250 = 0$

$C_y = 750\ N$

FBD of Section (Cut through BD, BC, AC)



$\Sigma F_y = -500 - T_{BC}\sin 60° = 0$

$\underline{\underline{T_{BC} = -577.4\ N}}$ (Compression)

$\circlearrowleft\!+\ \Sigma M_B = T_{AC}(\sqrt{3}) + 500(1) = 0$

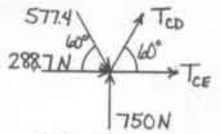
$T_{AC} = -288.7\ N$ (Compression)

$\Sigma F_x = T_{BD} + T_{AC} + T_{BC}\cos 60° = 0$

$T_{BD} = 577.4\ N$ (Tension)

(continued)

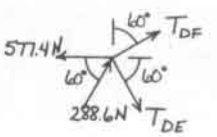
FBD of Joint C



$\Sigma F_y = -577.4 \sin 60° + T_{CD} \sin 60° + 750 = 0$

$\underline{T_{CD} = -288.6\ N}$ (Compression)

FBD of Joint D



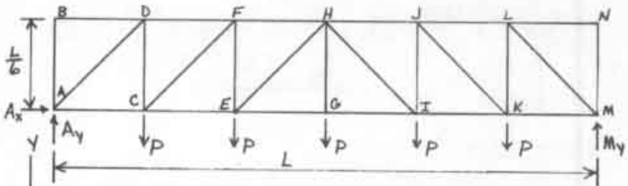
$\Sigma F_x = -577.4 + 288.6 \cos 60° + T_{DF} \sin 60° + T_{DE} \cos 60° = 0$

$\Sigma F_y = T_{DF} \cos 60° - T_{DE} \sin 60° + 288.6 \sin 60° = 0$

$\underline{T_{DF} = 250\ N}$ (Tension)

6.60

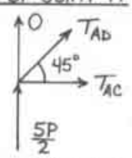
FBD of entire truss



$\Sigma F_x = A_x = 0$

From symmetry: $A_y = M_y = \frac{5P}{2}$

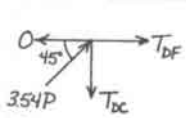
FBD of Joint A



$\Sigma F_y = T_{AD} \sin 45° + \frac{5}{2}P = 0$

$\underline{T_{AD} = -3.54P}$ (Compression)

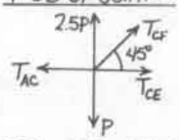
FBD of Joint D



$\Sigma F_y = -T_{DC} + 3.54 \sin 45° = 0$

$T_{DC} = 2.5P$ (Tension)

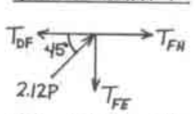
FBD of Joint C



$\Sigma F_y = 1.5P + T_{CF} \sin 45° = 0$

$\underline{T_{CF} = -2.12P}$ (Compression)

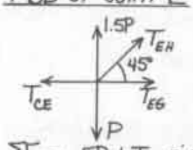
FBD of Joint F



$\Sigma F_y = 2.12P \sin 45° - T_{FE} = 0$

$T_{FE} = 1.5P$

FBD of Joint E



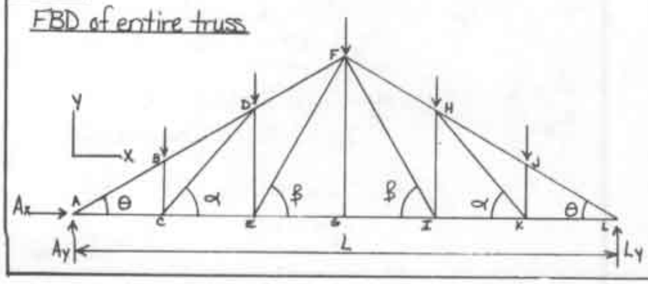
$\Sigma F_y = .5P + T_{EH} \sin 45° = 0$

$\underline{T_{EH} = -0.707P}$ (Compression)

From symmetry all diagonal members are in compression.

6.61

FBD of entire truss



$\tan \theta = 7/12$, $\theta = 30.26°$

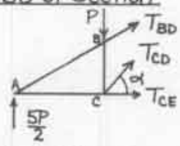
$\tan \alpha = 14/12$, $\alpha = 49.40°$

$\tan \beta = 21/12$, $\beta = 60.26°$

By symmetry: $A_y = L_y = \frac{5P}{2}$

$A_x = 0$

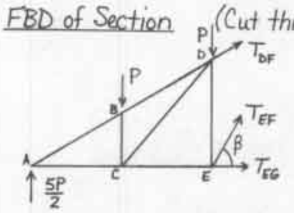
FBD of Section (Cut through BD, CD, CE)



$\circlearrowleft + \Sigma M_A = -P(\frac{L}{6}) + T_{CD} \sin\alpha (\frac{L}{6}) = 0$

$\underline{T_{CD} = 1.317P}$ (Tension)

FBD of Section (Cut through DF, EF, EG)

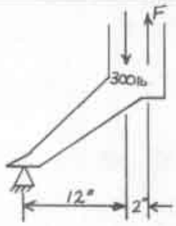


$\Sigma M_A = -P(\frac{L}{6}) - P(\frac{L}{3}) + T_{EF} \sin\beta (\frac{L}{3}) = 0$

$\underline{T_{EF} = 1.728P}$ (Tension)

By symmetry, FI and HK are in tension too.

7.1



Find the force the tendon must withstand.

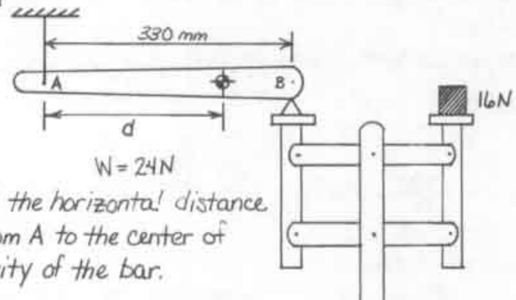
The foot is a second class lever.

$$\frac{L}{E} = \frac{a}{b} \quad (\text{Eqn. } 7.1)$$

$$\frac{300\,lb.}{F} = \frac{14''}{12''}$$

$$\underline{\underline{F = 257\,lb.}}$$

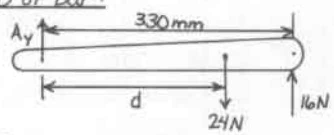
7.2



W = 24N

Find the horizontal distance d from A to the center of gravity of the bar.

FBD of bar:



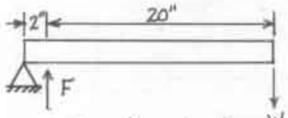
$$\circlearrowleft^{+} \Sigma M_A = -24(d) + 16(330) = 0$$

$$\underline{\underline{d = 220\,mm}}$$

7.3

Find the weight W so that the valve will start to open at 300 lb/in².

FBD of rod:



$$F = P \cdot A = P(\pi r^2) \quad (r = 1)$$

$$F = 300\,\text{lb/in}^2\left(\pi (1\,in)^2\right)$$

$$F = 942.5\ lb.$$

$$\frac{L}{E} = \frac{a}{b} \quad (\text{Eqn. } 7.1)$$

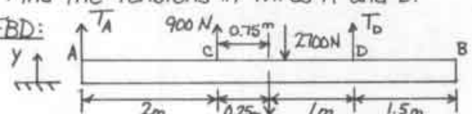
$$\frac{L}{E} = \frac{942.5}{W} \qquad \frac{a}{b} = \frac{22''}{2''} = 11 \qquad \frac{942.5\,lb}{W} = 11$$

$$\underline{\underline{W = 85.7\,lb.}}$$

7.4

Find the tensions in wires A and D.

FBD:



$$W_{AB} = (600\ \text{N/m})(5.5m) = 3{,}300\,N$$

$$\circlearrowleft^{+} \Sigma M_A = 900(2) - 3300(2.75) - 2700(3) + T_D(4) = 0$$

$$\underline{\underline{T_D = 3844\,N = 3.84\,kN}}$$

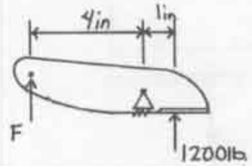
$$\Sigma F_y = T_A + 900 - 3300 - 2700 + 3843.75 = 0$$

$$\underline{\underline{T_A = 1256\,N = 1.256\,kN}}$$

7.5

Find the magnitude of P if 1200lb. is required to cut the bolt.

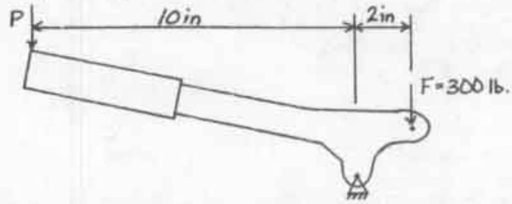
FBD:



$$\frac{L}{E} = \frac{a}{b} \quad (\text{Eqn. } 7.1) \ (\text{First-class})$$

$$\frac{1200\,lb.}{F} = \frac{4''}{1''} \qquad F = 300\,lb.$$

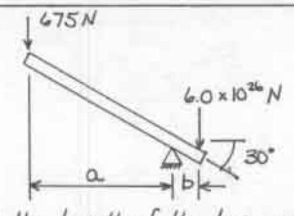
FBD:



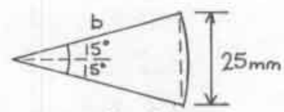
$$\frac{L}{E} = \frac{a}{b} \quad (\text{First - Class})$$

$$\frac{300}{P} = \frac{10''}{2''} \qquad \underline{\underline{P = 60\,lb.}}$$

7.6



Find the length of the lever needed to move the Earth 25mm.



$$b(\sin 15^\circ) = \frac{25}{2}\,mm$$

$$b = 48.3\,mm$$

$$\frac{L}{E} = \frac{a}{b} \quad (\text{Eqn } 7.1) \ (\text{First-Class})$$

$$\frac{6.0 \times 10^{26}\,N}{675\,N} = \frac{a}{48.3mm}$$

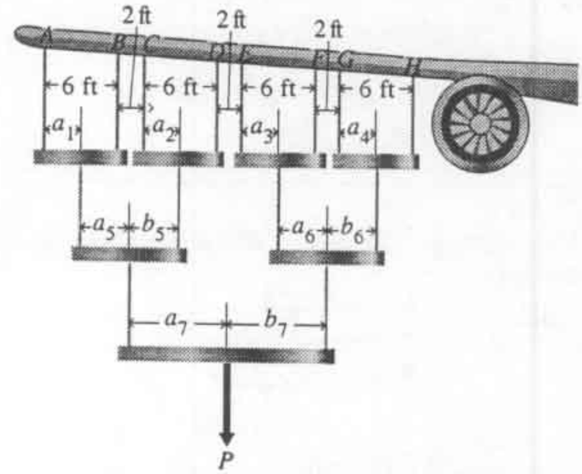
$$a = 4.29 \times 10^{25}\,mm = 4.29 \times 10^{19}\,km$$

Length, $L = a + b$

$$L = 4.29 \times 10^{19}\,km + 48.3\,mm$$

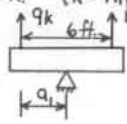
$$\underline{\underline{L = 4.29 \times 10^{19}\,km}}$$

7.7



Find the required lever arms $a_1, a_2, a_3, a_4, a_5, a_6, a_7, b_5, b_6$, and b_7 if $P = 196\,k$, $A = 9k$, $B = 13k$, $C = 18\,k$, $D = 22\,k$, $E = 27k$, $F = 31k$, $G = 36\,k$ and $H = 40\,k$. (k = kips)

FBD:



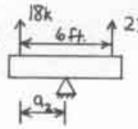
$\frac{L}{E} = \frac{a}{b}$ (Eqn. 7.1) (First-Class)

$\frac{13k}{9k} = \frac{a_1}{(6-a_1)}$

$78 - 13a_1 = 9a_1$

$\underline{\underline{a_1 = 3.55 \text{ ft.}}}$

FBD:



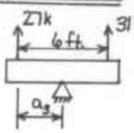
$\frac{L}{E} = \frac{a}{b}$ (First-Class)

$\frac{22k}{18k} = \frac{a_2}{6-a_2}$

$132 - 22a_2 = 18a_2$

$\underline{\underline{a_2 = 3.30 \text{ ft.}}}$

FBD:



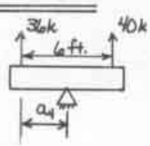
$\frac{L}{E} = \frac{a}{b}$ (First-Class)

$\frac{31k}{27k} = \frac{a_3}{6-a_3}$

$186 - 31a_3 = 27a_3$

$\underline{\underline{a_3 = 3.21 \text{ ft.}}}$

FBD:



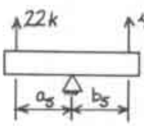
$\frac{L}{E} = \frac{a}{b}$ (First-Class)

$\frac{40k}{36k} = \frac{a_4}{6-a_4}$

$240 - 40a_4 = 36a_4$

$\underline{\underline{a_4 = 3.16 \text{ ft.}}}$

FBD:



$a_5 + b_5 = (6-a_1) + 2 + a_2$

$a_5 + b_5 = 7.76 \text{ ft.}$

$\frac{L}{E} = \frac{a}{b}$ (First-Class)

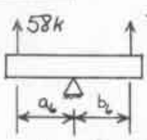
$\frac{40k}{22k} = \frac{a_5}{b_5} = \frac{a_5}{(7.76-a_5)}$

$310.2 - 40a_5 = 22a_5$

$\underline{\underline{a_5 = 5.00 \text{ ft.}}}$

$\underline{\underline{b_5 = 2.76 \text{ ft.}}}$

FBD:



$a_6 + b_6 = (6-a_3) + 2 + a_4 = 7.95 \text{ ft}$

$\frac{L}{E} = \frac{a}{b}$ (First-Class)

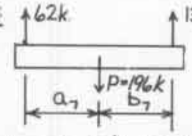
$\frac{76k}{58k} = \frac{a_6}{(7.95-a_6)}$

$604.2 - 76a_6 = 58a_6$

$\underline{\underline{a_6 = 4.51 \text{ ft.}}}$

$\underline{\underline{b_6 = 3.44 \text{ ft.}}}$

FBD:



$a_7 + b_7 = b_5 + (6-a_2) + 2 + a_3 + a_6 = 15.18$

$\frac{L}{E} = \frac{a}{b}$ (First-Class)

$\frac{134k}{62k} = \frac{a_7}{(15.18-a_7)}$

$\underline{\underline{a_7 = 10.38 \text{ ft.}}}$

$\underline{\underline{b_7 = 4.80 \text{ ft.}}}$

7.8

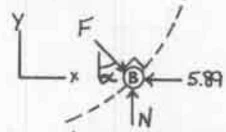
$p = 3\ \text{N/mm}^2$ = Pressure on the end of the cylinder

Find the weight W that can be held in the designated position.

Force exerted by the pressure p is: $P = (\pi r^2)p = \pi(25^2)(3) = 5890\text{N}$

$P = 5.89 \text{ kN}$

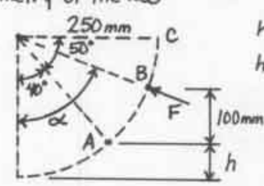
FBD of B:



$\Sigma F_x = F\sin\alpha - 5.89 = 0$

$F = \frac{5.89}{\sin\alpha}$ (a)

Geometry of the Rod



$h = 250 - 250\cos 40^\circ = 58.489 \text{ mm}$

$h + 100 = 250 - 250\cos\alpha = 158.489 \text{ mm}$

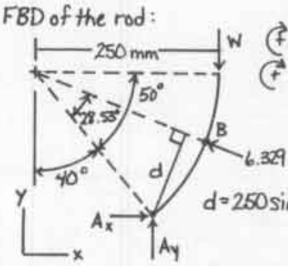
$\therefore \cos\alpha = 0.3660 \quad \alpha = 68.528^\circ$

$\sin\alpha = 0.9306$ (b)

By equations (a) and (b),

$F = 6.329 \text{ kN}$

FBD of the rod:



$\circlearrowleft^{+} \Sigma M_A = 0$

$\circlearrowleft^{+} \Sigma M_A = 250W(1-\cos 50^\circ) - 6.329(250\sin 28.53)$

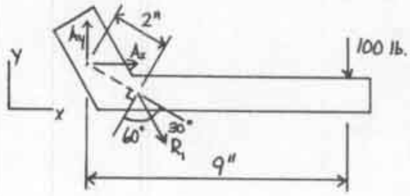
$89.303\,W = 755.66 \text{ kN}\cdot\text{mm}$

$\therefore \underline{\underline{W = 8.462 \text{ kN}}}$

$d = 250\sin 28.53^\circ$

7.9

Find the force F of the clamp at A.



Note: The 2" long arm is a 2-force member.

$\circlearrowleft^{+} \Sigma M_A = -R_1\cos 60^\circ(2) - 100(9) = 0$

$R_1 = -900 \text{ lb.}$

$\Sigma F_y = -100 + 900\cos 30^\circ + A_y = 0$

$A_y = -679.4 \text{ lb.}$

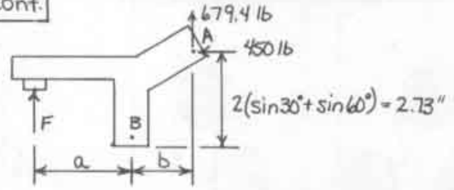
$\Sigma F_x = A_x - 900\sin 30^\circ = 0$

$A_x = 450 \text{ lb.}$

(continued)

7.9 Cont.

FBD:



$b = 4 - 2\cos 60° - 2\cos 30° = 1.268''$

$a = 4 - b = 2.73''$

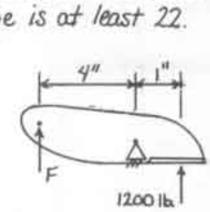
$\circlearrowright + \Sigma M_B = 679.4(1.268) + 450(2.732) - F(2.732) = 0$

$\underline{F = 765 \text{ lb}}$

7.10

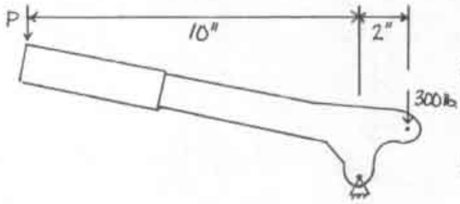
Modify the system to ensure that the mechanical advantage is at least 22.

FBD:



$\frac{L}{E} = \frac{a}{b}$ (Eqn. 7.1) (First-Class)

$\frac{1200}{F} = \frac{4}{1}$ $F = 300$ lb.



$\frac{L}{E} = \frac{a}{b}$

$\frac{300}{P} = \frac{10}{2}$

$P = 60$ lb

$m = \frac{1200 \text{ lb}}{60 \text{ lb}} = 20$

For $m = 22$,

$22 = \frac{1200 \text{ lb}}{P}$ $P = 54.5$ lb

One of the 4 lengths must be changed so that $P = 54.5$ lb.
Possible solutions are:

$\frac{300 \text{ lb}}{54.5 \text{ lb}} = \frac{a}{b} = 5.5$

Either change 2" to 1.818" or change 10" to 11"

$\frac{F}{54.5} = \frac{10}{2}$ $F = 272.7$ lb.

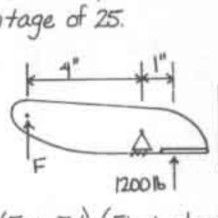
or $\frac{1200 \text{ lb}}{272.7 \text{ lb}} = \frac{a}{b} = 4.4$

Either change 4" to 4.4" or change 1" to 0.909".
The preferred option is likely the one that results in a bolt cutter that uses less material and is smaller.
Therefore, change 2" to 1.818".

7.11

a) Modify the system to ensure a mechanical advantage of 25.

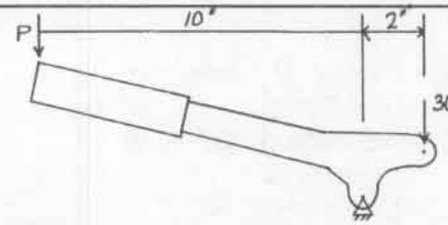
FBD:



$\frac{L}{E} = \frac{a}{b}$ (Eqn. 7.1) (First-class)

$\frac{1200 \text{ lb}}{F} = \frac{4''}{1''}$ $F = 300$ lb.

FBD:



$\frac{300}{P} = \frac{10''}{2''}$ $P = 60$ lb, as originally designed

We want:

$\frac{1200 \text{ lb}}{P} = m = 25$ $P = 48$ lb.

One of the 4 lengths must be changed so $P = 48$ lb.

$\frac{300 \text{ lb}}{48 \text{ lb}} = \frac{a}{b} = 6.25$

Either change 2" to 1.6" or 10" to 12.5".

or $\frac{F}{48 \text{ lb}} = \frac{10''}{2''}$ $F = 240$ lb

Either change 4" to 5" or 1" to 0.8"

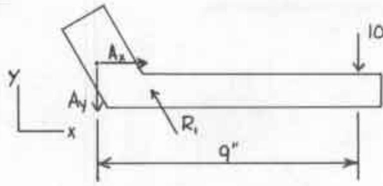
b) Compare the advantages and disadvantages

All the new designs make the cutter a more powerful tool. Changing 10" to 12.5" makes the tool bigger, heavier, and more awkward to handle. Changing 2" to 1.6" makes the handle have a fairly sharp angle. Changing 1" to 0.8" limits the cutting ability of large objects because the fulcrum is so close to the cutting point.

7.12

Modify the system to ensure a mechanical advantage of 10. The 9" dimension is not to be changed more than 1".

FBD:

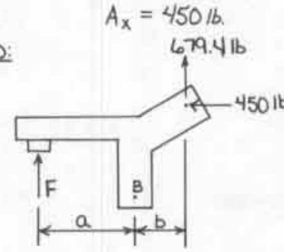


$R_1 = 900$ lb

$A_y = 679.4$ lb. (From Problem 7.9)

$A_x = 450$ lb.

FBD:



$a = 2.73''$

$b = 1.268''$ (From Problem 7.9)

$F = 765$ lb.

$m = \frac{L}{E} = \frac{F}{100 \text{ lb}}$ (Eqn. 7.1)

$m = \frac{765}{100} = 7.65$

For $m = 10$, $F = 1000$ lb.

Adjust 9" dimension to 10". That alone won't be enough to get $m = 10$. We must adjust something else as well.

(continued)

7.12 cont.

$\circlearrowleft + \Sigma M_A = R_1 \cos 60°(2) - 100(10) = 0$

$R_1 = 1000 \text{ lb.}$

$\Sigma F_y = -100 + 1000\cos 30° - A_y = 0$

$A_y = 766 \text{ lb.}$

$\Sigma F_x = A_x - 1000 \sin 30° = 0$

$A_x = 500 \text{ lb.}$

Let's also adjust the length of a.

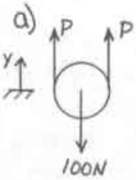
$\circlearrowleft + \Sigma M_B = 766(1.268) + 500(2.73) - 1000(a) = 0$

$a = 2.34''$

By shortening the length of the cutter arm (a) by 0.4" and lengthening the handles by 1", the mechanical advantage is increased to 10.

7.13

Find the magnitude P of the force required to maintain equilibrium in each system.

a)


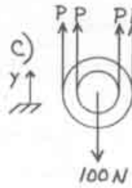
$\Sigma F_y = 2P - 100 = 0$

$\underline{P = 50 N}$

b)

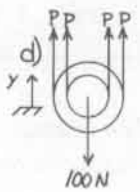

$\Sigma F_y = 2P - 100 = 0$

$\underline{P = 50 N}$

c)


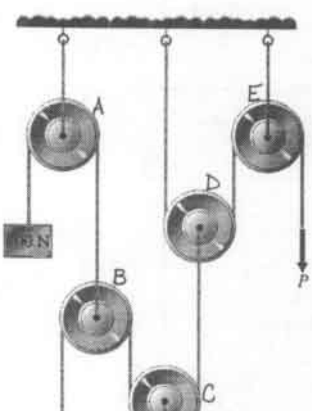
$\Sigma F_y = 4P - 100 = 0$

$\underline{P = 25 N}$

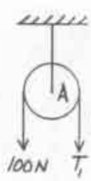
d)


$\Sigma F_y = 4P - 100 = 0$

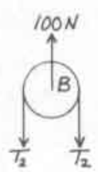
$\underline{P = 25 N}$

e)


(e)

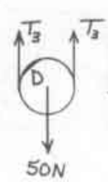


$T_1 = 100 N$ (Simple Pulley)



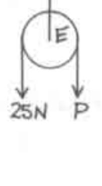
$2T_2 = 100$

$T_2 = 50 N$

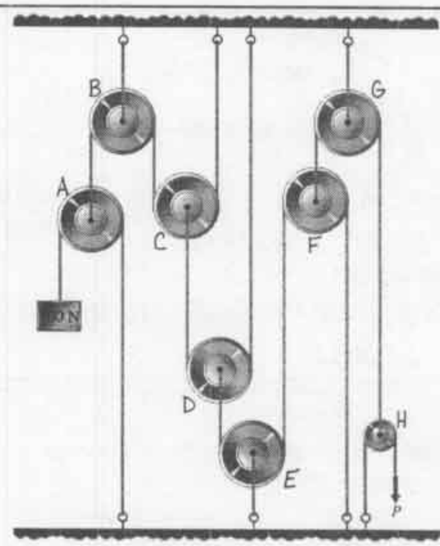


$2T_3 = 50$

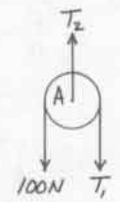
$T_3 = 25 N$



$\underline{P = 25 N}$ (Simple Pulley)

f)


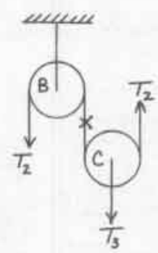
(f)

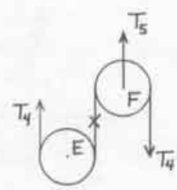


$T_1 = 100 N$

$T_2 = 2(100)$

$T_2 = 200 N$





$T_3 = 2T_2 = 400 N$ $\quad T_4 = 2T_3 = 800 N$ $\quad T_5 = 2T_4 = 1600 N$

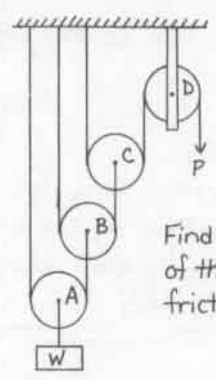
$T_5 = T_6$

$T_6 = 1600 N$

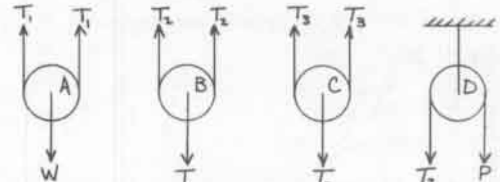
$2P = 1600$

$\underline{P = 800 N}$

7.14



Find the mechanical advantage of the system ($m = \frac{W}{P}$). Neglect friction.

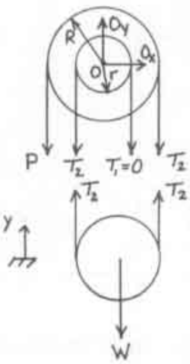


$2T_1 = W$ $\quad 2T_2 = \frac{W}{2}$ $\quad 2T_3 = \frac{W}{4}$ $\quad P = T_3 = \frac{W}{8}$

$T_1 = \frac{W}{2}$ $\quad T_2 = \frac{W}{4}$ $\quad T_3 = \frac{W}{8}$ $\quad \underline{m = \frac{W}{P} = 8}$

7.15

a) Draw separate free-body diagrams of the sprocket unit and the lower pulley.



Because the chain hangs slack from the right side of the smaller pulley, $T=0$ for that chain.

$\Sigma F_y = 2T_2 - W = 0$

$T_2 = \frac{W}{2}$

b) Show that the mechanical advantage is $\frac{W}{P} = \frac{2R}{R-r}$

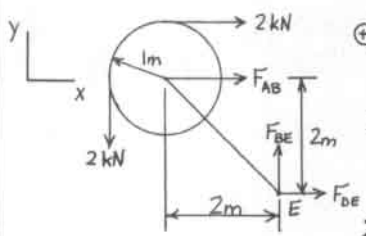
$\circlearrowleft + \Sigma M_o = -\frac{W}{2}(R) + \frac{W}{2}(r) + P(R) = 0$

$PR = \frac{W}{2}(R-r)$

$\underline{\underline{\frac{W}{P} = \frac{2R}{(R-r)}}}$

7.16

Find the forces in members AB, BE, and DE. Use the method of sections.



$\circlearrowleft + \Sigma M_E = -F_{AB}(2) + 2(3) - 2(3) = 0$

$\underline{F_{AB} = 0}$

$\Sigma F_x = 2 + F_{DE} = 0$

$\underline{F_{DE} = -2\text{ kN (Compression)}}$

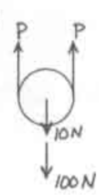
$\Sigma F_y = -2 + F_{BE} = 0$

$\underline{F_{BE} = 2\text{ kN (Tension)}}$

7.17

Find the magnitude of P to maintain equilibrium in each system if each pulley weighs 10 N.

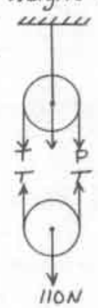
a)



$2P = 110\text{ N}$

$\underline{\underline{P = 55\text{ N}}}$

b)

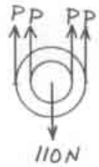


$T = P$

$2T = 110\text{N}$

$\underline{\underline{P = 55\text{N}}}$

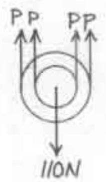
c)



$4P = 110\text{N}$

$\underline{\underline{P = 27.5\text{N}}}$

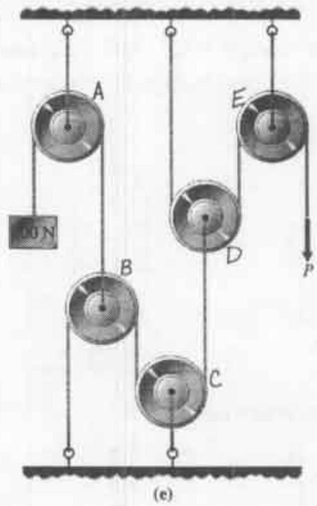
d)



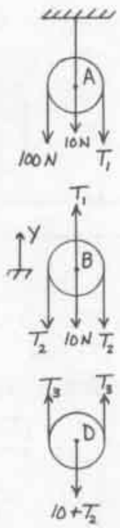
$4P = 110\text{ N}$

$\underline{\underline{P = 27.5\text{ N}}}$

e)



(e)



$T_1 = 100\text{ N}$

$\Sigma F_y = 0$

$\Sigma F_y = 100 - 10 - 2T_2$

$T_2 = 45\text{N}$

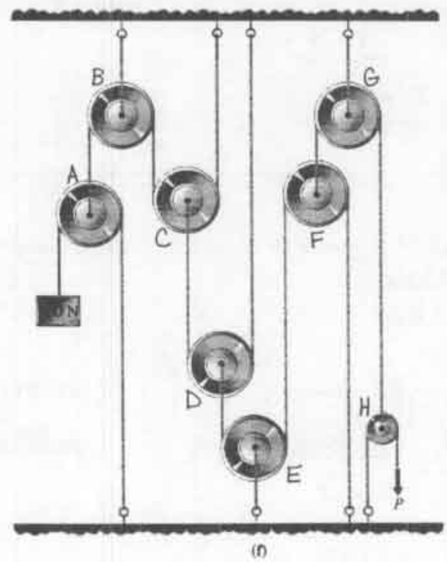
$2T_3 = 10 + 45$

$T_3 = 27.5\text{ N}$

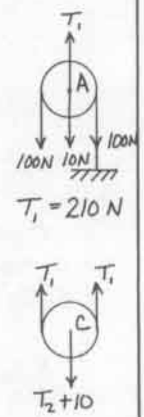
$P = T_3$

$\underline{\underline{P = 27.5\text{ N}}}$

f)



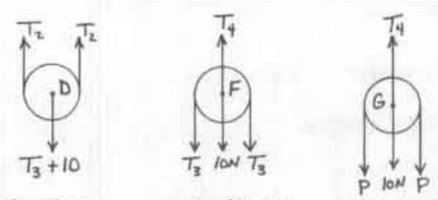
(f)



$T_1 = 210\text{ N}$

$2T_1 = T_2 + 10$

$T_2 = 410\text{ N}$



$2T_2 = T_3 + 10$

$T_3 = 810\text{N}$

$T_4 = 2T_3 + 10$

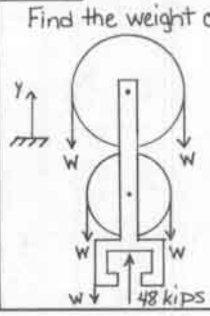
$T_4 = 1630\text{ N}$

$T_4 = 2P + 10$

$\underline{\underline{P = 810\text{N}}}$

7.18

Find the weight of the cage.

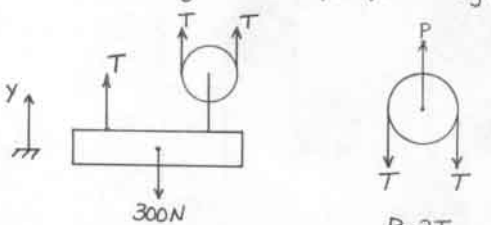


$\Sigma F_y = 48 - 5W = 0$

$\underline{\underline{W = 9.6\text{ kips}}}$

7.19

a) Find the pull P that the worker must exert to support the box if the weights of the pulleys are negligible.



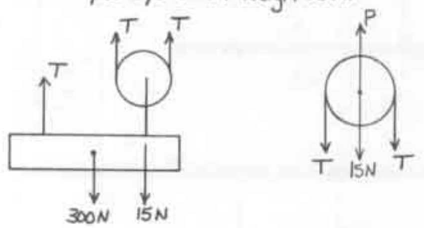
$\Sigma F_y = 3T - 300 = 0$

$T = 100N$

$P = 2T$

$\underline{\underline{P = 200\ N}}$

b) Find P if the pulleys each weigh 15N.



$3T = 315\ N$

$T = 105\ N$

$P = 2T + 15$

$\underline{\underline{P = 225\ N}}$

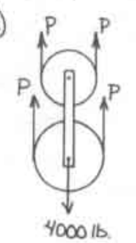
7.20

Find the pull P required to raise the 4000 lb weight with

a) Fig P7.20a

b) Fig P7.20b

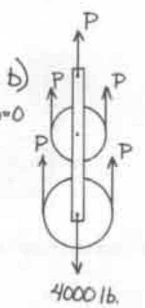
a)



$\Sigma F_y = 4P - 4000 = 0$

$\underline{\underline{P = 1000\ lb.}}$

b)

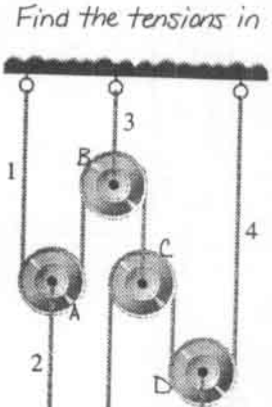


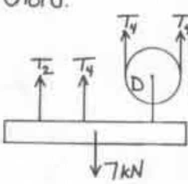
$\Sigma F_y = 5P - 4000\ lb = 0$

$\underline{\underline{P = 800\ lb.}}$

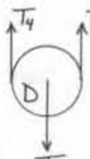
7.21

Find the tensions in each chord.

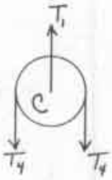




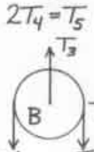
$T_2 + 3T_4 = 7kN$ (1)



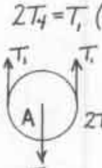
$2T_4 = T_5$ (2)



$2T_4 = T_1$ (3)



$T_3 = 2T_1$ (4)



$2T_1 = T_2$ (5)

Subtract (3) from (2)

$T_1 = T_5$ (6)

Sub for T_1 from (6) into (5).

$2T_5 = T_2$ (7)

Sub for T_5 from (7) into (2)

$T_2 = 4T_4$ (8)

Sub for T_2 from (8) into (1)

$T_4 = 1\ kN$

It follows that...

$\underline{\underline{T_1 = 2kN}}$ $\underline{\underline{T_4 = 1\ kN}}$

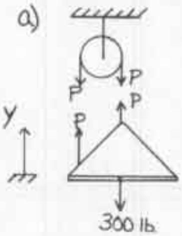
$\underline{\underline{T_2 = 4kN}}$ $\underline{\underline{T_5 = 2\ kN}}$

$\underline{\underline{T_3 = 4kN}}$

7.22

Determine the pull that the worker must exert to maintain equilibrium for each arrangement.

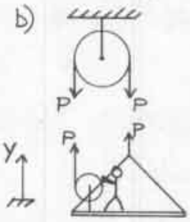
a)



$\Sigma F_y = 2P - 300 = 0$

$\underline{\underline{P = 150\ lb}}$

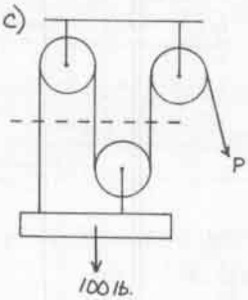
b)

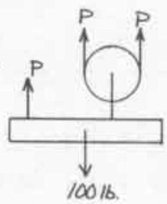


$\Sigma F_y = 2P - 300 = 0$

$\underline{\underline{P = 150\ lb}}$

c)





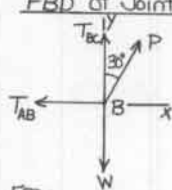
$3P = 100\ lb$

$\underline{\underline{P = 33.3\ lb.}}$

7.23

Find the mechanical advantage ($m = \frac{W}{P}$) of the system.

<u>FBD of Joint B:</u>



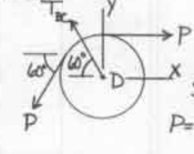
$\Sigma F_y = P\cos 30° + T_{BC} - W = 0$

$T_{BC} = W - P\cos 30°$ (1)

$\Sigma F_x = P\sin 30° - T_{AB} = 0$

$T_{AB} = P\sin 30°$

<u>FBD:</u>



$\Sigma F_y = -P\sin 60° + T_{BC}\sin 60° = 0$

$T_{BC} = P$

Sub for T_{BC} into eqn. (1):

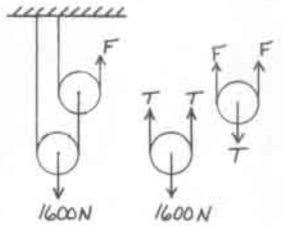
$P = W - P\cos 30°$ $W = P(1 + \cos 30°)$ $\underline{\underline{m = \frac{W}{P} = 1.866}}$

7.24

Design a pulley system for which a person, with minimum effort, can raise a 1600 N Crate using:

a) Two pulleys

- Use an Archimedes system of pulleys



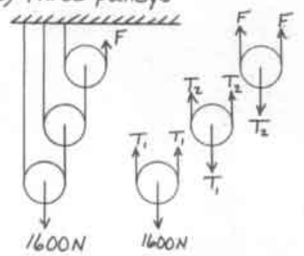
$2T = 1600N$

$T = 800N$

$2F = T$

$\underline{F = 400N}$

b) Three pulleys



$2T_1 = 1600N$

$2T_2 = T_1$

$2F = T_2$

$8F = 1600N$

$\underline{F = 200N}$

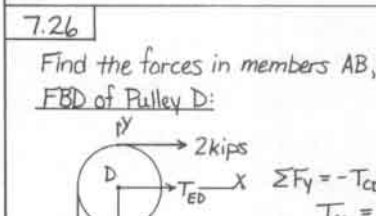
c) Because the crate is being raised slowly, equilibrium conditions apply. The crate cannot be raised an indefinite distance because of the ceiling, but for raising the crate a definite distance, this system uses minimum effort. The average worker could raise the 1600 N crate a short distance using either system since 400 N is about 90 lbs. and 200 N is about 45 lbs.

7.25

Find the forces in members BF and EF.

Using the method of sections (Chapter 6):

FBD of Section: (Cut through AB, BF, EF)



$\Sigma F_y = T_{BF} - 400 = 0$

$\underline{T_{BF} = 400N}$

$\circlearrowleft + \ \Sigma M_A = 400(2) + 400(1) - 400(1) + T_{EF}(2) = 0$

$2T_{EF} = -800$

$\underline{T_{EF} = -400N}$

7.26

Find the forces in members AB, ED, and EC.

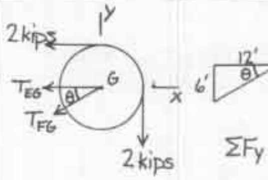
FBD of Pulley D:

$\Sigma F_y = -T_{CD} - 2 = 0$

$T_{CD} = -2$ kips (Compression)

$\Sigma F_x = 2 + T_{ED} = 0$

$\underline{T_{ED} = -2 \text{ kips (compression)}}$



$\tan\theta = \frac{6}{12}$

$\theta = 26.57°$

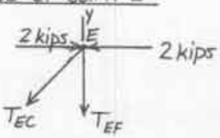
$\Sigma F_y = -T_{FG}\sin\theta - 2 = 0$

$T_{FG} = -4.47$ kips (Compression)

$\Sigma F_x = -2 - T_{FG}\cos\theta - T_{EG} = 0$

$T_{EG} = -2$ kips (Compression)

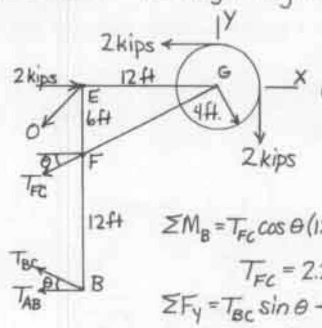
FBD of Joint E:



$\Sigma F_x = 2 - 2 - T_{EC}\cos 45° = 0$

$\underline{T_{EC} = 0}$

FBD of Section: (Cutting through ED, EC, FC, BC, AB)



$\theta = 26.57°$

$\Sigma M_B = T_{FC}\cos\theta(12) - 2(18) + 2(22) - 2(16) = 0$

$T_{FC} = 2.24$ kips (Tension)

$\Sigma F_y = T_{BC}\sin\theta - 2.24\sin\theta - 2 = 0$

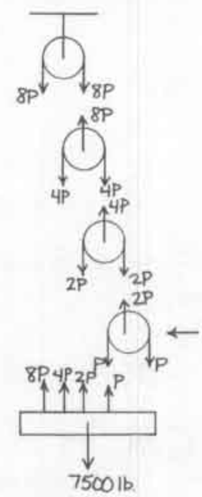
$T_{BC} = 6.71$ kips (Tension)

$\Sigma F_x = -2 + 2 - 6.71\cos\theta - 2.24\cos\theta - T_{AB} = 0$

$\underline{T_{AB} = -8 \text{ kips (Compression)}}$

7.27

a) Find the pull P required to hold a 7500 lb weight.

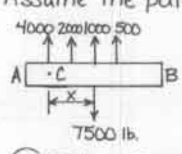


← Start Here

$8P + 4P + 2P + P = 7500\,lb$

$\underline{P = 500\,lb.}$

b) Find the distance x so the bar stays horizontal. Assume the pulleys are 2 ft. in diameter.



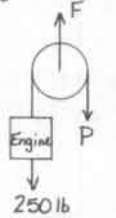
The ropes are evenly spaced at 1 ft.

$\circlearrowleft + \ \Sigma M_C = 2000(1) + 1000(2) + 500(3) - 7500(x) = 0$

$\underline{x = 0.733 \text{ ft.}}$

7.28

Answers B and C are both correct choices. The roof truss would not support the engine if they were using a simple pulley



$P = 250$ lb.

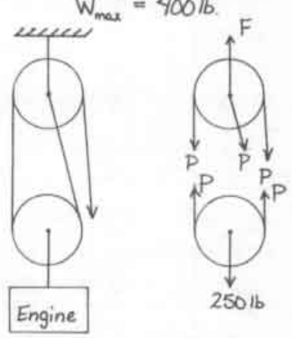
$F = 250 + P = 500$ lb (Failure!)

In choosing between B and C, the status of your friendship will be the governing factor.

7.29

Design a block and tackle system to hoist the engine in problem 7.28.

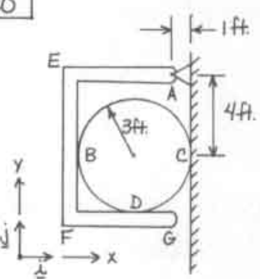
$W_{Engine} = 250$ lb.
$W_{max} = 400$ lb.



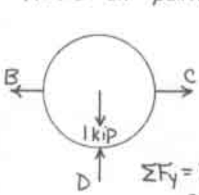
$F = 3P$

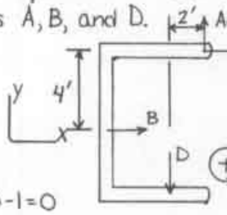
$2P = 250$ lb.
$P = 125$ lb.
$F = 375$ lb (Less than 400 lb) ✓

7.30



Find the (x,y) components of the forces on the frame AEFG at points A, B, and D.





$\Sigma F_y = D - 1 = 0$
$D = 1$ kip
$\therefore \; \underline{D = -1\underline{j} \text{ kip}}$
on the frame

$\Sigma F_y = A_y - 1 = 0$
$A_y = 1$ kip

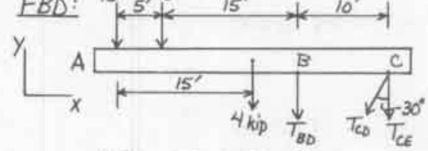
$\circlearrowleft\!+ \Sigma M_A = B(4) + 1(2) = 0$
$B = -0.5$ kip
$\underline{B = -0.5\underline{i} \text{ kip}}$

$\Sigma F_x = A_x + B = 0$
$A_x = 0.5$ kip
$\underline{A = 0.5\underline{i} + 1\underline{j} \text{ kip}}$

7.31

a) Find the forces in members BD, CD, and CE in example 7.8 if the platform weighs 4 kips. Also find the reactions at D and E.

FBD:



$\Sigma F_x = -T_{CD}\sin 30° = 0 \qquad \underline{T_{CD} = 0}$

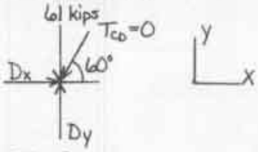
$\Sigma F_y = -10 - 10 - 4 - T_{BD} - T_{CE} = 0$
$T_{BD} + T_{CE} = -24$ kip

$\circlearrowleft\!+ \Sigma M_c = T_{BD}(10) + 4(15) + 10(25) + 10(30) = 0$

$\underline{T_{BD} = -61 \text{ kips (Compression)}}$

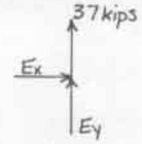
$\underline{T_{CE} = 37 \text{ kips (Tension)}}$

FBD of Joint D:



$\Sigma F_x = D_x = 0$
$\Sigma F_y = D_y - 61 = 0$
$\underline{D_x = 0}$
$\underline{D_y = 61 \text{ kips}}$

FBD of Joint E:

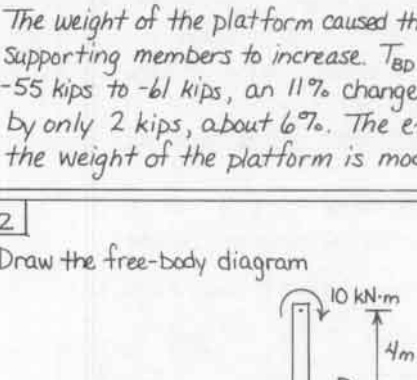


$\underline{E_x = 0}$
$\underline{E_y = -37 \text{ kips}}$

b) The weight of the platform caused the forces in the supporting members to increase. T_{BD} increased from -55 kips to -61 kips, an 11% change. T_{CE} changed by only 2 kips, about 6%. The effect of including the weight of the platform is modest.

7.32

a) Draw the free-body diagram



b) Find the support reactions.

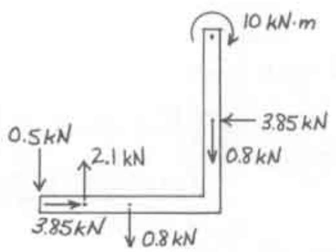
$\Sigma F_y = B_y - 0.5 - 0.8 - 0.8 = 0$
$\underline{B_y = 2.1 \text{ kN}}$

$\circlearrowleft\!+ \Sigma M_B = -10 + 0.5(2) - 0.8(2) + D(4) - 0.8(6) = 0$
$\underline{D = 3.85 \text{ kN}}$

$\Sigma F_x = -B_x - D = 0$
$\underline{B_x = -3.85 \text{ kN}}$

(continued)

c)

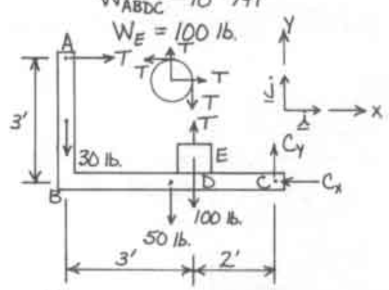


7.33

Find the tension in cord AE and the forces acting on ABDC.

$W_{ABDC} = 10\ \text{lb/ft}$

$W_E = 100\ \text{lb.}$



$$\circlearrowleft\!+\ \Sigma M_c = 100(2) + 50(2.5) + 30(5) - T(2) - T(3) = 0$$

$$\underline{\underline{T = 95\ \text{lb.}}}$$

$$\Sigma F_x = 95 - C_x = 0$$

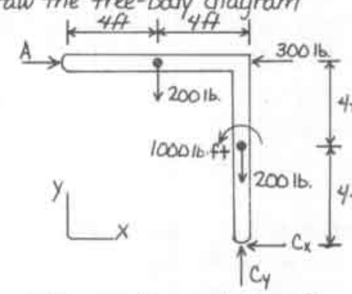
$$\underline{\underline{C_x = 95\ \text{lb.}}}$$

$$\Sigma F_y = T + C_y - 100 - 50 - 30 = 0$$

$$\underline{\underline{C_y = 85\ \text{lb.}}}$$

7.34

a) Draw the free-body diagram



b) Find the reactions at A and C.

$$\Sigma F_y = C_y - 200 - 200 = 0$$

$$\underline{\underline{C_y = 400\ \text{lb.}}}$$

$$\circlearrowleft\!+\ \Sigma M_c = 1000 + 300(8) + 200(4) - A(8) = 0$$

$$\underline{\underline{A = 525\ \text{lb}}}$$

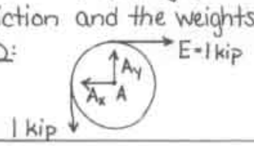
$$\Sigma F_x = A - 300 - C_x = 0$$

$$\underline{\underline{C_x = 225\ \text{lb.}}}$$

7.35

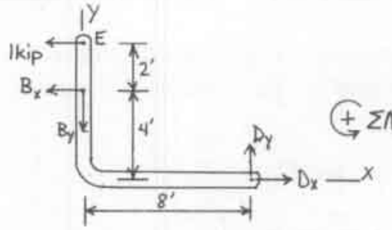
Find the (x,y) projections of the forces on ABC. Neglect friction and the weights of the pulley and bars.

FBD:



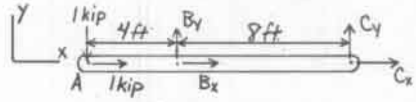
$A_x = 1\ \text{kip}$

$A_y = 1\ \text{kip}$



$$\circlearrowleft\!+\ \Sigma M_D = 1(6) + B_x(4) + B_y(8) = 0$$

$$4B_x + 8B_y = -6 \quad (1)$$



$$\Sigma F_y = B_y + C_y - 1 = 0$$

$$\circlearrowleft\!+\ \Sigma M_c = 1(12) - B_y(8) = 0$$

$$B_y = 1.5\ \text{kips}$$

$$C_y = -0.5\ \text{kips}$$

$$\Sigma F_x = B_x + C_x + 1 = 0$$

From (1): $4B_x + 8(1.5) = -6$

$$B_x = -4.5\ \text{kips}$$

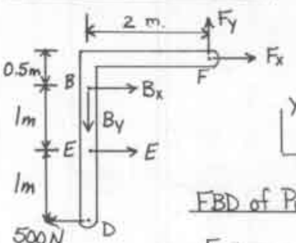
$$C_x = 3.5\ \text{kips}$$

$\underline{\underline{\underline{A} = \underline{i} - \underline{j}\ (\text{kips})}}$ $\quad$ $\underline{\underline{\underline{B} = -4.5\underline{i} + 1.5\underline{j}\ (\text{kips})}}$ $\quad$ $\underline{\underline{\underline{C} = 3.5\underline{i} - 0.5\underline{j}\ (\text{kips})}}$

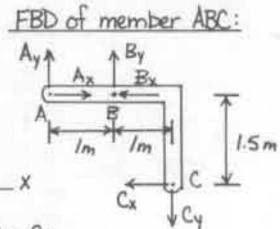
7.36

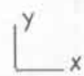
a) Draw free-body diagrams of bars ABC and DEBF and of pulley C.

FBD of member DEBF:

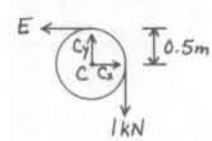


FBD of member ABC:





FBD of Pulley C:



b) Find the forces that act on ABC.

$E = 1\ kN$

$C_x = 1\ kN$

$C_y = 1\ kN$

$$\circlearrowleft\!+\ \Sigma M_A = B_y(1) - 1(1.5) - 1(2) = 0$$

$$B_y = 3.5\ kN$$

$$\Sigma F_y = A_y + 3.5 - 1 = 0$$

$$A_y = -2.5\ kN$$

$$\Sigma F_x = A_x - B_x - 1 = 0$$

Using DEBF:

$$\circlearrowleft\!+\ \Sigma M_F = 3.5(2) + 1(1.5) + B_x(0.5) - 0.5(2.5) = 0$$

$$B_x = -14.5\ kN$$

$$A_x = -13.5\ kN$$

$$\underline{\underline{\underline{A} = -13.5\underline{i} - 2.5\underline{j}\ (kN)}}$$

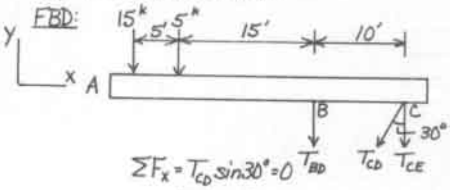
$$\underline{\underline{\underline{B} = 14.5\underline{i} + 3.5\underline{j}\ (kN)}}$$

$$\underline{\underline{\underline{C} = -\underline{i} - \underline{j}\ (kN)}}$$

7.37

a) Find the forces in members BD, CD, and CE and find the reactions at D and E. 3/4 of the load is carried by the front wheels.

FBD:



$\Sigma F_x = T_{CD} \sin 30° = 0$

$\underline{T_{CD} = 0}$

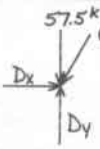
$\circlearrowleft + \Sigma M_C = 15(30) + 5(25) + T_{BD}(10) = 0$

$\underline{T_{BD} = -57.5 \text{ kips (Compression)}}$

$\Sigma F_y = -15 - 5 + 57.5 - T_{CE} = 0$

$\underline{T_{CE} = 37.5 \text{ kips (Tension)}}$

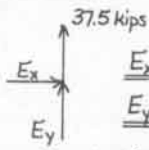
FBD of Joint D:



$\underline{D_x = 0}$

$\underline{D_y = 57.5 \text{ kips}}$

FBD of Joint E:

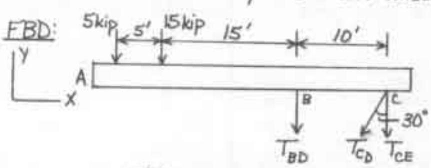


$\underline{E_x = 0}$

$\underline{E_y = -37.5 \text{ kips}}$

b) Find the same forces and reactants if 1/4 of the load was carried by the front wheels.

FBD:



$\Sigma F_x = T_{CD} \sin 30° = 0$

$\underline{T_{CD} = 0}$

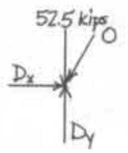
$\Sigma M_C = 5(30) + 15(25) + T_{BD}(10) = 0$

$\underline{T_{BD} = -52.5 \text{ kips (Compression)}}$

$\Sigma F_y = -5 - 15 + 52.5 - T_{CE} = 0$

$\underline{T_{CE} = 32.5 \text{ kips (Tension)}}$

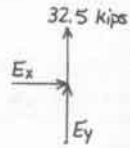
FBD of Joint D:



$\underline{D_x = 0}$

$\underline{D_y = 52.5 \text{kips}}$

FBD of Joint E:



$\underline{E_x = 0}$

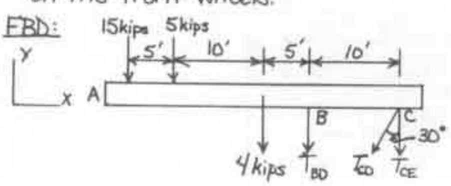
$\underline{E_y = -32.5 \text{ kips}}$

c) Arrangement (b) is preferred because it results in lower forces in the two columns.

7.38

a) Find the forces in the members and the reactants if the platform weighs 4 kips and 3/4 of the load is on the front wheels.

FBD:



$\Sigma F_x = T_{CD} \sin 30° = 0 \qquad \underline{T_{CD} = 0}$

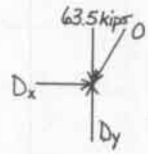
$\circlearrowleft + \Sigma M_C = 15(30) + 5(25) + 4(15) + T_{BD}(10) = 0$

$\underline{T_{BD} = -63.5 \text{ kips (Compression)}}$

$\Sigma F_y = -15 - 5 - 4 + 63.5 - T_{CE} = 0$

$\underline{T_{CE} = 39.5 \text{ kips (Tension)}}$

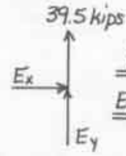
FBD of Joint D:



$\underline{D_x = 0}$

$\underline{D_y = 63.5 \text{ kips}}$

FBD of Joint E:

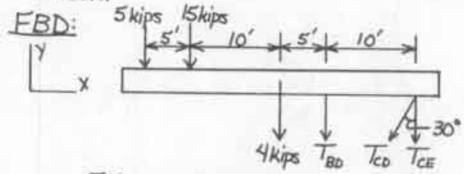


$\underline{E_x = 0}$

$\underline{E_y = -39.5 \text{ kips}}$

b) Solve part (a) but 1/4 of the load is on the front wheels.

FBD:



$\Sigma F_x = T_{CD} \sin 30° = 0$

$\underline{T_{CD} = 0}$

$\circlearrowleft + \Sigma M_C = 5(30) + 15(25) + 4(15) + T_{BD}(10) = 0$

$\underline{T_{BD} = -58.5 \text{ kips (Compress)}}$

$\Sigma F_y = -5 - 15 - 4 + 58.5 - T_{CE} = 0$

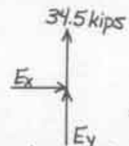
$\underline{T_{CE} = 34.5 \text{ kips (Tension)}}$

FBD of Joint D:

$\underline{D_x = 0}$

$\underline{D_y = 58.5 \text{ kips}}$

FBD of Joint E:



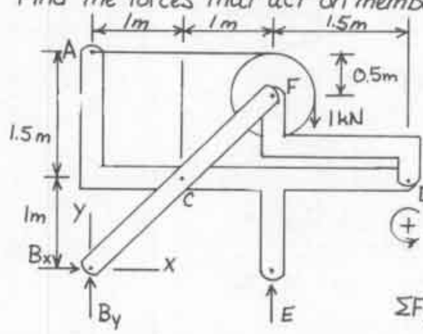
$\underline{E_x = 0}$

$\underline{E_y = -34.5 \text{ kips}}$

c) The weight distribution in part (b) is preferable because the magnitudes of the forces are smaller. The weight of the platform increases the tension in member CE.

7.39

Find the forces that act on member FD.



$\Sigma F_x = B_x = 0$

$B_x = 0$

$\circlearrowleft + \Sigma M_B = E(2) - 1(2.5) = 0$

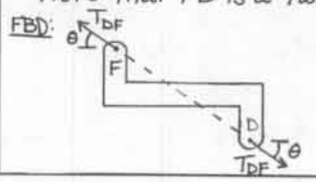
$E = 1.25 \text{ kN}$

$\Sigma F_y = B_y + 1.25 - 1 = 0$

$B_y = -0.25 \text{ kN}$

Note that FD is a two-force member.

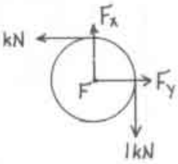
FBD:



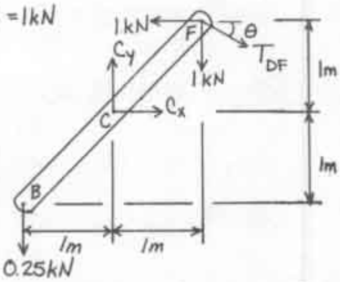
$\tan\theta = \frac{1}{1.5}$

$\theta = 33.69°$

FBD:



$F_x = F_y = 1 kN$



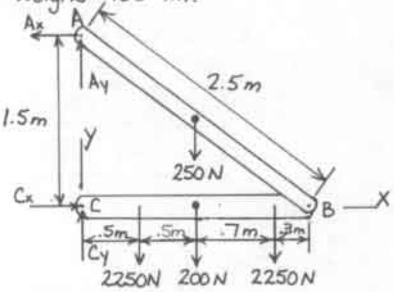
$$\circlearrowleft + \Sigma M_C = .25(1) - 1(1) + 1(1) - T_{DF}\cos\theta(1) - T_{DF}\sin\theta(1) = 0$$

$$0.25 = 1.387\, T_{DF}$$

$$\underline{\underline{T_{DF} = 0.1803\ kN = 180.3\ N}}$$

7.40

Find the forces in bar AB and at pins B and C if each bar weighs 100 N/m.



$$\Sigma F_y = A_y + C_y - 2250 - 200 - 250 - 2250 = 0 \quad (1)$$

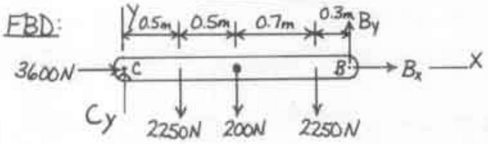
$$\circlearrowleft + \Sigma M_A = -250(1) - 200(1) - 2250(.5) - 2250(1.7) + C_x(1.5) = 0$$

$$\underline{C_x = 3600\ N}$$

$$\Sigma F_x = C_x - A_x = 0$$

$$\underline{A_x = 3600\ N}$$

FBD:



$$\Sigma F_x = 3600 + B_x = 0 \qquad \underline{B_x = -3600\ N}$$

$$\circlearrowleft + \Sigma M_B = 2250(.3) + 200(1) + 2250(1.5) - C_y(2) = 0$$

$$\underline{C_y = 2125\ N}$$

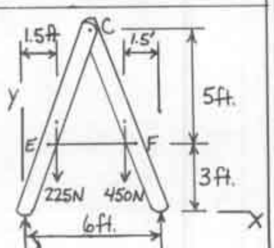
From eqn. (1): $\underline{A_y = 2825\ N}$

$$\Sigma F_y = 2125 - 4700 + B_y = 0$$

$$\underline{B_y = 2575\ N}$$

7.41

Find the force in cable EF.

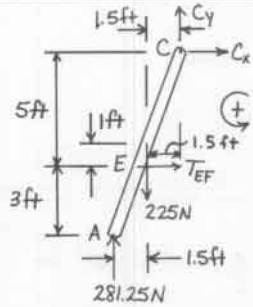


$$\circlearrowleft + \Sigma M_A = -1.5(225) - 4.5(450) + B(6) = 0$$

$$B = 393.75\ N$$

$$\Sigma F_y = A + B - 225 - 450 = 0$$

$$A = 281.25\ N$$



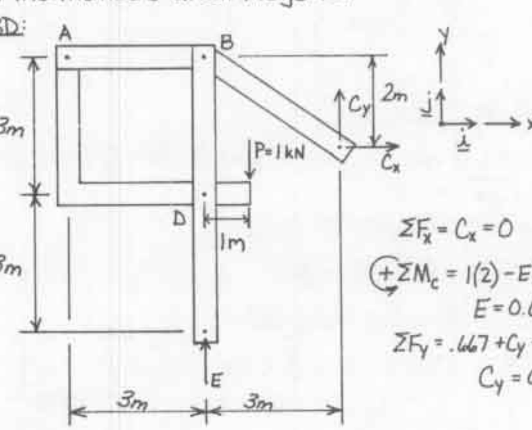
$$\circlearrowleft + \Sigma M_C = 225(1.5) + T_{EF}(5) - 281.25(3) = 0$$

$$\underline{\underline{T_{EF} = 101.3\ N}}$$

7.42

Find the (x, y) components of the forces exerted on all of the members at all the joints.

FBD:



$$\Sigma F_x = C_x = 0$$

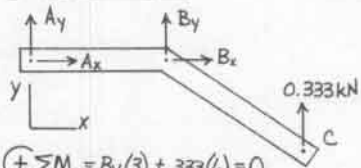
$$\circlearrowleft + \Sigma M_C = 1(2) - E(3) = 0$$

$$E = 0.667\ kN$$

$$\Sigma F_y = .667 + C_y - 1 = 0$$

$$C_y = 0.333\ kN$$

FBD of member ABC:



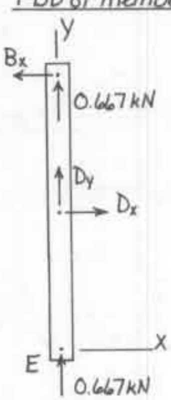
$$\circlearrowleft + \Sigma M_A = B_y(3) + .333(6) = 0 \qquad \Sigma F_x = A_x + B_x = 0$$

$$B_y = -0.667\ kN$$

$$\Sigma F_y = A_y + B_y + 0.333 = 0$$

$$A_y = 0.333\ kN$$

FBD of member BDE:



$$\Sigma F_x = D_x - B_x = 0$$

$$\Sigma F_y = 0.667 + 0.667 + D_y = 0$$

$$D_y = -1.333\ kN$$

$$\circlearrowleft + \Sigma M_B = D_x(3) = 0 \qquad D_x = 0$$

$$\therefore B_x = 0$$

$$\therefore A_x = 0$$

Member AD:

$\underline{A} = -0.333\underline{j}$ (kN)

$\underline{D} = 1.333\underline{j}$ (kN)

$\underline{P} = -\underline{j}$ (kN)

Member ABC:

$\underline{A} = 0.333\underline{j}$ (kN)

$\underline{B} = -0.667\underline{j}$ (kN)

$\underline{C} = 0.333\underline{j}$ (kN)

(continued)

Member BDE:

$\underline{B} = 0.667\underline{j}\ (kN)$

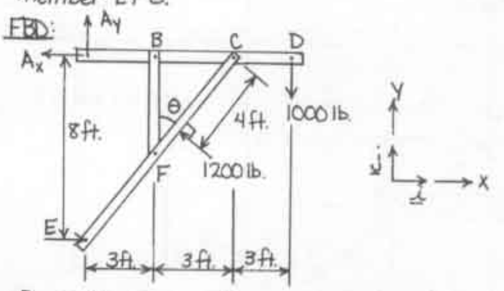
$\underline{D} = -1.333\underline{j}\ (kN)$

$\underline{E} = 0.667\underline{j}\ (kN)$

7.43

Find the (x,y) projections of the forces that act on member EFC.

FBD:



By pythagorean theorem, member EFC has a length of $\sqrt{8^2+6^2} = 10\ ft.$

By similar triangles, FC is 5' long.

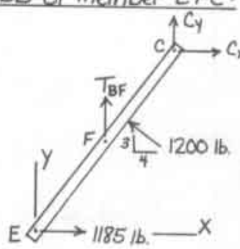
$\sin\theta = \frac{3}{5} \quad \cos\theta = \frac{4}{5}$

$\Sigma F_y = A_y - 1000 + 1200(\frac{3}{5}) = 0 \quad A_y = 280\ lb.$

$(+)\Sigma M_c = -1200(4) - 1000(3) + E(8) - A_y(6) = 0$

$E = 1185\ lb.$

FBD of member EFC:



$\Sigma F_x = C_x + 1185 - 1200(\frac{4}{5}) = 0$

$C_x = -225\ lb.$

$(+)\Sigma M_c = -T_{BF}(3) - 1200(4) + 1185(8) = 0$

$T_{BF} = 1560\ lb.$

$\Sigma F_y = C_y + 1560 + 1200(\frac{3}{5}) = 0$

$C_y = -2280\ lb.$

$\underline{C} = -225\underline{i} - 2280\underline{j}\ (lb)$

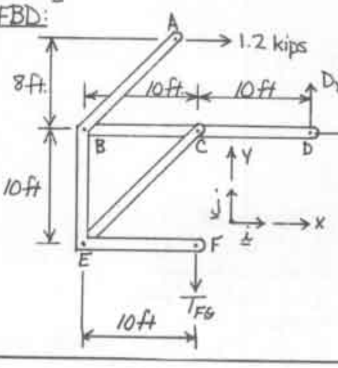
$\underline{E} = 1185\underline{i}\ (lb)$

$\underline{T_{BF}} = 1560\underline{j}\ (lb.)$

7.44

Find the forces that act on each member in terms of $\underline{i}$ and $\underline{j}$.

FBD:



$\Sigma F_x = 1.2 + D_x = 0$

$D_x = -1.2\ kips$

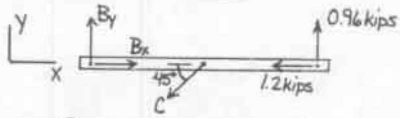
$(+)\Sigma M_D = T_{FG}(10) - 1.2(8) = 0$

$T_{FG} = 0.96\ kips$

$\Sigma F_y = D_y - 0.96 = 0$

$D_y = 0.96\ kips$

FBD of member BCD:



$(+)\Sigma M_c = 0.96(10) - B_y(10) = 0$

$B_y = 0.96\ kips$

$\Sigma F_y = 0.96 + 0.96 - C\sin 45° = 0$

$C = 2.72\ kips$

$\Sigma F_x = B_x - 1.2 - 2.72\cos 45° = 0$

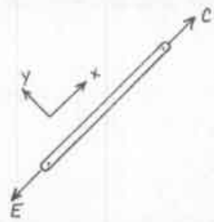
$B_x = 3.12\ kips$

$\underline{B} = 3.12\underline{i} + 0.96\underline{j}\ (kips)$

$\underline{C} = -1.92\underline{i} - 1.92\underline{j}\ (kips)$

$\underline{D} = -1.2\underline{i} + 0.96\underline{j}\ (kips)$

FBD of member CE:



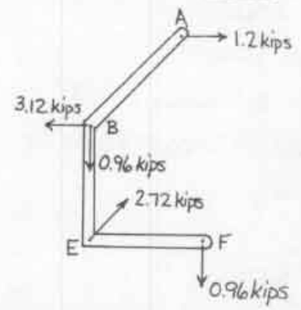
$\Sigma F_x = C - E = 0$

$E = 2.72\ kips$

$\underline{C} = 1.92\underline{i} + 1.92\underline{j}$

$\underline{E} = -1.92\underline{i} - 1.92\underline{j}$

FBD of member ABEF:



$\underline{A} = 1.2\underline{i}\ (kips)$

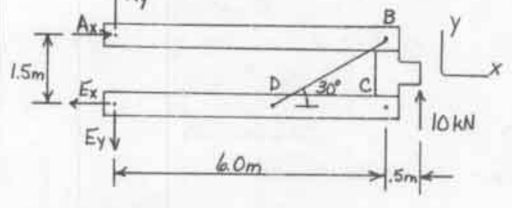
$\underline{B} = -3.12\underline{i} - 0.96\underline{j}\ (kips)$

$\underline{E} = 1.92\underline{i} + 1.92\underline{j}\ (kips)$

$\underline{F} = -0.96\underline{j}\ (kips)$

7.45

a) Find the tension in rod BD and the forces on each member.



$\Sigma F_y = A_y - E_y + 10 = 0 \quad (1)$

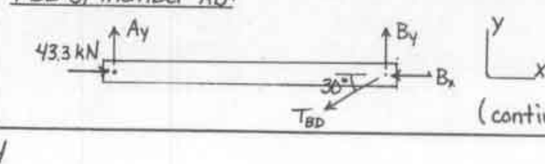
$(+)\Sigma M_A = 10(6.5) - E_x(1.5) = 0$

$\underline{E_x = 43.3\ kN}$

$\Sigma F_x = A_x - E_x = 0$

$\underline{A_x = 43.3\ kN}$

FBD of member AB:



(continued)

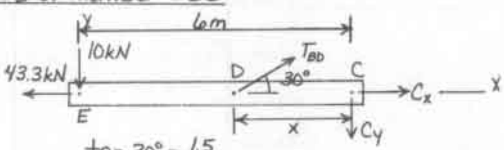
$\overset{+}{\curvearrowleft}\Sigma M_B = A_y(6) = 0 \qquad \underline{A_y = 0}$

From eqn. (1): $\underline{E_y = 10\ kN}$

$\Sigma F_x = 43.3 - B_x - T_{BD}\cos 30° = 0 \quad (2)$

$\Sigma F_y = B_y - T_{BD}\sin 30° = 0 \quad (3)$

FBD of member EDC:



$\tan 30° = \frac{1.5}{x}$

$x = 2.60 m$

$\overset{+}{\curvearrowleft}\Sigma M_C = -T_{BD}\sin 30°(2.60) + 10(6) = 0$

$\underline{T_{BD} = 46.2\ kN}$

$\Sigma F_x = -43.3 + 46.2\cos 30° + C_x = 0$

$\underline{C_x = 3.33\ kN}$

$\Sigma F_y = -10 + 46.2\sin 30° - C_y = 0$

$\underline{C_y = 13.09\ kN}$

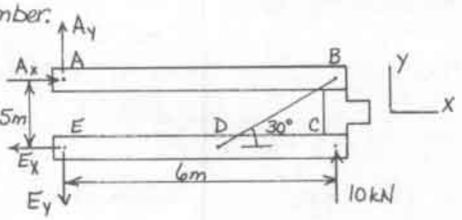
From eqn. (2): $\underline{B_x = 3.33\ kN}$

From eqn. (3): $\underline{B_y = 23.09\ kN}$

b) This structure is not a frame, because member AB is free to move if separated from the supports.

7.46

Find the tension in rod BD and the forces on each member.

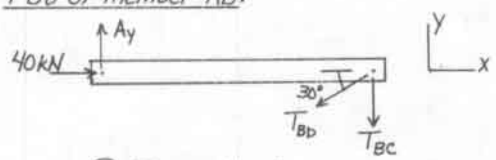


$\Sigma F_y = 10 + A_y - E_y = 0 \quad (1)$

$\overset{+}{\curvearrowleft}\Sigma M_E = 10(6) - A_x(1.5) = 0 \qquad \underline{A_x = 40\ kN}$

$\Sigma F_x = A_x - E_x = 0 \qquad \underline{E_x = 40\ kN}$

FBD of member AB:



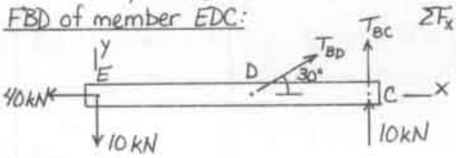
$\overset{+}{\curvearrowleft}\Sigma M_B = A_y(6) = 0$

$\underline{A_y = 0}$

From eqn. (1): $\underline{E_y = 10\ kN}$

$\Sigma F_y = -T_{BD}\sin 30° - T_{BC} = 0$

FBD of member EDC:



$\Sigma F_x = -40 + T_{BD}\cos 30° = 0$

$\underline{T_{BD} = 46.2\ kN}$ (T)

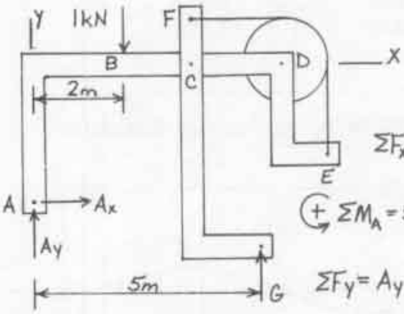
$\Sigma F_y = 10 + T_{BC} - 10 + T_{BD}\sin 30° = 0$

$\underline{T_{BC} = -23.09\ kN}$ (C)

7.47

Find the force at point C on member ABCDE.

FBD:



$\Sigma F_x = A_x = 0$

$A_x = 0$

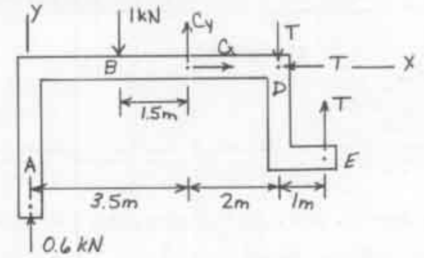
$\overset{+}{\curvearrowleft}\Sigma M_A = 5G - 2(1) = 0$

$G = 0.40\ kN$

$\Sigma F_y = A_y + G - 1 = 0$

$A_y = 0.60\ kN$

FBD of member ABCDE:



$\overset{+}{\curvearrowleft}\Sigma M_C = -0.6(3.5) + 1(1.5) - 2T + 3T = 0$

$T = 0.60 kN$

$\Sigma F_x = C_x - T = 0$

$C_x = 0.60 kN$

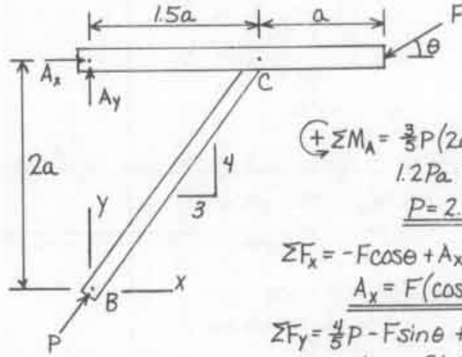
$\Sigma F_y = C_y + 0.60 - 1.0 + T - T = 0$

$C_y = 0.40 kN$

$\underline{\underline{C} = 0.60\underline{i} + 0.40\underline{j}\ (kN)}$

7.48

a) Derive formulas for the reactions A_x, A_y, and the force P in member BC, all in terms of F, a, and θ.



$\overset{+}{\curvearrowleft}\Sigma M_A = \frac{3}{5}P(2a) - F\sin\theta(2.5a) = 0$

$1.2Pa = 2.5aF\sin\theta$

$\underline{P = 2.083 F\sin\theta}$

$\Sigma F_x = -F\cos\theta + A_x + \frac{3}{5}P = 0$

$\underline{A_x = F(\cos\theta - 1.25\sin\theta)}$

$\Sigma F_y = \frac{4}{5}P - F\sin\theta + A_y = 0$

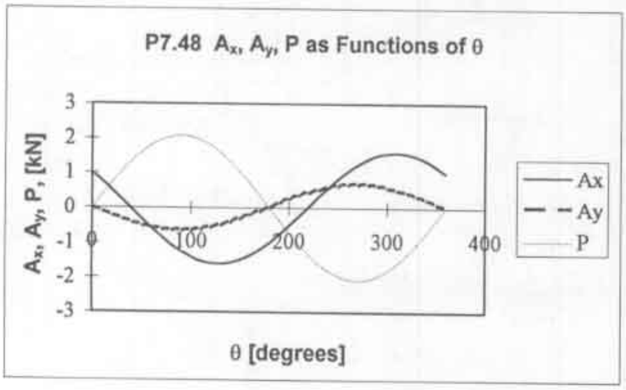
$\underline{A_y = -\tfrac{2}{3}F\sin\theta}$

b) Plot A_x, A_y, and P as functions of θ for $0° \le \theta \le 360°$ if F = 1kN and a = 1m.

(continued)

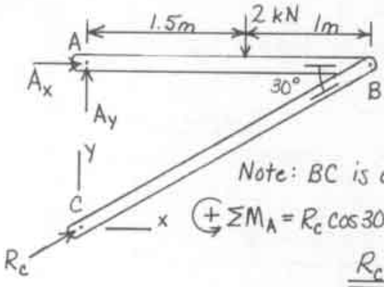
Sample Calculations:

θ (degrees)	A_x (kN)	A_y (kN)	P (kN)
0	1	0	0
10	0.768	-0.116	0.362
20	0.512	-0.228	0.713
30	0.241	-0.333	1.042
40	-0.037	-0.429	1.339
50	-0.315	-0.511	1.596



7.49

a) Find the forces that act on members AB and BC.



Note: BC is a two-force member.

$(+\circlearrowleft)\ \Sigma M_A = R_c \cos 30° (1.443) - 2(1.5) = 0$

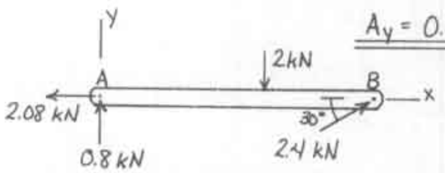
$\underline{\underline{R_c = 2.4 \text{ kN}}}$

$\Sigma F_x = A_x + 2.4 \cos 30° = 0$

$\underline{\underline{A_x = -2.08 \text{ kN}}}$

$\Sigma F_y = A_y - 2 + 2.4 \sin 30° = 0$

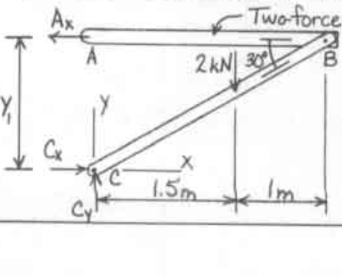
$\underline{\underline{A_y = 0.8 \text{ kN}}}$



b) This structure is not a frame, because it would collapse if the supports were removed.

7.50

a) Find the forces that act on members AB and BC



$\tan 30° = \frac{y_1}{2.5}$

$y_1 = 1.443 \text{ m}$

$\Sigma F_y = C_y - 2 = 0$

$\underline{\underline{C_y = 2 \text{ kN}}}$

$(+\circlearrowleft)\ \Sigma M_c = A_x(1.443) - 2(1.5) = 0$

$\underline{\underline{A_x = 2.08 \text{ kN}}}$

$\Sigma F_x = C_x - 2.08 = 0$

$\underline{\underline{C_x = 2.08 \text{ kN}}}$

$\underline{\underline{B_x = 2.08 \text{ kN}}}$

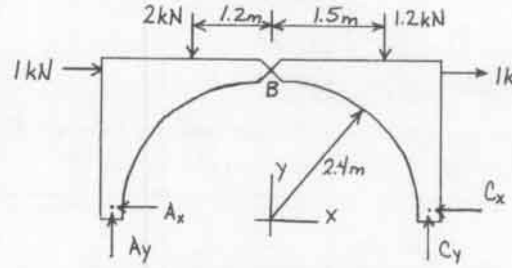
$\underline{\underline{B_y = 0}}$

$\underline{\underline{A_y = 0}}$

b) This result does not violate the principle of transmissibility. A force is moved along its line of action and equilibrium is still maintained. The net reaction forces are also unchanged.

7.51

a) Find the forces that act on the two parts.



$\Sigma F_x = 1 + 1 - A_x - C_x = 0$

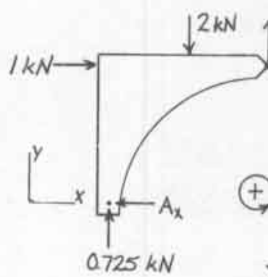
$A_x + C_x = 2 \quad (1)$

$(+\circlearrowleft)\ \Sigma M_A = -1(2.4) - 2(1.2) - 1.2(3.9) - 1(2.4) + C_y(4.8) = 0$

$\underline{\underline{C_y = 2.48 \text{ kN}}}$

$\Sigma F_y = -2 - 1.2 + A_y + C_y = 0$

$\underline{\underline{A_y = 0.725 \text{ kN}}}$



$\Sigma F_y = 0.725 + B_y - 2 = 0$

$\underline{\underline{B_y = 1.275 \text{ kN}}}$

$(+\circlearrowleft)\ \Sigma M_A = -1(2.4) - 2(1.2) + B_y(2.4) - B_x(2.4) = 0$

$\underline{\underline{B_x = -0.725 \text{ kN}}}$

$\Sigma F_x = 1 + B_x - A_x = 0$

$\underline{\underline{A_x = 0.275 \text{ kN}}}$

From eqn. (1): $A_x + C_x = 2$

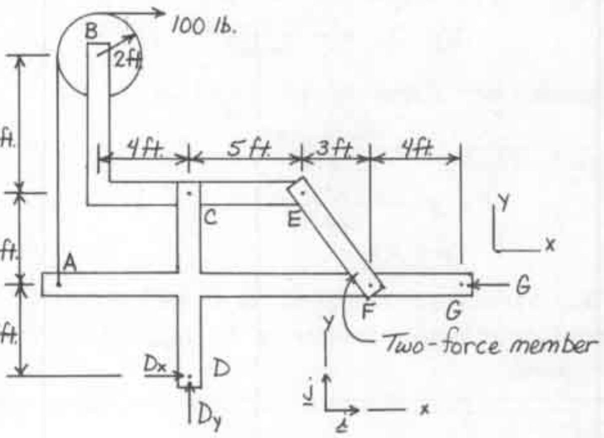
$\underline{\underline{C_x = 1.725 \text{ kN}}}$

b) This structure is a structural mechanism because it would collapse if the supports were removed.

7.52

Find the forces exerted on all the members at all the joints and pulleys in terms of $\underline{i}$ and $\underline{j}$.

(continued)



$\Sigma F_y = D_y = 0$ $\underline{\underline{D_y = 0}}$

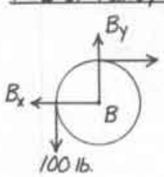
$\Sigma M_D = G(4) - 100(16) = 0$

$\underline{\underline{G = 400 \text{ lb.}}}$

$\Sigma F_x = 100 + D_x - G = 0$

$\underline{\underline{D_x = 300 \text{ lb.}}}$

FBD of Pulley B:

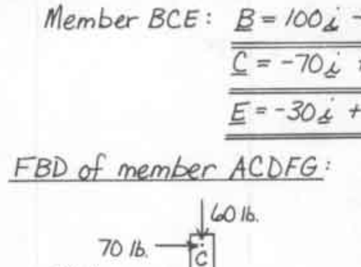


$B_x = 100$ lb.

$B_y = 100$ lb.

$\underline{\underline{\mathbf{B} = -100\,\mathbf{i} + 100\,\mathbf{j} \text{ (lb)}}}$

FBD of member BCE:



$\Sigma M_E = -C_y(5) - 100(6) + 100(9) = 0$

$C_y = 60$ lb.

$\Sigma F_y = C_y - 100 - \left(\frac{4}{5}\right)EF = 0$

$EF = -50$ lb.

$\Sigma F_x = 100 - C_x + \left(\frac{3}{5}\right)EF = 0$

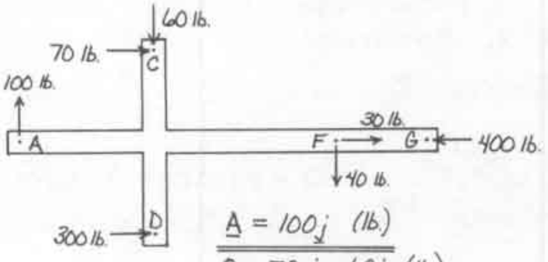
$C_x = 70$ lb.

Member BCE: $\underline{\underline{\mathbf{B} = 100\,\mathbf{i} - 100\,\mathbf{j} \text{ (lb.)}}}$

$\underline{\underline{\mathbf{C} = -70\,\mathbf{i} + 60\,\mathbf{j} \text{ (lb.)}}}$

$\underline{\underline{\mathbf{E} = -30\,\mathbf{i} + 40\,\mathbf{j} \text{ (lb.)}}}$

FBD of member ACDFG:



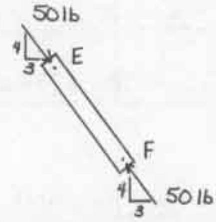
$\underline{\underline{\mathbf{A} = 100\,\mathbf{j} \text{ (lb.)}}}$

$\underline{\underline{\mathbf{C} = 70\,\mathbf{i} - 60\,\mathbf{j} \text{ (lb.)}}}$

$\underline{\underline{\mathbf{D} = 300\,\mathbf{i} \text{ (lb.)}}}$

$\underline{\underline{\mathbf{F} = 30\,\mathbf{i} - 40\,\mathbf{j} \text{ (lb.)}}}$ $\underline{\underline{\mathbf{G} = -400\,\mathbf{i} \text{ (lb.)}}}$

FBD of member EF:

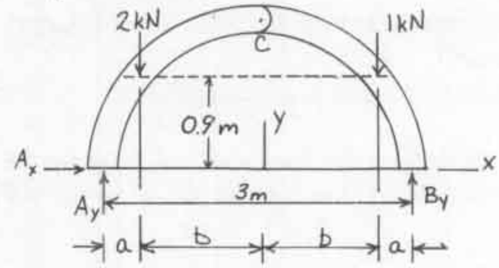


$\underline{\underline{\mathbf{E} = 30\,\mathbf{i} - 40\,\mathbf{j} \text{ (lb.)}}}$

$\underline{\underline{\mathbf{F} = -30\,\mathbf{i} + 40\,\mathbf{j} \text{ (lb.)}}}$

7.53

a) Find the tension in the tie rod and the forces acting on AC and BC.



$b = \sqrt{(1.5)^2 - (0.9)^2}$ (Pythagorean Theorem)

$b = 1.2$ m

$a = 1.5 - 1.2 = 0.3$ m

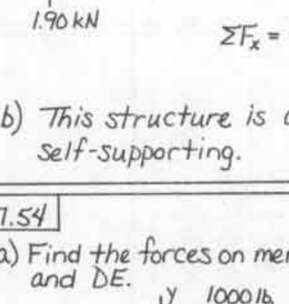
$\circlearrowleft\!+\ \Sigma M_A = B_y(3) - 1(2.7) - 2(0.3) = 0$

$\underline{\underline{B_y = 1.10 \text{ kN}}}$

$\Sigma F_x = A_x = 0$ $\underline{\underline{A_x = 0}}$

$\Sigma F_y = A_y + 1.10 - 2 - 1 = 0$

$\underline{\underline{A_y = 1.90 \text{ kN}}}$



$\Sigma F_y = 1.90 - 2 - C_y = 0$

$\underline{\underline{C_y = -0.1 \text{ kN}}}$

$\circlearrowleft\!+\ \Sigma M_C = 2(1.2) - 1.90(1.5) + T(1.5) = 0$

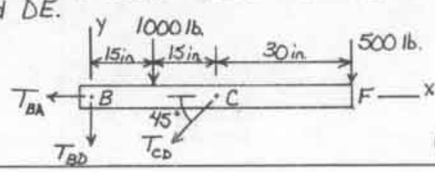
$\underline{\underline{T = 0.3 \text{ kN}}}$

$\Sigma F_x = T - C_x = 0$ $\underline{\underline{C_x = 0.3 \text{ kN}}}$

b) This structure is a frame because it can be self-supporting.

7.54

a) Find the forces on member BCF and in members AD and DE.



(continued)

$\circlearrowleft + \Sigma M_C = T_{BD}(30) + 1000(15) - 500(30) = 0$

$\underline{\underline{T_{BD} = 0}}$

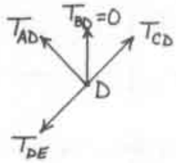
$\Sigma F_y = -1000 - 500 - T_{CD}\sin 45° = 0$

$\underline{\underline{T_{CD} = -2121 \text{ lb. (Comp.)}}}$

$\Sigma F_x = -T_{BA} - T_{CD}\cos 45° = 0$

$\underline{\underline{T_{BA} = 1500 \text{ lb. (Tension)}}}$

<u>FBD of Joint D:</u>



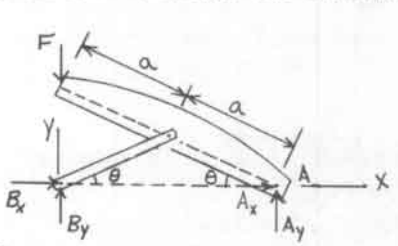
By inspection, $\underline{T_{AD} = 0}$ and

$\underline{\underline{T_{DE} = T_{CD} = -2121 \text{ lb. (Compression)}}}$

b) This structure is not a frame because member ED is free to rotate about D if the supports are removed.

7.55

Derive expressions for the reactions of support A and the thrust force P in member BC in terms of F and θ.

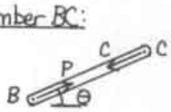


$\circlearrowleft + \Sigma M_B = A_y(2a\cos\theta) = 0 \qquad \underline{\underline{A_y = 0}}$

$\Sigma F_y = B_y - F = 0 \qquad B_y = F$

$\Sigma F_x = B_x + A_x = 0 \qquad A_x = -B_x$

<u>FBD of member BC:</u>



$B_x = P\cos\theta$

$B_y = P\sin\theta$

$F = P\sin\theta$

$\underline{\underline{P = \frac{F}{\sin\theta}}}$

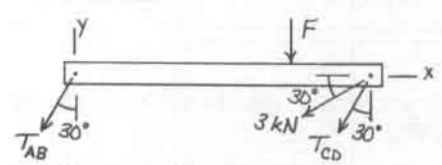
$A_x = -P\cos\theta = -\left(\frac{F}{\sin\theta}\right)\cos\theta$

$\underline{\underline{A_x = -\frac{F}{\tan\theta}}}$

7.56

a) Find the magnitude of F for $T_{AC} = 3$ kN.

<u>FBD of member BC:</u>



$\Sigma F_x = (-T_{AB} - T_{CD})\sin 30° - 3\cos 30° = 0$

$T_{AB} + T_{CD} = \frac{-3}{\tan 30°} \quad (1)$

$\Sigma F_y = -(T_{AB} + T_{CD})\cos 30° - F - 3\sin 30° = 0$

$T_{AB} + T_{CD} = \frac{-F - 3\sin 30°}{\cos 30°} \quad (2)$

Equate right side of (1) with right side of (2)

$\frac{-3}{\tan 30°} = \frac{-F - 3\sin 30°}{\cos 30°}$

$F = 3\,\frac{\cos^2 30° - \sin^2 30°}{\sin 30°}$

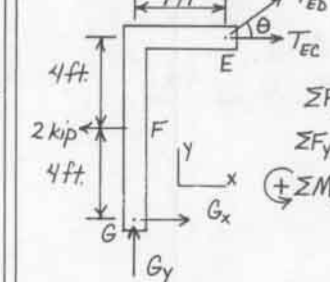
$\underline{\underline{F = 3 \text{ kN}}}$

b) This structure is a structural mechanism because it would collapse if the supports were removed.

7.57

Find the force in member BC.

<u>FBD of member EFG:</u>



$\sin\theta = \frac{3}{5}, \ \cos\theta = \frac{4}{5}$

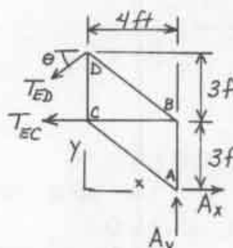
$\Sigma F_x = T_{EC} + \frac{4}{5}T_{ED} + G_x - 2 = 0 \quad (1)$

$\Sigma F_y = G_y + \frac{3}{5}T_{ED} = 0 \quad (2)$

$\circlearrowleft + \Sigma M_E = 8G_x - 2(4) - G_y(4) = 0$

$G_y = 2G_x - 2 \quad (3)$

<u>Truss section:</u>



$\circlearrowleft + \Sigma M_A = 3T_{EC} + 6\left(\frac{4}{5}T_{ED}\right) + 4\left(\frac{3}{5}T_{ED}\right) = 0$

$T_{EC} = -2.4\,T_{ED} \quad (4)$

$\Sigma F_x = A_x - T_{EC} - \frac{4}{5}T_{ED} = 0 \quad (5)$

$\Sigma F_y = A_y - \frac{3}{5}T_{ED} = 0 \quad (6)$

From (3), sub. for G_y in (2): $G_x = 1 - 0.3\,T_{ED} \quad (7)$

From (4) and (7), sub. for T_{EC} and G_x in (1).

$T_{ED} = -0.526$ kips (Comp)

With successive substitutions in (2), (4), (3), (6), and (5), you get

$G_y = 0.3158$ kips

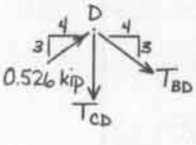
$T_{EC} = 1.263$ kips (Tension)

$G_x = 1.1579$ kips

$A_y = -0.3158$ kips

$A_x = 0.8420$ kips

<u>FBD of Joint D:</u>

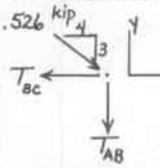


$\Sigma F_x = \frac{4}{5}(0.526) + \frac{4}{5}(T_{BD}) = 0$

$\underline{\underline{T_{BD} = -0.526 \text{ kips (Comp.)}}}$

(continued)

FBD of Joint B:

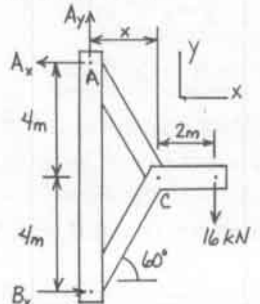


$\Sigma F_x = \frac{4}{5}(0.526) - T_{BC} = 0$

$\underline{T_{BC} = 0.421 \text{ kips (Tension)}}$

7.58

Find the (x,y) projections of all the forces acting on the members.



$\Sigma F_y = A_y - 16 = 0$

$\underline{A_y = 16 \text{ kN}}$

$\tan 60° = \frac{4}{x}$

$x = 2.31 \text{ m}$

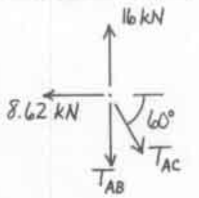
$\circlearrowleft + \Sigma M_A = -16(2+2.31) + B_x(8) = 0$

$\underline{B_x = 8.62 \text{ kN}}$

$\Sigma F_x = B_x - A_x = 0$

$\underline{A_x = 8.62 \text{ kN}}$

FBD of Joint A:



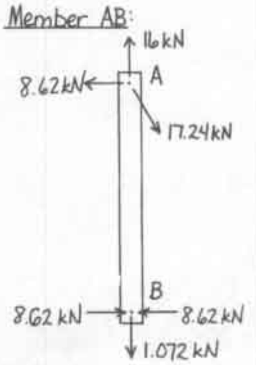
$\Sigma F_x = T_{AC} \cos 60° - 8.62 = 0$

$\underline{T_{AC} = 17.24 \text{ kN (Tension)}}$

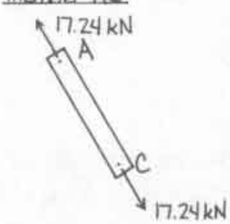
$\Sigma F_y = 16 - T_{AB} - 17.24 \sin 60° = 0$

$\underline{T_{AB} = 1.072 \text{ kN (Tension)}}$

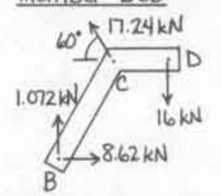
Member AB:



Member AC:

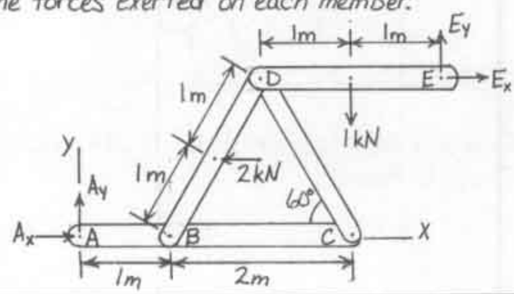


Member BCD:



7.59

Find the forces exerted on each member.

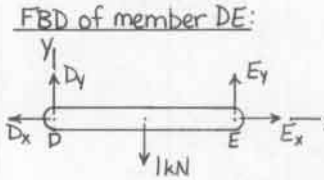


$\Sigma F_x = A_x + E_x - 2 = 0 \quad (1)$

$\Sigma F_y = A_y + E_y - 1 = 0 \quad (2)$

$\circlearrowleft + \Sigma M_A = 2(1\sin 60°) + E_y(4) - 1(3) - E_x(2\sin 60°) = 0 \quad (3)$

FBD of member DE:



$\circlearrowleft \Sigma M_D = -1(1) + E_y(2) = 0$

$E_y = 0.5 \text{ kN}$

$\Sigma F_y = D_y + E_y - 1 = 0$

$D_y = 0.5 \text{ kN}$

$\Sigma F_x = E_x - D_x = 0$

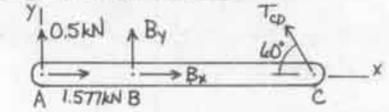
$D_x = 0.423 \text{ kN}$

From (2): $A_y = 0.5 \text{ kN}$

From (3): $E_x = 0.423 \text{ kN}$

From (1): $A_x = 1.577 \text{ kN}$

FBD of member ABC:



$\circlearrowleft \Sigma M_B = -0.5(1) + T_{CD}\sin 60°(2) = 0 \quad T_{CD} = 0.289 \text{ kN}$

$\Sigma F_y = 0.5 + B_y + .289 \sin 60° = 0 \quad B_y = -0.75 \text{ kN}$

$\Sigma F_x = 1.577 - .289\cos 60° + B_x = 0 \quad B_x = -1.433 \text{ kN}$

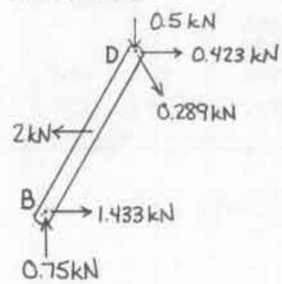
Member ABC:

$\underline{A} = 1.577\underline{i} + 0.5\underline{j} \text{ (kN)}$

$\underline{B} = -1.433\underline{i} - 0.75\underline{j} \text{ (kN)}$

$\underline{C} = -0.1444\underline{i} + 0.25\underline{j} \text{ (kN)}$

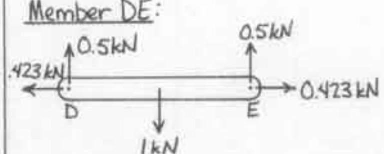
Member BD:



$\underline{B} = 1.433\underline{i} + .75\underline{j} \text{ (kN)}$

$\underline{D} = 0.567\underline{i} - .75\underline{j} \text{ (kN)}$

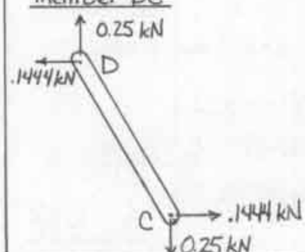
Member DE:



$\underline{D} = -0.423\underline{i} + 0.5\underline{j} \text{ (kN)}$

$\underline{E} = 0.423\underline{i} + 0.5\underline{j} \text{ (kN)}$

Member DC:

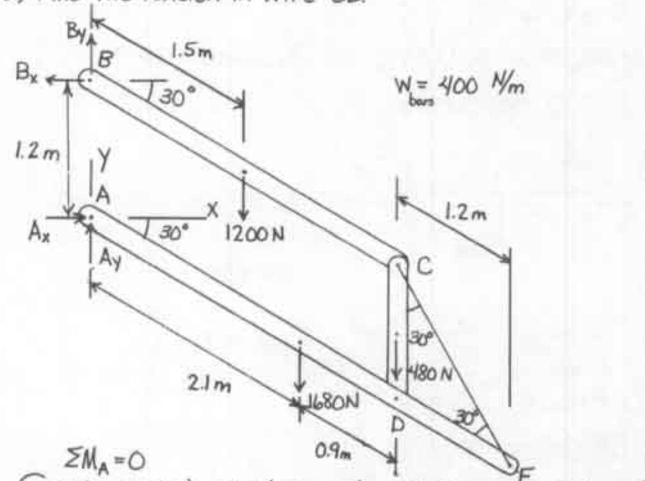


$\underline{C} = 0.1444\underline{i} - 0.25\underline{j} \text{ (kN)}$

$\underline{D} = -0.1444\underline{i} + 0.25\underline{j} \text{ (kN)}$

7.60

a) Find the tension in wire CE.



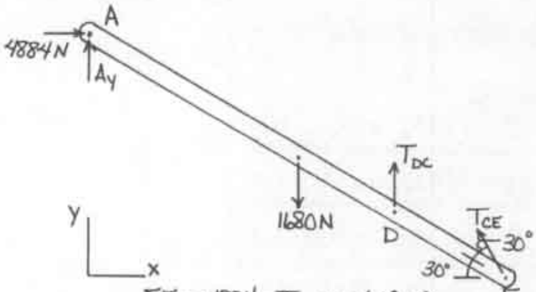
$\Sigma M_A = 0$

$\circlearrowleft + \ \Sigma M_A = B_x(1.2) - 1200(1.5\cos 30°) - 1680(2.1\cos 30°) - 480(3\cos 30°)$

$B_x = 4884\ N$

$\Sigma F_x = A_x - B_x = 0 \qquad A_x = 4884\ N$

FBD of member ADE:



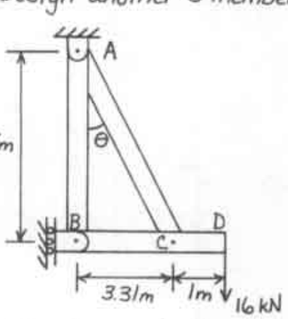
$\Sigma F_x = 4884 - T_{CE}\cos 60° = 0$

$\underline{T_{CE} = 9769\ N}$

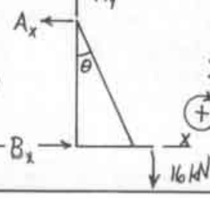
b) This structure is not a frame. The structure would collapse when supports were removed.

7.61

a) Design another 3-member frame



b) Find the projections (x,y) of all the forces.



$\tan\theta = \frac{3.31}{8} \qquad \theta = 22.5°$

$\Sigma F_y = A_y - 16 = 0 \qquad \underline{A_y = 16\ kN}$

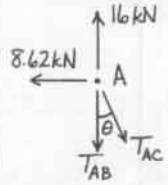
$\circlearrowleft + \ \Sigma M_A = 16(4.31) - B_x(8) = 0$

$\underline{\underline{B_x = 8.62\ kN}}$

$\Sigma F_x = B_x - A_x = 0$

$\underline{A_x = 8.62\ kN}$

FBD of Joint A:



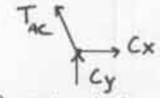
$\Sigma F_x = T_{AC}\sin 22.5° - 8.62 = 0$

$T_{AC} = 22.6\ kN$ (Tension)

$\Sigma F_y = -T_{AC}\cos 22.5° - T_{AB} + 16 = 0$

$\underline{T_{AB} = -4.88\ kN}$ (Comp.)

FBD of Joint C:



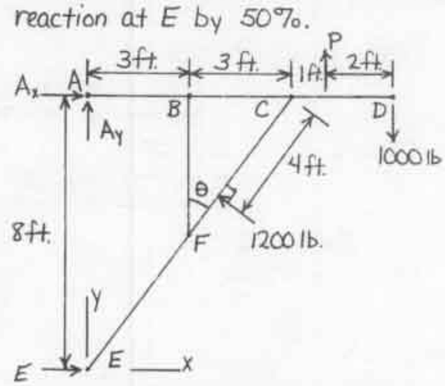
$\underline{\underline{C_x = T_{AC}\sin 22.5° = 8.65\ kN}}$

$\underline{\underline{C_y = -T_{AC}\cos 22.5° = -20.88\ kN}}$

c) This design causes an increased tension in all of the members. Stronger materials would have to be used; material cost would go up. This frame, however, uses straight members only and may be more practical to build.

7.62

Add a concentrated load P to decrease the support reaction at E by 50%.



By similar triangles, FC = 5'

$\sin\theta = \frac{3}{5} \qquad \cos\theta = \frac{4}{5}$

From problem 7.43, E = 1185 lb.

Thus, $E(.50) = 592.5$ lb.

We place P 2 feet left of D pointing upward.

$\circlearrowleft + \ \Sigma M_c = -1200(4) - A_y(6) + 592.5(8) - 1000(3) + P(1) = 0$

$P = 3060 + 6(A_y) \qquad (1)$

$\Sigma F_y = A_y + P - 1000 + 1200(\frac{3}{5}) = 0$

$A_y = 280 - P \qquad (2)$

From (2), sub. for A_y in (1).

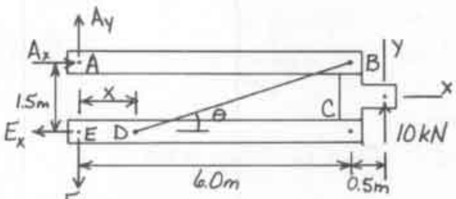
$P = 3060 + 6(280 - P)$

$\underline{P = 677.1\ lb}$

7.63

Find the best location for pin D along member CE. Include reasons.

(continued)

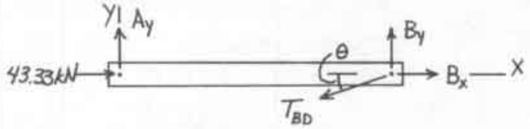


$$\Sigma F_y = A_y - E_y + 10 = 0 \quad (1)$$

$$\circlearrowleft^{+} \Sigma M_E = 10(6.5) - A_x(1.5) = 0 \qquad A_x = 43.33\ kN$$

$$\Sigma F_x = A_x - E_x = 0 \qquad E_x = 43.33\ kN$$

FBD of member AB:



$$\circlearrowleft^{+} \Sigma M_B = A_y(6) = 0 \qquad A_y = 0$$

From (1): $E_y = 10\ kN$

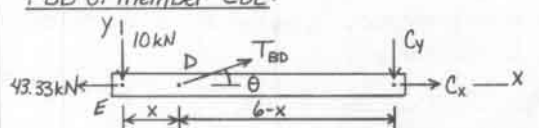
The support reactions will not change as pin D is moved. We now find B_x, B_y, T_{BD}, C_x, and C_y in terms of θ.

$$\Sigma F_y = B_y - T_{BD}\sin\theta = 0 \qquad B_y = T_{BD}\sin\theta \quad (2)$$

$$\Sigma F_x = B_x + 43.33 - T_{BD}\cos\theta = 0$$

$$B_x = T_{BD}\cos\theta - 43.33 \quad (3)$$

FBD of member CDE:



$$\tan\theta = \frac{1.5}{6-x} \qquad 6 - x = \frac{1.5}{\tan\theta} = \frac{1.5\cos\theta}{\sin\theta}$$

$$\circlearrowleft^{+} \Sigma M_C = (10)6 - T_{BD}\sin\theta(6-x) = 60 - 1.5T_{BD}\cos\theta = 0$$

$$T_{BD} = \frac{40}{\cos\theta} \quad (4)$$

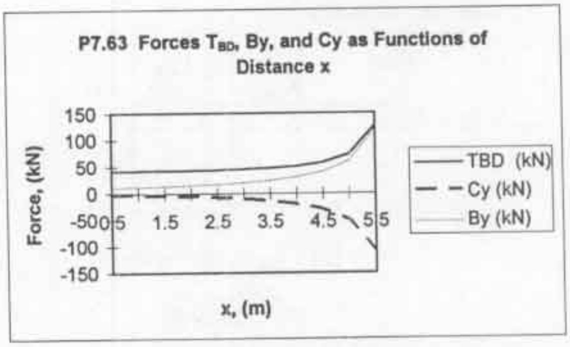
From (3): $B_x = -3.33\ kN$

$$\Sigma F_x = C_x - 43.33 + T_{BD}\cos\theta = 0 \qquad C_x = 3.33\ kN$$

$$\Sigma F_y = T_{BD}\sin\theta - 10 - C_y = 0 \qquad C_y = 40\tan\theta - 10 \quad (5)$$

Using EXCEL, graphs of equations (2), (4), and (5) with varying θ are created.

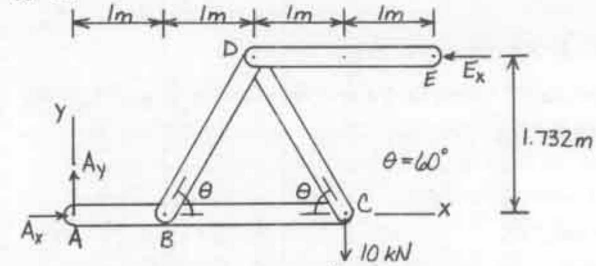
x (m)	θ(radians	T_{BD} (kN)	Cy (kN)	By (kN)
0.5	0.266	41.461	-0.91	10.91
1.0	0.291	41.761	-2.00	12.00
1.5	0.322	42.164	-3.33	13.33
2.0	0.359	42.720	-5.00	15.00
2.5	0.405	43.519	-7.14	17.14
3.0	0.464	44.721	-10.00	20.00
3.5	0.540	46.648	-14.00	24.00
4.0	0.644	50.000	-20.00	30.00
4.5	0.785	56.569	-30.00	40.00
5.0	0.983	72.111	-50.00	60.00
5.5	1.249	126.491	-110.00	120.00



From the graph, all the forces in the members are minimized as x and θ are minimized.
Thus, the best location for pin D is at E.

7.64

Determine if BD and CD will fail under a 10kN load at C.



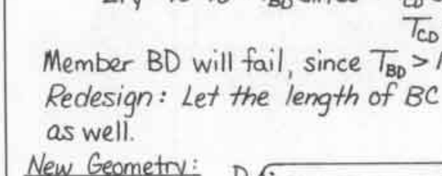
BD, CD, and DE are two-force members

$$\Sigma F_y = A_y - 10 = 0 \qquad A_y = 10\ kN$$

$$\circlearrowleft^{+} \Sigma M_A = 1.732\,E_x - 3(10) = 0 \qquad E_x = 17.32\ kN$$

$$\Sigma F_x = A_x - E_x = 0 \qquad A_x = 17.32\ kN$$

FBD of member ABC:



$$\circlearrowleft^{+} \Sigma M_C = 10(3) + T_{BD}\sin 60^\circ(2) = 0$$

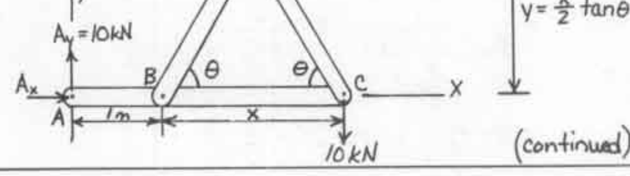
$$T_{BD} = -17.32\ kN\ (\text{Compression})$$

$$\Sigma F_y = 10 - 10 + T_{BD}\sin 60^\circ + T_{CD}\sin 60^\circ = 0$$

$$T_{CD} = 17.32\ kN\ (\text{Tension})$$

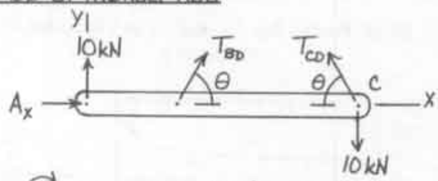
Member BD will fail, since $T_{BD} > 13kN$ (Compression).
Redesign: Let the length of BC be x. Let θ vary as well.

New Geometry:



(continued)

FBD of member ABC:



$$\circlearrowleft\!+\ \Sigma M_C = 10(x+1) + (T_{BD}\sin\theta)x = 0$$

$$x(10 + T_{BD}\sin\theta) = -10$$

$$x = \frac{-10}{(10 + T_{BD}\sin\theta)}$$

Let $T_{BD} = -13$ kN (Compression)

θ	x	y	Status
45°	-12.4m	—	No Good
60°	7.95m	6.88m	x is too big
75°	3.91m	7.30m	OK
80°	3.57m	10.12m	y is too big

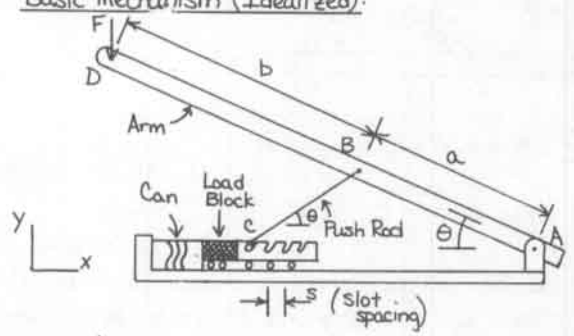
Use θ = 75° and x = 3.91 m This is a limiting case. Also, the length of DE does not enter.

7.65

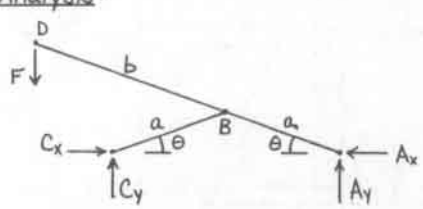
Design a soda can crusher that delivers at least 240 lb. of crushing force.

Approach: Modify the mechanism in Fig. P7.55 as shown. The average can is just less than 5 in. tall. Keep θ small so that the mechanical advantage is high. Design the mechanism to crush the can from 5 in. high to 1 in. high. Try for a mechanical advantage m = 6.0. Therefore, 40 lb. is required to operate the device. Also for m = 6.0, the operator force must move through 6(4) = 24" to crush the can. A ratchet device is needed to allow this movement in several segments.

Basic Mechanism (Idealized):



Force Analysis:

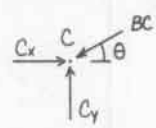


Note that length BC = a

$$\circlearrowleft\!+\ \Sigma M_A = F(a+b) - C_y(2a\cos\theta) = 0$$

$$C_y = \frac{F(a+b)}{2a\cos\theta}$$

FBD of Joint C:



$$\Sigma F_y = C_y - BC\sin\theta = 0$$

$$BC = \frac{C_y}{\sin\theta}$$

$$BC = \frac{F(a+b)}{2a\sin\theta\cos\theta}$$

$$\Sigma F_x = C_x - BC\cos\theta = 0$$

$$C_x = \frac{F(a+b)}{2a\sin\theta}$$

For $m = \frac{C_x}{F} = 6.0$ $\quad 6.0 = \frac{a+b}{2a\sin\theta}$

Choose b = 2a $\quad 6.0(2a\sin\theta) = 3a$

$\sin\theta = 1/4 \quad \theta = 14.48°$

Use θ = 15°

Choose s = 1 in.

For 1-in. crush per arm rotation of 15°, 4 rotations are required to crush the can.

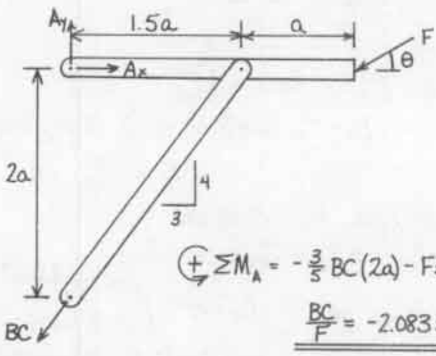
$$\cos\theta = \frac{2a-1}{2a}$$

$$a = \frac{1}{2(1-\cos\theta)} \quad \text{For } \theta = 15°,\ a = 14.67 \text{ in}$$

Use a = 15 in.

7.66

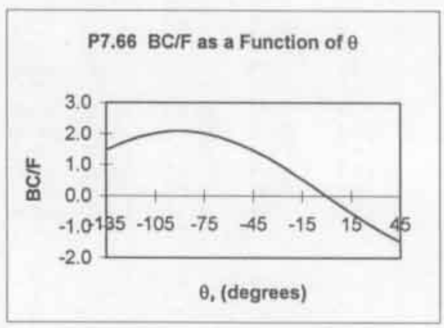
a) Plot the ratio of the force in member BC to the force F as a function of θ.



$$\circlearrowleft\!+\ \Sigma M_A = -\tfrac{3}{5}BC(2a) - F\sin\theta(2.5a) = 0$$

$$\frac{BC}{F} = -2.083\sin\theta$$

θ (degrees)	BC/F
-135	1.473
-125	1.707
-115	1.888
-105	2.012
-95	2.075
-85	2.075
-75	2.012
-65	1.888
-55	1.707
-45	1.473
-35	1.195
-25	0.880
-15	0.539
-5	0.182
5	-0.182
15	-0.539
25	-0.880
35	-1.195
45	-1.473



(continued)

b) Maximum tension occurs at $\theta = -90°$

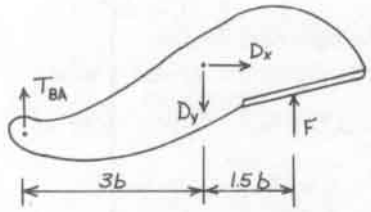
For $BC = 50{,}000$ lb, $F = \frac{50{,}000}{2.083} = 24{,}000$ lb.

Maximum compression occurs at $\theta = 45°$

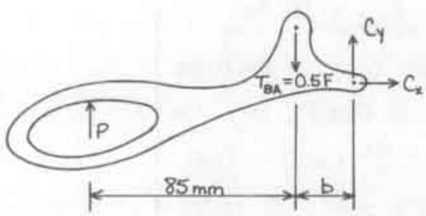
For $BC = -35{,}000$ lb, $F = \frac{-35{,}000}{-1.473} = 23{,}760$ lb.

7.67

a) Plot the mechanical advantage $m = \frac{F}{P}$ as a function of b for $12\text{mm} \le b \le 35\text{mm}$.



$\circlearrowleft + \Sigma M_D = F(1.5b) - T_{BA}(3b) = 0 \qquad T_{BA} = 0.5F$

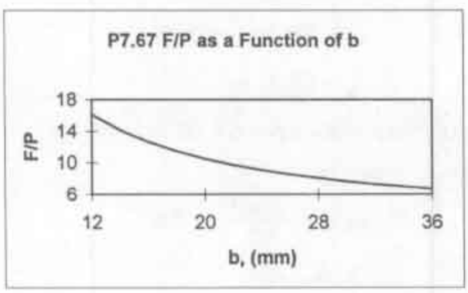


$\Sigma M_C = 0.5F(b) - P(85+b) = 0$

$0.5Fb = P(85+b)$

$$m = \frac{F}{P} = \frac{170}{b} + 2$$

b (mm)	F/P
12	16.167
14	14.143
16	12.625
18	11.444
20	10.500
22	9.727
24	9.083
26	8.538
28	8.071
30	7.667
32	7.313
34	7.000
36	6.722

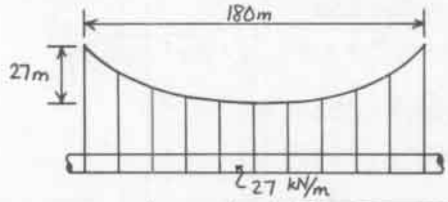


b) The shortest distance, $b = 12$ mm, gives the largest mechanical advantage because the fulcrum is closest to the output. For $b = 12$ mm, $m = 16.17$.

7.68

Find the maximum tension in the cable.

$$T_{max} = \frac{wa}{2}\sqrt{1 + \frac{1}{16}\left(\frac{a}{d}\right)^2} \quad \text{(Eqn. 7.10)}$$



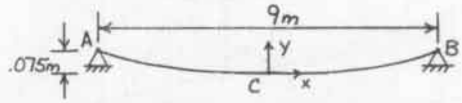
$a = 180$m
$d = 27$m
$w = 27$ kN/m

$$T_{max} = \frac{(27\text{ kN/m})(180\text{m})}{2}\sqrt{1 + \frac{1}{16}\left(\frac{180\text{m}}{27\text{m}}\right)^2}$$

$$T_{max} = 4723 \text{ kN}$$

7.69

a) Draw a sketch



b) Find the tension at either end.

$$T = \frac{wa}{2}\sqrt{1 + \frac{1}{16}\left(\frac{a}{d}\right)^2} \quad \text{(Eqn. 7.10)}$$

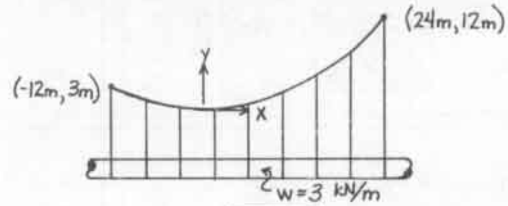
$a = 9$m
$d = 0.075$ m
$w = 1.50$ N/m

$$T_A = T_B = \frac{1.50(9)}{2}\sqrt{1 + \frac{1}{16}\left(\frac{9}{.075}\right)^2}$$

$$T_A = T_B = 203 \text{ N}$$

7.70

a) Find the maximum tension in the cable.



$$T_{max} = wx_1\sqrt{1 + \frac{x_1^2}{4y_1^2}} \quad \text{(Eqn. 7.7)}$$

$w = 3$ kN/m
$x_1 = 24$ m
$y_1 = 12$ m

$$T_{max} = 3(24)\sqrt{1 + \frac{(24)^2}{4(12)^2}}$$

$$T_{max} = 101.8 \text{ kN}$$

b) Find the length of the cable.

$ds = (\sec\theta)dx = \left(\sqrt{1 + \tan^2\theta}\right)dx$ (See Fig 7.17)

$\tan\theta = \frac{wx}{H}$ (See Fig. 7.15)

$H = \frac{wx_1^2}{2y_1}$ (Eqn. 7.5)

$\tan^2\theta = \frac{4y_1^2}{x_1^4}x^2 = \frac{4(12)^2}{(24)^4}x^2 = \frac{x^2}{576}$

$ds = \left(\sqrt{1 + \frac{x^2}{576}}\right)dx$

$$L = \int ds = \frac{1}{24}\int_{-12}^{24}(x^2 + 576)^{1/2}dx$$

(Continued)

From an integration table:

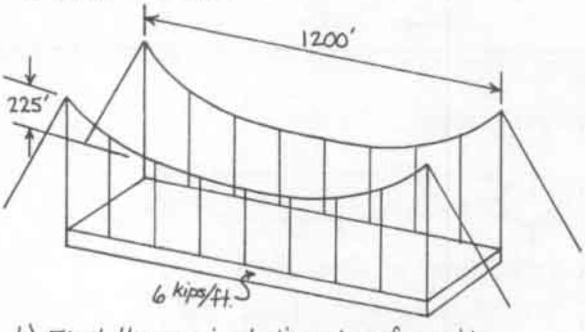
$$L = \frac{1}{48}\left[x(x^2+576)^{1/2} + 576 \ln\left(x + \sqrt{x^2+576}\right)\right]\Big|_{-12}^{24}$$

$$L = 65.68 - 25.65$$

$$\underline{L = 40.0\text{ m}}$$

7.71

a) Draw a sketch



b) Find the required diameter of a cable.

$$T_{max} = \frac{wa}{2}\sqrt{1+\frac{1}{16}\left(\frac{a}{d}\right)^2} \quad \text{(Eqn. 7.10)}$$

$a = 1200'$
$d = 225'$
$w = 6\ \frac{kip}{ft} = 3\ kip/ft/cable$

$T_{max} = 3{,}000$ kips

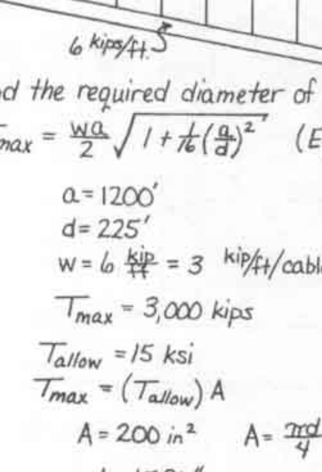
$T_{allow} = 15$ ksi
$T_{max} = (T_{allow})A$

$A = 200\ in^2 \qquad A = \frac{\pi d^2}{4}$

$\underline{d = 15.96''}$

7.72

a) Draw a sketch of the cable.



b) Find the maximum load (kN/m) that the cable can support.

$$T_{max} = \frac{wa}{2}\sqrt{1+\frac{1}{16}\left(\frac{a}{d}\right)^2} \quad \text{(Eqn. 7.10)}$$

$$160\ kN = \frac{w(100)}{2}\sqrt{1+\frac{1}{16}\left(\frac{100}{20}\right)^2}$$

$$\underline{w = 2.00\ kN/m}$$

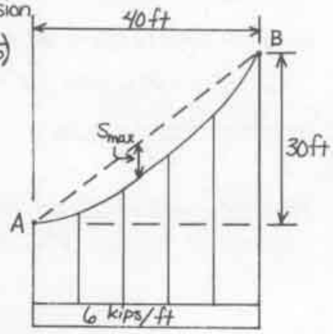
c) Find the length of the cable

$$L = \frac{a}{2}\left[\sqrt{1+16\left(\frac{d}{a}\right)^2} + \frac{1}{4}\left(\frac{a}{d}\right)\ln\left(4\left(\frac{d}{a}\right)+\sqrt{1+16\left(\frac{d}{a}\right)^2}\right)\right] \quad \text{(Eqn. 7.11)}$$

$$\underline{L = 109.8\text{ m}}$$

7.73

a) Find the minimum tension



$$T_{min} = H = \frac{wx_1^2}{2y_1} \quad \text{(Eqn. 7.5)}$$

$w = 6$ kips/ft
$x_1 = 40'$
$y_1 = 30'$

$$T_{min} = \frac{6(40)^2}{2(30)}$$

$$\underline{T_{min} = 160\text{ kips}}$$

b) Find the maximum tension

Max occurs at the largest x value, $x = 40'$

$$T_{max} = wx_1\sqrt{1+\frac{x_1^2}{4y_1^2}} \quad \text{(Eqn 7.7)}$$

$w = 6$ kips/ft
$x_1 = 40'$
$y_1 = 30'$

$$T_{max} = 6(40)\sqrt{1+\frac{(40)^2}{4(30)^2}}$$

$$\underline{T_{max} = 288\text{ kips}}$$

c) Find the length of the cable.

$ds = \sec\theta\, dx$ (See Fig. 7.17) $\quad ds = \sqrt{1+\tan^2\theta}\, dx$ (Trig. Funct.)

$\tan\theta = \frac{wx}{H}$ (See Fig. 7.15)

$$\tan^2\theta = \left(\frac{4y_1^2}{x_1^4}\right)x^2 = \frac{4(30)^2}{(40)^4}x^2 = \frac{x^2}{711.11}$$

$$ds = \left(1+\frac{x^2}{711.11}\right)^{1/2}dx$$

$$L = \frac{1}{26.67}\int_{0'}^{40'}(x^2+711.11)^{1/2}dx$$

From an integration table:

$$L = \frac{1}{53.33}\left[x(x^2+711.11)^{1/2} + 711.11\ln\left(x+(x^2+711.11)^{1/2}\right)\right]\Big|_0^{40}$$

$$L = 95.76 - 43.78$$

$$\underline{L = 52.0\text{ ft}}$$

d) Find the slope at B.

$$\tan\theta = \frac{wx}{H}$$

$$\tan\theta = \frac{6(40)}{160} = 1.5$$

$$\underline{\text{Slope} = 1.5}$$

e) Find S_{max}.

We must find the equations for both the line and the parabola.

Line: $y = mx + b \quad b = 0 \quad m = \frac{30}{40} = 0.75$

$$y = 0.75x$$

Parabola:

$$y = ax^2 \qquad a = \frac{1}{53.33} = 0.01875$$

$$y = 0.01875x^2$$

Subtract the parabola equation from the line equation, differentiate, and set to zero to find the maximum. (Continued)

$S = 0.75x - 0.01875x^2$

$S' = 0.75 - 0.0375x = 0$

$x = 20 \text{ ft}$

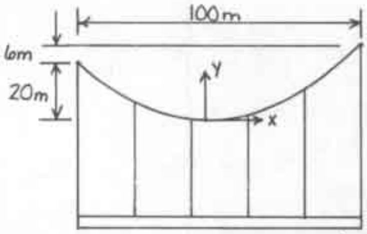
Plug in to find y values

$S_{max} = 0.75(20) - 0.01875(20)^2$

$\underline{S_{max} = 7.5 \text{ ft.}}$

7.74

a) Draw a sketch



$T_{allow} = 160 \text{ kN}$

b) Find the maximum load (kN/m) that the cable can support.

$T_{max} = wx_1\sqrt{1 + \frac{x_1^2}{4y_1^2}}$ (Eqn. 7.7)

Finding x_1: $H = \frac{wx_0^2}{2y_0} = \frac{wx_1^2}{2y_1}$ $y_0 = 20m$ $y_1 = 26m$

$\frac{x_0^2}{20} = \frac{x_1^2}{26}$ (1)

$|x_0| + |x_1| = 100$ (2)

$\frac{(100-x_1)^2}{20} = \frac{x_1^2}{26}$

$x_1 = 53.275 \text{ m}$

$x_0 = -46.725 \text{ m}$

$T_{max} = w(53.275)\sqrt{1 + \frac{(53.275)^2}{4(26)^2}}$

$T_{max} = 160 \text{ kN} = 76.27w$

$\underline{w = 2.10 \text{ kN/m}}$

c) Find the length of the cable.

$ds = \sec\theta \, dx = \sqrt{1 + \tan^2\theta}\, dx$

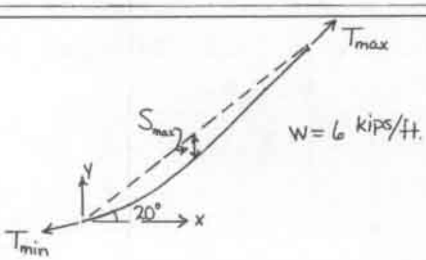
$\tan^2\theta = \left(\frac{wx}{H}\right)^2 = \left(\frac{4y_1^2}{x_1^4}\right)x^2 = \frac{x^2}{2979}$

$L = \frac{1}{54.58}\int_{-46.725}^{53.275} (x^2 + 2979)^{1/2}\, dx$

$L = \frac{1}{109.16}\left[x(x^2+2979)^{1/2} + 2979\ln\left(x + (x^2+2979)^{1/2}\right)\right]_{-46.7}^{53.3}$

$\underline{L = 112.74 \text{ m}}$

7.75



$w = 6 \text{ kips/ft.}$

a) Find the maximum tension.

Find the equation of the parabola:

$y = ax^2 + bx + c$

at $x = 0$, $y = 0 \longrightarrow c = 0$

$\frac{dy}{dx} = 2ax + b$

at $x = 0$, $\frac{dy}{dx} = \tan 20° \longrightarrow b = \tan 20° = 0.364$

at $x = 40'$, $y = 30'$

$30 = a(40)^2 + 0.364(40)$

$a = 0.00965$

$y = 0.00965x^2 + .364x$

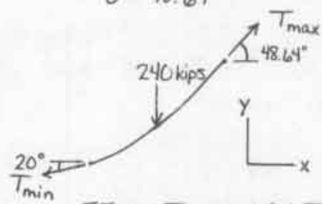
The slope at 40':

$\frac{dy}{dx} = 2(.00965)(40) + .364$

$\frac{dy}{dx} = 1.136$

$\theta = 48.64°$

FBD:



$\Sigma F_x = -T_{min}\cos 20° + T_{max}\cos 48.64° = 0$

$\Sigma F_y = -T_{min}\sin 20° + T_{max}\sin 48.64° - 240 = 0$

$T_{min} = 331 \text{ kips}$

$\underline{T_{max} = 470. \text{ kips}}$

b) Find the length of the cable

$ds = \sec\theta\, dx = \sqrt{1 + \tan^2\theta}\, dx$ (See Fig. 7.17)

$\tan\theta = \frac{dy}{dx}$

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$L = \int_0^{40}\sqrt{1 + (2(.00965)x + .364)^2}\, dx$

Using Derive:

$\underline{L = 50.6 \text{ ft}}$

c) Find S_{max}.

Find the equations for the line and the parabola.

Line:

$y = mx + b$ $b = 0$ $m = \frac{30}{40} = 0.75$

$y = 0.75x$

Parabola:

$y = .00965x^2 + .364x$ (From Part A)

Subtract, Differentiate, and set to zero to find the maximum.

$S = 0.75x - 0.00965x^2 - 0.364x$

$S' = 0.386 - 0.0193x = 0$

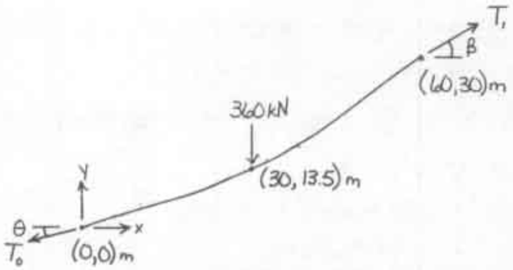
$x = 20 \text{ ft}$

$S_{max} = 0.75(20) - 0.00965(20)^2 - 0.364(20)$

$\underline{S_{max} = 3.86 \text{ ft}}$

7.76

a) Draw a free-body diagram.



b) Find the tensions T_0 and T_1

Find the equation of the parabola.

$$y = ax^2 + bx + c$$

at $(0,0)$: $c = 0$

at $(30, 13.5)$: $13.5 = a(30)^2 + b(30)$ (1)

at $(60,30)$: $30 = a(60)^2 + b(60)$ (2)

Solving (1) and (2):

$$a = 0.001667$$
$$b = 0.4$$
$$y = 0.001667x^2 + 0.4x$$
$$\frac{dy}{dx} = 0.003333x + 0.4$$

at $(0,0)$, $\frac{dy}{dx} = 0.4$ $\quad \theta = \tan^{-1}(0.4) = 21.801°$

at $(60,30)$, $\frac{dy}{dx} = 0.6$ $\quad \beta = \tan^{-1}(0.6) = 30.964°$

$\Sigma F_x = -T_0 \cos 21.801° + T_1 \cos 30.964° = 0$

$\Sigma F_y = -T_0 \sin 21.801° - 360 + T_1 \sin 30.964° = 0$

$$\underline{\underline{T_0 = 1939 \text{ kN}}}$$

$$\underline{\underline{T_1 = 2099 \text{ kN}}}$$

7.77

a) Find the minimum length of cable required.

$$w = \frac{180 \text{ N/m} + 150 \text{ N/m}}{2 \text{ cables}} = 165 \text{ N/m/cable}$$

$T_{max} = 50$ kN

$T_{max} = wx_1\sqrt{1 + \frac{x_1^2}{4y_1^2}}$ (Eqn. 7.7)

$50 = 0.165x_1\sqrt{1 + \frac{x_1^2}{4y_1^2}}$ (1)

$-x_0 + x_1 = 150$ (2)

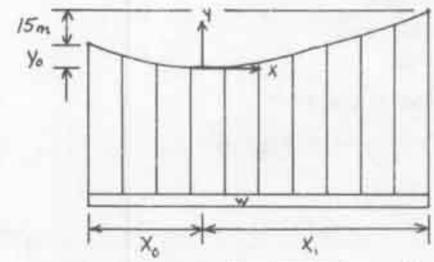
$y_1 = 15 + y_0$ (3)

$\frac{x_0^2}{y_0} = \frac{x_1^2}{y_1}$ (4) (From Eqn. 7.5)

4 Equations ; 4 Unknowns

Using TKSolver:

$x_0 = -46.519$ m
$x_1 = 103.484$ m
$y_0 = 3.7989$ m
$y_1 = 18.7989$ m



$ds = \sec\theta \, dx = \sqrt{1 + \tan^2\theta}\, dx$ (See Fig. 7.17)

$\tan\theta = \frac{wx}{H}$

$\tan^2\theta = \left(\frac{4y_0^2}{x_0^4}\right)x^2 = \left[\frac{4(3.80)^2}{(46.52)^4}\right]x^2 = \frac{x^2}{81123}$

$ds = \frac{1}{284.8}(x^2 + 81123)^{1/2}\,dx$

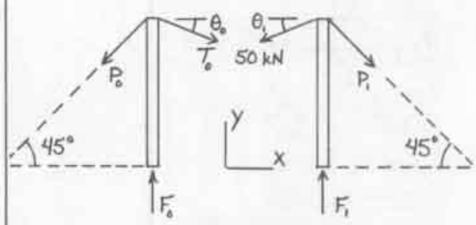
$L = \frac{1}{284.8}\int_{-46.519}^{103.484}(x^2 + 81123)^{1/2}\,dx$

From an integration table:

$$L = \frac{1}{569.6}\left[x(x^2 + 81123)^{1/2} + 81123\ln\left(x + (x^2 + 81123)^{1/2}\right)\right]\Bigg|_{-46.519}^{103.484}$$

$$\underline{L = 152.4 \text{ m}}$$

b) Find the vertical force in each pair of towers and the tension in each of the tie-back cables.



$\tan\theta_0 = \frac{wx_0}{H}$ $\quad H = \frac{wx_0^2}{2y_0}$ (Eqn. 7.5)

$H = 47.0$ kN (For one tower)

$\theta_0 = \tan^{-1}\left(\frac{0.165(46.519)}{47.0}\right)$

$\theta_0 = 9.28°$

$\theta_1 = \tan^{-1}\left(\frac{0.165(103.484)}{47.0}\right)$

$\theta_1 = 19.97°$

$T_0 = w\sqrt{x^2 + \frac{x_0^4}{4y_0^2}} = 0.165\sqrt{(-46.519)^2 + \frac{(46.519)^4}{4(3.7989)^2}}$

$T_0 = 47.6$ kN

Left tower:

$\Sigma F_x = 47.6 \cos 9.28° - P_0 \cos 45° = 0$

$\Sigma F_y = F_0 - 47.6 \sin 9.28° - P_0 \sin 45° = 0$

$$\underline{\underline{P_0 = 66.5 \text{ kN}}}$$

$$\underline{\underline{F_0 = 54.7 \text{ kN}}}$$

(Continued)

Right tower:

$$\Sigma F_x = P_1 \cos 45^\circ - 50 \cos 19.97^\circ = 0$$

$$\Sigma F_y = F_1 - P_1 \sin 45^\circ - 50 \sin 19.97^\circ = 0$$

$$\underline{\underline{P_1 = 66.5\ kN}}$$

$$\underline{\underline{F_1 = 64.1\ kN}}$$

7.78

Find the value of $\frac{d}{a}$ for a 5% error between equations 7.11 and 7.12.

7.12 is an approximation of 7.11 and is a lower bound on 7.11.

Eqn. 7.11:

$$L = \frac{a}{2}\left[\sqrt{1+16\left(\frac{d}{a}\right)^2} + \frac{1}{4}\left(\frac{a}{d}\right)\ln\left[4\left(\frac{d}{a}\right)+\sqrt{1+16\left(\frac{d}{a}\right)^2}\right]\right]$$

Eqn. 7.12 (three terms)

$$L = a\left[1 + \frac{8}{3}\left(\frac{d}{a}\right)^2 - \frac{32}{5}\left(\frac{d}{a}\right)^4\right]$$

Let $u = \frac{d}{a}$ and $\frac{1}{u} = \frac{a}{d}$

$$(0.95)\frac{a}{2}\left[\sqrt{1+16u^2} + \frac{1}{4u}\ln\left[4u + \sqrt{1+16u^2}\right]\right] = a\left[1 + \frac{8}{3}u^2 - \frac{32}{5}u^4\right]$$

Using Derive:

$$\underline{\underline{u = \frac{d}{a} = 0.3947}}$$

7.79

Determine the design sags and corresponding lengths for $10{,}000\ kN \le T_{max} \le 20{,}000\ kN$.

$a = 300m$, $w = 30\ kN/m$

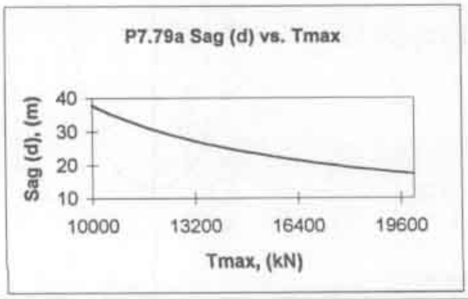
$$T_{max} = \frac{wa}{2}\sqrt{1+\frac{1}{16}\left(\frac{a}{d}\right)^2} \quad (\text{Eqn. } 7.10)$$

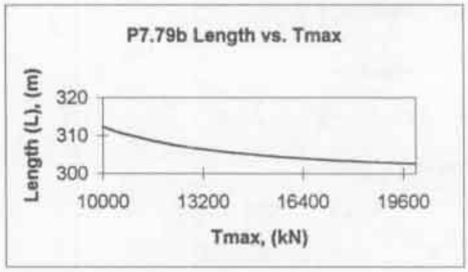
$$T_{max} = \frac{30(300)}{2}\sqrt{1+\frac{1}{16}\left(\frac{300}{d}\right)^2}$$

$$d = 337{,}500\sqrt{\frac{1}{T_{max}^2 - 4500^2}} \quad (m)$$

$$L = \frac{a}{2}\left[\sqrt{1+16\left(\frac{d}{a}\right)^2} + \frac{1}{4}\left(\frac{a}{d}\right)\ln\left[4\left(\frac{d}{a}\right)+\sqrt{1+16\left(\frac{d}{a}\right)^2}\right]\right] \quad (\text{Eqn } 7.11)$$

T max (kN)	d (m)	L (m)
10000	37.79	312.25
12000	30.34	307.99
14000	25.46	305.67
16000	21.98	304.24
18000	19.36	303.30
20000	#DIV/0!	#DIV/0!





7.80

a) Determine the required sag (d) for 10 kip/ft ≤ w ≤ 20 kip/ft

$a = 3000$ ft.

$T_{max} = 30{,}000$ kips

$$T_{max} = \frac{wa}{2}\sqrt{1+\frac{1}{16}\left(\frac{a}{d}\right)^2} \quad (\text{Eqn. } 7.10)$$

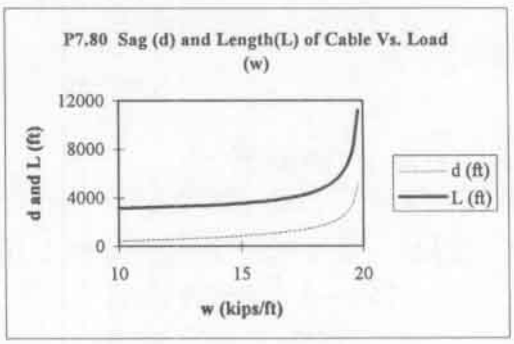
Solve for d in terms of a and T_{max}.

$$\left(\frac{2T_{max}}{wa}\right)^2 = 1 + \frac{1}{16}\left(\frac{a}{d}\right)^2$$

$$\frac{64T_{max}^2}{w^2a^2} - 16 = \left(\frac{a}{d}\right)^2$$

$$d = \sqrt{\frac{a^2}{\frac{64T_{max}^2}{w^2a^2} - 16}} = \frac{9.0(10^6)\,w}{\sqrt{(57.6(10^9) - 144(10^6)w^2)}}$$

w (kips/ft)	d (ft)	L (ft)
10	433.01	3159.19
12	562.50	3261.29
14	735.15	3427.66
16	1000.00	3735.94
18	1548.56	4510.78
20	#DIV/0!	#DIV/0!



b) Find the corresponding lengths in terms of a and d

$$L = \frac{a}{2}\left[\sqrt{1+16\left(\frac{d}{a}\right)^2} + \frac{1}{4}\left(\frac{a}{d}\right)\ln\left[4\left(\frac{d}{a}\right)+\sqrt{1+16\left(\frac{d}{a}\right)^2}\right]\right]$$

See the graph above.

7.81

Show that $C=0$ and $K=-H$ if the lowest point of the catenary is at (0,0).

$$\frac{dy}{dx} = \sinh\left(\frac{wx}{H} + C\right) \quad (\text{Eqn. } 7.16)$$

At $(0,0)$, $\frac{dy}{dx} = 0$:

(Continued)

$$0 = \sinh\left(\frac{w(0)}{H} + C\right) = \sinh(C)$$

$$\underline{\underline{C = 0}}$$

$$wy = H\cosh\left(\frac{wx}{H} + C\right) + K \quad \text{(Eqn. 7.17)}$$

At $(0,0)$, $y = 0$:

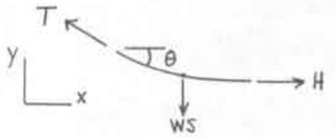
$$w(0) = H\cosh\left(\frac{w(0)}{H} + 0\right) + K$$

$$0 = H\cosh(0) + K$$

$$\underline{\underline{K = -H}}$$

7.82

a) Find the minimum tension (H) in the line.



$T = 3\text{ kN}$

$$\theta = 2^\circ$$

$$\Sigma F_x = H - 3\cos 2^\circ = 0$$

$$\underline{\underline{H = 2.998\text{ kN}}}$$

b) Find the maximum allowable angle for which H is 50% of T; that is, $H = 1.5\text{ kN}$.

$$\Sigma F_x = 1.5 - 3\cos\theta = 0$$

$$\cos\theta = 0.5$$

$$\underline{\underline{\theta_{max} = 60^\circ}}$$

7.83

Find the maximum and minimum tensions.

$L = 750\text{ m}$ $\quad x_0 = -360\text{ m}$

$a = 720\text{ m}$ $\quad x_1 = 360\text{ m}$

$w = \frac{500\text{ N}}{750\text{ m}} = \frac{2}{3}\text{ N/m}$

At $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$

$\therefore C = 0$ (See Eqn. 7.16)

$$L = \frac{H}{w}\left[\sinh\left(\frac{w}{H}x_1 + C\right) - \sinh\left(\frac{w}{H}x_0 + C\right)\right] \quad \text{(Eqn. 7.21)}$$

$$750 = \frac{H}{w}\left[2\sinh\left(\frac{w}{H}\right)(360)\right]$$

$$375\frac{w}{H} = \sinh\left(360\frac{w}{H}\right)$$

Using TKSolver:

$$\frac{w}{H} = \frac{2}{3H} = 0.00138$$

$$\underline{\underline{H = 483\text{ N}}}$$

$$T = H\cosh\left(\frac{wx}{H}\right) \quad \text{(Eqn. 7.23)}$$

$$T = 483\cosh(.00138x)$$

T_{max} occurs at $x = \pm 360\text{ m}$

$$T_{max} = 483\cosh(.4968)$$

$$\underline{\underline{T_{max} = 544\text{ N}}}$$

7.84

Find the minimum span a.

$L = 20\text{ m}$ $\quad d = 6\text{ m}$

$w = 100\text{ N/m}$

$$d = \frac{H}{w}\left[\cosh\left(\frac{wa}{2H}\right) - 1\right] \quad \text{(Eqn. 7.24)}$$

$$6 = \frac{H}{100}\left[\cosh\left(\frac{100a}{2H}\right) - 1\right] \quad (1)$$

$$L = \frac{H}{w}\left[\sinh\left(\frac{w}{H}x_1 + C\right) - \sinh\left(\frac{w}{H}x_0 + C\right)\right] \quad \text{(Eqn. 7.21)}$$

$C = 0$ Due to symmetry.

$$20 = \frac{H}{100}\left(2\sinh\left(\frac{100}{H}x\right)\right)$$

$$a = 2x$$

$$20 = \frac{H}{50}\sinh\left(\frac{50}{H}a\right) \quad (2)$$

Using TKSolver to solve equations (1) and (2):

$$H = 533\text{ N}$$

$$\underline{\underline{a = 14.79\text{ m}}}$$

7.85

Find the length and the sag of the wire.

$a = 750\text{ m}$ $\quad w = 3\text{ N/m}$

$T_{max} = 4500\text{ N}$

By symmetry, the minimum point lies at $x = 0$.

At $x = 0$, $\frac{dy}{dx} = 0 \quad \therefore C = 0$

$$T_{max} = H\cosh\left(\frac{w}{H}x\right) \quad \text{(Eqn. 7.23)}$$

T_{max} occurs at $x = \pm 375\text{ m}$.

$$4500 = H\cosh\left(\frac{3(375)}{H}\right)$$

Using TKSolver to find H:

$$H = 4354\text{ N}$$

$$L = \frac{H}{w}\left[\sinh\left(\frac{w}{H}x_1 + C\right) - \sinh\left(\frac{w}{H}x_0 + C\right)\right] \quad \text{(Eqn. 7.21)}$$

$$\frac{wL}{2H} = \sinh\left(\frac{w}{H}(375)\right)$$

$$\frac{3L}{2(4354)} = \sinh\left(\frac{3(375)}{4354}\right)$$

$$\underline{\underline{L = 758\text{ m}}}$$

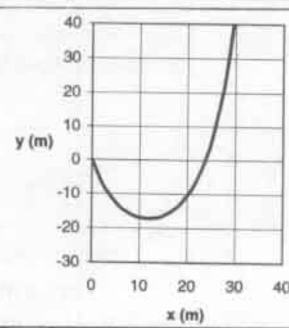
$$d = \frac{H}{w}\left[\cosh\left(\frac{wa}{2H}\right) - 1\right] \quad \text{(Eqn. 7.24)}$$

$$d = \frac{4354}{3}\left[\cosh\left(\frac{3(750)}{2(4354)}\right) - 1\right]$$

$$\underline{\underline{d = 48.7\text{ m}}}$$

7.86

a) Draw a sketch



$L = 90\text{ m}$

(continued)

b) Determine $\frac{K}{H}$ from the equation of the curve. Write two equations for $\frac{W}{H}$ and C.

$$\frac{W}{H}y = \cosh\left(\frac{W}{H}x + C\right) + \frac{K}{H} \quad \text{(Eqn. 7.17)}$$

At (0,0):
$$0 = \cosh(C) + \frac{K}{H}$$
$$\frac{K}{H} = -\cosh(C) \quad (1)$$

At (30,45):
$$\frac{45W}{H} = \cosh\left(\frac{30W}{H} + C\right) + \frac{K}{H} \quad (2)$$

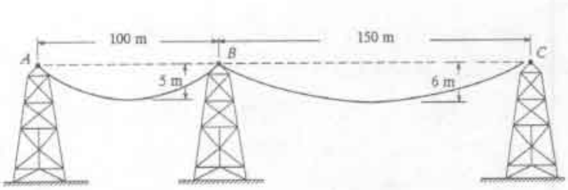
$$L = \frac{H}{W}\left[\sinh\left(\frac{W}{H}x_1 + C\right) - \sinh\left(\frac{W}{H}x_0 + C\right)\right] \quad \text{(Eqn. 7.21)}$$
$$90 = \frac{H}{W}\left[\sinh\left(\frac{W}{H}(30) + C\right) - \sinh(C)\right] \quad (3)$$

c) Solve for $\frac{W}{H}$, $\frac{K}{H}$, and C.
Using TKSolver to solve (1), (2), and (3):

$$\underline{\underline{\frac{K}{H} = -4}}$$
$$\underline{\underline{C = -2.07}}$$
$$\underline{\underline{\frac{W}{H} = 0.1743\ m^{-1}}}$$

7.87

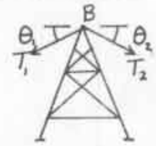
Find the resultant force that the cable exerts on Tower B.



$$d = \frac{H}{w}\left[\cosh\left(\frac{wa}{2H}\right) - 1\right] \quad \text{(Eqn. 7.24)}$$

A-B:
$d_1 = 5\text{m}$ $\quad w = 100\ \text{N/m}$
$a_1 = 100\text{m}$
$$5 = \frac{H_1}{100}\left[\cosh\left(\frac{5000}{H_1}\right) - 1\right]$$
$$H_1 = 25\,083\ N$$
T_{max} will occur at $x = 50\text{m}$.
$$T = H\cosh\left(\frac{wx}{H}\right) \quad \text{(Eqn. 7.23)}$$
$$T_1 = 25\,083\cosh\left(\frac{5000}{25083}\right)$$
$$T_1 = 25\,583\ N$$

B-C:
$d_2 = 6\text{m}$ $\quad w = 100\ \text{N/m}$
$a_2 = 150\text{m}$
$$6 = \frac{H_2}{100}\left[\cosh\left(\frac{7500}{H_2}\right) - 1\right]$$
$$H_2 = 46\,975\ N$$
T_{max} will occur at $x = -75\text{m}$
$$T_2 = H_2\cosh\left(\frac{wx}{H_2}\right) = 46\,975\cosh\left(\frac{7500}{46975}\right)$$
$$T_2 = 47\,575\ N$$

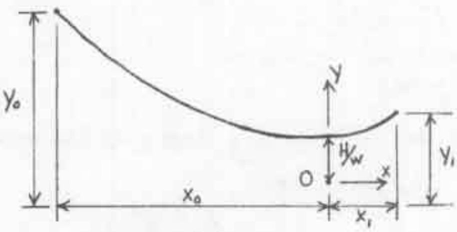


$T\cos\theta = H$ (From Fig. 7.18b)
$\theta_1 = 11.346^\circ$
$\theta_2 = 9.109^\circ$

$$\Sigma F_x = (-25\,583)\cos 11.346^\circ + (47\,575)\cos 9.109^\circ = 21\,892\ N$$
$$\Sigma F_y = (-25\,583)\sin 11.346^\circ - (47\,575)\sin 9.109^\circ = -12\,564\ N$$
$$\underline{\underline{\underline{R} = 21\,892\,\underline{i} - 12\,564\,\underline{j}\ \ (N)}}$$

7.88

Find the minimum length required.



$w = 0.3\ \text{lb/ft}$
$T_{max} = 1500\ \text{lb}$
T_{max} occurs at the left end.
$$T = H\cosh\left[\frac{wx}{H}\right] \quad \text{(Eqn. 7.23)}$$
$$1500 = H\cosh\left(\frac{0.3x_0}{H}\right) \quad (1)$$
$$wy = H\cosh\left(\frac{wx}{H}\right) \quad \text{(Eqn. 7.19)}$$
$$y_1 = \frac{H}{0.3}\cosh\left(\frac{.3x_1}{H}\right) \quad (2)$$
$$y_0 = \frac{H}{0.3}\cosh\left(\frac{.3x_0}{H}\right) \quad (3)$$
$$y_0 - y_1 = 700 \quad (4)$$
$$-x_0 + x_1 = 2450 \quad (5)$$

Using TKSolver with the 5 equations:
$H = 1290\ \text{lb}$
$x_0 = -2421.58\ \text{ft}$. $\quad x_1 = 28.42\ \text{ft}$
$y_0 = 5000\ \text{ft}$. $\quad y_1 = 4300\ \text{ft}$

$$L = \frac{H}{w}\left[\sinh\left(\frac{w}{H}x_1 + C\right) - \sinh\left(\frac{w}{H}x_0 + C\right)\right] \quad \text{(Eqn. 7.21)}$$
With the origin located as shown, C=0
$$L = \frac{1290}{0.3}\left[\sinh\left[\frac{0.3}{1290}(28.42)\right] - \sinh\left[\frac{0.3}{1290}(-2421.58)\right]\right]$$
$$\underline{L = 2580\ \text{ft}}$$

7.89

Find the length of the over-hanging chain so the chain remains in equilibrium.
$w = 2\ \text{lb/ft}$ $\quad d = 2.5\ \text{ft}$
$a = 10\ \text{ft}$ $\quad x = \frac{a}{2} = 5\ \text{ft}$ (Symmetry)
$$d = \frac{H}{w}\left[\cosh\left(\frac{wa}{2H}\right) - 1\right] \quad \text{(Eqn. 7.24)}$$
$$2.5 = \frac{H}{2}\left[\cosh\left(\frac{20}{2H}\right) - 1\right]$$

(continued)

$$H = 10.74 \text{ lb.}$$

T_{max} will occur at $x = 5$ ft

$$T = H\cosh\left(\frac{wx}{H}\right) \quad (\text{Eqn. } 7.23)$$

$$T_{max} = 10.74\cosh\left(\frac{10}{10.74}\right)$$

$$T_{max} = 15.74 \text{ lb.}$$

The tension at the pin is equal to the weight of the over-hanging chain.

$$L = \frac{T_{max}}{w} = \frac{15.74 \text{ lb.}}{2 \text{ lb/ft}}$$

$$\underline{\underline{L = 7.87 \text{ ft.}}}$$

7.90

$$y = 8\cosh\left(\frac{x}{8}\right)$$

$$x_0 = -10 \text{ ft} \quad x_1 = 30 \text{ ft}$$

$$w = 2 \text{ lb/ft}$$

a) Calculate the ordinates y_0 and y_1 of the end points.

$$y_0 = 8\cosh\left(\frac{-10}{8}\right) \qquad \underline{\underline{y_0 = 15.11 \text{ ft}}}$$

$$y_1 = 8\cosh\left(\frac{30}{8}\right) \qquad \underline{\underline{y_1 = 170.2 \text{ ft}}}$$

b) Calculate the length and the weight of the chain.

$$L = \frac{H}{w}\left[\sinh\left(\frac{x_1}{H/w} + C\right) - \sinh\left(\frac{x_0}{H/w} + C\right)\right] \quad (\text{Eqn. } 7.21)$$

where $H/w = 8$ and $C = 0$

$$L = 8\left[\sinh\left(\frac{x_1}{8}\right) - \sinh\left(\frac{x_0}{8}\right)\right]$$

$$\underline{\underline{L = 182.8 \text{ ft}}}$$

$$W = L \cdot w = 182.8 \cdot 2$$

$$\underline{\underline{W = 365.6 \text{ lb.}}}$$

c) Calculate T_0, T_1, and H

$$H/w = 8$$

$$\underline{\underline{H = 2(8) = 16 \text{ lb.}}}$$

$$T = H\cosh\left(\frac{wx}{H}\right) \quad (\text{Eqn. } 7.23)$$

$$T_0 = 16\cosh\left(\frac{-10}{8}\right) \qquad \underline{\underline{T_0 = 30.2 \text{ lb.}}}$$

$$T_1 = 16\cosh\left(\frac{30}{8}\right) \qquad \underline{\underline{T_1 = 340 \text{ lb.}}}$$

7.91

$$x = a\sinh^{-1}\left(\frac{s}{a}\right)$$

$$y = \sqrt{a^2 + s^2} + \text{constant}$$

$$a = \frac{H}{w}$$

Show that these equations are the parametric equations for a catenary.

$$s = \tfrac{1}{2}L \quad (\text{By inspection})$$

$$L = \frac{H}{w}\left[\sinh\left(\frac{w}{H}x_1 + C\right) - \sinh\left(\frac{w}{H}x_0 + C\right)\right] \quad (\text{Eqn. } 7.21)$$

$$-x_0 = x_1\,; \quad C = 0 \quad (\text{symmetry})$$

$$L = 2\frac{H}{w}\sinh\left(\frac{w}{H}x\right)$$

$$s = \frac{H}{w}\sinh\left(\frac{wx}{H}\right) \quad (1)$$

$$a = \frac{H}{w}$$

$$s = a\sinh\left(\frac{x}{a}\right)$$

$$\frac{s}{a} = \sinh\left(\frac{x}{a}\right)$$

$$\underline{\underline{x = a\sinh^{-1}\left(\frac{s}{a}\right)}}$$

$$wy = H\cosh\left(\frac{w}{H}x + \cancel{C}\right) + K \quad (\text{Eqn. } 7.17)$$

$$\frac{wy}{H} = \cosh\left(\frac{wx}{H}\right) + K/H$$

Using a hyperbolic identity,

$$\frac{wy}{H} = \sqrt{1 + \sinh^2\left(\frac{wx}{H}\right)} + K/H$$

From equation (1) above;

$$\sinh\left(\frac{wx}{H}\right) = \frac{sw}{H}$$

$$\frac{wy}{H} = \sqrt{1 + \frac{s^2w^2}{H^2}} + K/H$$

$$a = \frac{H}{w}, \text{ so } \frac{y}{a} = \sqrt{1 + \frac{s^2}{a^2}} + K/H$$

$$\underline{\underline{y = \sqrt{a^2 + s^2} + K/H}}$$

7.92

Plot the design sag as a function of T_{max}, $15 \text{ kN} \le T_{max} \le 23 \text{ kN}$.

$$a = 250\text{m} \qquad w = 75 \text{ N/m} = 0.075 \text{ kN/m}$$

$$x_0 = -125\text{m} \qquad x_1 = 125\text{ m}$$

$$C = 0 \quad (\text{Symmetry})$$

$$T_{max} = H\cosh\left(\frac{wx}{H}\right) \quad (\text{Eqn. } 7.23)$$

$$d = \frac{H}{w}\left[\cosh\left(\frac{wa}{2H}\right) - 1\right] \quad (\text{Eqn. } 7.24)$$

$$a = 2x$$

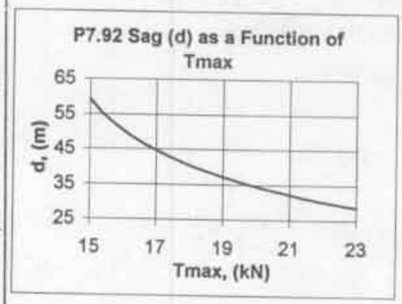
$$d = \frac{H}{w}\left[\cosh\left(\frac{wx}{H}\right) - 1\right]$$

Sub. for T_{max};

$$d = \frac{1}{w}\left[T_{max} - H\right]$$

$$d = \frac{T_{max} - H}{w}$$

Tmax (kN)	H (kN)	d (m)
15.0	10.556	59.253
15.4	11.271	55.054
15.8	11.919	51.748
16.2	12.524	49.010
16.6	13.100	46.671
17.0	13.653	44.630
17.4	14.188	42.822
17.8	14.710	41.199
18.2	15.220	39.730
18.6	15.721	38.389
19.0	16.213	37.158
19.4	16.698	36.021
19.8	17.178	34.966
20.2	17.651	33.983
20.6	18.120	33.064
21.0	18.585	32.202
21.4	19.046	31.391
21.8	19.503	30.626
22.2	19.957	29.904
22.6	20.409	29.219
23.0	20.857	28.569



7.93

Plot the sag as a function of w for $0.036\ ^{kN}/_{m} \le w \le 0.086\ ^{kN}/_{m}$.

$a = 250\ m \qquad T_{max} = 16\ kN$

$x = 125\ m$

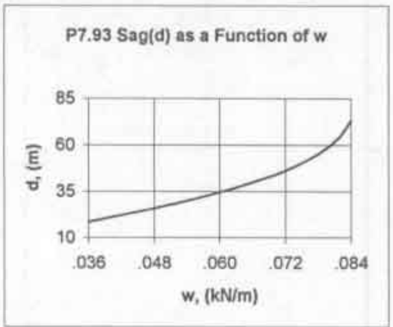
$T_{max} = H\cosh\left(\frac{wx}{H}\right)$ (Eqn. 7.23)

$16 = H\cosh\left(\frac{125\,w}{H}\right)$

$d = \frac{H}{w}\left[\cosh\left(\frac{wa}{H}\right) - 1\right]$ (Eqn. 7.24)

$d = \frac{16 - H}{w}$

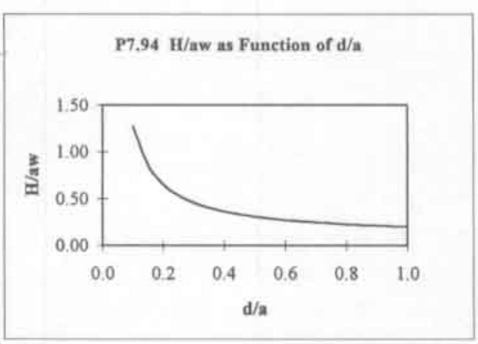
w (N/m)	H (kN)	d (m)
0.036	15.335	18.47
0.038	15.254	19.62
0.040	15.168	20.79
0.042	15.077	21.98
0.044	14.979	23.21
0.046	14.875	24.46
0.048	14.764	25.75
0.050	14.646	27.08
0.052	14.521	28.45
0.054	14.387	29.86
0.056	14.245	31.34
0.058	14.094	32.87
0.060	13.932	34.47
0.062	13.759	36.14
0.064	13.573	37.92
0.066	13.374	39.79
0.068	13.158	41.80
0.070	12.923	43.96
0.072	12.665	46.31
0.074	12.381	48.91
0.076	12.061	51.83
0.078	11.695	55.20
0.080	11.26	59.25
0.082	10.708	64.54
0.084	9.8619	73.07



7.94

a) Plot $\frac{H}{aw}$ as a function of $\frac{d}{a}$.

$\frac{d}{a} = \frac{H}{wa}\left[\cosh\left(\frac{wa}{2H}\right) - 1\right]$ (Eqn. 7.24)



d/a	H/aw
0.10	1.266
0.15	0.857
0.20	0.656
0.25	0.537
0.30	0.459
0.35	0.405
0.40	0.365
0.45	0.334
0.50	0.309
0.55	0.290
0.60	0.273
0.65	0.260
0.70	0.248
0.75	0.238
0.80	0.229
0.85	0.221
0.90	0.214
0.95	0.208
1.00	0.203

b) As d/a goes to zero, $\frac{H}{aw}$ goes to infinity. This occurs because there is no sag and the minimum tension, H, increases rapidly for constant span and cable weight.

7.95

A couple can be displaced anywhere in a plane and maintain the same moment. Therefore, if the couple was moved, the bar would remain in equilibrium with the 5 kN force.

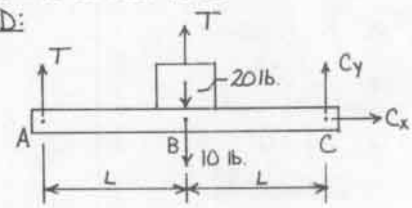
7.96

a) First-class lever
b) Second-class lever
c) Second-class lever
d) Third-class lever
e) Third-class lever

7.97

Find the contact force N.

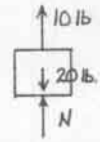
FBD:



$\circlearrowleft + \Sigma M_c = 10L + 20L - TL - 2TL = 0$

$T = 10\ lb.$

FBD:

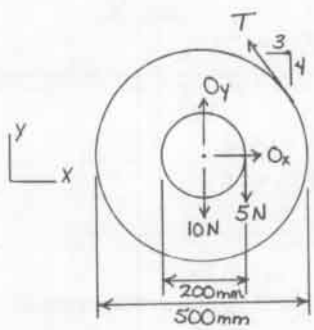


$\Sigma F = N + 10 - 20 = 0$

$\underline{\underline{N = 10\ lb.}}$

7.98

Find the reactions at O.



$\circlearrowleft + \Sigma M_O = -5(.100) + T(.250) = 0$

$T = 2\ N$

$\Sigma F_x = O_x - 2\left(\frac{3}{5}\right) = 0$

$\underline{\underline{O_x = 1.2\ N}}$

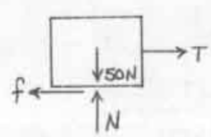
$\Sigma F_y = O_y + T\left(\frac{4}{5}\right) - 15 = 0$

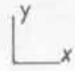
$\underline{\underline{O_y = 13.4\ N}}$

7.99

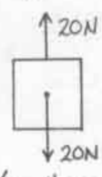
a) Draw free-body diagrams of A and B.

FBD of A:





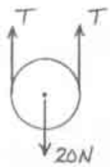
FBD of B:



(continued)

b) Find T and the forces on block A.

FBD of pulley:



$\underline{\underline{T = 10\ N}}$

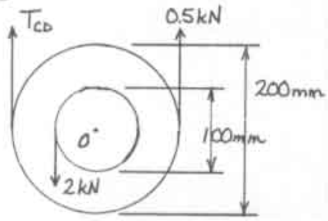
From the FBD of A:

$\Sigma F_x = 10 - f = 0 \qquad \underline{\underline{f = 10\ N}}$

$\Sigma F_y = N - 50 = 0 \qquad \underline{\underline{N = 50\ N}}$

7.100

Find T_{CD}.



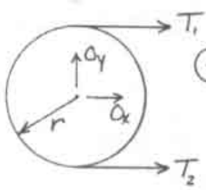
$\circlearrowleft + \Sigma M_o = -T_{CD}(.100) + 0.5(.100) + 2(.050) = 0$

$\underline{\underline{T_{CD} = 1.5\ kN}}$

7.101

Show that belt tensions are equal.
No friction on the shaft at O.

FBD of pulley:



$\circlearrowleft + \Sigma M_o = T_2 r - T_1 r = 0$

$\underline{\underline{T_1 = T_2}}$

7.102

a) Mechanism
b) Mechanism
c) Truss
d) Frame
e) Frame

7.103

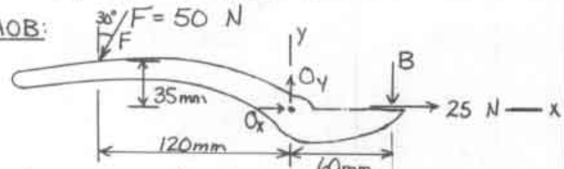
a) Find the force that pin O applies on member AOB.

From the FBD of the pliers (See Fig. P7.103)

$\Sigma F_x = 50 - 2F\sin 30° = 0$

$F = 50\ N$

FBD of AOB:



(By symmetry, the 50 N force in the wire is split into two 25 N pieces, one at B and one at D.)

$\circlearrowleft + \Sigma M_o = 50\cos 30°(120) - B(60) + 50(\sin 30°)(35) = 0$

$B = 101.2\ N$

$\Sigma F_x = 25 - 50\sin 30° + O_x = 0$

$\underline{\underline{O_x = 0}}$

$\Sigma F_y = O_y - 50\cos 30° - B = 0$

$\underline{\underline{O_y = 144.5\ N}}$

7.106

Find T_{max}.

$w = 0.20\ lb/ft$

$a = 100\ ft$

$d = 3\ ft$

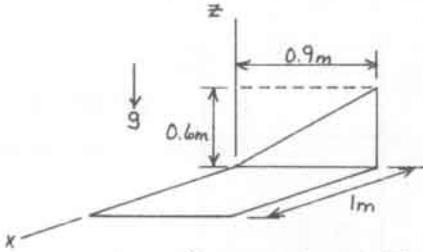
$T_{max} = \frac{wa}{2}\sqrt{1 + \frac{1}{16}\left(\frac{a}{d}\right)^2} \quad$ (Eqn. 7.10)

$T_{max} = \frac{.2(100)}{2}\sqrt{1 + \frac{1}{16}\left(\frac{100}{3}\right)^2}$

$\underline{\underline{T_{max} = 83.9\ lb.}}$

8.1

Find the gravity axis for the sheet metal object shown. Gravity acts in the $-z$ direction.



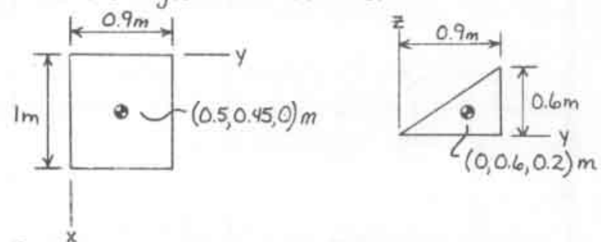
Let γ = specific weight [N/m³]

t = sheet thickness [m]

$w = \gamma t$ [N/m²]

$\Sigma F_z = -\frac{1}{2}(0.9)(0.6)w - (0.9)(1.0)w = -1.17w$ [N]

The gravity axes of the triangle and the rectangle act at their geometric centers.



$\circlearrowleft + \Sigma M_x = -(0.9)(1.0)w(.45) - \frac{1}{2}(0.9)(0.6)w(0.6) = -0.567w$

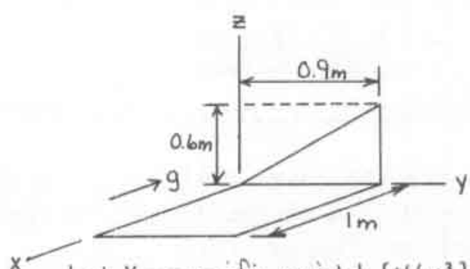
$\circlearrowleft + \Sigma M_y = (0.9)(1.0)w(0.5) = 0.45w$

Determine the location of the gravity axis.

$\Sigma M_x = \bar{y}\,\Sigma F_z$	$\Sigma M_y = -\bar{x}\,\Sigma F_z$
$-0.567w = \bar{y}(-1.17w)$	$0.45w = (1.17w)\bar{x}$
$\underline{\bar{y} = 0.485\text{ m}}$	$\underline{\bar{x} = 0.385\text{ m}}$

8.2

Find the gravity axis for the sheet metal object shown. Gravity acts in the $-x$ direction



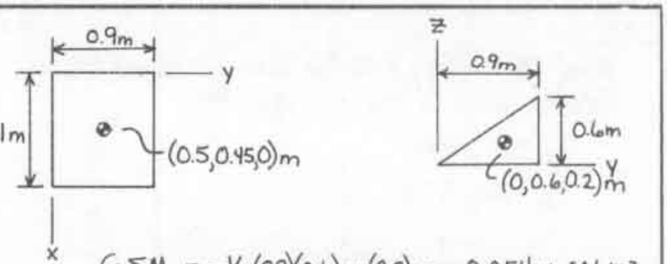
Let γ = specific weight [N/m³]

t = sheet thickness [m]

$w = \gamma t$ [N/m²]

$\Sigma F_x = -(0.9)(1.0)w - \frac{1}{2}(0.9)(0.6)w = -1.17w$ [N]

The gravity axes of the triangle and the rectangle act at their geometric centers.



$\circlearrowleft + \Sigma M_y = -\frac{1}{2}(0.9)(0.6)w(0.2) = -0.054w$ [N·m]

$\circlearrowleft + \Sigma M_z = (0.9)(1.0)w(0.45) + \frac{1}{2}(0.9)(0.6)w(0.6) = 0.567w$ [N·m]

Determine the location of the gravity axis.

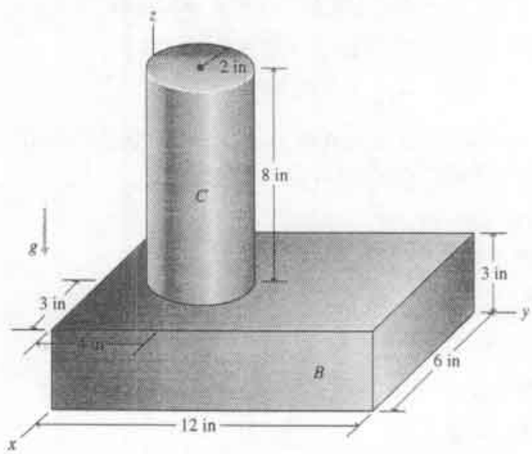
$\circlearrowleft + \Sigma M_y = \bar{z}\,\Sigma F_x$	$\circlearrowleft + \Sigma M_z = -\bar{y}\,\Sigma F_x$
$-0.054w = (-1.17w)\bar{z}$	$0.567w = -\bar{y}(-1.17w)$
$\underline{\bar{z} = 0.0462\text{ m}}$	$\underline{\bar{y} = 0.485\text{ m}}$

8.3

Find the gravity axis for the solid object shown. Gravity acts in the $-z$ direction.

$\gamma_C = 0.3$ lb/in³

$\gamma_B = 0.1$ lb/in³



$W_C = \gamma_C V_C = (0.3)(\pi(2)^2(8)) = 30.16$ lb.

$W_B = \gamma_B V_B = (0.1)(6 \cdot 3 \cdot 12) = 21.60$ lb.

$\Sigma F_z = -W_C - W_B = -51.76$ lb.

The individual gravity axes of B and C will act at their geometric centers.

The center of B is at $(3, 6, 1.5)$ in.

The center of C is at $(3, 4, 7)$ in.

$\circlearrowleft + \Sigma M_x = -30.16(4) - 21.6(6) = -250.24$ lb·in

$\circlearrowleft + \Sigma M_y = 30.16(3) + 21.6(3) = 155.28$ lb·in

Determine the location of the gravity axis.

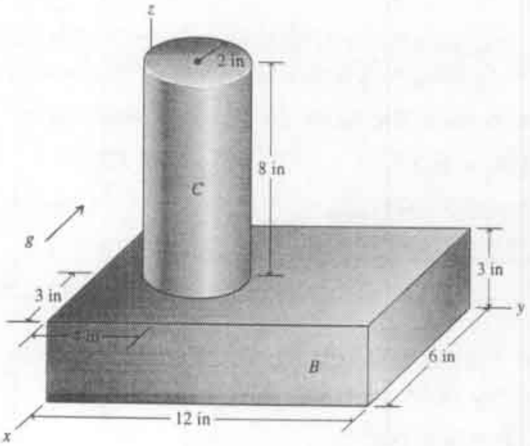
$\circlearrowleft + \Sigma M_x = \bar{y}\,\Sigma F_z$	$\circlearrowleft + \Sigma M_y = -\bar{x}\,\Sigma F_z$
$-250.24 = \bar{y}(-51.76)$	$155.28 = -\bar{x}(-51.76)$
$\underline{\bar{y} = 4.83\text{ in.}}$	$\underline{\bar{x} = 3.00\text{ in.}}$

8.4

Find the gravity axis for the solid object shown. Gravity acts in the $-x$ direction.

$\gamma_C = 0.3\ \text{lb/in}^3$

$\gamma_B = 0.1\ \text{lb/in}^3$



$W_C = \gamma_C V_C = 0.3\left(\pi(2)^2(8)\right) = 30.16\ \text{lb.}$

$W_B = \gamma_B V_B = 0.1(6\cdot 3\cdot 12) = 21.60\ \text{lb.}$

$\Sigma F_x = -W_C - W_B = -51.76\ \text{lb.}$

The individual gravity axes of B and C will act at their geometric centers.

The center of B is at $(3, 6, 1.5)$ in.

The center of C is at $(3, 4, 7)$ in.

$\Sigma M_y = -30.16(7) - 21.6(1.5) = -243.52\ \text{lb-in}$

$\Sigma M_z = 30.16(4) + 21.6(6) = 250.24\ \text{lb-in}$

Determine the location of the gravity axis.

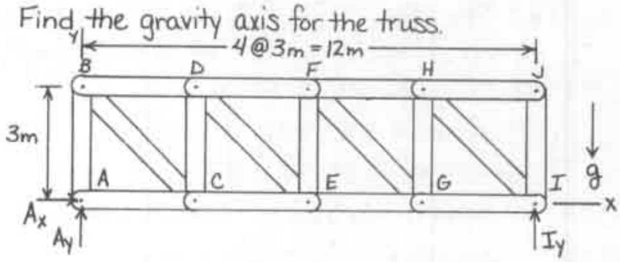
$\Sigma M_y = \bar{z}\,\Sigma F_x \qquad \Sigma M_z = -\bar{y}\,\Sigma F_x$

$-243.52 = \bar{z}(-51.76) \qquad 250.24 = -\bar{y}(-51.76)$

$\underline{\bar{z} = 4.70\ \text{in.}} \qquad \underline{\bar{y} = 4.83\ \text{in.}}$

8.5

Find the gravity axis for the truss.

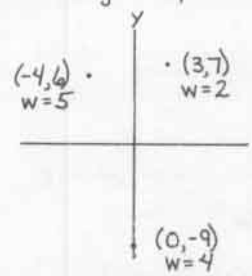


By using symmetry the gravity axis is located at:

$\underline{\bar{x} = 6\ \text{m}} \qquad \underline{\bar{y} = 1.5\ \text{m}}$

8.6

Find the gravity axis for a system of weights with individual gravity axes as shown.



$\Sigma F_z = -5 - 2 - 4 = -11$

$\Sigma M_x = 4(9) - 2(7) - 5(6) = -8$

$\Sigma M_y = 2(3) - 5(4) = -14$

Determine the location of the gravity axis.

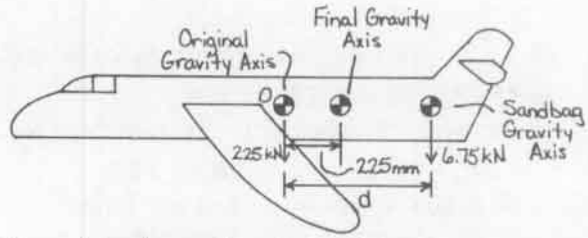
$\Sigma M_x = \bar{y}\,\Sigma F_z \qquad \Sigma M_y = -\bar{x}\,\Sigma F_z$

$-8 = \bar{y}(-11) \qquad -14 = -\bar{x}(-11)$

$\underline{\bar{y} = 0.73} \qquad \underline{\bar{x} = -1.273}$

8.7

Find the location of the sandbags (6750 N) required to shift the gravity axis 225 mm towards the tail. The plane weighs 225 kN.



The original gravity axis is at O. The plane weight plus sand bags have a moment of

$M_o = 6.75d + 225(0) = 6.75d$

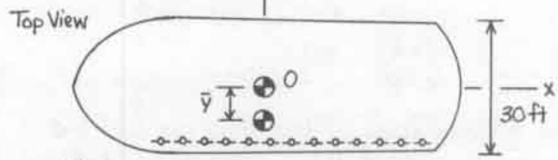
For the resultant,

$M_o = (225 + 6.75)(0.225) = 52.14\ \text{kN·m}$

$\therefore\ d = \dfrac{52.14}{6.75} = 7.725\ \text{m}$

8.8

A ship weighs 500 tons (1×10^6 lb). It carries 15,000 lb of passengers. Find the gravity axis if all the passengers are along one rail.



$\Sigma F_z = -W_B - W_P = -1{,}015{,}000\ \text{lb.}$

Initially the gravity axis is at O.

$\Sigma M_o = 15000(15) = 225{,}000\ \text{lb·ft}$

(continued)

8.8 cont.

Determine the location of the gravity axis.

$$\circlearrowleft^{+} \Sigma M_o = \bar{y}\, \Sigma F_z$$

$$225{,}000 = -1{,}015{,}000\, \bar{y}$$

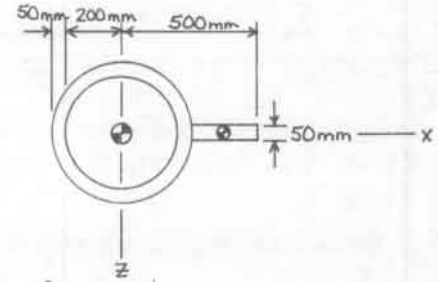
$$\underline{\bar{y} = -0.222 \text{ ft}}$$

8.9

Find the location of an eye-bolt so a casting can be lifted without rotating.

In order for the casting to remain horizontal the eye-bolt should be located on the casting's gravity axis.

Top View:



γ = specific wt. of material

$$W_{cyl} = \gamma\left[\pi(.250)^2(0.400) - \pi(0.200)^2(0.350)\right] = 0.03456\,\gamma$$

$$W_{fin} = \gamma\,(0.05)(0.300)(0.250) = 0.00375\,\gamma$$

$$\Sigma F_y = -W_{cyl} - W_{fin} = -0.03831\,\gamma$$

$$\circlearrowleft^{+} \Sigma M_z = 0.00375\,\gamma\,(0.375) = -0.00141\,\gamma$$

Determine the location of the gravity axis.

$$\circlearrowleft^{+} \Sigma M_z = \bar{x}\, \Sigma F_y$$

$$-0.00141\,\gamma = \bar{x}\,(-0.03831\,\gamma)$$

$$\underline{\bar{x} = 0.0367 \text{ m} = 36.7 \text{ mm}}$$

8.10

A 4-engine plane weighs 360 kN, including engines at 8.1 kN each. Find the new gravity axis if new replacement engines weighing 7.2 kN each are installed at new locations.

The weight of the plane without engines is $W_o = 360 - 4(8.1) = 327.6$ kN

Let $\bar{x}_1$ = the location of the gravity axis without engines.

Figure A is a schematic that indicates the weights with the original engines.

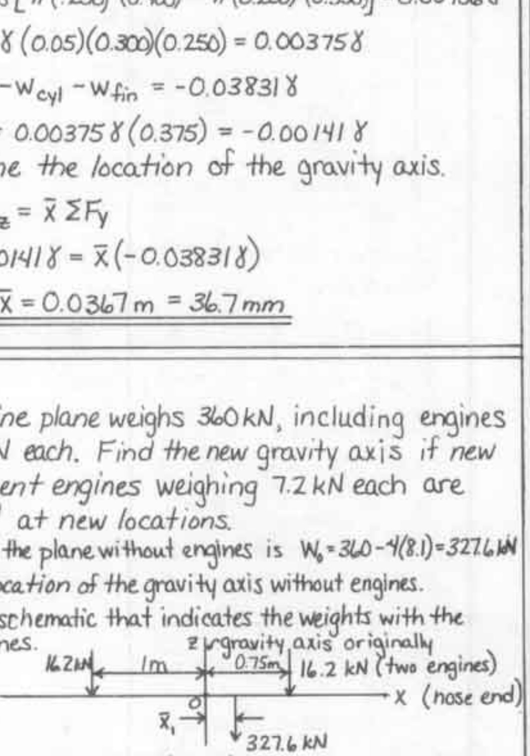


Figure A

From Fig. A,

$$\circlearrowleft^{+} \Sigma M_o = 16.2(1) - 16.2(0.75) - 327.6\,\bar{x}_1 = 0 \quad \therefore \bar{x}_1 = 0.01236\text{m}$$

Figure B is a schematic that indicates the weights with the new engines.

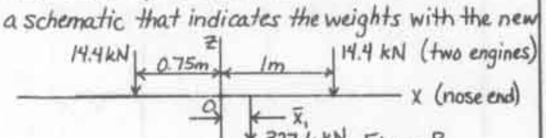


Figure B

From Fig. B,

$$\circlearrowleft^{+} \Sigma M_o = (14.4)(0.75) - 14.4(1.0) - (327.6)(0.01236) = -(14.4 + 14.4 + 327.6)\,\bar{x}$$

$$\bar{x} = \frac{7.649}{356.4} = 0.0215 \text{ m} = 21.5 \text{ mm}$$

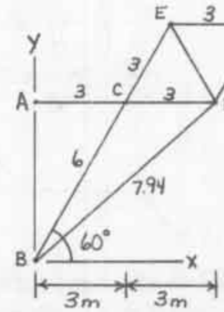
Hence, the gravity axis has moved 21.5mm toward the nose of the plane.

8.11

Find the gravity axis for the truss shown.

A = cross-sectional area
γ = specific weight
} Equivalent for all members

Determine the length of each member.



$$\Sigma F_y = -\gamma A\,(3(6) + 6 + 7.94) = -31.94\,\gamma A$$

$$\circlearrowleft^{+} \Sigma M_B = -\gamma A\left[6(1.5) + 3(1.5) + 3(4.5) + 3(3.75) + 3(5.25) + 3(6) + 3(6.75) + 7.94(3)\right] = -116.06\,\gamma A$$

$$\circlearrowleft^{+} \Sigma M_B = \bar{x}\, \Sigma F_y$$

$$-116.06\,\gamma A = \bar{x}(-31.94\,\gamma A)$$

$$\underline{\underline{\bar{x} = 3.63 \text{ m}}}$$

8.12

Wheels A, B, and C of a plane carry the loads shown. Will the 9kN load at D cause the plane to tip backwards?

The original weight of the airplane is

$$W_o = 135 + 157.5 + 22.5 = 315 \text{ kN}$$

A schematic of the original loads is shown in Figure A.

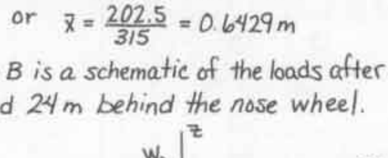


Figure A

By Fig. A, the location $\bar{x}$ of the original gravity axis is given by $\quad \Sigma M_o = (22.5)(9) = W_o(\bar{x})$

$$\text{or} \quad \bar{x} = \frac{202.5}{315} = 0.6429 \text{ m}$$

Figure B is a schematic of the loads after the 9kN load is placed 24 m behind the nose wheel.

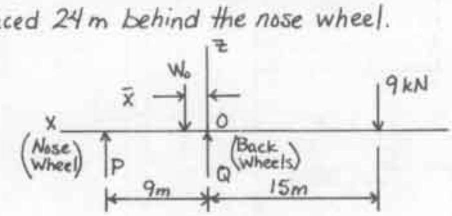


Figure B

(continued)

8.12 cont. If the plane is to fall on its tail, the load of the nose wheel would be P=0. With P=0, by Fig. B, we have

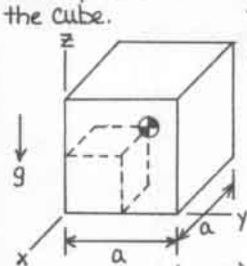
$\circlearrowleft + \Sigma M_o = W_o\bar{x} - 9(15) = 315(0.6429) - 135 = 67.5\ kN \cdot m > 0$

Hence, the plane would not tip.

8.13

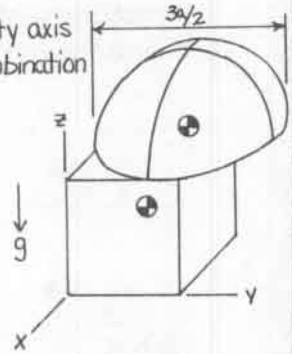
A hemisphere rests on a cube. Find the distance between the gravity axes of the cube and the combination of the two solids.

Gravity axis of the cube.



Gravity axis at $(-\frac{a}{2}, \frac{a}{2})$

The gravity axis of the combination



The gravity axis of the hemisphere is $\frac{3}{4}a$ for both the x and y directions.

$$\Sigma F_z = -\gamma a^3 - \gamma(\tfrac{1}{2})(\tfrac{4}{3}\pi[\tfrac{1}{2}(\tfrac{3}{2}a)]^3) = -1.884\gamma a^3$$

$$\circlearrowleft + \Sigma M_x = -\gamma a^3(\tfrac{1}{2}a) - \gamma(\tfrac{1}{2})(\tfrac{4}{3}\pi(\tfrac{3}{4}a)^3(\tfrac{3}{4}a) = -1.163\gamma a^4$$

$$\circlearrowleft + \Sigma M_y = \gamma a^3(\tfrac{1}{2}a) + \gamma(\tfrac{1}{2})(\tfrac{4}{3}\pi(\tfrac{3}{4}a)^3(\tfrac{3}{4}a) = 1.163\gamma a^4$$

Determine the location of the gravity axes.

$\circlearrowleft + \Sigma M_x = \bar{y}\,\Sigma F_z \qquad \circlearrowleft + \Sigma M_y = -\bar{x}\,\Sigma F_z$

$-1.163\gamma a^4 = \bar{y}(-1.884\gamma a^3) \qquad 1.163\gamma a^4 = -\bar{x}(-1.884\gamma a^3)$

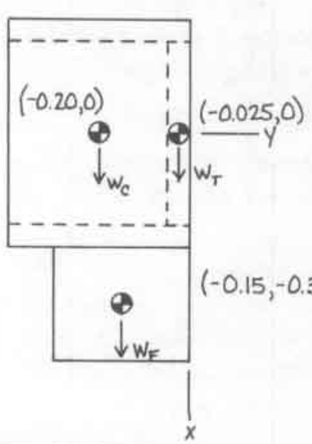
$\bar{y} = 0.6171a \qquad \bar{x} = 0.6171a$

$$D = \sqrt{(0.6171a - 0.5a)^2 + (0.6171a - 0.5a)^2}$$

$$\underline{\underline{D = 0.1656a}}$$

8.14

To locate the coordinate of the eye-bolt, first rotate the cast on its side. Therefore, the force of gravity is in a different direction.



γ = specific weight

$$W_T = \gamma(\pi(0.20)^2(0.05)) = 0.006283\gamma$$
$$W_F = \gamma(0.05)(0.30)(0.25) = 0.003758\gamma$$
$$W_c = \gamma\pi[(0.25^2 - 0.20^2)(0.40)] = 0.02828\gamma$$
$$\Sigma F_x = -W_T - W_F - W_c = -0.0383\gamma$$
$$\circlearrowleft + \Sigma M_o = W_T(0.025) + W_F(0.150) + W_c(0.200)$$
$$\circlearrowleft + \Sigma M_o = 0.006374\gamma$$

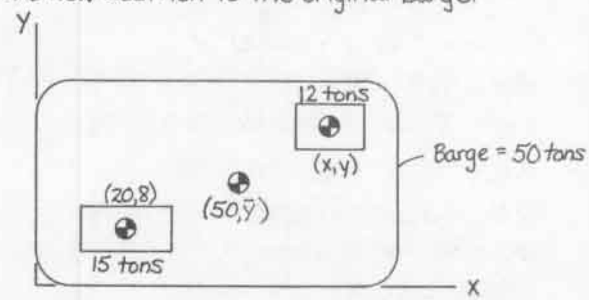
Determine the location of the eye-bolt.

$$\circlearrowleft + \Sigma M_o = \bar{y}\,\Sigma F_x$$
$$0.006374\gamma = \bar{y}(-0.0383\gamma)$$
$$\underline{\underline{\bar{y} = -0.1664\ m = -166.4\ mm}}$$

8.15

Locate a second transformer on the barge of example 8.2 so that the gravity axis passes through $\bar{x} = 50$ ft.

The new addition to the original barge:



In order to find the $\bar{x}$ value only the ΣM_y is necessary.

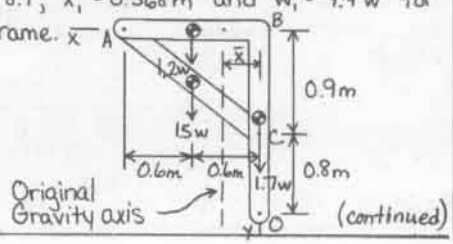
$$\circlearrowleft + \Sigma M_y = 12(x) + 50(50) + 15(20) = 2800 + 12x \text{ (ton-ft)}$$
$$\Sigma F_z = -15 - 50 - 12 = -77 \text{ tons}$$
$$\circlearrowleft + \Sigma M_y = -\bar{x}\,\Sigma F_z \qquad \bar{x} = 50\text{ ft.}$$
$$2800 + 12x = -50(-77)$$
$$\underline{x = 87.5\text{ ft.}}$$

The second transformer should be placed so that the x coordinate is 87.5 ft. The $\bar{y}$ coordinate is not asked for in this problem.

8.16

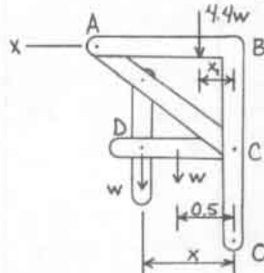
Add two members of weight w (N/m) to the frame of Example 8.1 so the new gravity axis is at $\bar{x} = 0.50$ m.

From Example 8.1, $\bar{x}_1 = 0.368$ m and $W_1 = 4.4w$ for the original frame.



(continued)

New Frame - Add one horizontal member that extends to the left from Joint C. Hang a member from AC; adjust its position to have the desired effect. Let each member be 1 m long.



Total Frame Weight:

$W_2 = W_1 + 2w = 6.4w$

Moments about O:

$\circlearrowleft \Sigma M_o = (4.4w)\bar{x}_1 + w(0.5) + w(x)$

$\circlearrowleft \Sigma M_o = (2.119 + x)w$

New gravity axis has $\Sigma M_o = W_2\,\bar{x}_2$.

$(2.119 + x)w = (6.4w)\bar{x}_2$

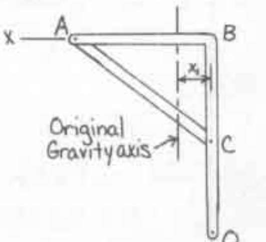
with $\bar{x}_2 = 0.5\,m$, $x = 1.08\,m$

Choose a location $x = 1.0m$ for the second new member so that it can support the new horizontal member. The resulting gravity axis is located at $\bar{x}_2 = 0.487m$ (within 3% of target). This is one of many reasonable solutions.

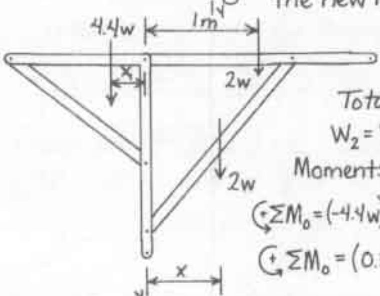
8.17

Add two members of weight w (N/m) to the frame of Example 8.1 so the new gravity axis is at $\bar{x} = 0.20\,m$ to the right of the vertical member.

From example 8.1, $\bar{x}_1 = 0.368\,m$ (Left of vertical member)
$W_1 = 4.4\,w$ for the original frame.



For the new frame, let the new members each have a length of 2m. Extend one horizontally to the right from B. Incline the other from BCO to support the new horizontal member.



Total Frame Weight:

$W_2 = W_1 + 4w = 8.4w$

Moments about O:

$\circlearrowleft \Sigma M_o = (-4.4w)\bar{x}_1 + 2w(1) + (2w)x$

$\circlearrowleft \Sigma M_o = (0.3808 + 2x)w$

New gravity axis has $\Sigma M_o = W_2\,\bar{x}_2$

$(0.3808 + 2x)w = (8.4w)\bar{x}_2$

with $\bar{x}_2 = 0.2m$, $x = 0.65m$

The new inclined member joins the new horizontal member 1.3m to the right of B. It joins BCO at 1.52 m below B. There many other possible choices.

8.18

Locate the center of gravity of the cylinder-block system in Problem #3 using the technique used in Problems #3 and #4. Check answers with Equation (8.1).

From Problem #3:

$\Sigma F_z = -\gamma_c V_c - \gamma_B V_B = -51.76\,lb.$

$\circlearrowleft \Sigma M_x = -30.16(4) - 21.6(6) = -250.24$ lb-in

$\circlearrowleft \Sigma M_y = 30.16(3) + 21.6(3) = 155.28$ lb-in

$\circlearrowleft \Sigma M_x = \bar{y}\,\Sigma F_z \qquad \Sigma M_y = -\bar{x}\,\Sigma F_z$

$\underline{\bar{y} = 4.83\text{ in.}} \qquad \underline{\bar{x} = 3.00\text{ in}}$

From Problem #4:

$\Sigma F_x = -\gamma_c V_c - \gamma_B V_B = -51.76$ lb.

$\circlearrowleft \Sigma M_y = -30.16(7) - 21.6(1.5) = -243.52$ lb-in

$\circlearrowleft \Sigma M_z = 30.16(4) + 21.6(6) = 250.24$ lb-in

$\circlearrowleft \Sigma M_y = \bar{z}\,\Sigma F_x \qquad \circlearrowleft \Sigma M_z = -\bar{y}\,\Sigma F_x$

$\underline{\bar{z} = 4.70\text{ in}} \qquad \underline{\bar{y} = 4.83\text{ in.}}$

Now using Eqn. (8.1)

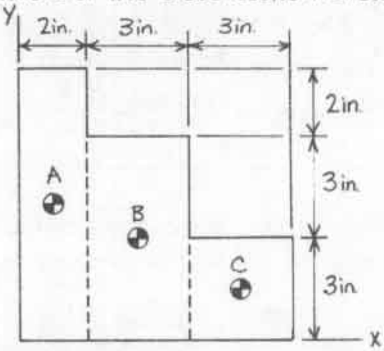
$$\bar{x} = \frac{1}{W}\Sigma w_i x_i = \frac{21.6\cdot 3\cdot w + 30.16\cdot 3\cdot w}{51.76w} = \underline{3.00\text{ in.}}$$

$$\bar{y} = \frac{1}{W}\Sigma w_i y_i = \frac{21.6\cdot 6\cdot w + 30.16\cdot 4\cdot w}{51.76w} = \underline{\underline{4.83\text{ in.}}}$$

$$\bar{z} = \frac{1}{W}\Sigma w_i z_i = \frac{21.6\cdot 1.5\cdot w + 30.16\cdot 7\cdot w}{51.76w} = \underline{\underline{4.70\text{ in.}}}$$

8.19

Find the centroid of the cross-section shown.



Total weight of the casting:

$W = W_A + W_B + W_C = (2\cdot 8)\gamma + (3\cdot 6)\gamma + (3\cdot 3)\gamma$

$W = 43\gamma$

From equation (8.1):

$$\bar{x} = \frac{1}{W}\Sigma w_i x_i = \frac{(2\cdot 8\cdot 1)w + (3\cdot 6\cdot 3.5)w + (3\cdot 3\cdot 6.5)w}{43w}$$

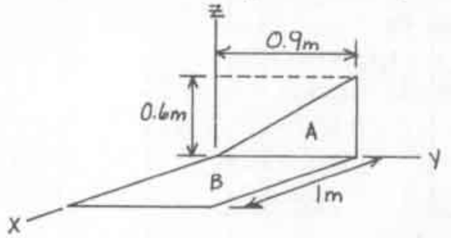
$\underline{\underline{\bar{x} = 3.20\text{ in.}}}$

$$\bar{y} = \frac{1}{W}\Sigma w_i y_i = \frac{(2\cdot 8\cdot 4)w + (3\cdot 6\cdot 3)w + (3\cdot 3\cdot 1.5)w}{43w}$$

$\underline{\underline{\bar{y} = 3.06\text{ in.}}}$

8.20

Locate the coordinates $(\bar{x}, \bar{y})$ of the center of gravity of the sheet of metal in Problem #1 using the technique of Problems #1 and #2. Check answers with equation (8.1).



From Problem #1:

$\Sigma F_z = -\frac{1}{2}(0.9)(0.6)w - (0.9)(1.0)w = -1.17w$

$\circlearrowleft \Sigma M_x = -(0.9)(1.0)w(0.45) - \frac{1}{2}(0.9)(0.6)w(0.6) = -0.567w$

$\circlearrowleft \Sigma M_y = (0.9)(1.0)w(0.5) = 0.45w$

$\circlearrowleft \Sigma M_x = \bar{y}\,\Sigma F_z \qquad \circlearrowleft \Sigma M_y = -\bar{x}\,\Sigma F_z$

$\underline{\bar{y} = 0.485\text{m}} \qquad \underline{\bar{x} = 0.385\text{m}}$

From Problem #2:

$\Sigma F_x = -(0.9)(1.0)w - \frac{1}{2}(0.9)(0.6)w = -1.17w$

$\circlearrowleft \Sigma M_y = -\frac{1}{2}(0.9)(0.6)w(0.2) = -0.054w$

$\circlearrowleft \Sigma M_z = (0.9)(1.0)w(0.45) + \frac{1}{2}(0.9)(0.6)w(0.6) = 0.567w$

$\circlearrowleft \Sigma M_y = \bar{z}\,\Sigma F_x \qquad \circlearrowleft \Sigma M_z = -\bar{y}\,\Sigma F_x$

$\underline{\bar{z} = 0.0462m} \qquad \underline{\bar{y} = 0.485m}$

Now using eqn. (8.1):

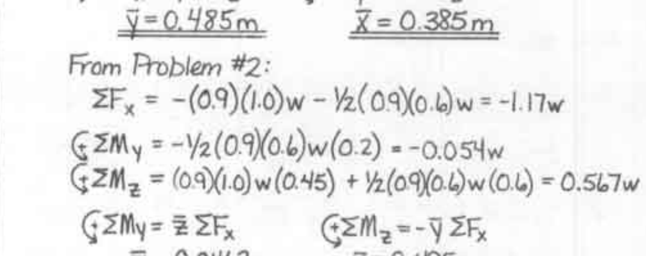
$$\bar{x} = \frac{1}{W}\Sigma w_i x_i = \frac{0.5 \cdot 0.9 \cdot w}{1.17w} = \underline{0.385\,m}$$

$$\bar{y} = \frac{1}{W}\Sigma w_i y_i = \frac{\frac{1}{2}(0.6)(0.9)w(0.6) + (0.9)w(0.45)}{1.17w} = \underline{\underline{0.485m}}$$

$$\bar{z} = \frac{1}{W}\Sigma w_i z_i = \frac{\frac{1}{2}(0.6)(0.9)w(0.2)}{1.17w} = \underline{0.0462\,m}$$

8.21

Find the elevation of the center of gravity of the homogeneous wooden bench shown.



Total weight of the table:

$\Sigma F = 4W_{leg} + W_{top} + 2W_{end} + 2W_{side}$

$= 4 \cdot 4 \cdot 4 \cdot 28 \cdot \gamma + 26 \cdot 50 \cdot 2 \cdot \gamma + 2 \cdot 24 \cdot 6 \cdot \gamma + 2 \cdot 44 \cdot 6 \cdot \gamma$

$\Sigma F = 5208\gamma$

Consider the origin to be at the bottom of the legs.

$\circlearrowleft \Sigma M = (4 \cdot 4 \cdot 28)(4)(14)\gamma + [2(24 \cdot 6) + 2(44 \cdot 6)]\,25\,\gamma + (50 \cdot 26 \cdot 2)(29)\gamma$

$\circlearrowleft \Sigma M = 120{,}888\gamma$

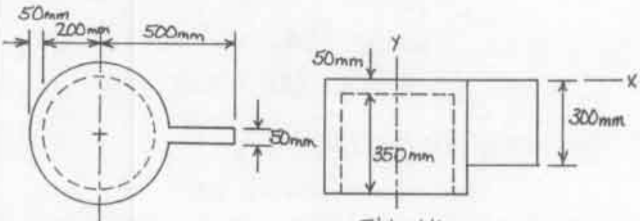
$\circlearrowleft \Sigma M = \bar{z}\,\Sigma F$

$120{,}888\gamma = 5208\gamma\,\bar{z}$

$\underline{\underline{\bar{z} = 23.2 \text{ in.}}}$

8.22

a) Locate the center of gravity of the casting using the technique used in Problems #8.9 and #8.14. Check answers with equation (8.1).



Top View Side View

From Problem #8.9:

$\Sigma F_y = -\gamma\left[\pi(0.25)^2(0.40) - \pi(0.20)^2(0.35)\right] - \gamma(0.05)(0.30)(0.25)$

$\Sigma F_y = -0.03831\gamma$

$\circlearrowleft \Sigma M_z = -\gamma(0.05)(0.30)(0.25)(0.375) = -0.00141\gamma$

$\circlearrowleft \Sigma M_z = \bar{x}\,\Sigma F_y$

$\underline{\bar{x} = 0.0367\,m = 36.7mm}$

From Problem #8.14:

$\Sigma F_x = -\gamma\left(\pi(0.20)^2(0.05)\right) - \gamma(0.05)(0.30)(0.25) - \gamma\pi\left[(0.25)^2 - (0.20)^2\right.$ $\left.(0.40)\right] = -0.03831\gamma$

$\circlearrowleft \Sigma M_o = \gamma\left(\pi(0.20)^2(0.05)(0.025)\right) + \gamma(0.05)(0.30)(0.25)(0.15) + \gamma\pi\left[(0.25)^2 - (0.20)^2)(0.40)\right](0.20) = 0.006374\gamma$

$\circlearrowleft \Sigma M_o = \bar{y}\,\Sigma F_x$

$\underline{\bar{y} = -0.1664\,m = -166.4\text{ mm}}$

Now using equation (8.1):

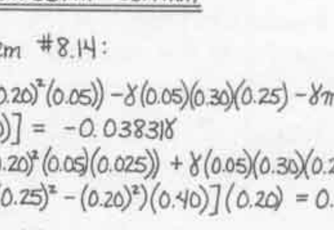
$$\bar{x} = \frac{1}{W}\Sigma w_i x_i = \frac{-0.00375\gamma(0.375)}{-0.03831\gamma} = 0.0367\,m$$

$$\bar{y} = \frac{1}{W}\Sigma w_i y_i = \frac{(.006283\gamma(.025) + .00375\gamma(.15) + .02828\gamma(.20))}{-0.03831\gamma}$$

$\bar{z} = 0$ (symmetry) $\qquad \bar{y} = -0.1664m$

<u>Center of Gravity: $(36.71, -166.4, 0)$ mm</u>

b) Find the distance from line $\overline{AB}$ to the gravity axis if the casting is hung so $\overline{AB}$ is vertical.



The perpendicular distance S from point $(\bar{x}, \bar{y})$ to the line $y = mx + b$ is:

$$S = \frac{|m\bar{x} - \bar{y} + b|}{\sqrt{1 + m^2}} \qquad m = \frac{300}{250} = 1.2$$

(continued)

8.22 cont.

$100 = 1.2(250) + b \qquad b = -200$

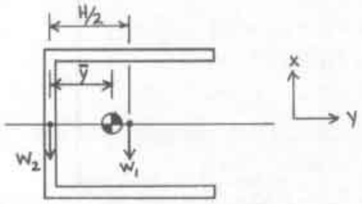
$$s = \frac{|(1.2)(36.71) - (233.6) + (-200)|}{\sqrt{1+(1.2)^2}}$$

$\underline{\underline{S = 249 \text{ mm}}}$

8.23

Show that $\bar{y} = \frac{2H^2}{4H+D}$ for a cylindrical tank that is open on top.

Hanging the tank by its center as shown yields:



Cylindrical wall: $W_1 = \pi D H \gamma t$ Circular base: $W_2 = \frac{\pi D^2}{4}\gamma t$

$$\circlearrowleft\!+ \Sigma M_{C.G.} = \frac{\pi D^2 \gamma t \bar{y}}{4} - \pi D H \gamma t (H/2 - \bar{y}) = 0$$

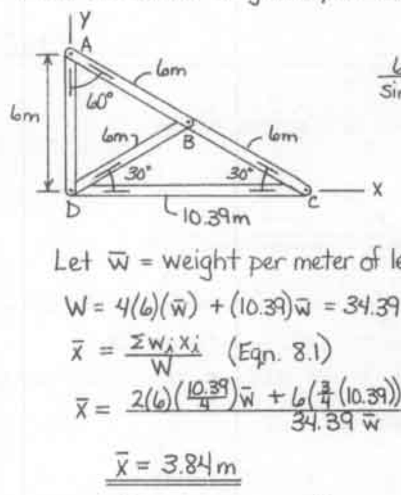
$$\frac{\pi D H^2 \cancel{\gamma}\cancel{t}}{2} = \left(\frac{\pi D^2 \cancel{\gamma}\cancel{t}}{4} + \pi D H \cancel{\gamma}\cancel{t}\right)\bar{y}$$

$$2\cancel{\pi}\cancel{D}H^2 = (\cancel{\pi}D^{\cancel{2}} + 4\cancel{\pi}\cancel{D}H)\bar{y}$$

$$\underline{\underline{\bar{y} = \frac{2H^2}{4H+D}}}$$

8.24

Find the center of gravity of the truss shown.



$\frac{6}{\sin 30°} = \frac{DC}{\sin 120°}$

$DC = 10.39 \text{ m}$

Let $\bar{w}$ = weight per meter of length of each bar.

$W = 4(6)(\bar{w}) + (10.39)\bar{w} = 34.39\,\bar{w}$

$\bar{x} = \frac{\Sigma w_i x_i}{W}$ (Eqn. 8.1)

$$\bar{x} = \frac{2(6)\left(\frac{10.39}{4}\right)\bar{w} + 6\left(\frac{3}{4}(10.39)\right)\bar{w} + 10.39\left(\frac{10.39}{2}\right)\bar{w}}{34.39\,\bar{w}}$$

$\underline{\underline{\bar{x} = 3.84 \text{ m}}}$

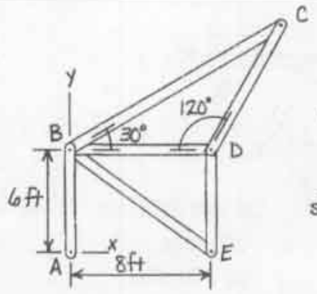
$\bar{y} = \frac{\Sigma w_i y_i}{W}$ (Eqn. 8.1)

$$\bar{y} = \frac{6(3)\bar{w} + 2(6)\left(\frac{6}{4}\right)\bar{w} + 6\left(\frac{3}{4}\right)6\bar{w}}{34.39\,\bar{w}}$$

$\underline{\underline{\bar{y} = 1.832 \text{ m}}}$

8.25

Find the center of gravity of the truss shown.



$AB = DE = 6 \text{ ft.}$

$BD = CD = 8 \text{ ft.}$

$BE = \sqrt{6^2 + 8^2} = 10 \text{ ft.}$

$\frac{8}{\sin 30°} = \frac{BC}{\sin 120°}$

$BC = 13.856 \text{ ft.}$

Let $\bar{w}$ = weight per foot of length of each bar.

$W = 2(6)\bar{w} + 2(8)\bar{w} + 10\bar{w} + 13.856\bar{w} = 51.86\,\bar{w}$

$\bar{x} = \frac{\Sigma w_i x_i}{W}$ (Eqn. 8.1)

$$\bar{x} = \frac{(6)(8)\bar{w} + 10(4)\bar{w} + 8(4)\bar{w} + 8(10)\bar{w} + 13.856(6)\bar{w}}{51.86\,\bar{w}}$$

$\underline{\underline{\bar{x} = 5.46 \text{ ft.}}}$

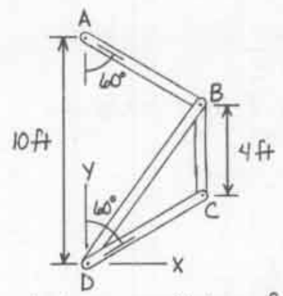
$\bar{y} = \frac{\Sigma w_i y_i}{W}$ (Eqn. 8.1)

$$\bar{y} = \frac{2(6)(3)\bar{w} + 8(6)\bar{w} + 10(3)\bar{w} + 8(9.464)\bar{w} + 13.856(9.464)\bar{w}}{51.86\,\bar{w}}$$

$\underline{\underline{\bar{y} = 6.19 \text{ ft.}}}$

8.26

Find the center of gravity of the truss shown.



$BC = 4 \text{ ft}$

$AB = DC = 6 \text{ ft}$

$BD = 8.72 \text{ ft.}$

Let $\bar{w}$ = weight per foot of length of each bar.

$W = 2(6)\bar{w} + 4\bar{w} + 8.72\bar{w} = 24.72\,\bar{w}$

$\bar{x} = \frac{\Sigma w_i x_i}{W}$ (Eqn. 8.1)

$$\bar{x} = \frac{2(6)(2.598)\bar{w} + 8.72(2.598)\bar{w} + 4(5.196)\bar{w}}{24.72\,\bar{w}}$$

$\underline{\underline{\bar{x} = 3.02 \text{ ft.}}}$

$\bar{y} = \frac{\Sigma w_i y_i}{W}$ (Eqn. 8.1)

$$\bar{y} = \frac{6(3/2)\bar{w} + 4(5)\bar{w} + 8.72\left(\frac{7}{2}\right)\bar{w} + 6(7 + 3/2)\bar{w}}{24.72\,\bar{w}}$$

$\underline{\underline{\bar{y} = 4.47 \text{ ft.}}}$

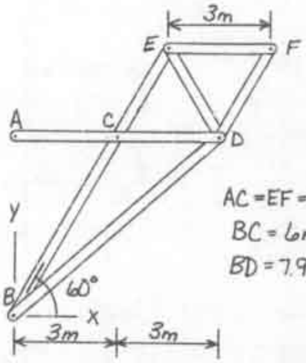
8.27

Find the center of gravity of the truss shown.

Let $\bar{w}$ = weight per meter of length of each bar.

(Continued)

8.27 cont.



AC = EF = ED = CE = CD = DF = 3m
BC = 6m
BD = 7.937m

$W = (6)(3)\bar{w} + 6\bar{w} + 7.937\bar{w} = 31.94\bar{w}$

$\bar{x} = \frac{\Sigma w_i x_i}{W}$ (Eqn. 8.1)

$$\bar{x} = \frac{[6(1.5)\bar{w} + 7.937(3)\bar{w} + 3(1.5)\bar{w} + 3(4.5\bar{w}) + 3(3.75)\bar{w} + 3(5.25)\bar{w} + 3(6)\bar{w} + 3(6.75)\bar{w}]}{31.94\bar{w}}$$

$\underline{\bar{x} = 3.63\ m}$

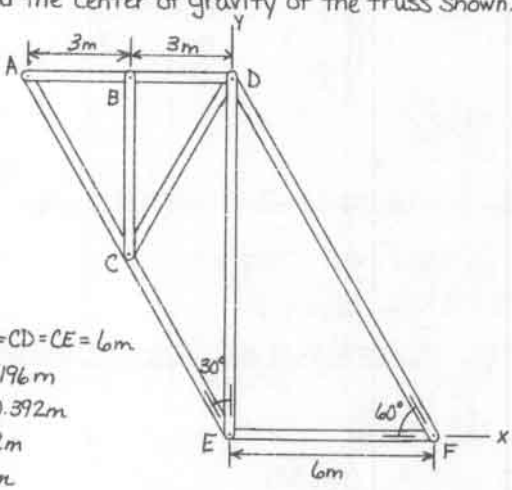
$\bar{y} = \frac{\Sigma w_i y_i}{W}$ (Eqn. 8.1)

$$\bar{y} = \frac{[2(3)(5.196)\bar{w} + 6\left(\frac{5.196}{2}\right)\bar{w} + 7.937\left(\frac{5.196}{2}\right)\bar{w} + 3(3)(6.50)\bar{w} + 3(7.79)\bar{w}]}{31.94\bar{w}}$$

$\underline{\bar{y} = 4.68m}$

8.28

Find the center of gravity of the truss shown.



AD = AC = CD = CE = 6m
BC = 5.196 m
DE = 10.392m
DF = 12m
EF = 6m

Let $\bar{w}$ = weight per meter of length of each bar.

$W = (4)(6)\bar{w} + 5.196\bar{w} + 10.392\bar{w} + 12\bar{w} + 6\bar{w} = 57.59\bar{w}$

$\bar{x} = \frac{\Sigma w_i x_i}{W}$ (Eqn. 8.1)

$$\bar{x} = \frac{[6(3)\bar{w} + 12(3)\bar{w} - 2(6)(1.5)\bar{w} - 6(3)\bar{w} - 5.196(3)\bar{w} - 6(4.5)\bar{w}]}{57.59\bar{w}}$$

$\underline{\bar{x} = -0.427m}$

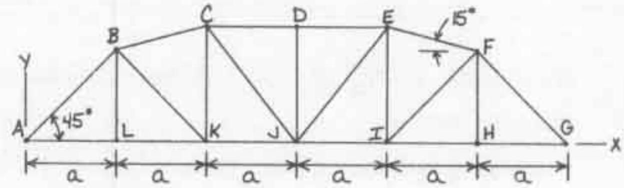
$\bar{y} = \frac{\Sigma w_i y_i}{W}$ (Eqn. 8.1)

$$\bar{y} = \frac{[10.392\left(\frac{10.392}{2}\right)\bar{w} + 12\left(\frac{10.392}{2}\right)2\bar{w} + 6(7.79)\bar{w} + 5.196(7.79)\bar{w} + 6(10.39)\bar{w}]}{57.59\bar{w}}$$

$\underline{\bar{y} = 5.70\ m}$

8.29

Find the center of gravity of the truss shown.



$AB = FG = BK = FI = \sqrt{2}a$

$BL = FH = a$

$BC = EF = \frac{a}{\cos 15°} = 1.035a$

$CK = EI = DJ = a + (a)\tan 15° = 1.268a$

$CD = DE = a$

$CJ = EJ = \sqrt{a^2 + (a + (a)\tan 15°)^2} = 1.615a$

From the symmetry in the truss

$\underline{\bar{x} = 3a}$

Let $\bar{w}$ = weight per unit length of each bar.

$W = [4(\sqrt{2}a)\bar{w} + 4a\bar{w} + 2(1.035a)\bar{w} + 3(1.268a)\bar{w} + 2(1.615a\bar{w}) + 6a\bar{w}] = 24.76a\bar{w}$

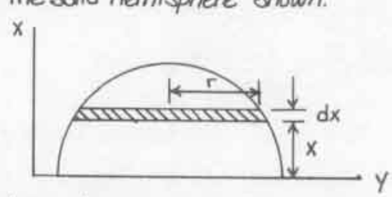
$\bar{y} = \frac{\Sigma w_i y_i}{W}$ (Eqn. 8.1)

$$\bar{y} = \frac{[4(\sqrt{2}a)\left(\frac{a}{2}\right) + 2a\left(\frac{a}{2}\right) + 2(1.035a)\left(a + \frac{a\tan 15°}{2}\right) + 3(1.268a)\left(\frac{1.268a}{2}\right) + 2a(1.268a) + 2(1.615a)\left(\frac{1.268a}{2}\right)]\bar{w}}{24.76a\bar{w}}$$

$\underline{\bar{y} = 0.532a}$

8.30

Find $\bar{x}$ for the solid hemisphere shown.



Take the volume element as a circular disk perpendicular to the x-axis with a thickness dx.

In cartesian coordinates:

$dV = \pi r^2 dx$

$r^2 = a^2 - x^2$

$V = \int_0^a \pi(a^2 - x^2)dx$

$V = \left(\pi a^2 x - \frac{\pi x^3}{3}\right)\Big|_0^a = \pi a^3 - \frac{\pi a^3}{3} = \frac{2\pi a^3}{3}$ (a)

Now find Q_{yz}.

$Q_{yz} = \int x\,dV$ (Eqn. 8.10)

$Q_{yz} = \pi\int_0^a (a^2x - x^3)\,dx$ (continued)

$Q_{yz} = \pi\left(\frac{a^2x^2}{2} - \frac{x^4}{4}\right)\Big|_0^a = \frac{\pi a^4}{2} - \frac{\pi a^4}{4} = \frac{\pi a^4}{4}$ (b)

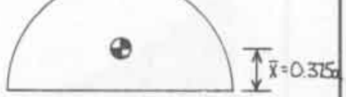
8.30 cont.

$\bar{x}$ can now be found using (a) and (b).

$\bar{x} = \dfrac{Q_{yz}}{V}$ (Eqn. 8.9)

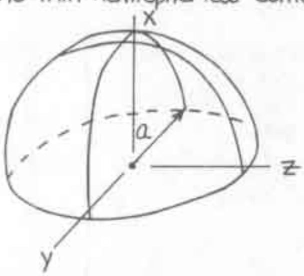
$\bar{x} = \dfrac{\frac{\pi a^4}{4}}{\frac{2\pi a^3}{3}} = \dfrac{3a}{8}$

$\underline{\underline{\bar{x} = 0.375a}}$



8.31

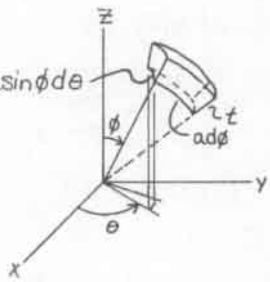
Find $\bar{x}$ for the thin hemispherical dome shown.



Choose a volume element with the dimensions:

t, $ad\phi$, $a\sin\phi\, d\theta$ in spherical coordinates

where ϕ is measured from $+x$-axis and θ from $+y$-axis. (See Fig. 8.12)



Find the Volume.

$dV = ta^2 \sin\phi\, d\phi\, d\theta$

$V = \int_0^{2\pi}\int_0^{\pi/2} ta^2 \sin\phi\, d\phi\, d\theta = ta^2(2\pi)\left[-\cos\phi\Big|_0^{\pi/2}\right]$

$V = 2\pi t a^2(0+1) = 2\pi t a^2$

Find the first area moment.

$Q_{yz} = \int x\, dV$ where $x = a\cos\phi$

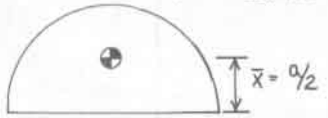
$Q_{yz} = \int_0^{2\pi}\int_0^{\pi/2} ta^2(a\cos\phi)\sin\phi\, d\phi\, d\theta$

$Q_{yz} = 2\pi t a^3 \int_0^{\pi/2} \sin\phi \cos\phi\, d\phi$

$Q_{yz} = 2\pi t a^3 \int_0^{\pi/2} \tfrac{1}{2}\sin 2\phi\, d\phi$

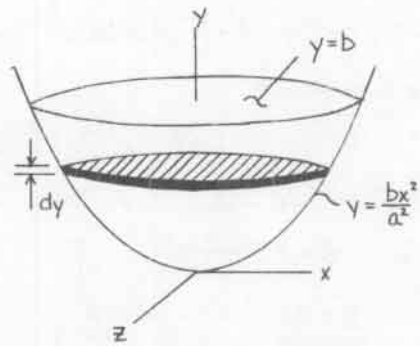
$Q_{yz} = 2\pi t a^3 (\tfrac{1}{2})\left[\dfrac{-\cos 2\phi}{2}\right]\Big|_0^{\pi/2} = \pi t a^3(\tfrac{1}{2}+\tfrac{1}{2}) = \pi t a^3$

Therefore, $\bar{x} = \dfrac{Q_{yz}}{V} = \dfrac{\pi t a^3}{2\pi t a^2} = a/2$



8.32

Find $\bar{y}$ for the paraboloidal solid shown.



Take the volume element as a circular disk perpendicular to the x-axis with a thickness dy.

$dV = \pi x^2 dy$ where $x^2 = \left(y\frac{a^2}{b}\right)$

$V = \dfrac{\pi a^2}{b}\int_0^b y\, dy$

$V = \dfrac{\pi a^2}{b}\cdot \dfrac{y^2}{2}\Big|_0^b = \dfrac{\pi a^2 b}{2}$ (a)

Now find Q_{xz}

$Q_{xz} = \int y\, dy$ (Eqn. 8.10)

$Q_{xz} = \dfrac{\pi a^2}{b}\int_0^b y^2 dy$

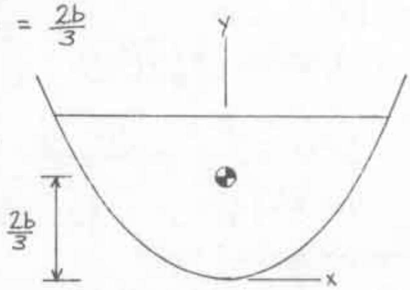
$Q_{xz} = \dfrac{\pi a^2}{b}\cdot\dfrac{y^3}{3}\Big|_0^b = \dfrac{\pi a^2 b^2}{3}$ (b)

Now find Q_{xz} using (a) and (b).

$\bar{y} = \dfrac{Q_{xz}}{V}$ (Eqn. 8.9)

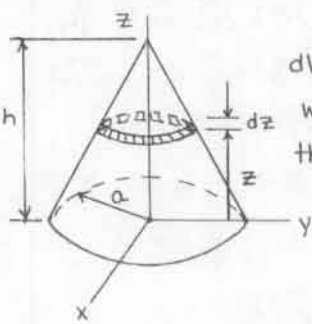
$\bar{y} = \dfrac{\frac{\pi a^2 b^2}{3}}{\frac{\pi a^2 b}{2}} = \dfrac{2b}{3}$

$\underline{\underline{\bar{y} = \dfrac{2b}{3}}}$



8.33

Find $\bar{z}$ for the thin-walled hollow circular cone shown.



$dV = 2\pi x\, dz\, t$

where $z = h - \frac{h}{a}x$

then $x = a\left(1 - \frac{z}{h}\right)$

(continued)

8.33 cont. Find the volume.

$$V=\int_0^h 2\pi x t\,dz = 2\pi t a\int_0^h (1-z/h)\,dz$$

$$V = 2\pi t a\left[z-\frac{z^2}{2h}\right]_0^h = \pi t a h$$

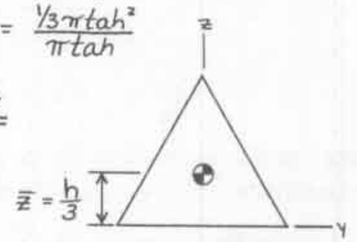
Find the area moment.

$$Q_{xy}=\int_0^h (2\pi x t) z\,dz = 2\pi t a\int_0^h z(1-z/h)\,dz$$

$$Q_{xy} = 2\pi t a\left[\frac{z^2}{2}-\frac{z^3}{3h}\right]_0^h = \tfrac{1}{3}\pi t a h^2$$

so $\bar{z} = \dfrac{Q_{xy}}{V} = \dfrac{\tfrac{1}{3}\pi t a h^2}{\pi t a h}$

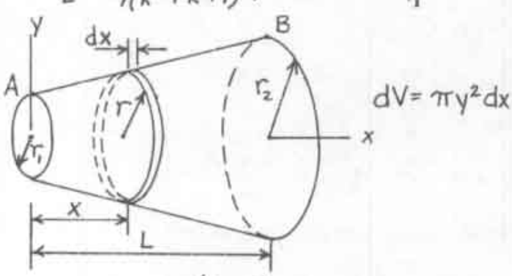
$$\underline{\underline{\bar{z} = \frac{h}{3}}}$$



8.34

For the solid tapered shaft, show that

$$\frac{\bar{x}}{L} = \frac{3k^2+2k+1}{4(k^2+k+1)}, \text{ where } k=\frac{r_2}{r_1}$$



$dV = \pi y^2 dx$

Line AB: $y = \left(\frac{r_2-r_1}{L}\right)x + r_1$

Find the volume:

$$V=\int_0^L \pi y^2 dx = \pi\int_0^L \left[\left(\frac{r_2-r_1}{L}\right)x + r_1\right]^2 dx$$

$$V = \pi\left[\left(\frac{r_2-r_1}{L}\right)^2\frac{x^3}{3} + \frac{r_1(r_2-r_1)}{L}x^2 + r_1^2 x\right]_0^L$$

$$V = \frac{\pi L}{3}\left[r_2^2 + r_1 r_2 + r_1^2\right] \quad (a)$$

Find the moment Q_{yz}:

$$Q_{yz} = \int_0^L x(\pi y^2 dx) = \pi\int_0^L\left[\left(\frac{r_2-r_1}{L}\right)x + r_1\right]^2 x\,dx$$

$$Q_{yz} = \pi\left[\left(\frac{r_2-r_1}{L}\right)^2\frac{x^4}{4} + \frac{r_1(r_2-r_1)}{L}\left(\frac{2x^3}{3}\right) + r_1^2\left(\frac{x^2}{2}\right)\right]_0^L$$

$$Q_{yz} = \frac{\pi L^2}{12}\left[3r^2 + 2r_1 r_2 + r_1^2\right] \quad (b)$$

Substitute $k = \frac{r_2}{r_1}$ into (a) and (b):

$$V = \frac{\pi L r_1^2}{3}\left[k^2+k+1\right]$$

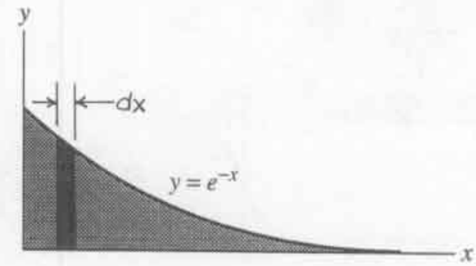
$$Q_{yz} = \frac{\pi L^2 r_1^2}{12}\left[3k^2+2k+1\right]$$

$$\bar{x} = \frac{Q_{yz}}{V} = \frac{L}{4}\left[\frac{3k^2+2k+1}{k^2+k+1}\right]$$

$$\underline{\underline{\frac{\bar{x}}{L} = \frac{3k^2+2k+1}{4[k^2+k+1]}}}$$

8.35

Find the centroid of the area shown.



Consider the small elemental area of width dx and height y.

$$dA = y\,dx$$

Substituting $y = e^{-x}$ into the above, integrate to find the area.

$$A = \int dA = \int y\,dx = \int_0^\infty e^{-x}dx$$

$$A = -e^{-x}\Big|_0^\infty = \left(\lim_{x\to\infty} -e^{-x}\right) - (-e^0)$$

$$A = 1$$

$$Q_y = \int x\,dA = \int_0^\infty x e^{-x} dx$$

Integration by parts

$u = x \quad du = dx$

$dv = e^{-x}dx \quad v = -e^{-x}$

$$Q_y = -xe^{-x} - \int_0^\infty(-e^{-x})dx$$

$$\left(-xe^{-x} - e^{-x}\right)\Big|_0^\infty$$

$$Q_y = 1$$

$$\bar{x} = \frac{Q_y}{A} = \frac{1}{1} \qquad \underline{\underline{\bar{x} = 1}}$$

$$Q_x = \int \frac{y}{2}dA$$

$$Q_x = \int_0^\infty \frac{y^2}{2}dx$$

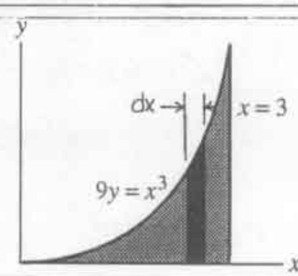
$$Q_x = \int_0^\infty \frac{e^{-2x}}{2}dx = -\frac{e^{-2x}}{4}\Big|_0^\infty = \frac{1}{4}$$

$$\bar{y} = \frac{Q_x}{A} = \frac{1/4}{1} \qquad \underline{\underline{\bar{y} = \frac{1}{4}}}$$

$$\underline{\underline{(\bar{x},\bar{y}) = (1, 1/4)}}$$

8.36

Find the centroid of the area shown.



(continued)

8.36 cont.

Consider the small elemental area with a height y and a width dx

Find the area.

$$dA = y\,dx$$
$$A = \int y\,dx = \int_0^3 \frac{x^3}{9}\,dx$$
$$A = \frac{x^4}{36}\Big|_0^3 = \frac{81}{36}$$
$$A = 2.25$$

Find $\bar{x}$.

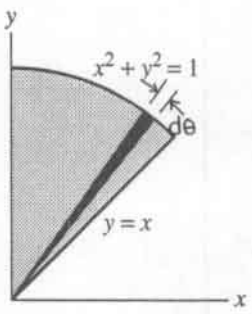
$$Q_y = \int x\,dA$$
$$= \int_0^3 \frac{x^4}{9}\,dA = \frac{x^5}{45}\Big|_0^3 = \frac{243}{45}$$
$$Q_y = 5.4$$
$$\bar{x} = \frac{Q_y}{A} \quad (\text{Eqn. } 8.16)$$
$$\bar{x} = \frac{5.4}{2.25} \qquad \underline{\underline{\bar{x} = 2.4}}$$

Find $\bar{y}$.

$$Q_x = \int \frac{y}{2}\,dA$$
$$= \int \frac{y^2}{2}\,dx = \int_0^3 \frac{x^6}{162}\,dx$$
$$Q_x = \frac{x^7}{1134}\Big|_0^3 = \frac{2187}{1134}$$
$$Q_x = 1.929$$
$$\bar{y} = \frac{1.929}{2.25} \qquad \underline{\underline{\bar{y} = 0.857}}$$
$$\underline{\underline{(\bar{x}, \bar{y}) = (2.4, 0.857)}}$$

8.37

Use polar coordinates to find the centroid of the area shown.



$$dA = \tfrac{1}{2} r(r\,d\theta)$$
$$dA = \tfrac{1}{2} r^2 d\theta = \tfrac{1}{2}\,d\theta$$
where $r=1$

Find A:

$$A = \int_{\pi/4}^{\pi/2} \tfrac{1}{2}\,d\theta = \tfrac{1}{2}\theta\Big|_{\pi/4}^{\pi/2} = \frac{\pi}{8} = 0.3927$$

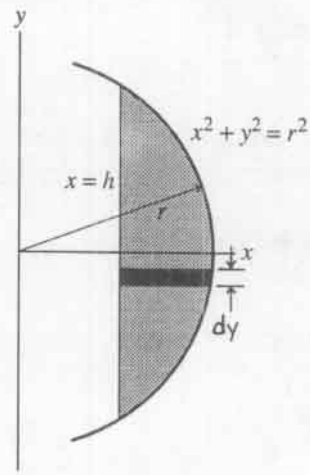
Find $\bar{x}$ and $\bar{y}$:

The centroid of the infinitesimal slice lies at $(\tfrac{2}{3} r\cos\theta, \tfrac{2}{3} r\sin\theta)$ where again $r=1$.

$$Q_y = \int x\,dA = \int_{\pi/4}^{\pi/2} (\tfrac{2}{3}\cos\theta)\tfrac{1}{2}\,d\theta = \tfrac{1}{3}\int_{\pi/4}^{\pi/2} \cos\theta\,d\theta$$
$$Q_y = \tfrac{1}{3}\sin\theta\Big|_{\pi/4}^{\pi/2} = \tfrac{1}{3}\left(1 - \tfrac{1}{\sqrt{2}}\right) = 0.09763$$
$$Q_x = \int y\,dA = \int_{\pi/4}^{\pi/2} (\tfrac{2}{3}\sin\theta)\tfrac{1}{2}\,d\theta = \tfrac{1}{3}\int_{\pi/4}^{\pi/2} \sin\theta\,d\theta$$
$$Q_x = \tfrac{1}{3}(-\cos\theta)\Big|_{\pi/4}^{\pi/2} = \tfrac{1}{3}\left(0 + \tfrac{1}{\sqrt{2}}\right) = 0.2357$$
$$\bar{x} = \frac{Q_y}{A} = \frac{0.09763}{0.3927} = 0.2486$$
$$\bar{y} = \frac{Q_x}{A} = \frac{0.2357}{0.3927} = 0.6002$$
$$\underline{\underline{(\bar{x}, \bar{y}) = (0.249, 0.600)}}$$

8.38

a) Find the centroid of the area shown.



$$dA = w\,dy$$
$$w = x - h = \sqrt{r^2-y^2} - h$$
$$dA = \left(\sqrt{r^2-y^2} - h\right)dy$$
$$A = \int_{-\sqrt{r^2-h^2}}^{+\sqrt{r^2-h^2}} \left(\sqrt{r^2-y^2} - h\right)dy$$

Using Integration Tables

$$A = \left[\tfrac{1}{2}\left(y\sqrt{r^2-y^2} + r^2\sin^{-1}\frac{y}{|r|}\right) - hy\right]\Bigg|_{-\sqrt{r^2-h^2}}^{\sqrt{r^2-h^2}}$$
$$A = -h\sqrt{r^2-h^2} + r^2\sin^{-1}\frac{\sqrt{r^2-h^2}}{|r|}$$

Now find Q_y:

$$Q_y = \int \frac{\left(h+\sqrt{r^2-y^2}\right)}{2}\,dA$$
$$Q_y = \int \tfrac{1}{2}\left(h+\sqrt{r^2-y^2}\right)\left(\sqrt{r^2-y^2} - h\right)dy$$
$$Q_y = \tfrac{1}{2}\int_{-\sqrt{r^2-h^2}}^{\sqrt{r^2-h^2}} (-h^2 + r^2 - y^2)\,dy$$
$$Q_y = \tfrac{1}{2}\left[-h^2 y + r^2 y - \frac{y^3}{3}\right]\Bigg|_{-\sqrt{r^2-h^2}}^{\sqrt{r^2-h^2}}$$
$$Q_y = -h^2\sqrt{r^2-h^2} + r^2\sqrt{r^2-h^2} - \frac{(r^2-h^2)^{3/2}}{3}$$
$$\underline{\underline{\bar{x} = \frac{Q_y}{A} = \frac{-h^2\sqrt{r^2-h^2} + r^2\sqrt{r^2-h^2} - \frac{(r^2-h^2)^{3/2}}{3}}{r^2\sin^{-1}\frac{\sqrt{r^2-h^2}}{r} - h\sqrt{r^2-h^2}}}}$$

By symmetry $\underline{\underline{\bar{y} = 0}}$

(continued)

b.) Show that part (a) is equivalent to:

$$\bar{x} = \frac{2(r^2-h^2)^{3/2}}{3[r^2 \arccos(h/r) - h\sqrt{r^2-h^2}]}$$

$$\bar{y} = 0$$

First look at the denominator of $\bar{x}$ in part (a).

$$\left[r^2 \sin^{-1}\left(\frac{\sqrt{r^2-h^2}}{r}\right) - h\sqrt{r^2-h^2} \right]$$

Observe that $\sin^{-1}\left(\frac{\sqrt{r^2-h^2}}{r}\right)$ gives the same angle as $\cos^{-1}\left(\frac{h}{r}\right)$.

From that observation, the two denominators are equal.

Now look at the numerator of $\bar{x}$ in part (a).

$$\left[-h^2\sqrt{r^2-h^2} + r^2\sqrt{r^2-h^2} - \frac{(r^2-h^2)^{3/2}}{3} \right]$$

$$= (r^2-h^2)\sqrt{r^2-h^2} - \frac{(r^2-h^2)^{3/2}}{3}$$

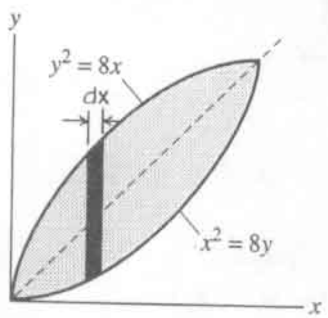
$$= (r^2-h^2)^{3/2} - \frac{(r^2-h^2)^{3/2}}{3}$$

$$= \frac{2(r^2-h^2)^{3/2}}{3}$$

The two answers are equivalent.

8.39

Find the centroid of the area shown.



In order to find the limits of integration, set the two equations equal to each other.

$$\sqrt{8x} = \frac{x^2}{8} \qquad \frac{y^2}{8} = \sqrt{8y}$$

$$8x = \frac{x^4}{64} \qquad \frac{y^4}{64} = 8y$$

$$x^3 = 512 \qquad y^3 = 512$$

$$x = 8 \text{ or } x = 0 \qquad y = 8 \text{ or } y = 0$$

Note: The centroid will lie on the line of symmetry. This line is given by $y = x$, thus $\bar{x} = \bar{y}$

$$dA = \left(\sqrt{8x} - \frac{x^2}{8}\right)dx$$

$$A = \int_0^8 \left(\sqrt{8x} - \frac{x^2}{8}\right)dx = \frac{2}{3}(\sqrt{8})\,x^{3/2} - \frac{x^3}{24}\Big|_0^8$$

$$A = \frac{64}{3} = 21.33$$

Now find Q_y.

$$Q_y = \int x\,dA$$

$$Q_y = \int_0^8 \left(x\sqrt{8x} - \frac{x^3}{8}\right)dx = \int_0^8 \left(\sqrt{8}\,x^{3/2} - \frac{x^3}{8}\right)dx$$

$$Q_y = \frac{2}{5}(\sqrt{8})\,x^{5/2} - \frac{x^4}{32}\Big|_0^8$$

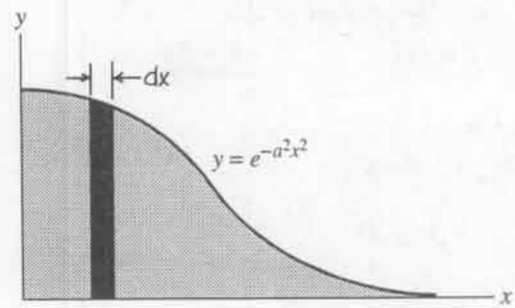
$$Q_y = 76.8$$

$$\bar{x} = \frac{Q_y}{A} = \frac{76.8}{21.33} = 3.6$$

$$\therefore \underline{(\bar{x}, \bar{y}) = (3.6, 3.6)}$$

8.40

Find the centroid of the area shown.



$$dA = e^{-a^2x^2}dx$$

$$A = \int_{x=0}^{x=\infty} e^{-a^2x^2}dx$$

Let $t = ax$ and $dt = a\,dx$

$$A = \frac{1}{a}\int_{t=0}^{t=\infty} e^{-t^2}dt = \frac{1}{a}\left(\frac{\sqrt{\pi}}{2}\right) = \underline{\frac{\sqrt{\pi}}{2a}}$$

Now find Q_x and Q_y.

$$Q_y = \int_0^\infty x\,dA$$

$$Q_y = \int_0^\infty x e^{-a^2x^2}dx$$

Let $t = a^2x^2$ and $dt = 2a^2x\,dx$

$$Q_y = \frac{1}{2a^2}\int_0^\infty e^{-t}dt = \frac{-e^{-t}}{2a^2}\Big|_0^\infty = \underline{\frac{1}{2a^2}}$$

$$Q_x = \int \frac{y}{2}dA$$

$$Q_x = \frac{1}{2}\int_{x=0}^{x=\infty}\left(e^{-a^2x^2}\right)\left(e^{-a^2x^2}\right)dx = \frac{1}{2}\int_{x=0}^{x=\infty} e^{-2a^2x^2}dx$$

Let $t = \sqrt{2}\,ax$ and $dt = \sqrt{2}\,a\,dx$

$$Q_x = \frac{1}{2\sqrt{2}a}\int_{t=0}^{t=\infty} e^{-t^2}dt$$

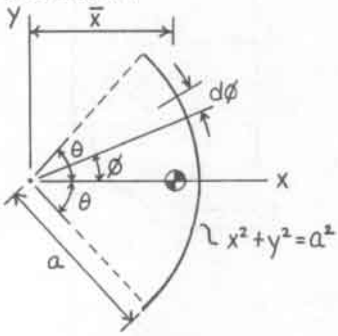
$$Q_x = \frac{\sqrt{\pi}}{2}\left(\frac{1}{2\sqrt{2}a}\right) = \frac{\sqrt{\pi}}{4\sqrt{2}a}$$

$$\bar{x} = \frac{Q_y}{A} = \frac{\frac{1}{2a^2}}{\frac{\sqrt{\pi}}{2a}} = \frac{1}{a\sqrt{\pi}} \qquad \underline{\underline{(\bar{x}, \bar{y}) = \left(\frac{1}{a\sqrt{\pi}}, \frac{1}{\sqrt{2}}\right)}}$$

$$\bar{y} = \frac{Q_x}{A} = \frac{\sqrt{\pi}/4\sqrt{2}a}{\sqrt{\pi}/2a} = \frac{1}{2\sqrt{2}}$$

8.41

a) Find the centroid coordinate $\bar{x}$ of the circular arc shown.



$L=\int ds \qquad ds = a\,d\phi$

$L=\int_{-\theta}^{\theta} a\,d\phi = a\phi\Big|_{-\theta}^{\theta} = 2a\theta$

$Q_y=\int x\,ds \qquad x = a\cos\phi$

$Q_y=\int_{-\theta}^{\theta} a^2\cos\phi\, d\phi = a^2\sin\phi\Big|_{-\theta}^{\theta}$

$Q_y = a^2(\sin\theta - \sin(-\theta)) = 2a^2\sin\theta$

$\bar{x} = \frac{Q_y}{L} = \frac{2a^2\sin\theta}{2a\theta} = \frac{a\sin\theta}{\theta}$

$$\underline{\underline{\bar{x} = \frac{a\sin\theta}{\theta}}}$$

b) Specialize the formula of part (a) for a quarter circle.

with $\theta = \frac{\pi}{4}$: $\frac{a\sin(\frac{\pi}{4})}{\frac{\pi}{4}} = \underline{\underline{\frac{2\sqrt{2}\,a}{\pi}}}$

c) Specialize the formula of part (a) for a semicircle.

with $\theta=\frac{\pi}{2}$: $\frac{a\sin(\frac{\pi}{2})}{\frac{\pi}{2}} = \underline{\underline{\frac{2a}{\pi}}}$

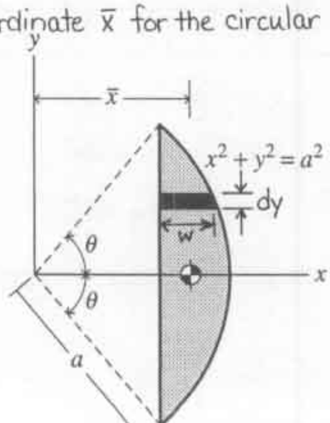
d) Specialize the formula of part (a) for a circle.

with $\theta=\pi$: $\frac{a\sin\pi}{\pi} = \underline{\underline{0}}$

Symmetry also leads to $\bar{x}=0$ for a circle

8.42

Find the centroid coordinate $\bar{x}$ for the circular segment shown.



$A=\frac{a^2}{2}(2\theta - \sin 2\theta)$

$dA = w\,dy$

$dA = (x - a\cos\theta)dy$

$= (\sqrt{a^2-y^2} - a\cos\theta)dy$

Find $Q_y=\int x\,dA$

$Q_y = \frac{1}{2}\int (a\cos\theta + \sqrt{a^2-y^2})(\sqrt{a^2-y^2} - a\cos\theta)\,dy$

$Q_y = \frac{1}{2}\int_{-a\sin\theta}^{a\sin\theta}(a^2-y^2 - a^2\cos^2\theta)dy$

$Q_y = \frac{1}{2}\left[a^2y - \frac{y^3}{3} - a^2\cos^2\theta\, y\right]\Big|_{-a\sin\theta}^{a\sin\theta}$

$Q_y = \frac{2a^3\sin^3\theta}{3}$

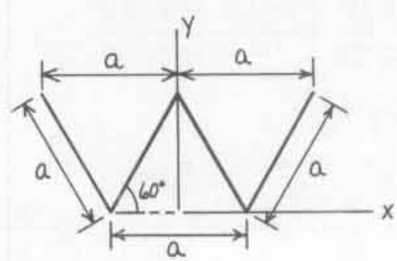
$\bar{x} = \frac{Q_y}{A}$

$\bar{x} = \frac{\frac{2a^3\sin^3\theta}{3}}{\frac{a^2}{2}(2\theta - \sin 2\theta)}$

$$\underline{\underline{\bar{x} = \frac{4a\sin^3\theta}{3(2\theta - \sin 2\theta)}}}$$

8.43

Find the centroid of the composite W-section shown.



By symmetry $\underline{\bar{x}=0}$

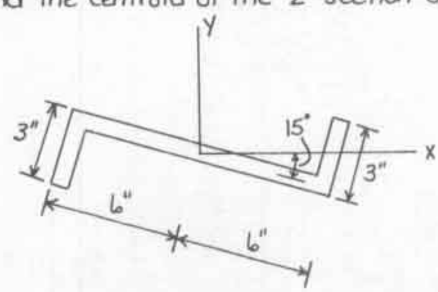
$\bar{y} = \frac{\Sigma L_i y_i}{L}$

$\bar{y} = \frac{4a(\frac{a}{2}\sin 60°)}{4a}$

$$\underline{\underline{\bar{y} = 0.433a}}$$

8.44

Find the centroid of the Z-section shown.

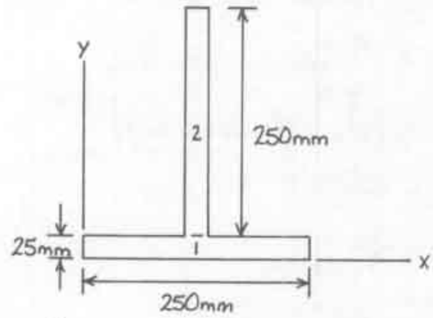


Due to the rotational symmetry about the z-axis the centroid of the section lies at the origin of the xy plane.

$$\underline{\underline{(\bar{x},\bar{y}) = (0,0)}}$$

8.45

Find the centroid of the inverted T-section shown.



By inspection, $\bar{x} = 125\ mm$

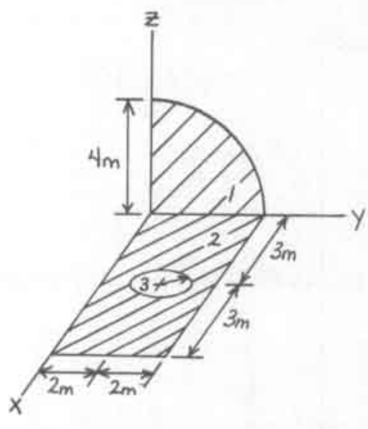
Centroid Table for T-Beam			
Part	Area	Centroid Distance	Moment Area
i	A_i [mm²]	y_i [mm]	$A_i y_i$ [mm³]
1	6 250	12.5	78 125
2	6 250	150	937 500
Σ	12 500	—	1 015 625

$$\bar{y} = \frac{\Sigma A_i y_i}{A}$$

$$\bar{y} = \frac{1\,015\,625}{12\,500} = 81.25\ mm$$

8.46

Find $\bar{y}$ for the composite area shown.

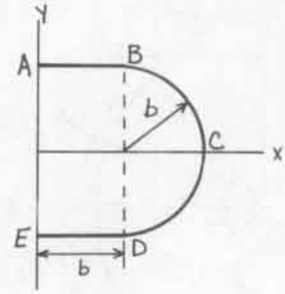


Centroid Table for Composite			
Part	Area	Centroid Distance	Moment of Area
i	A_i [m²]	y_i [m]	$A_i y_i$ [m³]
1	12.57	1.698	21.33
2	24.00	2.000	48.00
3	-3.14	2.000	- 6.28
Σ	33.425	—	63.05

$$\bar{y} = \frac{\Sigma A_i y_i}{A} = \frac{63.05}{33.425} \qquad \bar{y} = 1.886\ m$$

8.47

Find the centroid of the composite line shown.



By symmetry $\qquad \bar{y} = 0$

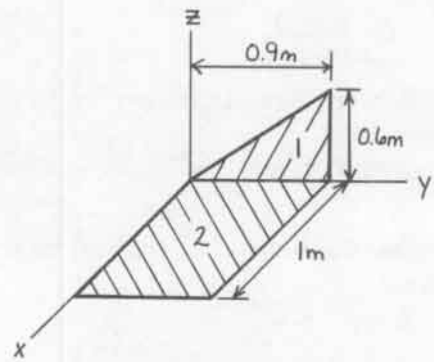
$$\bar{x} = \frac{\Sigma L_i x_i}{L}$$

$$\bar{x} = \frac{(2b)(\tfrac{1}{2}b) + \left(\frac{2b}{\pi} + b\right)(b\pi)}{2b + b\pi} = \frac{b^2(3+\pi)}{b(2+\pi)}$$

$$\bar{x} = 1.194\,b$$

8.48

Find the centroid of the composite area shown.



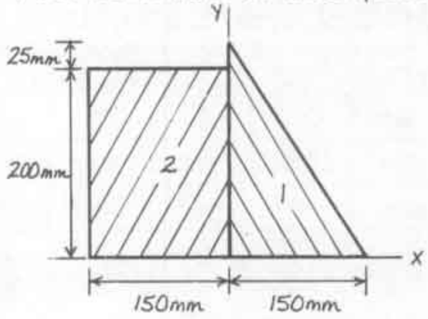
Centroid Table for Composite							
Part	Area	Centroid Distance			Moment of Area		
i	A_i (m²)	x_i (m)	y_i (m)	z_i (m)	$A_i x_i$ (m³)	$A_i y_i$ (m³)	$A_i z_i$ (m³)
1	0.27	0	0.60	0.20	0	0.162	0.054
2	0.90	0.50	0.45	0	0.450	0.405	0
Σ	1.17	—	—	—	0.450	0.567	0.054

$$\bar{x} = \frac{\Sigma A_i x_i}{A} = \frac{0.450}{1.17} \qquad \bar{x} = 0.385\ m$$

$$\bar{y} = \frac{\Sigma A_i y_i}{A} = \frac{0.567}{1.17} \qquad \bar{y} = 0.485\ m$$

$$\bar{z} = \frac{\Sigma A_i z_i}{A} = \frac{0.054}{1.17} \qquad \bar{z} = 0.0462\ m$$

8.49 Find the centroid of the composite area shown.



Centroid Table for Composite					
Part	Area	Centroid Distance		Moment of Area	
i	A_i (mm²)	x_i (mm)	y_i (mm)	A_ix_i (mm³)	A_iy_i (mm³)
1	16 875	50	75	843 750	1 265 625
2	30 000	-75	100	-2 250 000	3 000 000
Σ	46 875	—	—	-1 406 250	4 265 625

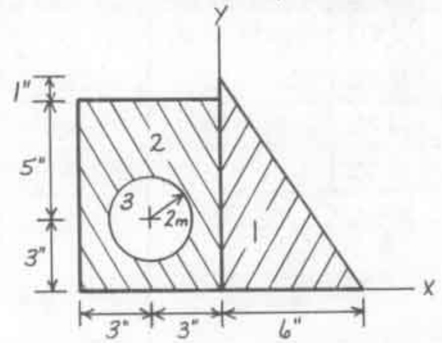
$\bar{x} = \frac{\Sigma A_i x_i}{A}$ $\quad \bar{x} = \frac{-1\,406\,250}{46\,875}$

$\underline{\bar{x} = -30 \text{ mm}}$

$\bar{y} = \frac{\Sigma A_i y_i}{A} = \frac{4\,265\,625}{46\,875}$

$\underline{\bar{y} = 91 \text{ mm}}$

8.50
Find the centroid of the composite area shown.

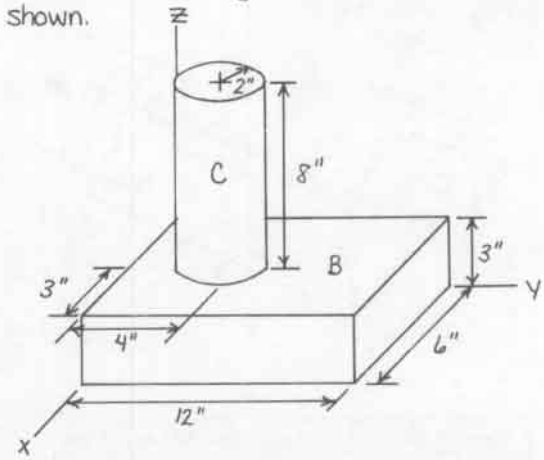


Centroid Table for Composite					
Part	Area	Centroid Distance		Moment of Area	
i	A_i (in²)	x_i (in)	y_i (in)	A_ix_i (in³)	A_iy_i (in³)
1	27	2	3	54	81
2	48	-3	4	-144	192
3	-12.57	-3	3	37.70	-37.70
Σ	62.43	—	—	-52.30	235.30

$\bar{x} = \frac{\Sigma A_i x_i}{A} = \frac{-52.30}{62.43}$ $\quad \underline{\bar{x} = -0.838 \text{ in.}}$

$\bar{y} = \frac{\Sigma A_i y_i}{A} = \frac{235.30}{62.43}$ $\quad \underline{\bar{y} = 3.77 \text{ in.}}$

8.51
Find the center of gravity of the composite body shown.



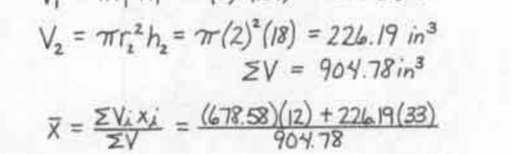
Center of Gravity Table for Composite								
Part	Volume	Weight	Centroid Distances			Moments of Weights		
i	V_i (in³)	$w_i = \gamma V_i$	x_i (in)	y_i (in)	z_i (in)	w_ix_i	w_iy_i	w_iz_i
B	216	21.6	3	6	1.5	64.8	129.6	32.4
C	100.53	30.159	3	4	7	90.478	120.6	211.12
Σ	—	51.759	—	—	—	155.28	250.2	243.5

$\bar{x} = \frac{155.28}{51.759}$ $\quad \underline{\bar{x} = 3.00 \text{ in.}}$

$\bar{y} = \frac{250.2}{51.759}$ $\quad \underline{\bar{y} = 4.83 \text{ in.}}$

$\bar{z} = \frac{243.5}{51.759}$ $\quad \underline{\bar{z} = 4.70 \text{ in.}}$

8.52



a.) Find the center of gravity of the stepped shaft shown.

$V_1 = \pi r_1^2 h_1 = \pi(3)^2(24) = 678.58 \text{ in}^3$

$V_2 = \pi r_2^2 h_2 = \pi(2)^2(18) = 226.19 \text{ in}^3$

$\Sigma V = 904.78 \text{ in}^3$

$\bar{x} = \frac{\Sigma V_i x_i}{\Sigma V} = \frac{(678.58)(12) + 226.19(33)}{904.78}$

$\underline{\bar{x} = 17.25 \text{ in.}}$

b) Repeat part (a) for a 2-in. diameter axial hole in the shaft.

$\Sigma V = 904.78 - \pi(1^2)(42) = 772.83 \text{ in}^3$

$\bar{x} = \frac{\Sigma V_i x_i}{\Sigma V} = \frac{(678.58)(12) + (226.19)(33) - (42\pi)(21)}{772.83}$

$\bar{x} = 16.61 \text{ in}$

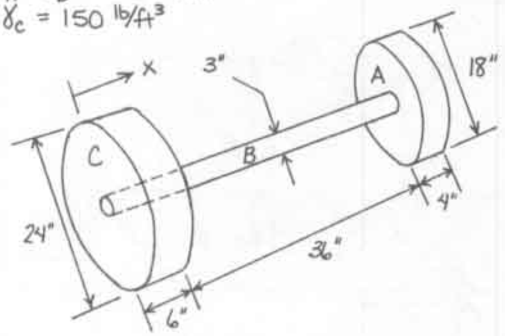
Now determine displacement $17.25 - 16.61 = \underline{d = 0.640 \text{ in.}} \leftarrow$

8.53

a) Find the center of gravity of the system shown.

$\gamma_A = \gamma_B = 490\ ^{lb}/_{ft^3}$

$\gamma_c = 150\ ^{lb}/_{ft^3}$



Center of Gravity Table for Above System				
Part	Volume	Weight	Centroid Distance	Moment of Weight
i	V_i (ft^3)	$w_i = \gamma V_i$ (lb)	x_i (ft)	$w_i x_i$ (lb-ft)
A	0.5891	288.7	3.667	1058.7
B	0.1718	84.18	1.75	147.3
C	1.5463	231.94	0.25	58.0
Σ	——	604.8	——	1264.0

$$\bar{x} = \frac{\Sigma w_i x_i}{\Sigma W} = \frac{1264.0}{604.8}$$

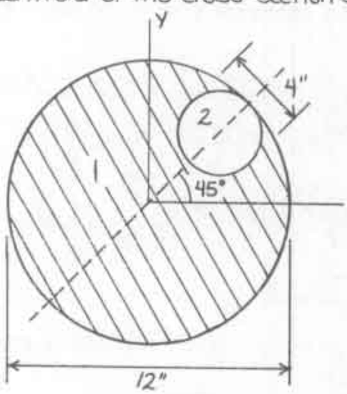
$$\underline{\bar{x} = 2.090 ft. = 25.08 in.}$$

b) Does the center of gravity coincide with the centroid of the volume? Explain.

No, the center of gravity is not located at the centroid of the volume because the system is not homogeneous.

8.54

Find the centroid of the cross-section shown.



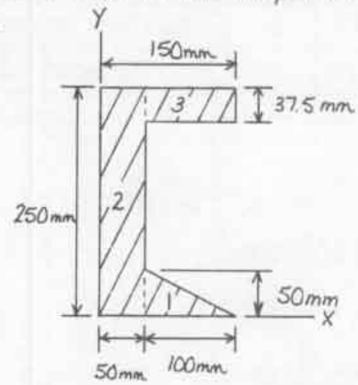
Centroid Table for Cross-Section					
Part	Area	Centroid Distance		Moments of Area	
i	A_i (in^2)	x_i (in)	y_i (in)	$A_i x_i$ (in^3)	$A_i y_i$ (in^3)
1	36π	0	0	0	0
2	-4π	2.828	2.828	-35.54	-35.54
Σ	32π	——	——	-35.54	-35.54

$$\bar{x} = \frac{\Sigma A_i x_i}{A} = \frac{-35.54}{32\pi} \qquad \bar{y} = \frac{\Sigma A_i y_i}{A} = \frac{-35.54}{32\pi}$$

$$\underline{\bar{x} = -0.354 in.} \quad \underline{\bar{y} = -0.354 in.}$$

8.55

Find the centroid for the composite cross-section shown.



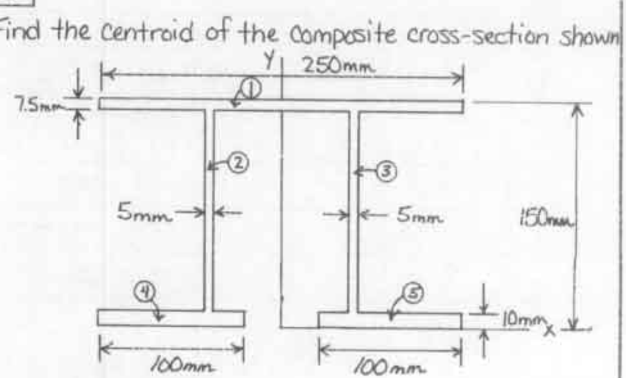
Centroid Table for Cross Section					
Part	Area	Centroid Distance		Moments of Area	
i	A_i (mm^2)	x_i (mm)	y_i (mm)	$x_i A_i$ (mm^3)	$y_i A_i$ (mm^3)
1	2 500	83.33	16.67	208 333	41 667
2	12 500	25	125	312 500	1 562 500
3	3 750	100	231.25	375 000	867 188
Σ	18 750			895 833	2 471 355

$$\bar{x} = \frac{\Sigma x_i A_i}{A} = \frac{895\,833}{18\,750} \qquad \bar{y} = \frac{\Sigma y_i A_i}{A} = \frac{2\,471\,355}{18\,750}$$

$$\underline{\bar{x} = 47.8 mm} \qquad \underline{\bar{y} = 131.8 mm}$$

8.56

Find the centroid of the composite cross-section shown



By symmetry, $\underline{\bar{x} = 0}$

(continued)

8.56 cont.

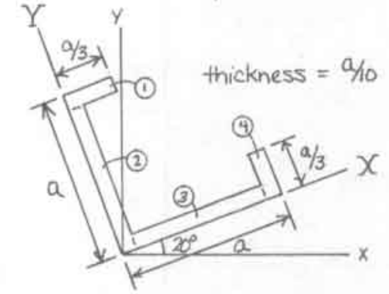
Centroid Table for Cross-Section			
Part	Area	Centroid Distance	Moments of Area
i	A_i (mm^2)	y_i (mm)	$A_i y_i$ (mm^3)
1	1875	146.25	274 219
2	662.5	76.25	50 516
3	662.5	76.25	50 516
4	1000	5	5000
5	1000	5	5000
Σ	5200	——	385 250

$$\bar{y} = \frac{\Sigma y_i A_i}{A} = \frac{385\,250}{5200}$$

$$\underline{\bar{y} = 74.1 \text{ mm}}$$

8.57

Find the centroid $(\bar{x}, \bar{y})$ of the cross-section shown.

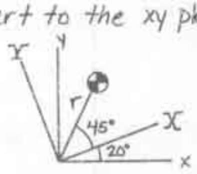


Centroid Table for X,Y Plane					
Part	Area	Centroid Distance		Moments of Area	
i	A_i	X_i	Y_i	$A_i X_i$	$A_i Y_i$
1	$0.0333a^2$	$0.167a$	$0.95a$	$0.00556a^3$	$0.03167a^3$
2	$0.09a^2$	$0.05a$	$0.45a$	$0.0045a^3$	$0.0405a^3$
3	$0.08a^2$	$0.50a$	$0.05a$	$0.04a^3$	$0.004a^3$
4	$0.0333a^2$	$0.95a$	$0.167a$	$0.03167a^3$	$0.00556a^3$
Σ	$0.237a^2$	——	——	$0.08173a^3$	$0.08173a^3$

$$\bar{X} = \frac{\Sigma X_i A_i}{A} = \frac{0.08173a^3}{0.237a^2} = 0.3449a$$

$$\bar{Y} = \frac{\Sigma Y_i A_i}{A} = \frac{0.08173a^3}{0.237a^2} = 0.3449a$$

Now convert to the xy plane



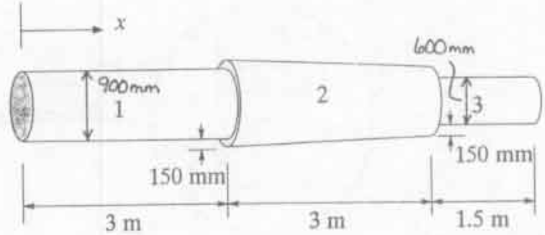
$$r = \sqrt{(.3449a)^2 + (.3449a)^2}$$
$$r = 0.4878a$$

$$\underline{\bar{x} = r\cos 65° = 0.206a}$$

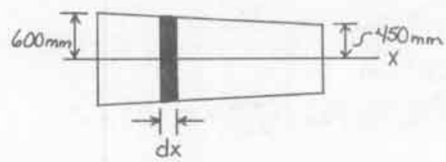
$$\underline{\bar{y} = r\sin 65° = 0.442a}$$

8.58

Find $\bar{x}$ for the center of gravity of the homogeneous shaft shown.



First find $\bar{x}$ of part 2, which is tapered.



$y = mx + b$

At $x = 0$, $y = 0.6$m $\quad 0.6 = m(0) + b$

At $x = 3$, $y = 0.450$m $\quad b = 0.6$m

$0.450 = m(3) + 0.6$

$m = -0.05$

$y = -0.05x + 0.60$

For an infinitesimal circular disk of thickness dx

$$dV = \pi y^2 dx$$

$$V = \int_0^3 \pi(-0.05x + 0.60)^2 dx = \pi \int_0^3 (0.0025x^2 - 0.06x + 0.36)dx$$

$$V = \pi\left[\frac{0.0025x^3}{3} - \frac{0.06x^2}{2} + 0.36x\right]\Bigg|_0^3$$

$$V = 2.615 \text{ m}^3$$

$$\Sigma M_o = \int_0^3 x\,dV = \pi\int_0^3 (0.0025x^3 - 0.06x^2 + 0.36x)dx$$

$$\Sigma M_o = \pi\left[\frac{0.0025x^4}{4} - \frac{0.06x^3}{3} + \frac{0.36x^2}{2}\right]\Bigg|_0^3$$

$$\Sigma M_o = 3.552 \text{ m}^4 \qquad \bar{x} = \frac{\Sigma M_o}{V}$$

$$\bar{x} = \frac{3.552}{2.615} = 1.358 \text{ m}$$

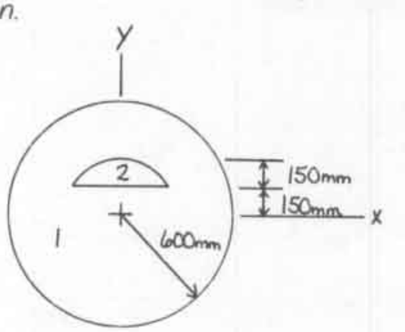
Now find $\bar{x}$ for the composite shaft.

Centroid Table for Shaft			
Part	Volume	Centroid Distance	Moment of Volume
i	V_i (m^3)	x_i (m)	$x_i V_i$ (m^4)
1	1.9085	1.50	2.863
2	2.6154	4.358	11.398
3	0.4241	6.75	2.863
Σ	4.948	——	17.124

$$\bar{x} = \frac{\Sigma x_i V_i}{V} = \frac{17.124}{4.948} \qquad \underline{\bar{x} = 3.46 \text{ m}}$$

8.59

Find the center of gravity of the homogeneous flywheel shown.



By symmetry, $\underline{\bar{x}=0}$

$\bar{y}_2 = \left[\frac{4r\sin^3\theta}{3(2\theta - \sin 2\theta)}\right]$ (From Problem 8.38)

$r = 300\text{mm}$, $\theta = \pi/3$

$\bar{y}_2 = 211.5\text{mm}$

$A_2 = \frac{300^2}{2}\left(2(\pi/3) - \sin\left(\frac{2\pi}{3}\right)\right) = 55\ 277\ \text{mm}^2$

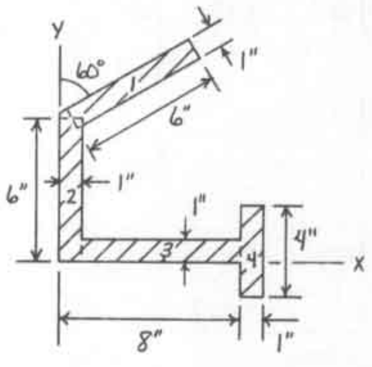
Center of Gravity Table for Wheel			
Part	Area	Centroid Distance	Moment of Area
i	$A_i\ (mm^2)$	$y_i\ (mm)$	$y_iA_i\ (mm^3)$
1	1 130 973	0	0
2	− 55 277	211.5	−11 691 085
Σ	1 075 696	——	−11 691 085

$\bar{y} = \frac{\Sigma y_i A_i}{A} = \frac{-11\ 691\ 085}{1\ 075\ 696}$

$\underline{\bar{y} = -10.87\ \text{mm}}$

8.60

Find the centroid of the cross-section shown.



For Area 1:

$\bar{x} = 0.5 + \frac{6\sin 60°}{2} = 3.098\ in.$

$\bar{y} = 6.0 + \frac{6\cos 60°}{2} = 7.5\ in.$

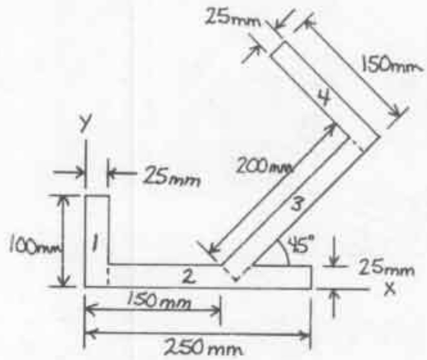
Centroid Table for Cross-Section					
Part	Area	Centroid Distances		Moments of Area	
i	$A_i\ (in^2)$	$x_i\ (in)$	$y_i\ (in)$	$x_iA_i\ (in^3)$	$y_iA_i\ (in^3)$
1	6.0	3.098	7.5	18.588	45.0
2	6.0	0.50	3.0	3.0	18.0
3	7.0	4.50	0.5	31.5	3.5
4	4.0	8.50	0.5	34.0	2.0
Σ	23.0	——	——	87.09	68.5

$\bar{x} = \frac{\Sigma x_i A_i}{A} = \frac{87.09}{23.0}$ $\qquad \underline{\bar{x} = 3.79\ in.}$

$\bar{y} = \frac{\Sigma y_i A_i}{A} = \frac{68.5}{23.0}$ $\qquad \underline{\bar{y} = 2.98\ in.}$

8.61

Find the centroid of the part shown.



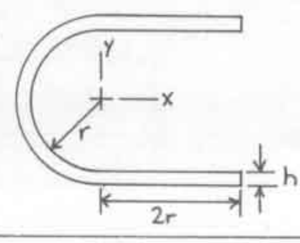
Centroid Table for Part					
Part	Area	Centroid Distances		Moments of Area	
i	$A_i\ (mm^2)$	$x_i\ (mm)$	$y_i\ (mm)$	$x_iA_i\ (mm^3)$	$y_iA_i\ (mm^3)$
1	2500	12.5	50.0	31 250	125 000
2	5625	137.5	12.5	773 437	70 313
3	5000	220.7	95.71	1 103 500	478 550
4	3750	273.7	219.45	1 026 375	822 938
Σ	16 875	——	——	2 934 562	1 496 801

$\bar{x} = \frac{\Sigma x_i A_i}{A} = \frac{2\ 934\ 562}{16\ 875}$ $\qquad \underline{\bar{x} = 173.9\ \text{mm}}$

$\bar{y} = \frac{\Sigma y_i A_i}{A} = \frac{1\ 496\ 801}{16\ 875}$ $\qquad \underline{\bar{y} = 88.7\ \text{mm}}$

8.62

Derive a formula for the centroid of the U-section shown.

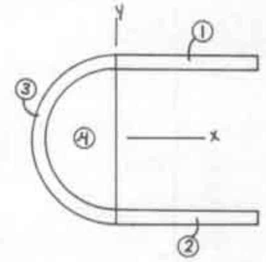


(continued)

8.62 cont.

By symmetry: $\bar{y} = 0$

For $\bar{x}$, break the area into 4 parts.



For the semicircular parts $\bar{x}_i = \frac{4R}{3\pi}$ (Table D.2)

Centroid Table for U-Section			
Part	Area	Centroid Distance	Moment of Area
i	A_i	$\bar{x}_i$	$A_i\bar{x}_i$
1	$2hr$	r	$2hr^2$
2	$2hr$	r	$2hr^2$
3	$\frac{\pi}{2}(r+h)^2$	$\frac{-4(r+h)}{3\pi}$	$-\frac{2}{3}(r+h)^3$
4	$-\frac{\pi r^2}{2}$	$-\frac{4r}{3\pi}$	$\frac{2r^3}{3}$
Σ	$4hr+\frac{\pi}{2}(h^2+2rh)$	——	$4hr^2-\frac{2}{3}(h^3+3h^2r+3r^2h)$

$$\bar{x} = \frac{\Sigma A_i \bar{x}_i}{A} = \frac{[4r^2 - \frac{2}{3}h^2 - 2hr - 2r^2]}{[4r + \frac{\pi h}{2} + \pi r]}$$

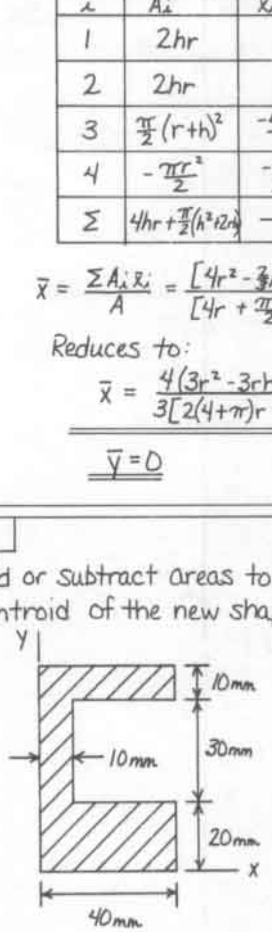
Reduces to:

$$\bar{x} = \frac{4(3r^2 - 3rh - h^2)}{3[2(4+\pi)r + \pi h]}$$

$\underline{\bar{y} = 0}$

8.63

Add or subtract areas to the C-shaped area so the centroid of the new shape is $(\bar{x},\bar{y}) = (20,20)$ mm.



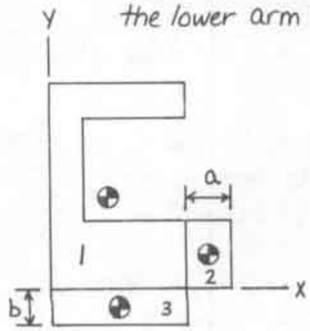
From Example 8.10,

$(\bar{x}_c, \bar{y}_c) = (17,27)$ mm

$A = 1500$ mm^2

∴ the centroid must be shifted right 3mm and down 7mm.

Approach: Add area to the lower arm of the section until $\bar{x} = 20$. Then thicken the lower arm until $\bar{y} = 20$.



Centroid Table for Adjusted C-section					
Part	Area	Centroid Distances		Moments of Area	
i	A_i	x_i (mm)	y_i (mm)	A_ix_i (mm^3)	A_iy_i (mm^3)
1	1500	17	27	25 500	40 500
2	$20a$	$40+\frac{a}{2}$	10	$800a+10a^2$	$200a$
3	$40b$	20	$-\frac{b}{2}$	$800b$	$-20b^2$
Σ	$1500+20a+40b$	——	——	$25500+10a^2+800a+800b$	$40500+200a-20b^2$

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{25\,500 + 800a + 10a^2 + 800b}{1500 + 20a + 40b} = 20$$

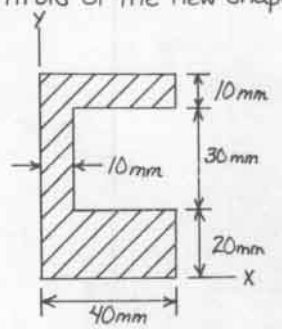
$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{40\,500 + 200a - 20b^2}{1500 + 20a^2 + 40b} = 20$$

Simultaneous solution of these equations gives:

$\underline{a = 9.155\text{ mm}, \quad b = 8.87\text{ mm}}$

8.64

Add or subtract area to the C-shaped area so the centroid of the new shape is $(\bar{x},\bar{y}) = (10,15)$ mm.



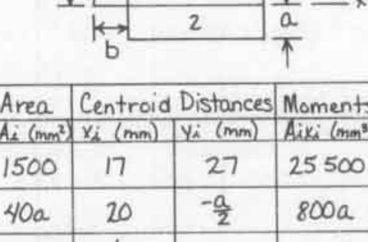
From Example 8.10, for the C-shaped area

$(\bar{x}_c, \bar{y}_c) = (17,27)$ mm

and $A = 1500$ mm^2

∴ the centroid must be shifted left 7mm and down 12mm.

Approach: Add a 40mm wide strip of area to the bottom of the lower arm to shift the centroid down. Then add a strip 30mm high to the left edge to shift the centroid left.



Part	Area	Centroid Distances		Moments of Area	
i	A_i (mm^2)	x_i (mm)	y_i (mm)	A_ix_i (mm^3)	A_iy_i (mm^3)
1	1500	17	27	25 500	40 500
2	$40a$	20	$-\frac{a}{2}$	$800a$	$-10a^2$
3	$30b$	$-\frac{b}{2}$	15	$-15b^2$	$450b$
Σ	$1500+40a+30b$	——	——	$25\,500+800a-15b^2$	$40\,500-10a^2+450b$

8.64 cont.

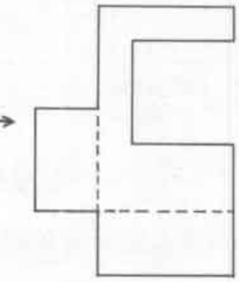
$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{25\,500 + 800a - 15b^2}{1500 + 40a + 30b} = 10$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{40\,500 - 10a^2 + 450b}{1500 + 40a + 30b} = 15$$

Simultaneous solution of these equations gives:

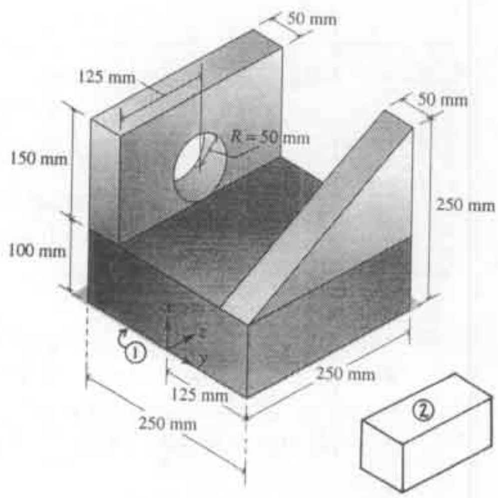
$\underline{a = 18.54\,mm\ ,\ b = 18.28\,mm}$

New Cross-Section →



8.65

Add or subtract a part to the object of Example 8.13 so the new center of gravity is located at $(\bar{x},\bar{y},\bar{z}) = (70, 10, 126.83)$ mm.



From Example 8.13, $(\bar{x},\bar{y},\bar{z}) = (63.66, -2.58, 126.83)$ mm and $W = 550.42\ N$.

$\therefore$ the center of gravity must be moved 6.34 mm in x, 12.58 mm in y, and no change in z.

Approach: Add a rectangular block with $\gamma = 78\ kN/m^3$. Find the req'd location of its centroid for an arbitrarily* chosen volume. Select $V = 0.0006\,m^3$

$\therefore W = 46.8\ N$

Part	Weight	Centroid Distances		Moments of Weight	
i	W_i (N)	x_i (mm)	y_i (mm)	$W_i x_i$ (N-mm)	$W_i y_i$ (N-mm)
1	550.42	63.66	−2.58	35 040	−1420
2	46.8	$\bar{x}_2$	$\bar{y}_2$	$46.8\bar{x}_2$	$46.8\bar{y}_2$
Σ	597.22	——	——	$35040 + 46.8\bar{x}_2$	$-1420 + 46.8\bar{y}_2$

$$\bar{x} = \frac{\Sigma W_i x_i}{\Sigma W_i} = \frac{35\,040 + 46.8\bar{x}_2}{597.22} = 70.0 \qquad \bar{x}_2 = 144.6\ mm$$

$$\bar{y} = \frac{\Sigma W_i y_i}{\Sigma W_i} = \frac{-1420 + 46.8\bar{y}_2}{597.22} = 10.0 \qquad \bar{y}_2 = 158.0\ mm$$

* Some iteration was required to select the proper volume of the additional part

Now choose proportions for the block.
With $(\bar{x}_2, \bar{y}_2) = (144.6, 158.0)$ mm, the block can be placed on the base of the original object.

Give the block dimensions (x, y, z).
Put the block on the base.

$\therefore\ \bar{x}_2 = \dfrac{100 + (100 + x)}{2} = 144.6 \Rightarrow \underline{x = 89.2\ mm}$

Move the side of the block against the triangular wedge.

$$\bar{y}_2 = \frac{200 + (200 - y)}{2} = 158.0 \qquad \underline{y = 84.0\,mm}$$

Find z to get the proper volume.

$$z = \frac{V}{x \cdot y} = \frac{0.0006}{(0.0892)(0.084)} = 0.0801\,m$$

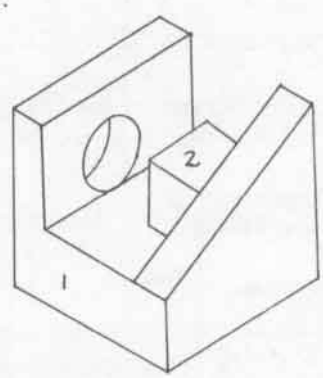
$\underline{z = 80.1\,mm}$

Set the block so that $\bar{z}_2 = 126.83$.

$$z_{min} = 126.83 - \frac{80.1}{2} = 86.8\,mm$$

$$z_{max} = 126.83 + \frac{80.1}{2} = 166.83\,mm$$

Final Object:



8.66

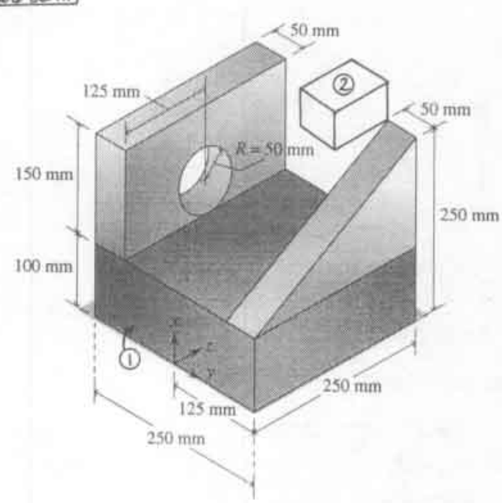
Add or subtract a part to the object in Example 8.13 so the new center of gravity is located at: $(\bar{x},\bar{y},\bar{z}) = (100, 0, 150)$ mm.

From Example 8.13, $(\bar{x},\bar{y},\bar{z}) = (63.66, -2.58, 126.83)$ mm and $W = 550.42\ N$

Approach: Add a rectangular block with $\gamma = 78\ kN/m^3$. Find the location of its centroid for an arbitrarily* chosen volume. Use $V_2 = 0.001\,m^3$

* Some iteration was required to select the proper volume of the additional part. (continued)

8.66 cont.



Part	Weight	Centroid Distances			Moments of Weight		
i	W_i	$\bar{x}_i$	$\bar{y}_i$	$\bar{z}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$	$A_i\bar{z}_i$
1	550.42	63.66	-2.58	126.83	35 040	-1 420	69 810
2	78	$\bar{x}_2$	$\bar{y}_2$	$\bar{z}_2$	$78\bar{x}_2$	$78\bar{y}_2$	$78\bar{z}_2$
Σ	628.42	——	——	——	$35040 + 78\bar{x}_2$	$-1420 + 78\bar{y}_2$	$69810 + 78\bar{z}_2$

$$\bar{X} = \frac{35\,040 + 78\bar{x}_2}{628.42} = 100 \qquad \bar{x}_2 = 356.4 \text{ mm}$$

$$\bar{Y} = \frac{-1420 + 78\bar{y}_2}{628.42} = 0 \qquad \bar{y}_2 = 18.2 \text{ mm}$$

$$\bar{Z} = \frac{69\,810 + 78\bar{z}_2}{628.42} = 150 \qquad \bar{z}_2 = 313.5 \text{ mm}$$

Now choose proportions for the block.
Place the block at the back edge of the 50×150×250 rectangular piece. Give the block dimensions (x, y, z).

Let $\underline{y = 36.4\text{mm}}$ so the block touches the x-z plane. Set z so the block touches the back face of the rectangular piece.

$$\bar{z}_2 = 250 + \frac{z}{2} = 313.5 \Rightarrow \underline{z = 127\text{mm}}$$

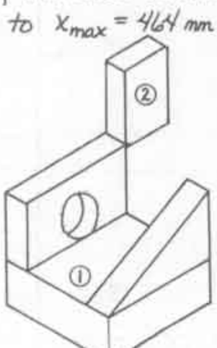
Find x to get the proper volume.

$$x = \frac{V}{y \cdot z} = \frac{0.001}{(0.0364)(0.127)} = 0.216 \text{ m}$$

$$\underline{x = 216 \text{ mm}}$$

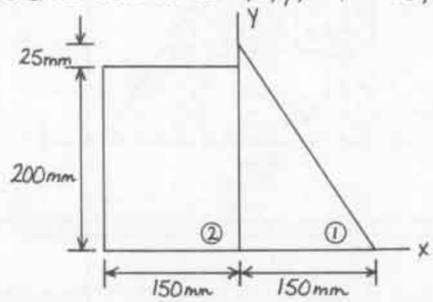
For $\bar{x}_2 = 356.4$ mm, the block extends from $x_{min} = 248$ mm to $x_{max} = 464$ mm

<u>Final Object</u>



8.67

Cut a simple shape from the area shown and add it at some other location so that the new centroid is located at $(\bar{x}, \bar{y}) = (-35.33, 107)$ mm.



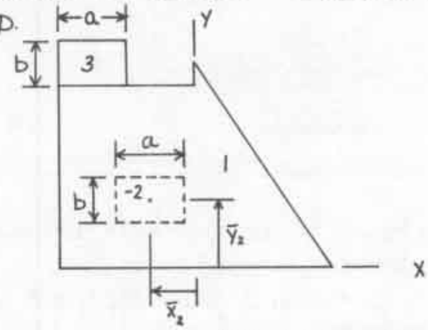
Find the centroid of the original shape.

Part	Area	Centroid Distances		Moments of Area	
i	A_i (mm²)	x_i (mm)	y_i (mm)	A_ix_i (mm³)	A_iy_i (mm³)
1	16 875	50	75	843 750	1 265 625
2	30 000	-75	100	-2 250 000	3 000 000
Σ	46 875	——	——	-1 406 250	4 265 625

$$\bar{X} = \frac{-1\,406\,250}{46\,875} = -30 \text{ mm}$$

$$\bar{Y} = \frac{4\,265\,625}{46\,875} = 91 \text{ mm}$$

Now take out a rectangular block and add it back as shown to move the centroid 5.33 mm left and 16 mm up.



Try $a = 100$ mm, $b = 50$ mm (Selected by trial and error)

Find the centroid of the modified shape.

Part	Area	Centroid Distances		Moments of Area	
i	A_i (mm²)	x_i (mm)	y_i (mm)	A_ix_i (mm³)	A_iy_i (mm³)
1	46 875	-30	91	-1 406 250	4 265 625
2	-5 000	$\bar{x}_2$	$\bar{y}_2$	$-5000\bar{x}_2$	$-5000\bar{y}_2$
3	5000	-100	225	-500 000	1 125 000
Σ	46 875	——	——	$-1\,906\,250 - 5000\bar{x}_2$	$5\,390\,625 - 5000\bar{y}_2$

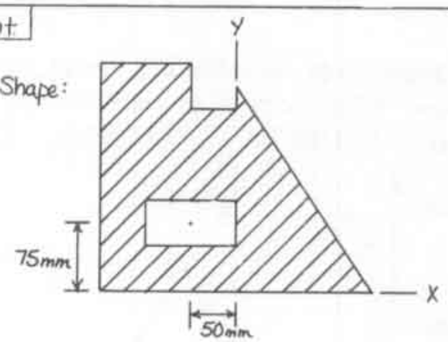
$$\bar{X} = \frac{-1\,906\,250 - 5000\bar{x}_2}{46\,875} = -35.33 \Rightarrow \underline{\underline{\bar{x}_2 = -50.0 \text{ mm}}}$$

$$\bar{Y} = \frac{5\,390\,625 - 5000\bar{y}_2}{46\,875} = 107 \Rightarrow \underline{\underline{\bar{y}_2 = 75.0 \text{ mm}}}$$

(continued)

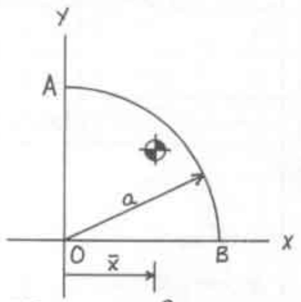
8.67 cont.

Final Shape:



8.68

By the Pappus theorem of surfaces, determine the area of the surface generated by revolving the arc AB of a circle of radius a about the x-axis through 2π radians.



$L = \frac{\pi a}{2}$ $\quad \bar{x} = \frac{2a}{\pi}$ $\quad \theta = 2\pi$

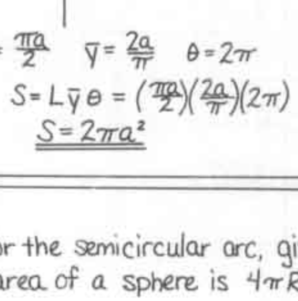
$S = L\bar{x}\theta$

$S = \left(\frac{\pi a}{2}\right)\left(\frac{2a}{\pi}\right)2\pi$

$\underline{S = 2\pi a^2}$

8.69

By the Pappus theorem of surfaces, determine the area of the surface generated by revolving the arc AB of a circle of radius a about the y-axis through 2π radians.



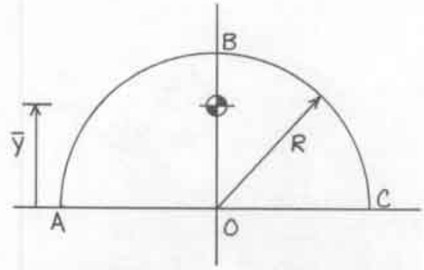
$L = \frac{\pi a}{2}$ $\quad \bar{y} = \frac{2a}{\pi}$ $\quad \theta = 2\pi$

$S = L\bar{y}\theta = \left(\frac{\pi a}{2}\right)\left(\frac{2a}{\pi}\right)(2\pi)$

$\underline{\underline{S = 2\pi a^2}}$

8.70

Find $\bar{y}$ for the semicircular arc, given that the surface area of a sphere is $4\pi R^2$.



$S = 4\pi R^2 \quad \theta = 2\pi \quad L = \pi R$

$S = L\bar{y}\theta$

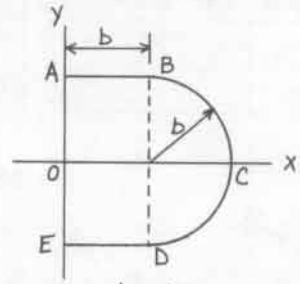
Now solve for $\bar{y}$.

$4\pi R^2 = (\pi R)(2\pi)\bar{y}$

$\underline{\underline{\bar{y} = \frac{2R}{\pi}}}$

8.71

Revolve the curve ABCDE about the y-axis through 2π radians. Find the area of the surface generated.



$L = 2b + \pi b = (2+\pi)b$

$\bar{x} = \frac{\Sigma L_i x_i}{\Sigma L_i} = \frac{2b(\frac{1}{2}b) + (\pi b)(b + \frac{2b}{\pi})}{(2+\pi)b}$

$\bar{x} = \frac{b(3+\pi)}{(2+\pi)}$

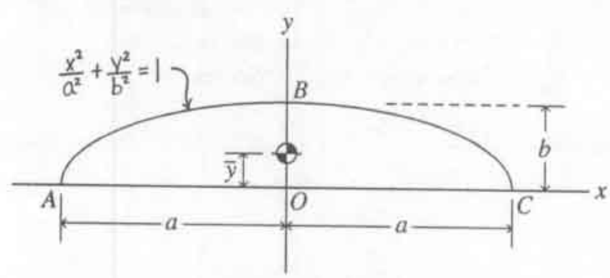
$\theta = 2\pi$

$S = L\bar{x}\theta = (2+\pi)b\left[\frac{b(3+\pi)}{2+\pi}\right](2\pi)$

$\underline{\underline{S = b^2(6\pi + 2\pi^2)}}$

8.72

The volume of an ellipsoid of revolution is $V = \left(\frac{4\pi}{3}\right)ab^2$. Find $\bar{y}$ for the semi-ellipse shown.

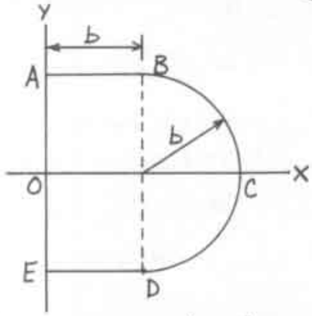


$V = \frac{4\pi ab^2}{3} \quad A = \frac{\pi ab}{2} \quad \theta = 2\pi$

$V = A\bar{y}\theta \quad \frac{4\pi ab^2}{3} = \left(\frac{\pi ab}{2}\right)\bar{y}(2\pi) \quad \underline{\underline{\bar{y} = \frac{4b}{3\pi}}}$

8.73

Revolve line ABCDE through π radians about the x-axis. Find the area of the surface generated.



To apply the Pappus theorem of surfaces, the line cannot cross the x-axis. So by symmetry, we can revolve line ABC through 2π radians about the x-axis to obtain the same surface.

$$L = b + \frac{\pi b}{2} = \left(1+\frac{\pi}{2}\right)b$$

$$\theta = 2\pi$$

$$\bar{y} = \frac{\Sigma L_i y_i}{\Sigma L_i} = \frac{b(b) + \frac{\pi b}{2}\left(\frac{2b}{\pi}\right)}{b(1+\pi/2)} = \frac{4b}{2+\pi}$$

$$S = L\bar{y}\theta = \left[\left(1+\frac{\pi}{2}\right)b\right]2\pi\left[\frac{4b}{2+\pi}\right]$$

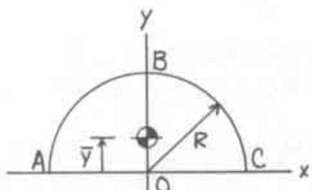
$$\underline{\underline{S = 4\pi b^2}}$$

8.74

The volume of a sphere is $V = \frac{4\pi R^3}{3}$.

a.) Find $\bar{y}$ for a semicircle.

b.) Find $\bar{y}$ for a quarter circle.



a.) Revolve the semicircle about the x-axis

$$V = \frac{4\pi R^3}{3} \quad A = \frac{\pi R^2}{2} \quad \theta = 2\pi$$

$$V = A\bar{y}\theta$$

$$\frac{4\pi R^3}{3} = \frac{\pi R^2}{2}(\bar{y})(2\pi)$$

$$\underline{\underline{\bar{y} = \frac{4R}{3\pi}}}$$

b.) Revolve the quarter circle ABOA about the x-axis.

$$V = \frac{1}{2}\left(\frac{4\pi R^3}{3}\right) = \frac{2\pi R^3}{3} \quad A = \frac{\pi R^2}{4} \quad \theta = 2\pi$$

$$V = A\bar{y}\theta$$

$$\frac{2\pi R^3}{3} = \frac{\pi R^2}{4}(\bar{y})(2\pi)$$

$$\underline{\underline{\bar{y} = \frac{4R}{3\pi}}}$$

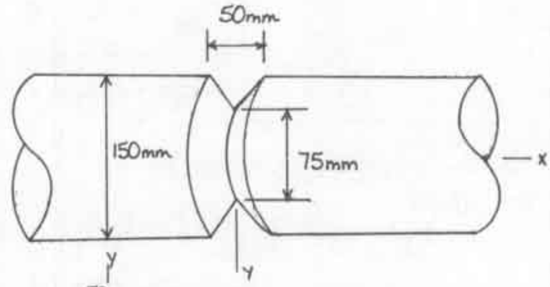
Note: By inspection, we could observe that $\bar{y}$ for the semicircle is the same as that for the quarter circle.

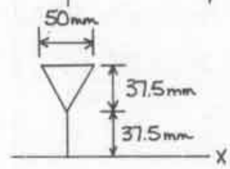
8.75

A V-shaped groove is cut from a circular shaft.

a.) Find the volume of the *material* removed.

b.) Find the surface area of the groove.





a.)

$$A = \tfrac{1}{2}(50)(37.5) = 937.5\text{ mm}^2$$

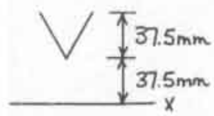
$$\bar{y} = \tfrac{2}{3}(37.5) + 37.5 = 62.5\text{ mm}$$

$$\theta = 2\pi$$

$$V = A\bar{y}\theta = 937.5(62.5)(2\pi) = 368\,155\text{ mm}^3$$

$$\underline{\underline{V = 3.68\times10^5\text{ mm}^3}}$$

b.)



$$L = 2\sqrt{25^2 + 37.5^2} = 90.1\text{ mm}$$

$$\bar{y} = \tfrac{1}{2}(37.5) + 37.5 = 56.25\text{ mm}$$

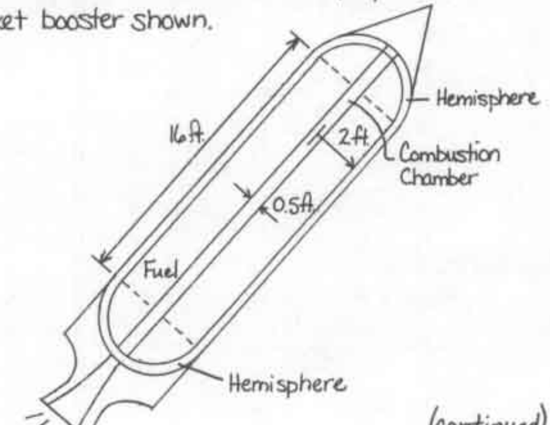
$$\theta = 2\pi$$

$$S = L\bar{y}\theta = 90.1(56.25)(2\pi) = 31\,858\text{ mm}^2$$

$$\underline{\underline{S = 3.19\times10^4\text{ mm}^2}}$$

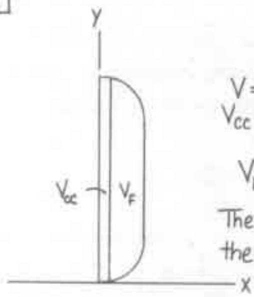
8.76

Find the volume of the solid-fuel propellant in the rocket booster shown.



(continued)

8.76 cont.



V = Volume of solid fuel cylinder
V_{cc} = Volume of Combustion Chamber

$V_{Fuel} = V - V_{cc}$ (a)

The volume is found by revolving the areas around the y-axis

$A = \frac{1}{2}\pi(2)^2 + 2(16) = 38.28\ ft^2$

$\theta = 2\pi$

$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{\left(\frac{4(2)}{3\pi}\right)\left(\frac{1}{2}\pi(2)^2\right) + 2(16)(1)}{38.28} = 0.9753\ ft$

$V = A\bar{x}\theta = 38.28(0.9753)(2\pi) = 234.6\ ft^3$ (b)

$V_{cc} = \pi(.25)^2(20) = 3.926\ ft^3$ (c)

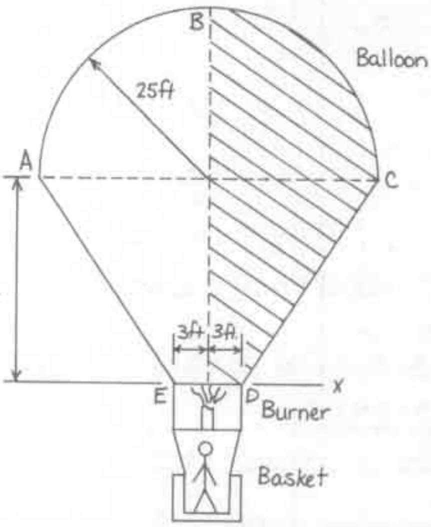
Sub (b) and (c) into (a)

$V_{Fuel} = 234.6 - 3.926$

$\underline{V_{Fuel} = 231\ ft^3}$

8.77

Find the volume of the hot-air balloon formed by a hemisphere and a truncated cone.



Revolve the shaded area about the y-axis through 2π radians

Volume of air = $V_{Hemisphere} + V_{cone}$

Hemisphere:

$A = \frac{1}{4}\pi(25)^2 \quad \bar{x} = \frac{4(25)}{3\pi} \quad \theta = 2\pi$

$V_H = \frac{1}{4}\pi(25)^2\left(\frac{4(25)}{3\pi}\right)2\pi = 32{,}725\ ft^3$

Truncated Cone:

$A = \frac{1}{2}(22)(30) + 3(30) = 420\ ft^2$

$\bar{x} = \frac{\frac{22(30)}{2}\left(\frac{22}{3} + 3\right) + 90(1.5)}{420} = 8.44\ ft$

$\theta = 2\pi$

$V_c = 420(8.44)2\pi = 22{,}274\ ft^3$

$V_{AIR} = V_H + V_c = 32{,}725 + 22{,}274$

$\underline{\underline{V_{AIR} = 55{,}000\ ft^3}}$

8.78

a.) Find the volume V of a torus with central radius R and circular cross-section diameter D.

$A_T = \frac{\pi D^2}{4} \quad \theta = 2\pi$

$V_{Tunnel} = A_T R\theta = \frac{\pi D^2(R)(2\pi)}{4}$

$\underline{\underline{V_T = \frac{\pi^2 D^2 R}{2}}}$

b.) Find the surface area S of a torus with central radius R and cross-section diameter D.

$L = \pi D \quad \theta = 2\pi$

$S = LR\theta = \pi D(R)(2\pi)$

$\underline{\underline{S = 2\pi^2 DR}}$

c.) Evaluate V and S for R = 43 km and D = 5m.

$V = \frac{\pi^2(5^2)(43000)}{2} = 5.30 \times 10^6\ m^3$

$\underline{\underline{V = 5.30 \times 10^6\ m^3}}$

$S = 2\pi^2(5)(43{,}000)$ $\quad \underline{\underline{S = 4.24 \times 10^6\ m^2}}$

d.) Find the cost of construction at \$10/m³ for excavation and \$3/m² for lining 30% of the tunnel wall.

Cost of excavation:

\$10.00/m³ (5.30×10⁶ m³) = <u>\$53,000,000</u>

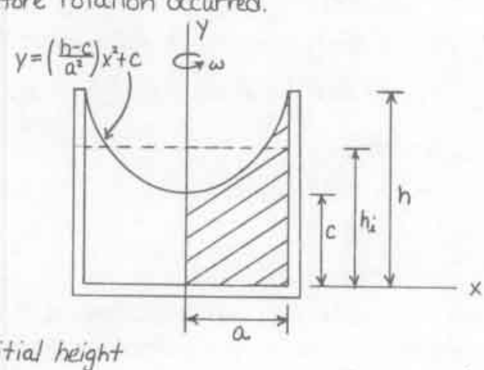
Cost of lining:

(0.3) \$3.00/m² (4.24×10⁶ m²) = <u>\$3,820,000</u>

<u>Total Cost:</u> <u>\$56,820,000</u>

8.79

Under rotation, liquid in a cylinder forms a paraboloidal surface. Find the height h_i of the liquid before rotation occurred.



h_i = Initial height

(continued)

8.79 cont.

Find the volume of the liquid by rotating the shaded area about the y-axis through 2π radians.

$$A=\int_0^a\left(\left(\frac{h-c}{a^2}\right)x^2+c\right)dx = \frac{(h-c)a}{3}+ca$$

$$A=\underline{\frac{a(h+2c)}{3}}$$

$$Q_y=\int x\,dA$$

$$Q_y=\int_0^a\left(\left(\frac{h-c}{a^2}\right)x^3+cx\right)dx = \frac{(h-c)a^2}{4}+\frac{ca^2}{2}$$

$$Q_y=\underline{\frac{a^2(h+c)}{4}}$$

$$\bar{x}=\frac{Q_y}{A}=\frac{\frac{a^2(h+c)}{4}}{\frac{a(h+2c)}{3}}=\frac{3a(h+c)}{4(h+2c)}$$

$$\theta=2\pi$$

$$V=A\bar{x}\theta$$

$$V=\left[\frac{a(h+2c)}{3}\right]\left[\frac{3a(h+c)}{4(h+2c)}\right]2\pi=\frac{\pi a^2(h+c)}{2} \quad (a)$$

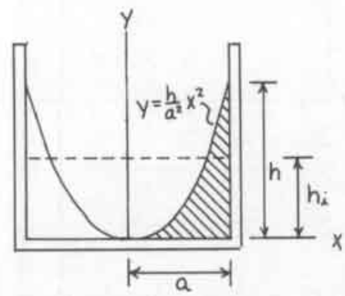
The volume did not change thus set (a) equal to $\pi a^2 h_i$ and solve for h_i.

$$\pi a^2 h_i=\frac{\pi a^2(h+c)}{2}$$

$$\underline{\underline{h_i=\frac{(h+c)}{2}}}$$

8.80

Liquid in a rotating cylinder forms a paraboloidal surface. Find the height h_i of the liquid before rotation occurred.



The volume can be found by rotating the shaded area about the y-axis through 2π radians.

$$A=\int_0^a\frac{h}{a^2}x^2dx=\frac{ah}{3}$$

$$Q_y=\int_0^a x\,dA=\int_0^a\frac{h}{a^2}x^3dx=\frac{a^2h}{4}$$

$$\bar{x}=\frac{Q_y}{A}=\frac{\frac{a^2h}{4}}{\frac{ah}{3}}=\frac{3a}{4} \qquad \theta=2\pi$$

$$V=A\bar{x}\theta=\left(\frac{ah}{3}\right)\left(\frac{3a}{4}\right)2\pi=\frac{\pi a^2h}{2}$$

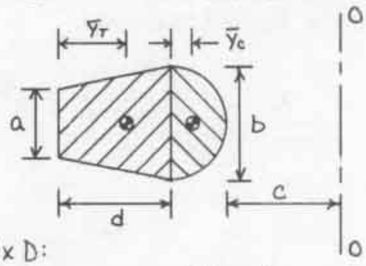
Rotation does not cause the volume to change, thus

$$\pi a^2h_i=\frac{\pi a^2h}{2}$$

$$\underline{\underline{h_i=\frac{h}{2}}}$$

8.81

a.) Find the volume V of the circular part ($\theta=\pi$) of a crane hook with the cross-section shown.



From Appendix D:

For the trapezoid: $\bar{y}_T=\frac{d(2b+a)}{3(b+a)}$, $A_T=\left(\frac{b+a}{2}\right)d$

For the semicircle: $\bar{y}_c=\frac{2b}{3\pi}$, $A_c=\frac{\pi b^2}{8}$

For the revolution of this area about axis O-O through $\theta=\pi$:

$$\underline{V=\pi\left[\left(\frac{b+a}{2}\right)d\left(d+\frac{b}{2}+c-\frac{d(2b+a)}{3(b+a)}\right)+\left(\frac{\pi b^2}{8}\right)\left(\frac{b}{2}+c-\frac{2b}{3\pi}\right)\right]}$$

It is likely that this equation can be simplified, but it is not necessary.

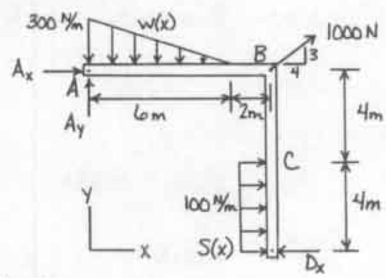
b) *Evaluate V for $a=15$mm, $b=25$mm, $c=25$mm, and $d=25$mm*

$$V=\pi\left[\left(\frac{25+15}{2}\right)25\left(25+\frac{25}{2}+25-\frac{25(2(25)+15)}{3(25+15)}\right)+\left(\frac{\pi(25)^2}{8}\right)\left(\frac{25}{2}+25-\frac{50}{3\pi}\right)\right]$$

$$\underline{V=101\ 728\ mm^3}$$

8.82

a.) Find the resultants and lines of action of the distributed loads shown.



Triangular load:

$$w(x)=-50x+300$$

$$R=\int_0^6(-50x+300)dx = \frac{-50(6)^2}{2}+300(6)$$

$$\underline{R=900\ N}$$

$$M=\int_0^6(-50x^2+300x)dx=\frac{-50(6)^3}{3}+\frac{300(6)^2}{2}$$

$$M=1800\ N\cdot m$$

$$\bar{x}=\frac{M}{R}=\frac{1800}{900} \qquad \underline{\bar{x}=2m}$$

Constant load:

$$S(x)=100\ N/m$$

$$R=\int_0^4 100dy = 100(4) \qquad \underline{R=400\ N}$$

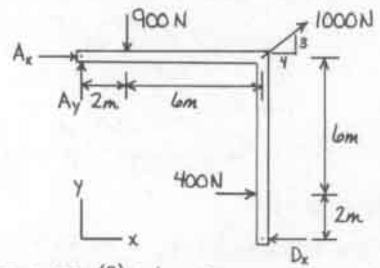
$$M=\int_0^4 100y\,dy=\frac{100(4)^2}{2} \qquad M=800\ N\cdot m$$

(continued)

8.82 cont.

$\bar{y} = \frac{M}{R} = \frac{800}{400}$ $\underline{\bar{y} = 2m}$

b.) Find the support reactions.



$\Sigma F_y = -900 + 1000(\frac{3}{5}) + A_y = 0$

$\underline{A_y = 300N}$

$(+\Sigma M_A = -900(2) + 1000(\frac{3}{5})(8) + 400(6) - D_x(8) = 0$

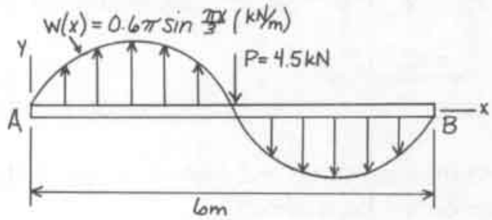
$\underline{D_x = 675N}$

$\Sigma F_x = A_x + 1000(\frac{4}{5}) + 400 - 675 = 0$

$\underline{A_x = -525N}$

8.83

Find the magnitude and line of action of the loads on a propeller shaft.

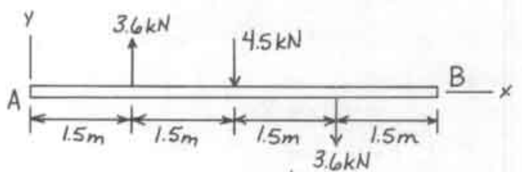


$R_{0\text{-}3} = \int_0^3 0.6\pi \sin\frac{\pi x}{3}\,dx = -0.6\pi\cos\left(\frac{\pi x}{3}\right)\left(\frac{3}{\pi}\right)\Big|_0^3$

$R_{0\text{-}3} = -1.8\cos\frac{\pi x}{3}\Big|_0^3 = 3.6\,kN$

By symmetry of loading,

$R_{3\text{-}6} = -R_{0\text{-}3} = -3.6kN$



Resultant is $\underline{R = 4.5\,kN\downarrow}$

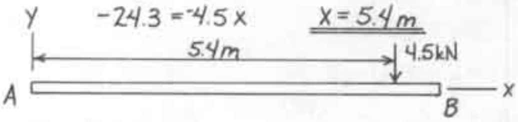
Sum moments about A and find the line of action

$(+\Sigma M_A = 3.6(1.5) - 4.5(3) - 3.6(4.5) = 0$

$\Sigma M_A = -24.3\ kN\cdot m$

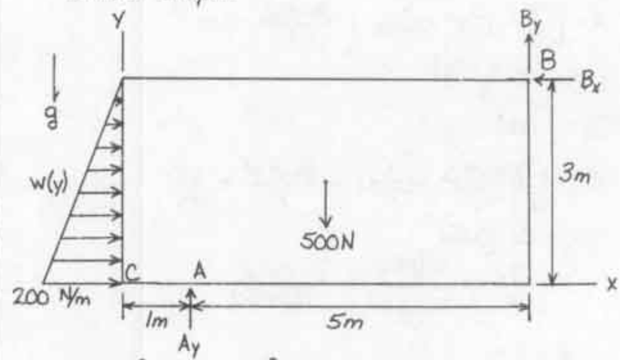
$\Sigma M_A = Rx$

$-24.3 = -4.5x$ $\underline{x = 5.4m}$



8.84

a.) Replace the distributed load by a force at C and a couple.



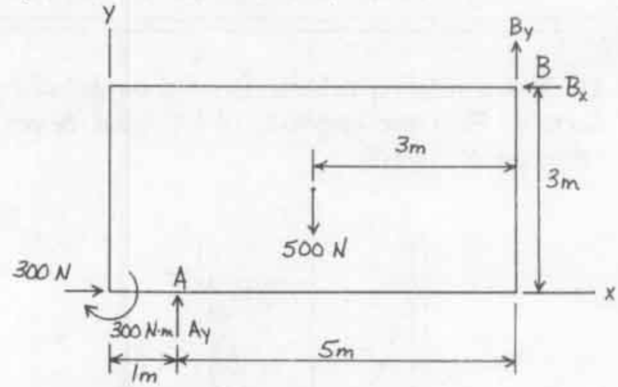
$R = \int_0^3 w(y)\,dy = \int_0^3\left(-\frac{200}{3}y + 200\right)dy$

$R = 200y - \frac{200y^2}{6}\Big|_0^3$ $\underline{R = 300N}$

The line of action is at the centroid of the triangle. (1m above C)

$\underline{M = -300(1) = -300\,N\cdot m}$

b.) Find the reactions at A and B.



$\Sigma F_x = 300 - B_x = 0$

$\underline{B_x = 300\,N}$

$(+\Sigma M_B = 500(3) - A_y(5) + 300(3) - 300 = 0$

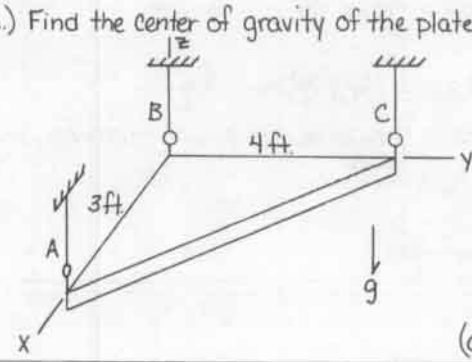
$\underline{A_y = 420\,N}$

$\Sigma F_y = 420 - 500 + B_y = 0$

$\underline{B_y = 80\,N}$

8.85

a.) Find the center of gravity of the plate shown.



(continued)

8.85 cont.

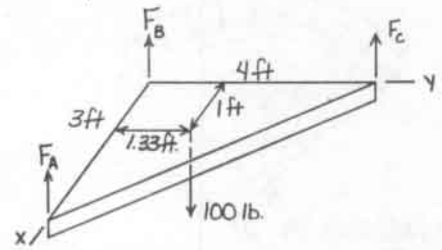
Because the plate is homogeneous, the center of gravity is at the centroid of the triangular area.

$\bar{x} = \frac{1}{3}(3) = 1\,ft \qquad \bar{y} = \frac{1}{3}(4) = 1.33\,ft$

$\underline{CG = (1, 1.33)\ ft}$

b.) Find the force in each rope.

FBD of plate:



Equilibrium Equations

$\circlearrowleft \Sigma M_{B_y} = -F_A(3) + 100(1) = 0 \qquad \underline{\underline{F_A = 33.3\ lb.}}$

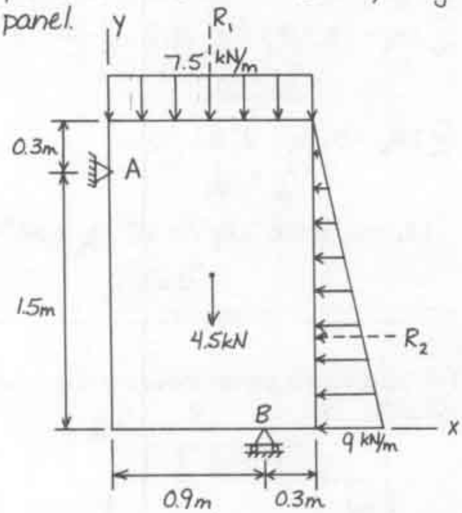
$\circlearrowleft \Sigma M_{B_x} = F_C(4) - 100(1.33) = 0 \qquad \underline{\underline{F_C = 33.3\ lb.}}$

$\Sigma F_y = -100 + 33.3 + 33.3 + F_B = 0$

$\underline{\underline{F_B = 33.3\ lb.}}$

8.86

a.) Find the resultants of the distributed loads on the panel and draw the free-body diagram of the panel.



$R_1 = \int_0^{1.2} 7.5\,dx = 7.5x \Big|_0^{1.2} \qquad \underline{\underline{R_1 = 9\ kN}}$

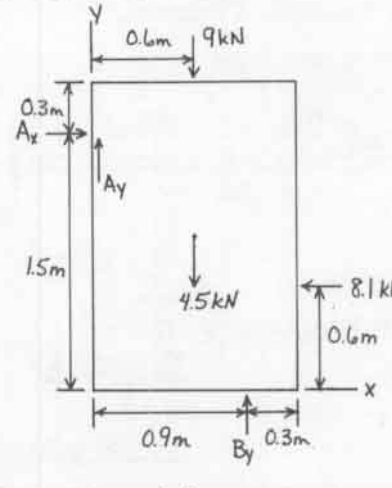
By symmetry, the line of action $\underline{\underline{x = 0.6m}}$

$R_2 = \int_0^{1.8} 5x\,dx = \frac{5x^2}{2}\Big|_0^{1.8} \qquad \underline{\underline{R_2 = 8.1\ kN}}$

The line of action is at the centroid of the triangle.

$y = \frac{1}{3}(1.8) \qquad \underline{\underline{y = 0.6m}}$

FBD with resultants and reactions:



b.) Find the reactions at A and B.

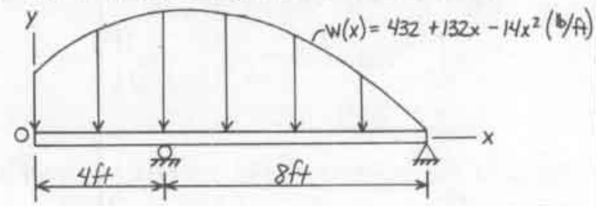
$\Sigma F_x = -8.1 + A_x = 0 \qquad \underline{\underline{A_x = 8.1\ kN}}$

$\circlearrowright + \Sigma M_A = -9(0.6) - 8.1(0.9) + B_y(0.9) - 4.5(0.6) = 0 \qquad \underline{\underline{B_y = 17.1\ kN}}$

$\Sigma F_y = -9 - 4.5 + A_y + 17.1 = 0 \qquad \underline{\underline{A_y = -3.6\ kN}}$

8.87

a.) Find the resultant of the distributed load.



$R = \int_0^{12} (432 + 132x - 14x^2)\,dx = \left(432x + 66x^2 - \frac{14x^3}{3}\right)\Big|_0^{12}$

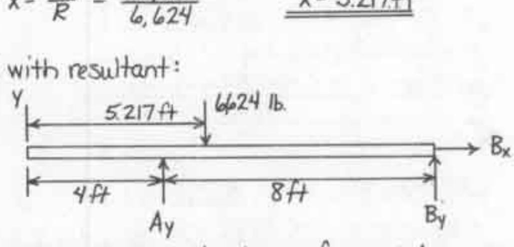
$\underline{R = 6{,}624\ lb.}$

$M_O = \int_0^{12} (432x + 132x^2 - 14x^3)\,dx = \left(216x^2 + 44x^3 - \frac{14x^4}{4}\right)\Big|_0^{12}$

$M = 34{,}560\ lb{\cdot}ft$

$\bar{x} = \frac{M_O}{R} = \frac{34{,}560}{6{,}624} \qquad \underline{\underline{\bar{x} = 5.217\,ft}}$

FBD with resultant:



b.) Replace the resultant by a force at A and a couple.

$\underline{\underline{R_A = 6{,}624\ lb}} \qquad M = (6{,}624)(5.217 - 4)$

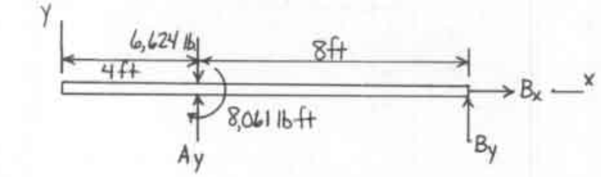
$\underline{\underline{M = 8{,}061\ lb{\cdot}ft}}$

(continued)

8.87 cont.

c.) Find the support reactions.

FBD with R_A and couple:



$\Sigma F_x = B_x = 0 \qquad \underline{B_x = 0}$

$\circlearrowleft + \Sigma M_A = -8061 + B_y(8) = 0$

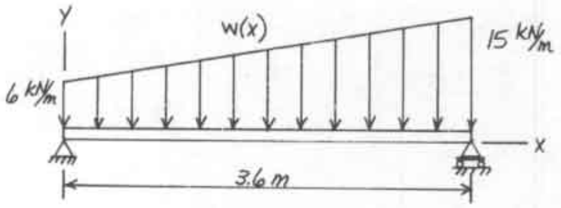
$\underline{B_y = 1008 \text{ lb.}}$

$\Sigma F_y = -6{,}624 + A_y + 1008 = 0$

$\underline{A_y = 5616 \text{ lb.}}$

8.88

a.) Find the resultant of the distributed load.



$w(x) = \frac{9}{3.6}x + 6 = (2.5x + 6) \text{ kN/m}$

$R = \int_0^{3.6}(2.5x+6)dx = \left(\frac{2.5x^2}{2} + 6x\right)\Big|_0^{3.6}$

$\underline{R = 37.8 \text{ kN}}$

b.) Find the moment of the distributed load about A.

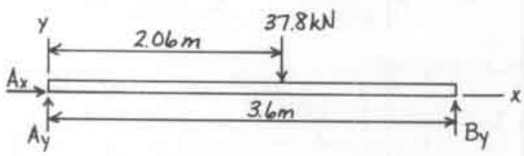
$M = \int_0^{3.6}(2.5x^2 + 6x)dx = \left(\frac{2.5x^3}{3} + \frac{6x^2}{2}\right)\Big|_0^{3.6}$

$\underline{M = 77.8 \text{ kN}\cdot\text{m}}$

c.) Find the support reactions.

$\bar{x} = \frac{M}{R} = \frac{77.76}{37.8} = 2.06 \text{m}$

FBD of bar:



$\Sigma F_x = A_x = 0 \qquad \underline{A_x = 0}$

$\circlearrowleft + \Sigma M_A = -77.76 + B_y(3.6) = 0 \qquad \underline{B_y = 21.6 \text{ kN}}$

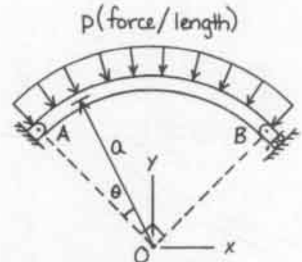
$\Sigma F_y = A_y - 37.8 + 21.6 = 0 \qquad \underline{A_y = 16.2 \text{ kN}}$

8.89

Pressure p is applied to a 90° arc of a fuselage ring.

a.) Find the resultant force.

b.) Find the reactions.



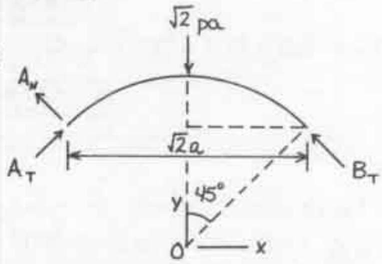
a.) $dF = -p(a d\theta)\cos(\pi/4 - \theta)$

$F = -2\int_0^{\pi/4} pa\cos(\pi/4 - \theta)d\theta$

$F = 2pa\sin(\pi/4 - \theta)\Big|_0^{\pi/4} = 2pa\sin(0) - 2pa\sin(\frac{\pi}{4})$

$\underline{F = -\sqrt{2}\,pa}$

b.) FBD:



$\circlearrowleft + \Sigma M_A = (B_T/\sqrt{2})\sqrt{2}a - (\sqrt{2}pa)\frac{a}{\sqrt{2}} = 0$

$\underline{B_T = pa}$

$\circlearrowleft + \Sigma M_O = B_T(a) - A_T(a) = 0$

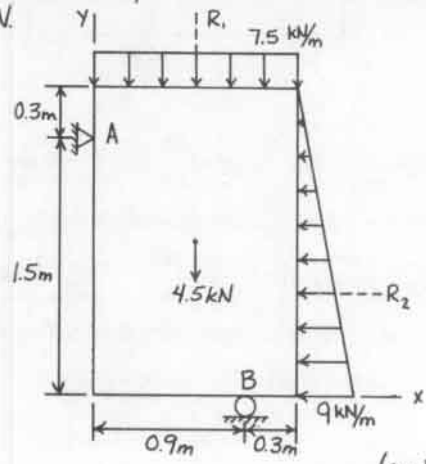
$\underline{A_T = pa}$

$\Sigma F_x = A_T \cos 45° - B_T \cos 45° - A_N \sin 45° = 0$

$\underline{A_N = 0}$

8.90

Add a load to the panel shown so the reaction at B is 14 kN.



(continued)

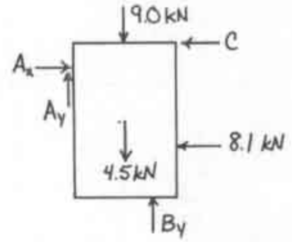
8.90 cont.

Find the distributed load resultants

$R_1 = (7.5)(1.2) = 9.0\,kN$ @ $x = 0.60\,m$

$R_2 = \frac{1}{2}(9)(1.8) = 8.1\,kN$ @ $y = 0.60\,m$

Find the reaction at B:



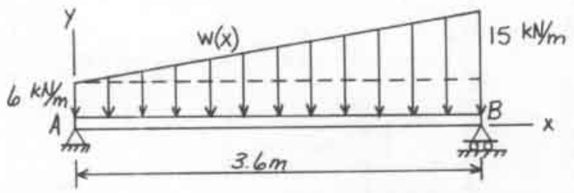
$$\circlearrowleft + \Sigma M_A = 0.9B_y - 0.6(9.0+4.5) - 0.9(8.1) = 0$$
$$0.9B_y = 15.39 \quad (a)$$
$$\underline{B_y = 17.1\,kN} > 14\,kN$$

Add a force C at the top of the panel to reduce B_y. Set $B_y = 14\,kN$ and add the effect of C to Eqn. (a):

$$\circlearrowleft + \Sigma M_A = 0.9(14) - 15.39 + 0.3C = 0$$
$$\underline{C = 9.3\,kN}$$

8.91

Add an additional load to the beam shown to reduce the reaction at B by 50%.

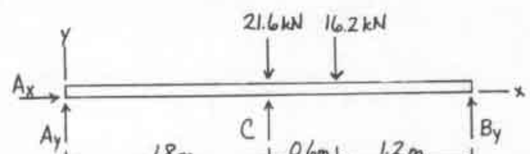


Find the resultant of the distributed load. Treat w(x) as a rectangular load plus a triangular load.

For the rectangular (uniform) load:
$$R_R = 6(3.6) = 21.6\,kN \text{ @ } x = 1.8\,m$$

For the triangular load:
$$R_T = \tfrac{1}{2}(15-6)(3.6) = 16.2\,kN \text{ @ } x = 2.4\,m$$

Find the reaction at B:



$$\circlearrowleft + \Sigma M_A = 3.6B_y - 21.6(1.8) - 16.2(2.4) = 0$$
$$3.6B_y - 77.76 = 0 \quad (a)$$
$$\underline{B_y = 21.6\,kN}$$

Add a force C at midspan to reduce B_y to $0.5(21.6) = 10.8\,kN$. Set $B_y = 10.8\,kN$ and add the effect of C to Eqn. (a):

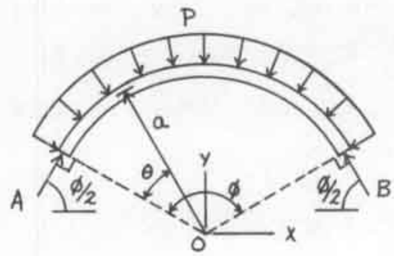
$$\circlearrowleft + \Sigma M_A = 3.6(10.8) - 77.76 + 1.8C = 0$$
$$\underline{C = 21.6\,kN}$$

8.92

For the ring loaded as shown, $a = 10\,ft$, $p = 20\,lb/in$.

a) Plot the reactions at A and B as functions of ϕ for $90° \le \phi \le 180°$.

b.) Make a recommendation based on a maximum vert. reaction of 2000 lb. at each support.

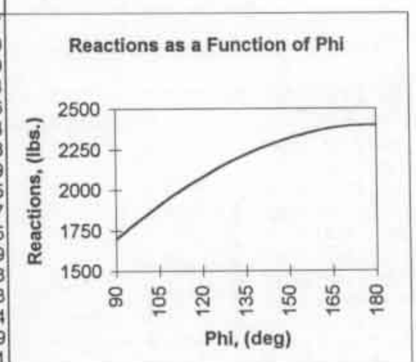


a) $p = 20\,lb/in = 240\,lb/ft$

By symmetry, $A = B$

$$\Sigma F_y = A_y + B_y - \int_0^{\phi} pa\cos(\phi/2 - \theta)\,d\theta = 0$$
$$2A_y + pa\left(\sin(\phi/2 - \theta)\right)\Big|_0^{\phi} = 0$$
$$2A_y + pa\left[\sin\left(\tfrac{-\phi}{2}\right) - \sin(\phi/2)\right] = 0$$
$$2A_y - 2pa\sin(\phi/2) = 0$$
$$\underline{A_y = B_y = pa\sin(\phi/2) = 2400\sin(\phi/2)}$$

φ Degrees	φ Radians	Reactions (lbs.)
90	1.571	1697
95	1.658	1769
100	1.745	1839
105	1.833	1904
110	1.920	1966
115	2.007	2024
120	2.094	2078
125	2.182	2129
130	2.269	2175
135	2.356	2217
140	2.443	2255
145	2.531	2289
150	2.618	2318
155	2.705	2343
160	2.793	2364
165	2.880	2379
170	2.967	2391
175	3.054	2398
180	3.142	2400



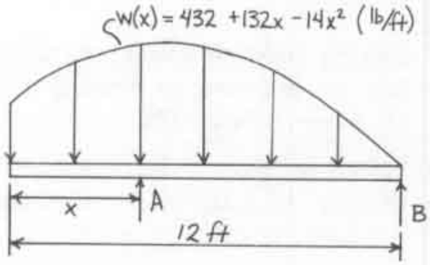
b.) From the plot, A_y and B_y exceed 2000 lbs for $\phi > 113°$. So the angle subtended by the ring should not exceed 113°.

8.93

For the beam shown:

a) Plot the reactions at A and B for $0 \le x \le 6$ ft where x is the location of the left support.

b) Recommend the "best" location for support A.



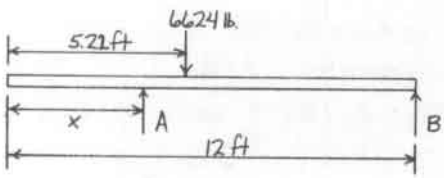
a.) Find the resultant of the distributed load.

$R = \int_0^{12} (432 + 132x - 14x^2)dx = 6624$ lb.

$M = \int_0^{12} (432x + 132x^2 - 14x^3)dx = 34{,}560$ lb·ft

$\bar{x} = \frac{M}{R} = \frac{34{,}560}{6624} = 5.22$ ft

Find the reactions at A and B



$\circlearrowleft + \Sigma M_B = 6624(12 - 5.22) - A(12 - x) = 0$

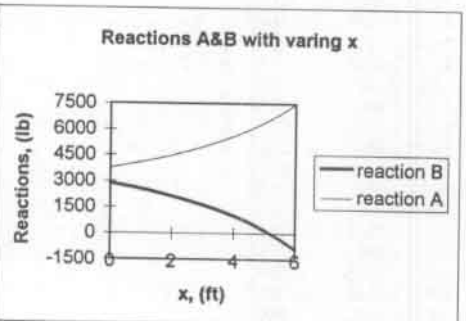
$$\underline{\underline{A = \left(\frac{44928}{12 - x}\right)}}$$

$\Sigma F_y = A + B - 6624 = 0$

$$\underline{\underline{B = 6624 - \left(\frac{44928}{12 - x}\right)}}$$

The plot of A and B for $0 \le x \le 6$ ft follows:

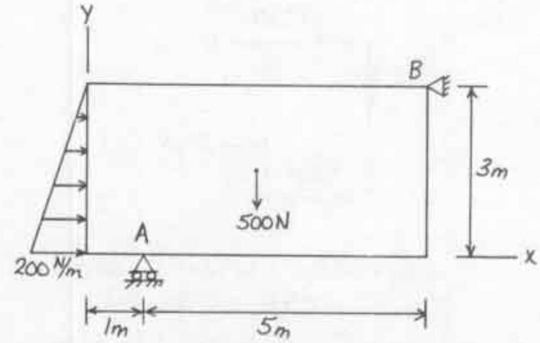
x(ft)	B (lb)	A (lb)
0.0	2880	3744
0.5	2717	3907
1.0	2539	4085
1.5	2345	4279
2.0	2131	4493
2.5	1894	4730
3.0	1632	4992
3.5	1338	5286
4.0	1008	5616
4.5	633	5991
5.0	205	6419
5.5	-288	6912
6.0	-864	7488



b.) Recommendation: Locate support A at x=0 so that the reactions at A and B are as close to being the same as possible.

8.94

Place an opening in the panel shown to reduce the panel's weight to 480 N and reduce the reaction at A by 20 N. The opening must not be within 0.5m of any panel edge.



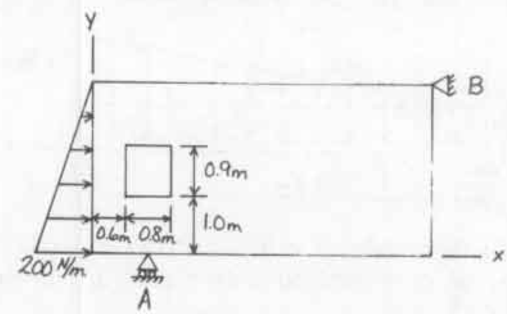
Approach: Since the panel weight must be reduced by 20N and the reaction at A must also be reduced by 20N, the opening must be centered directly above the support at A.

The panel weighs $\frac{500}{3(6)} = 27.8$ N/m²

$\therefore$ the opening has area $\frac{20}{27.8} = 0.72\ m^2$

Choose a rectangular opening 0.8m × 0.9m.

<u>Final Panel Configuration</u>



As usual in design problems, other possibilities exist.

9.1

Given: $P_g = 120\ kPa$ at some point in the ocean.
$\gamma = 10.0\ kN/m^3$

Find: The depth to the point below the surface

$$P_g = 120\ kPa = 120\ kN/m^2$$
$$P_g = \gamma h \qquad [Eq. (9.5)]$$
$$h = \frac{P_g}{\gamma} = \frac{120\ kN/m^2}{10.0\ kN/m^3}$$

$\underline{\underline{h = 12\ m}}$

9.2

Given: $\gamma = 64\ lb/ft^3$ for sea water

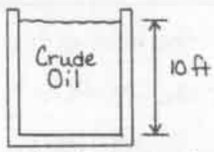
Find: the gage pressure (lb/in^2) at a depth of 3 miles below the ocean's surface.

$$P_g = \gamma h \quad [Eq. (9.5)]$$
$$P_g = (64\ lb/ft^3)(3\ mi.)\left(\frac{5280\ ft.}{1\ mi.}\right)\left(\frac{1\ ft^2}{144\ in^2}\right) = 7040\ lb/in^2$$

$\underline{\underline{P_g = 7040\ psi}}$

9.3

Given: $\gamma_{OIL} = 54\ lb/ft^3$



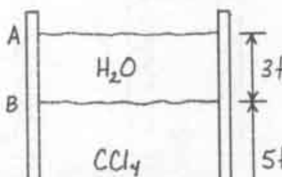
Find: the pressure in the oil 10 ft. below the surface.

$$P_g = \gamma h \quad [Eq. (9.5)]$$
$$P_g = (54\ lb/ft^3)(10\ ft) = 540\ psf\left(\frac{1\ ft^2}{144\ in^2}\right)$$

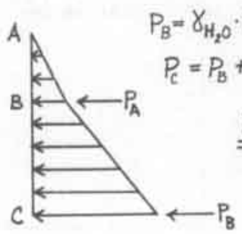
$\underline{\underline{P_g = 3.75\ psi}}$

9.4

Given: $\gamma_{H_2O} = 62.4\ lb/ft^3 \qquad h_{H_2O} = 3\ ft.$
$\gamma_{CCl_4} = 99\ lb/ft^3 \qquad h_{CCl_4} = 5\ ft.$



Find: the pressure on the bottom of the vessel



$$P_B = \gamma_{H_2O} \cdot h_{H_2O} = 62.4(3) = 187.2\ lb/ft^2$$
$$P_C = P_B + \gamma_{CCl_4} \cdot h_{CCl_4} = 187.2 + 99(5)$$

$\underline{\underline{P_C = 682\ lb/ft^2 = 4.74\ psi}}$

9.5

Find: The equivalence of 14.696 psi in:
a) N/m^2
b) in Hg
c) m H_2O
d) ft H_2O

a) $14.696\ psi = 101.3\ kPa = \underline{\underline{101\,300\ N/m^2}}$

b) $14.696\ psi = \underline{\underline{29.92\ in\ Hg}}$ See Standard Atmosperic Conditions

c) $sg_{Hg} = 13.595$
$$14.696\ psi = 760\ mm\ Hg \times \frac{(13.595\ mm\ H_2O)}{1\ mm\ Hg} \times \frac{(1\ m)}{(1000\ mm)}$$
$\underline{\underline{14.696\ psi = 10.33\ m H_2O}}$

d) $14.696\ psi = 10.33\ m\ H_2O \times \frac{3.281\ ft}{1\ m} = \underline{\underline{33.9\ ft\ H_2O}}$

9.6

Given: Atmospheric pressure at sea level is
$P_{atm} = 14.696\ psi$
$\gamma_{Air} = 0.07651\ lb/ft^3$

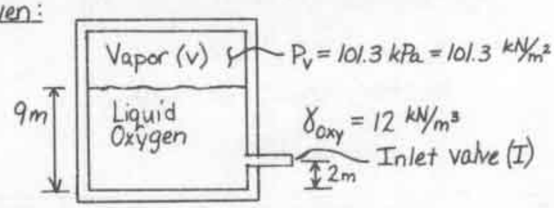
Find: the atmospheric pressure at h = 700 ft.

$$P_g = P_{atm} - \gamma h$$
$$P_g = 14.696 - (0.07651)(700)\left(\frac{1\ ft^2}{144\ in^2}\right)$$

$\underline{\underline{P_g = 14.324\ lb/in^2}}$

9.7

Given:



Find: the pressure at the inlet valve

$$P_I = P_v + \gamma h$$
$$P_I = 101.3\ kN/m^2 + (12\ kN/m^3)(7\ m)$$

$\underline{\underline{P_I = 185.3\ kPa}}$

9.8

Find: the equivalent of $2880\ lb/ft^2$ in:
a) in Hg
b) ft H_2O

a) By Standard Atmospheric Conditions
$$2880\ lb/ft^2 \times \frac{29.92\ in\ Hg}{2116.2\ lb/ft^2} = \underline{\underline{40.72\ in\ Hg}}$$

b) $2880\ lb/ft^2 = (62.4\ lb/ft^3)h$

$\underline{\underline{h = 46.2\ ft\ H_2O}}$

9.9

Find: The equivalent of $P = 120$ kPa in
a) mm Hg
b) m H_2O

a) By Standard Atmospheric Conditions,

$$120\,kPa \times \frac{(760\,mm\,Hg)}{(101.3\,kPa)} = \underline{900\ mm\ Hg}$$

b) From data in Eqn (9.2)

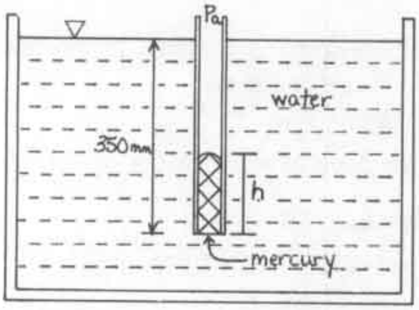
$$120\,kPa = 120\,kN/m^2 = \gamma_{H_2O}(h)$$

$$120\,kN/m^2 = (9.78\,kN/m^3)h$$

$$\underline{h = 12.27\,m\ H_2O}$$

9.10

Given:



Find: the length h of mercury in the glass tube

Observation: The pressure at the surface of both the mercury and water are equal.

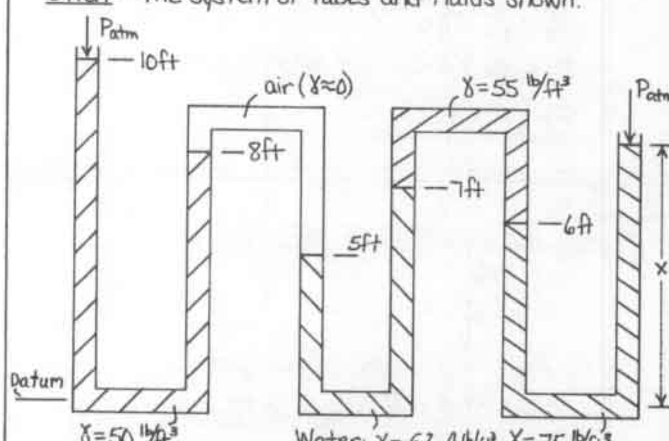
$$\gamma_{H_2O}(h_{H_2O}) = \gamma_{Hg}(h_{Hg})$$

$$(9.78\,kN/m^3)(350\,mm) = 133\,kN/m^3(h_{Hg})$$

$$\underline{h = 25.74\,mm}$$

9.11

Given: the system of tubes and fluids shown.



Find: The height x of the fluid in the right tube.

Start at the top of the left tube and accumulate pressures until the top of the right tube is reached.

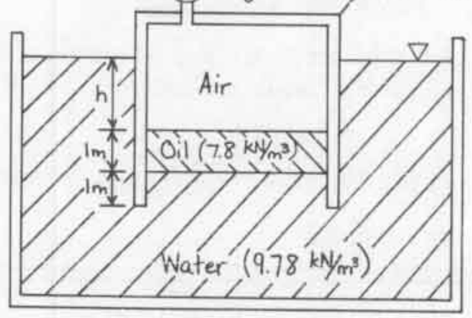
$$P_{atm} + (10-8)50 - (7-5)62.4 + (7-6)55 - (x-6)75 = P_{atm}$$

$$100 - 124.8 + 55 - 75x + 450 = 0$$

$$\underline{\underline{x = 6.40\,ft}}$$

9.12

Given: The gage pressure of the air in the tank is 20kPa.



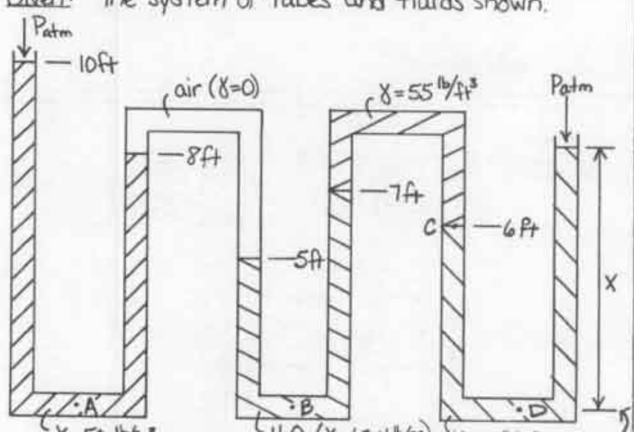
Find: the value of h

$$20\,kPa = 20\,kN/m^2 = (9.78)(h+1) - 7.8(1)$$

$$\underline{h = 1.843\,m}$$

9.13

Given: The system of tubes and fluids shown.



Find: the maximum pressure in each of the liquids.

The max pressure in each liquid exists at the level of the datum for A, B, and D and at 6ft above the datum for C.

$$P_A = P_{atm} + 50(10) = 2116 + 500$$

$$\underline{P_A = 2616\,lb/ft^2}$$

$$P_B = P_A - 8(50) + 5(62.4)$$

$$\underline{P_B = 2528\,lb/ft^2}$$

(continued)

9.13 (cont.)

$P_C = P_B - 62.4(7) + 55(1)$

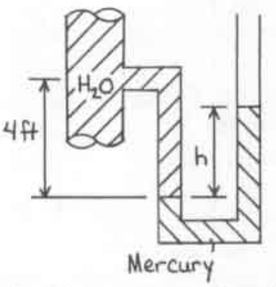
$\underline{\underline{P_C = 2146 \text{ lb/ft}^2}}$

$P_D = P_C + 75(6)$

$\underline{\underline{P_D = 2596 \text{ lb/ft}^2}}$

9.14

Given: A water pipe with a mercury gage as shown.



$h = 1.20$ ft

$\gamma_{H_2O} = 62.4 \text{ lb/ft}^3$

$\gamma_{Hg} = 846 \text{ lb/ft}^3$

Find: the gage pressure in the pipe adjacent to the pipe connection.

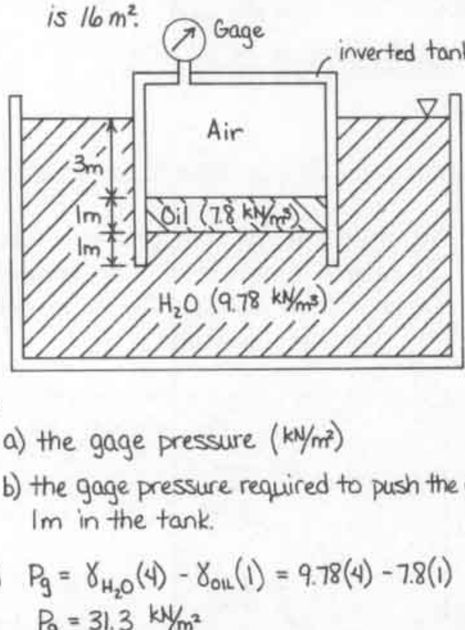
$P_g = \gamma_{Hg}(h) - \gamma_{H_2O}(4\text{ft})$

$P_g = (846)(1.20) - 62.4(4)$

$\underline{P_g = 766 \text{ lb/ft}^2}$

9.15

Given: The cross sectional area of the tank shown is 16 m^2.



Find:

a) the gage pressure (kN/m²)

b) the gage pressure required to push the oil down 1m in the tank.

a) $P_g = \gamma_{H_2O}(4) - \gamma_{OIL}(1) = 9.78(4) - 7.8(1)$

$\underline{P_g = 31.3 \text{ kN/m}^2}$

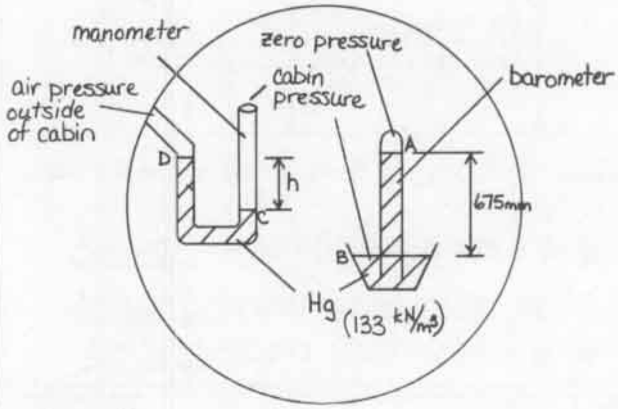
b) $P_g = \gamma_{H_2O}(5) - \gamma_{OIL}(1) = 9.78(5) - 7.8(1)$

$\underline{P_g = 41.1 \text{ kN/m}^2}$

Note: The tank area is superfluous information

9.16

Given: the cross section of the cabin of a blimp shown. The blimp is 300 m above Coors Field in Denver, CO. Air pressure at field level is $P_b = 625$ mmHg. Pressure in the cabin is 675 mmHg. $\gamma_{AIR} = 10.4 \text{ N/m}^3$.



Find: h

$P_A = 0$

$P_B = P_C$

$P_B = \gamma_{Hg}(0.675) = 133(0.675) = 89.775 \text{ kN/m}^2$

$P_D = 625 \text{ mm Hg} - (300\text{m})(0.0104 \text{ kN/m}^3)\left(\frac{760 \text{mmHg}}{101.3 \text{ kPa}}\right)$

$P_D = 601.59 \text{ mm Hg} = 80.19 \text{ kN/m}^2$

$P_D - P_C = 80.19 - 89.775 = 133(-h)$

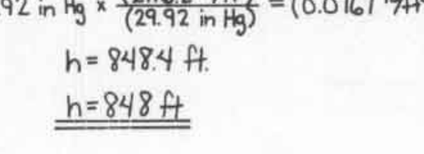
$h = 0.0721$ m

$\underline{h = 72.1 \text{ mm}}$

9.17

Given: The local atmospheric pressure at A is 29 in Hg. Atmospheric pressure at sea level is 29.92 in Hg. $\gamma_{AIR} = 0.0767 \text{ lb/ft}^3$

Find the elevation at A.



$P_1 = P_A + \gamma h$

$29.92 - 29.0 = (0.0767)h$

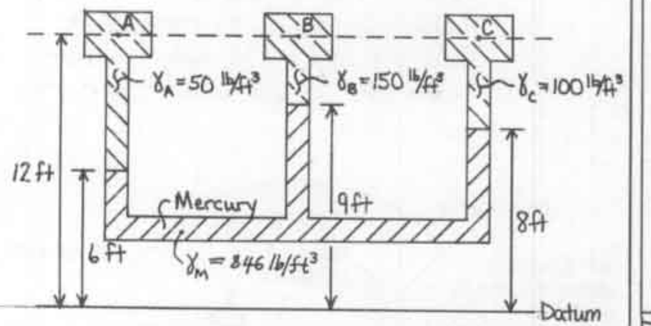
$0.92 \text{ in Hg} \times \frac{(2116.2 \text{ lb/ft}^2)}{(29.92 \text{ in Hg})} = (0.0767 \text{ lb/ft}^3)h$

$h = 848.4$ ft.

$\underline{\underline{h = 848 \text{ ft}}}$

9.18

Given: 3 tanks with different fluids as shown.



Find: $P_B - P_C$, $P_C - P_A$, $P_A - P_B$ at the level of the dashed line.

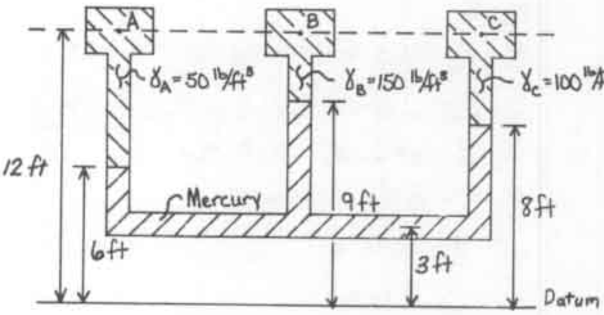
$$P_B - P_C = 150(-3) + 846(-1) + 100(4) = \underline{-896 \text{ lb/ft}^2}$$

$$P_C - P_A = 100(-4) + 846(-2) + 50(6) = \underline{-1792 \text{ lb/ft}^2}$$

$$P_A - P_B = 50(-6) + 846(3) + 150(3) = \underline{2688 \text{ lb/ft}^2}$$

9.19

Given: 3 tanks with different fluids as shown.
$P_A = 4000$ lb/in²



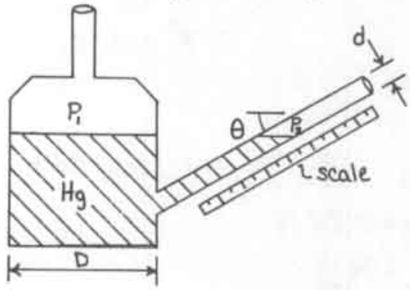
Find: the maximum pressure in the mercury.

$$P_{Hg} = 4000 \text{ lb/in}^2 \left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right) + (50 \text{ lb/ft}^3)(6 \text{ ft}) + (846 \text{ lb/ft}^3)(3 \text{ ft})$$

$$P_{Hg} = 578{,}838 \text{ lb/ft}^2 = \underline{4020 \text{ lb/in}^2}$$

9.20

Given: $P_1 = P_2 = P_{atm}$, $D = 50$ mm, $d = 5$ mm
2.2 N of Hg is added to the top tube. The scale reading increases by 200mm.



Find: θ

Find the volume of Hg added:

$$W_{Hg} = \gamma V \Rightarrow V = \frac{W}{\gamma} = \frac{2.2}{133\,000} = 1.654(10^{-5})\,m^3$$

$$\underline{V = 16\,541 \text{ mm}^3}$$

Find the split between additional Hg in the bowl and in the tube. h = increase in height of Hg in bowl.

$$V = h\left(\frac{\pi}{4}\right)D^2 + \frac{h}{\sin\theta}\left(\frac{\pi}{4}\right)d^2 \quad \text{where } \frac{h}{\sin\theta} = 200 \text{ mm}$$

$$V = \frac{\pi}{4}\left[(200\sin\theta)(50^2) + 200(5^2)\right]$$

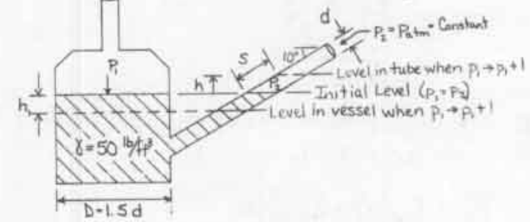
$$\sin\theta = \frac{\frac{4V}{\pi} - 200(5^2)}{200(50^2)} = \frac{\frac{4(16541)}{\pi} - 5000}{500\,000}$$

$$\sin\theta = 0.0321 \Rightarrow \underline{\theta = 1.841^\circ}$$

9.21

Given: Initially, $P_1 = P_2 = P_{atm}$. Then P_1 increases by 1.0 lb/ft² and P_2 does not change.

Find: The change S in the scale reading.



By the figure, $P_1 + 1 - 50(h_1 + h) = P_2$ (a)

or since $P_1 = P_2$, Eq. (a) yields: $50(h_1 + h) = 1$ (b)

Also, by the figure, $\left(\frac{\pi d^2}{4}\right)s = \frac{\pi}{4}(1.5d)^2 h_1$ or $h_1 = \frac{4}{9}s$ (c)

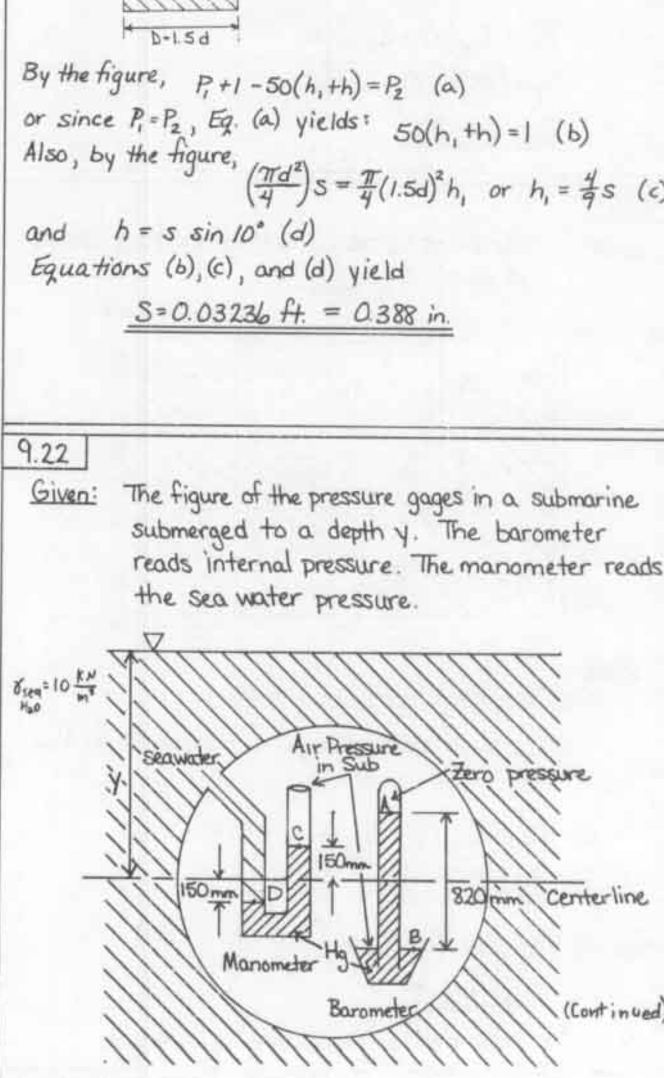
and $h = s \sin 10^\circ$ (d)

Equations (b), (c), and (d) yield

$$\underline{S = 0.03236 \text{ ft.} = 0.388 \text{ in.}}$$

9.22

Given: The figure of the pressure gages in a submarine submerged to a depth y. The barometer reads internal pressure. The manometer reads the sea water pressure.



(Continued)

9.22 (cont.)

Find: depth y

$P_A = 0$, $P_B = 0.820(133) = 109.06\ kPa$

$P_B = P_C$

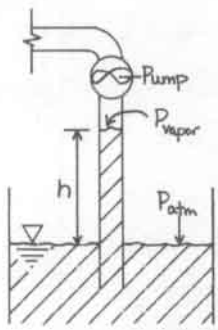
$P_D = P_C + (0.30)(133) = 109.06 + 39.9 = 148.96\ kPa$

$P_D = 10(y + 0.15) = 148.96\ kPa$

$\underline{y = 14.75\ m}$

9.23

Given: Water boils at 51.75 kPa absolute pressure. $\gamma = 9.49\ kN/m^3$



Find: the maximum height h that water can be lifted in a pipe.

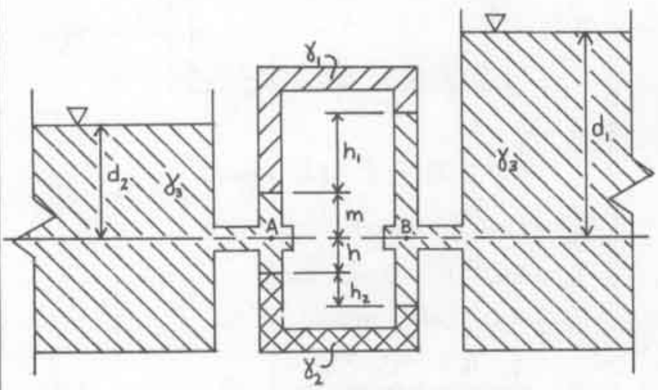
$P_{vapor} = P_{atm} - \gamma h$

$51.75 = 101.3 - 9.49h$

$\underline{h = 5.22\ m}$

9.24

Given: Two U-tubes connect two tanks as shown.



Find: γ_3 in terms of γ_1, γ_2, h_1, and h_2

For lower tube:

$P_A - P_B = -\gamma_3(h) + \gamma_2(-h_2) + \gamma_3(h + h_2)$

$P_A - P_B = h_2(\gamma_3 - \gamma_2)$

For upper tube:

$P_A - P_B = \gamma_3(m) + \gamma_1(h_1) - \gamma_3(m + h_1)$

$P_A - P_B = h_1(\gamma_1 - \gamma_3)$

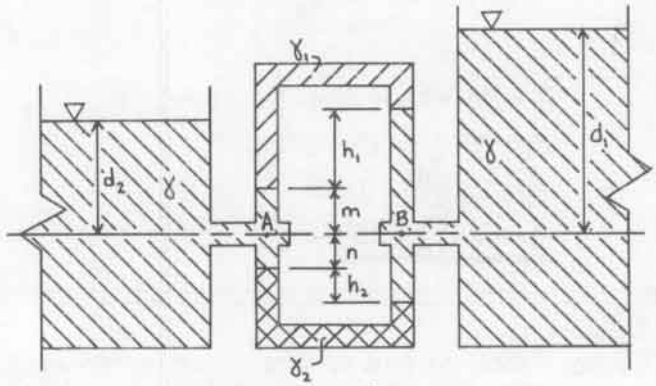
$h_2(\gamma_3 - \gamma_2) = h_1(\gamma_1 - \gamma_3)$

$\gamma_3(h_2 + h_1) = \gamma_2 h_2 + \gamma_1 h_1$

$$\underline{\underline{\gamma_3 = \frac{\gamma_2 h_2 + \gamma_1 h_1}{(h_2 + h_1)}}}$$

9.25

Given: Two U-tubes connect two tanks as shown.



Find: h_1 and h_2 in terms of $d_1, d_2, \gamma_1, \gamma_2$, and γ

$P_A = \gamma d_2$, $P_B = \gamma d_1$

$P_A - P_B = \gamma(d_2 - d_1)$

For the upper tube:

$P_A - P_B = \gamma m + \gamma_1 h_1 - \gamma(h_1 + m)$

$\gamma(d_2 - d_1) = h_1(\gamma_1 - \gamma)$

$$\underline{\underline{h_1 = \frac{\gamma(d_2 - d_1)}{(\gamma_1 - \gamma)}}}$$

For the lower tube:

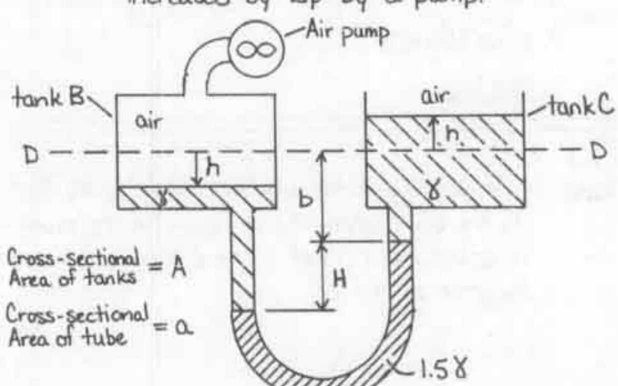
$P_A - P_B = -\gamma n - \gamma_2 h_2 + \gamma(n + h_2)$

$\gamma(d_2 - d_1) = h_2(\gamma - \gamma_2)$

$$\underline{\underline{h_2 = \frac{\gamma(d_2 - d_1)}{(\gamma - \gamma_2)}}}$$

9.26

Given: Initially the air in the two tanks shown is at atmospheric pressure. Pressure in tank B is increased by Δp by a pump.



Find: a.) h in terms of H, A, and a (continued)

b.) Δp in terms of H, A, a, and γ

9.26 (cont.)

a.) Volume of fluid pushed from tank B

$$V_B = Ah$$

Volume of fluid pushed into tube

$$V_t = \frac{aH}{2}$$

$$V_B = V_t \qquad \underline{\underline{h = \frac{aH}{2A}}}$$

b.)

$$P_c + 2\gamma h + \gamma b + (1.5\gamma)H - \gamma H - \gamma b = P_B$$

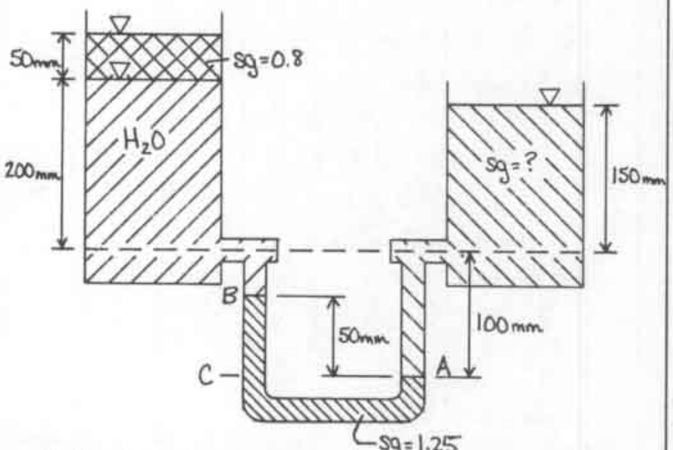
$$\Delta p = P_B - P_c = 2\gamma h + 0.5\gamma H$$

$$\Delta p = 2\gamma\left(\frac{aH}{2A}\right) + 0.5\gamma H$$

$$\underline{\underline{\Delta p = \gamma H\left(\frac{a}{A} + 0.5\right)}}$$

9.27

Given: *The right tank contains a fluid of unknown sg. It is connected to the left tube via a U-tube as shown.*



Find: the unknown sg

Let γ = specific weight of water

$$P_A = P_C$$

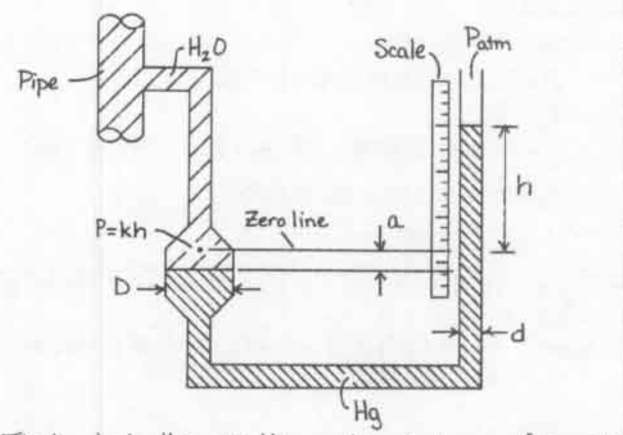
$$P_C = (0.8)(0.05)\gamma + (0.25)\gamma + (1.25)(0.05)\gamma = 0.3525\gamma$$

$$P_A = (sg)(0.25)\gamma$$

$$\underline{\underline{sg = 1.41}}$$

9.28

Given: *The mercury in the tube is aligned with that in the bowl when the pressure in the pipe is atmospheric. Let p be the pressure in the pipe at the zero line.*



Find: k in the equation p=kh, in terms of d and D.

Volume lost from bowl = Volume gained by tube

$$\frac{\pi}{4}D^2 a = \frac{\pi}{4}d^2 h$$

$$a = \frac{d^2 h}{D^2}$$

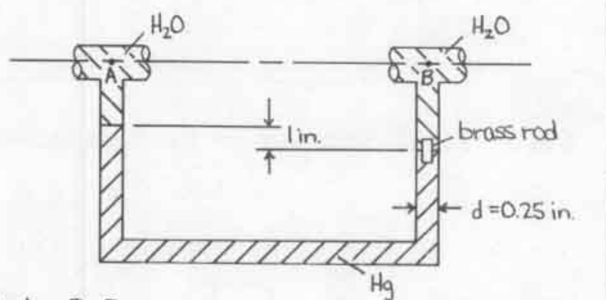
$$P = 133(h+a) - 9.78a$$

$$P = 133h + 123.22a = h\left(133 + 123.22\frac{d^2}{D^2}\right)$$

$$\underline{\underline{k = 133 + 123.22\frac{d^2}{D^2}}}$$

9.29

Given: *A small brass rod is dropped into a U-tube as shown.*



Find: $P_B - P_A$

What is $P_B - P_A$ if rod is removed?

$$\gamma_{H_2O} = 62.4 \text{ lb/ft}^3 = 0.03611 \text{ lb/in}^3$$

$$\gamma_{Hg} = 846 \text{ lb/ft}^3 = 0.490 \text{ lb/in}^3$$

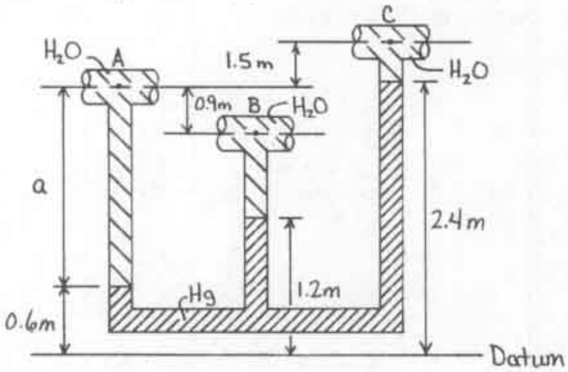
$$P_B - P_A = (1.0)(0.490 - 0.03611)$$

$$\underline{\underline{P_B - P_A = 0.453 \text{ lb/in}^2}}$$

The brass rod plays no role in the pressures in the pipes.

9.30

Given: the system of pipes and tubes shown



Find: $P_B - P_C$, $P_C - P_A$, and $P_A - P_B$

$\gamma_{H_2O} = 9.78 \ kN/m^3 = 9\,780 \ N/m^3$

$\gamma_{Hg} = 133 \ kN/m^3 = 133\,000 \ N/m^3$

Let a be the mercury level in the left tube below pipe A.

$$P_B = P_C + (1.5 + a + 0.6 - 2.4)\,9780 + 1.2(133000) - (a + 0.6 - 0.9 - 1.2)\,9780$$

$$\underline{\underline{P_B - P_C = 171\,300 \ N/m^2}}$$

$$P_C = P_A + a(9780) - (2.4 - 0.6)(133000) - (1.5 + a + 0.6 - 2.4)\,9780$$

$$\underline{\underline{P_C - P_A = -236\,500 \ N/m^2}}$$

$$P_A = P_B + (a + 0.6 - 1.2 - 0.9)(9780) + (1.2 - 0.6)(133000) - 9780a$$

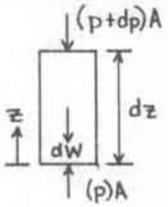
$$\underline{\underline{P_A - P_B = 65\,130 \ N/m^2}}$$

9.31

Given: The specific weight of air is $\gamma = kp$.

Find: Show that pressure p at altitude z is $p = p_o e^{-kz}$, where p_o is pressure at $z = 0$.

Draw a FBD of a vertical column of air.



$dW = \gamma A\,dz$

Sum the forces in the z-direction

$\Sigma F_z = pA - pA - dpA - dW = 0$

$A\,dp = -dW = -\gamma A\,dz = -kpA\,dz$

$\frac{dp}{p} = -k\,dz$

Integrate: $\ln p = -kz + c$

At $z = 0$, $p = p_o$

$\ln p_o = c$

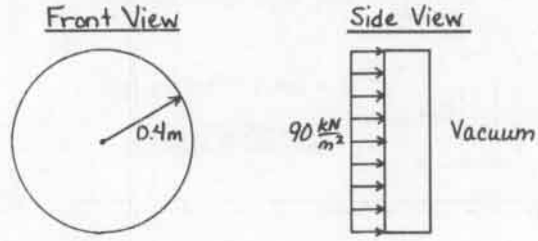
$\ln p = -kz + \ln p_o$

$\ln\left(\frac{p}{p_o}\right) = -kz$

$$\underline{\underline{p = p_o e^{-kz}}}$$

9.32

Find the force on a circular plate subjected to $90 \ kN/m^2$ on one side and vacuum on the other.



$F = P_o A$

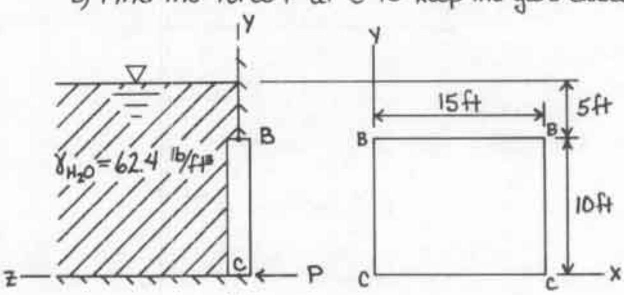
$F = (90)\pi(0.4^2)$ $\quad$ $\underline{F = 45.2 \ kN}$

9.33

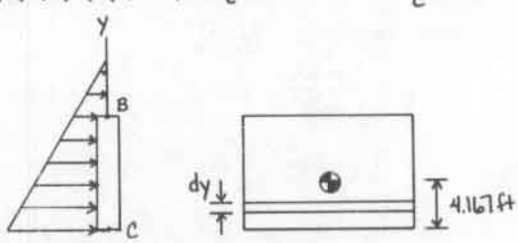
A 10 ft × 15 ft gate is hinged at B-B as shown

a) Find the center of pressure on the gate.

b) Find the force P at C to keep the gate closed.



a)



$P_B = 5\gamma = 5(62.4) = 312 \ lb/ft^2$

$P_C = 15\gamma = 15(62.4) = 936 \ lb/ft^2$

$F = \frac{1}{2}(312 + 936)(10)(15) = 93,600 \ lb.$ $\quad$ (a)

$y_p = -\frac{M_x}{F}$ $\quad$ (b)

By above figure

$M_x = -\int_0^{10} y\,(15)\,p\,dy$ $\quad$ (c)

To evaluate M_x, p must be a function of y.

$p = -62.4y + 936$ $\quad$ (d)

(Continued)

9.33 (cont.)

By equations (c) and (d):

$$M_x = -15\int_0^{10} y(-62.4y + 936)\,dy = -390{,}000 \text{ lbft} \quad (e)$$

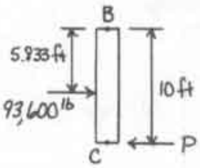
Combining (a), (b), and (e):

$$Y_p = \frac{-390{,}000 \text{ lbft}}{93{,}600 \text{ lb}} = 4.167 \text{ ft}$$

$$\underline{Y_p = 4.167 \text{ ft}}$$

By symmetry $\underline{X_p = 7.5 \text{ ft}}$

b.) Note that the moment about the hinge B-B of the forces acting on the gate must be zero.

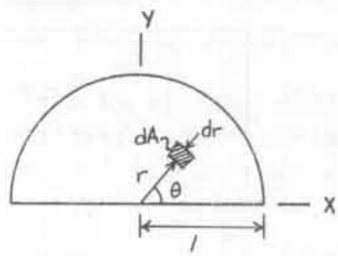


$$\Sigma M_B = -P(10) + 5.833(93{,}600) = 0$$

$$\underline{\underline{P = 54{,}600 \text{ lb.}}}$$

9.34

A semicircular plate carries pressure $p = 3r$. Find the resultant force and the center of pressure.



$dA = r\,dr\,d\theta$

$F = \int_A p\,dA \qquad F = \int_0^1\int_0^\pi 3r^2\,d\theta\,dr$

$$F = \int_0^1 3\pi r^2\,dr = \frac{3\pi r^3}{3}\Big|_0^1$$

$$\underline{F = \pi} \quad (a)$$

From Symmetry, $\underline{X_p = 0}$

$$Y_p = \frac{\int_A p y\,dA}{\int_A p\,dA} \quad (b)$$

$$\int_A p y\,dA = \int_0^1\int_0^\pi (3r^2)(r\sin\theta)\,d\theta\,dr$$

$$\int_0^1\int_0^\pi 3r^3\sin\theta\,d\theta\,dr = \int_0^1 -3r^3\left(\cos\theta\,\Big|_0^\pi\right)dr$$

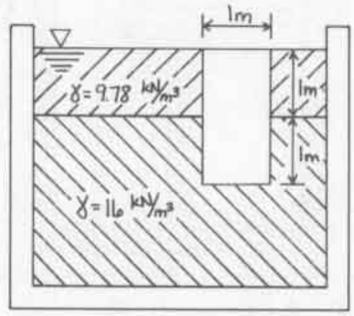
$$= \int_0^1 6r^3\,dr = \frac{6r^4}{4}\Big|_0^1 = \frac{3}{2} \quad (c)$$

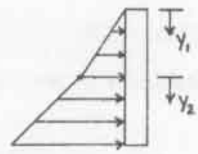
Now combining (a), (b), and (c);

$$Y_p = \frac{3/2}{\pi} \qquad \underline{\underline{Y_p = \frac{3}{2\pi}}}$$

9.35

Find the resultant hydrostatic force that acts on one side of the plate shown.





$$F = \int_0^1 9.78y_1\,dy_1 + \int_0^1 (16y_2 + 9.78)\,dy_2$$

$$= \frac{9.78}{2}y_1^2\Big|_0^1 + \left(\frac{16}{2}y_2^2 + 9.78y_2\right)\Big|_0^1$$

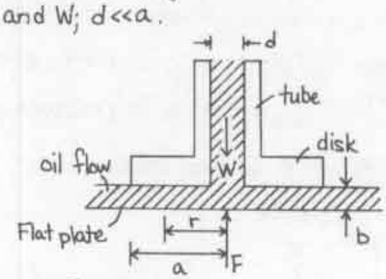
$$\underline{\underline{F = 22.7 \text{ kN}}}$$

9.36

Oil is pumped through a tube at a flow rate Q. The oil exits between a circular disk and a flat plate. Gage pressure is given by

$$p = \frac{6\mu_v Q}{\pi b^3}\left(\ln\frac{a}{r}\right)$$

The disk and tube weigh W. Find b in terms of μ_v, Q, a, and W; $d \ll a$.



$$F = \int_A p\,dA$$

$$p = \left(\frac{6\mu_v Q}{\pi b^3}\right)\ln\left(\frac{a}{r}\right), \quad dA = 2\pi r\,dr$$

$$F = \int_0^a \left(\frac{6\mu_v Q}{\pi b^3}\right)\ln\left(\frac{a}{r}\right)(2\pi r)\,dr$$

$$F = \frac{12\mu_v Q}{b^3}\int_0^a r\ln\left(\frac{a}{r}\right)dr$$

Use integration by parts

$u = \ln\left(\frac{a}{r}\right) \quad du = -\frac{1}{r}\,dr$

$dv = r\,dr \quad v = \frac{r^2}{2}$

(continued)

9.36 (cont.)

$$F=\frac{12\mu_v Q}{b^3}\left[\frac{r^2\ln\left(\frac{a}{r}\right)}{2}\Big|_0^a-\int_0^a-\frac{r}{2}\,dr\right]$$

$$F=\frac{12\mu_v Q}{b^3}\left[\frac{a^2}{4}\right]=\frac{3\mu_v Q a^2}{b^3}$$

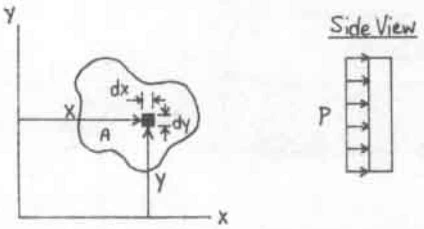
Now set $F=W$

$$\frac{3\mu_v Q a^2}{b^3}=W$$

$$\underline{\underline{b=\left(\frac{3\mu_v Q a^2}{W}\right)^{1/3}}}$$

9.37

Show that, for constant pressure on an area, the center of pressure coincides with the centroid.



$$X_p=\frac{\int_A p\,x\,dA}{\int_A p\,dA}=\frac{p\int_A x\,dA}{p\int_A dA} \qquad Y_P=\frac{\int_A p\,y\,dA}{\int_A p\,dA}=\frac{p\int_A y\,dA}{p\int_A dA}$$

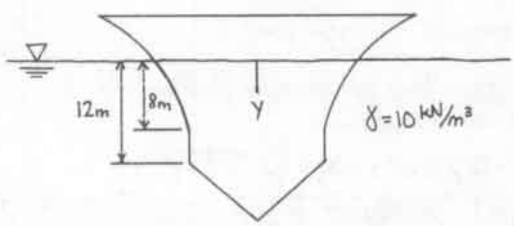
$$\underline{\underline{X_p=\frac{\int_A x\,dA}{\int_A dA}=\bar{X}}} \qquad \underline{\underline{Y_P=\frac{\int_A y\,dA}{\int_A dA}=\bar{Y}}}$$

9.38

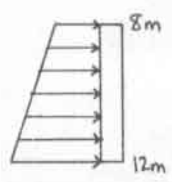
A 3m wide plate is 8m below water to its top edge and 12m below water to its bottom edge.

a) Find the pressure force on the plate.

b) Find the center of pressure.



a)



$$P_8=10(8)=80\ kN/m^2$$

$$P_{12}=10(12)=120\ kN/m^2$$

$$F=\tfrac{1}{2}(80+120)(3)(4)=1200\ kN$$

$$\underline{F=1200\ kN}$$

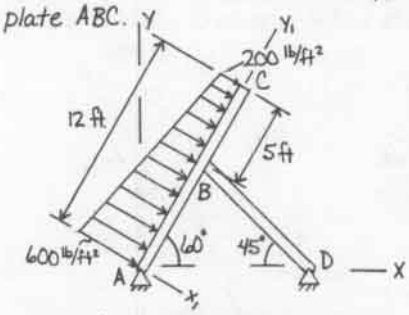
b) Y_P will coincide with the centroid of the trapezoid pressure diagram in part a.

Part	A_i	y_i	$A_i y_i$
1	80(4)	8+2	3200
2	½(40)(4)	8+⅔(4)	853
Total	400		4053

$\bar{Y}=\frac{4053}{400}$ $\quad$ $\underline{Y_P=10.13\,m}$ below water level

9.39

Find the force in rod BD that supports the rectangular plate ABC.



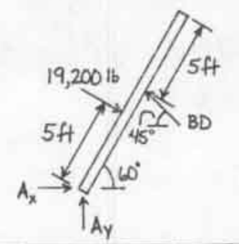
$$F=\tfrac{1}{2}(600+200)(12)(4)=19{,}200\ lb.$$

Center of pressure will be at the centroid of the trapezoidal area above.

Part	A_i	Y_i	$Y_i A_i$
1	200(12)	6	14,400
2	.5(400)(12)	4	9,600
Total	4800		24,000

$$\bar{Y}_{1i}=\frac{24{,}000}{4{,}800}=5\ ft$$

FBD of ABC



$$\Sigma M_A=0$$

$$\circlearrowleft+\ \Sigma M_A=-19{,}200(5)+BD(7)(\cos 15°)$$

$$BD=14{,}198\ lb$$

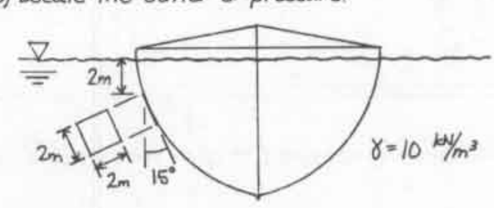
$$\underline{BD=14.20\ kips}$$

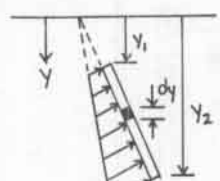
9.40

A 2m × 2m panel is located as shown.

a) Find the pressure force.

b) Locate the center of pressure.





a) $p=\gamma y$, $y_1=2\,m$

$$y_2=2+2\cos 15°=3.93\,m$$

$$dA=2\frac{dy}{\cos 15°}=2.071\,dy$$

$$F=\int_2^{3.93}\gamma y(2.071\,dy)$$

$$F=20.71\frac{y^2}{2}\Big|_2^{3.93}$$

$$\underline{F=118.5\ kN}$$

b) The center of pressure will coincide with the centroid of the area of the trapezoid in part a.

$p_1=2(10)=20\ kN/m^2$

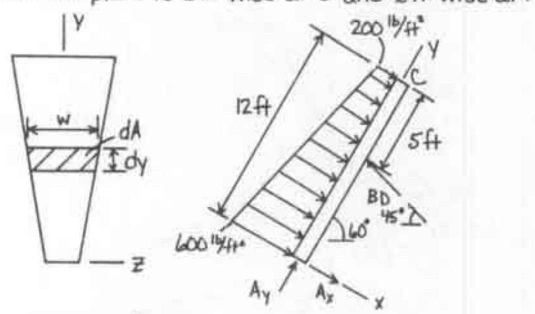
$p_2=(3.93)10=39.3\ kN/m^2$

Part	A_i	Y_i	$Y_i A_i$
1	20(2)	2+1(cos15°)	118.64
2	½(19.3)(2)	2+$\frac{4}{3}$(cos15)	63.46
Total	59.3		182.10

$\bar{Y}=\frac{182}{59.3}=\underline{3.07\,m\text{ below surface}}$

9.41

Find the force in rod BD that supports the trapezoidal plate ABC. The plate is 6 ft wide at C and 2 ft wide at A.



Find the pressure force.

$F=\int_A p\,dA \qquad p=600-33.3y$

$dA=w\,dy \qquad w=2+0.333y$

$F=\int_0^{12}(600-33.3y)(2+0.333y)\,dy$

$=\int_0^{12}(1200+133.33y-11.1y^2)\,dy$

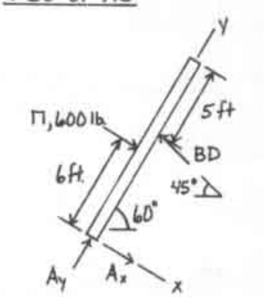
$=\left(1200y+133.33\frac{y^2}{2}-\frac{11.1y^3}{3}\right)\Big|_0^{12}=17{,}600\text{ lb.} \qquad (a)$

$Y_P=\frac{\int py\,dA}{F} \qquad \int py\,dA=\int_0^{12}(1200y+133.33y^2-11.1y^3)\,dy$

$=\frac{1200}{2}y^2+\frac{133.33}{3}y^3-\frac{11.1}{4}y^4\Big|_0^{12}=105{,}600\text{ lb·ft}$

$Y_P=\frac{105\,600\text{ lb·ft}}{17\,600\text{ lb}}=6.00\text{ ft}$

FBD of AC

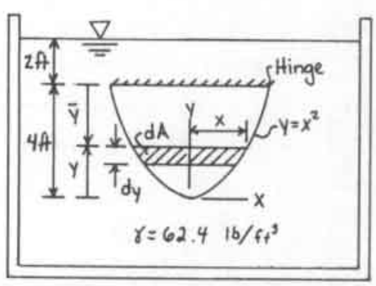


$\circlearrowleft\!+\ \Sigma M_A=-17600(6)+BD(7)\cos 15^\circ=0$

$\underline{\underline{BD=15{,}618\text{ lb.}=15.62\text{ kips}}}$

9.42

Find the moment required to open the parabolic gate shown.



To open the gate, the moment about the hinge must equal that due to hydrostatic pressure.

$dA=2x\,dy=2\sqrt{y}\,dy$

$p=62.4(6-y)=374.4-62.4y$

Moment arm for $p\,dA$ is $\bar{y}=4-y$.

$\therefore M=\int_0^4 p\bar{y}\,dA=\int_0^4 62.4(6-y)(4-y)(2\sqrt{y})\,dy$

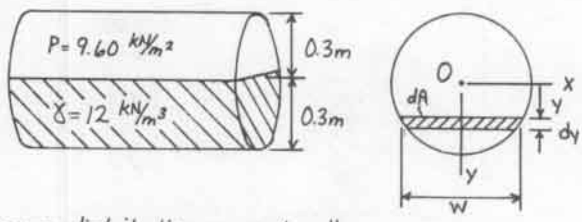
$M=124.8\int_0^4\left(24y^{1/2}-10y^{3/2}+y^{5/2}\right)dy$

$M=124.8\left(\frac{24y^{3/2}}{3/2}-\frac{10y^{5/2}}{5/2}+\frac{y^{7/2}}{7/2}\right)\Big|_0^4$

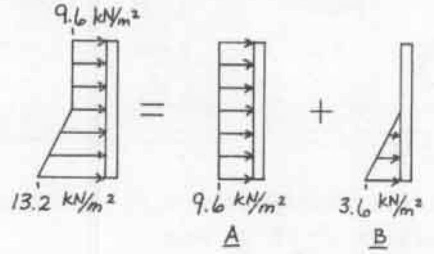
$\underline{\underline{M=4564\text{ lb·ft}}}$

9.43

The cylindrical tank is half full of liquid with $\gamma=12$ kN/m³. Air at 9.60 kN/m² fills the other half. Find the resultant force and the center of pressure on an end wall.



Pressure distribution on end wall



Find resultant of pressure distribution A:

$F_A=9.6\left(\frac{\pi}{4}\right)(0.6^2)=2.714\text{ kN}$ with center of pressure through point O.

Find resultant of pressure distribution B:
(Note: +y is down)

$p=12y\,,\ dA=w\,dy=\left(2\sqrt{0.3^2-y^2}\right)dy$

$F_B=\int_0^{0.3}(12y)\left(2\sqrt{0.3^2-y^2}\right)dy=24\int_0^{0.3}\left(y\sqrt{0.3^2-y^2}\right)dy$

$F_B=24\left[-\frac{1}{3}(0.3^2-x^2)^{3/2}\Big|_0^{0.3}\right]=0.216\text{ kN}$

Find center of pressure for F_B.

$M_B=\int yp\,dA=24\int_0^{0.3}\left(y^2\sqrt{0.3^2-y^2}\right)dy$

$M_B=24\left[-\frac{y}{4}(0.3^2-y^2)^{3/2}+\frac{0.3^2}{8}\left(y\sqrt{0.3^2-y^2}+0.3^2\sin^{-1}\frac{y}{0.3}\right)\right]_0^{0.3}$

$M_B=24\left[\frac{0.3^2}{8}(0.3^2)\frac{\pi}{2}\right]=0.03817$

$y_B=M_B/F_B=\frac{0.03817}{0.216}=0.1767\text{ m}$

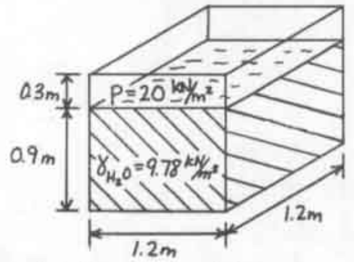
Find the resultant of the combined pressure distributions.

$F=F_A+F_B=2.714+0.216 \qquad \underline{F=2.93\text{ kN}}$

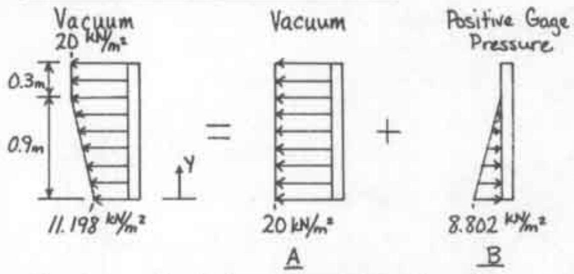
$y=\frac{F_Ay_A+F_By_B}{F_A+F_B}=\frac{2.714(0)+0.216(0.1767)}{2.93} \qquad \underline{y=0.01303\text{ m}}$

9.44

A closed cubical tank is 1.2 m high. It contains water ($\gamma_{H_2O} = 9.78\ kN/m^3$) to a depth of 0.9 m. Gas at $p = 20\ kN/m^2$ vacuum is above the water. Find the resultant force and the center of pressure on the tank sidewall.



Pressure distribution on sidewall:



Find the resultant of pressure distribution A:

$F_A = -20(1.2)(1.2) = -28.8$ kN with center of pressure at $y_A = 0.6$m.

Find the resultant of pressure distribution B:

$p = 8.802 - 9.78y, \quad dA = 1.2\,dy$

$F_B = \int_0^{0.9} (8.802 - 9.78y)1.2\,dy = 1.2\left[8.802y - 4.89y^2\right]_0^{0.9}$

$F_B = 4.75$ kN

Find the center of pressure for F_B.

$M_B = \int_0^{0.9} y\,p\,dA = 1.2\int_0^{0.9} y(8.802 - 9.78y)\,dy$

$M_B = 1.2\left[4.401y^2 - 3.26y^3\right]_0^{0.9} = 1.425$ kN·m

$y_B = \frac{M_B}{F_B} = \frac{1.425\ kN \cdot m}{4.75\ kN}$

$y_B = 0.30$ m

Find the resultant of the combined pressure distributions

$F = F_A + F_B = -28.8 + 4.75 \qquad \underline{F = -24.05\ kN}$

(Resultant is a net suction force)

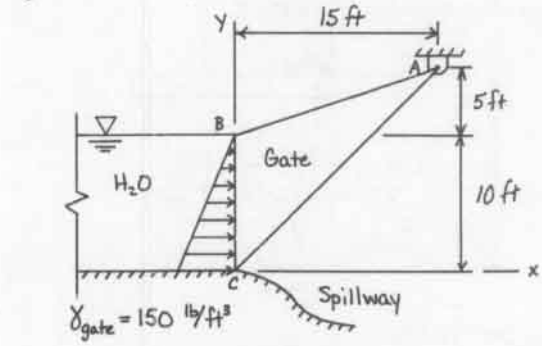
$y = \frac{F_A y_A + F_B y_B}{F}$

$y = \frac{-28.8(0.6) + 4.75(0.3)}{-24.05}$

$\underline{y = 0.659\ m}$

9.45

A 20 ft wide gate has a uniform specific gravity of 150 lb/ft³. Find the reaction at A and the force on the gate at C. Neglect friction.



$W_{gate} = (150)\left(\frac{10 \cdot 15}{2}\right)(20) = 225,000$ lb.

Gravity axis location:

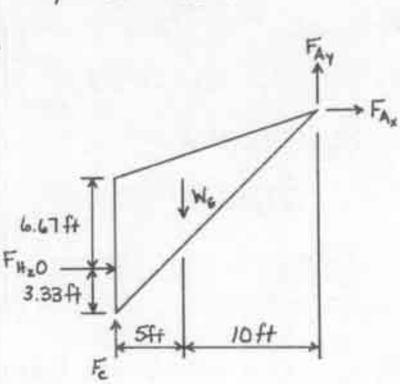
$x = 15/3 = 5$ ft.

$F_{H_2O} = 1/2\,(62.4)(10)(10)(20) = 62,400$ lb

Center of pressure:

$y = 10/3 = 3.33$ ft

FBD of gate:



$\Sigma F_x = F_{H_2O} + F_{A_x} = 0$

$62,400 + F_{A_x} = 0 \qquad \underline{\underline{F_{A_x} = -62,400\ lb}}$

$\circlearrowright\!+\ \Sigma M_A = F_{H_2O}(6.67 + 5) + W_G(10) - F_C(15) = 0$

$62400(11.67) + 225,000(10) = F_C(15)$

$\underline{\underline{F_C = 198,533\ lb}}$

$\Sigma F_y = -W_G + F_C + F_{A_y} = 0$

$-225,000 + 198,533 + F_{A_y} = 0$

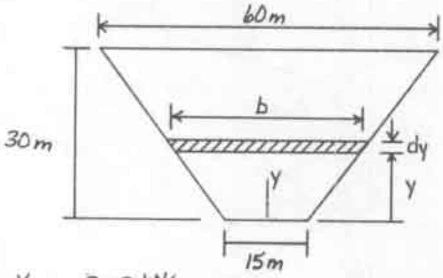
$\underline{\underline{F_{A_y} = 26,467\ lb}}$

9.46

A trapezoidal dam is 30 m high, 60 m wide at the top and 15 m wide at the bottom.

a) Find the total force on the dam.

b) Find the center of pressure.



a) $\gamma_{H_2O} = 9.78\ \text{kN/m}^3$

$$p = 9.78(30-y), \quad dA = b\,dy$$
$$b = 15 + 1.5y$$

$$F = \int p\,dA = \int_0^{30} 9.78(30-y)(15+1.5y)\,dy$$
$$F = 9.78\int_0^{30}(450 + 30y - 1.5y^2)\,dy$$
$$F = 9.78\left[450y + 15y^2 - 0.5y^3\right]_0^{30}$$
$$\underline{F = 132{,}030\ \text{kN}}$$

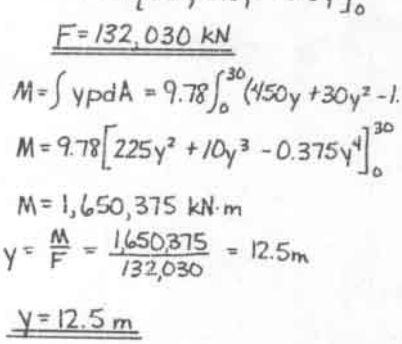
b) $$M = \int y\,p\,dA = 9.78\int_0^{30}(450y + 30y^2 - 1.5y^3)\,dy$$
$$M = 9.78\left[225y^2 + 10y^3 - 0.375y^4\right]_0^{30}$$
$$M = 1{,}650{,}375\ \text{kN}\cdot\text{m}$$
$$y = \frac{M}{F} = \frac{1{,}650{,}375}{132{,}030} = 12.5\text{m}$$
$$\underline{y = 12.5\ \text{m}}$$

9.47

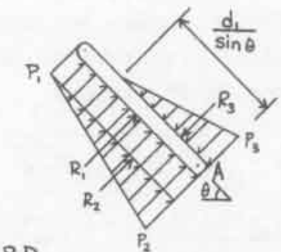
Gate AB is 2m × 4m and separates two resevoirs. It has weight W.

a) Find the force on the gate at B in terms of W, d_0, d_2, and θ.

b) For $\theta = 30°$, $d_2 = 6m$, $d_1 = 0$, find W required to cause the gate to open.



a) Pressure distributions and resultants on the gate



$P_1 = \gamma(d_2 - 2\sin\theta)$

$P_2 = \gamma d_2$

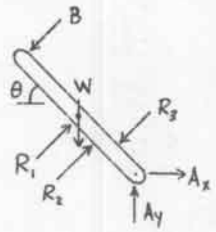
$P_3 = \gamma d_1$

$R_1 = (P_1)(2)(4) = 8P_1$ acting at 1.0m from A.

$R_2 = \frac{1}{2}(P_2 - P_1)(2)(4) = 4(P_2 - P_1)$ acting at 0.67m from A.

$R_3 = \frac{1}{2}(P_3)\left(\frac{d_1}{\sin\theta}\right)(4) = \frac{2d_1P_3}{\sin\theta}$ acting at $\frac{d_1}{3\sin\theta}$ from A.

Equilibrium of the gate:



$\circlearrowleft\!+\ \Sigma M_A = 0$

$$2B + W\cos\theta + R_3\left(\frac{d_1}{3\sin\theta}\right) - R_1 - \frac{2}{3}R_2 = 0$$
$$B = \frac{1}{2}R_1 + \frac{1}{3}R_2 - \frac{1}{2}W\cos\theta - \frac{R_3 d_1}{6\sin\theta}$$

Substitute for R_1, R_2, and R_3 to obtain:

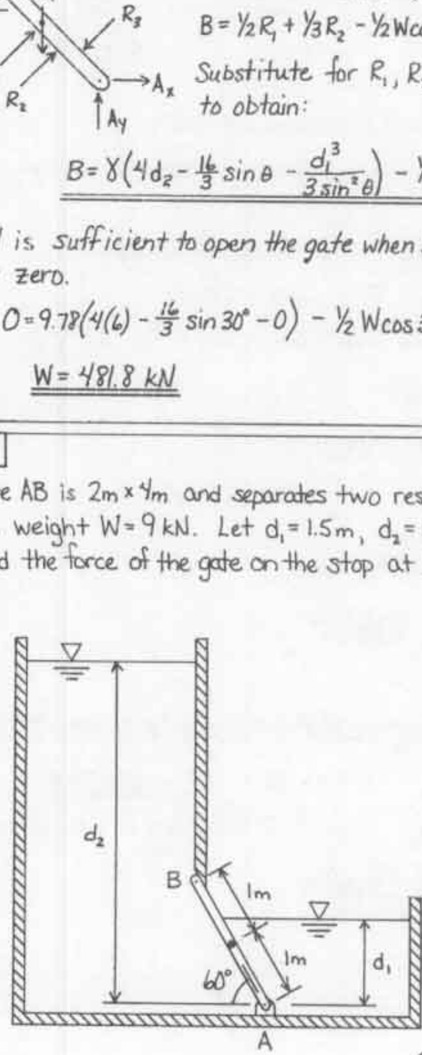
$$\underline{\underline{B = \gamma\left(4d_2 - \frac{16}{3}\sin\theta - \frac{d_1^3}{3\sin^2\theta}\right) - \frac{1}{2}W\cos\theta}} \quad (a)$$

b) W is *sufficient to open the gate when B in Eq. (a) is zero.*

$$0 = 9.78\left(4(6) - \frac{16}{3}\sin 30° - 0\right) - \frac{1}{2}W\cos 30°$$
$$\underline{\underline{W = 481.8\ \text{kN}}}$$

9.48

Gate AB is 2m × 4m and separates two reserviors. It has weight W = 9 kN. Let $d_1 = 1.5m$, $d_2 = 6m$, and $\theta = 60°$. Find the force of the gate on the stop at B. $\gamma = 9.78\ \text{kN/m}^3$



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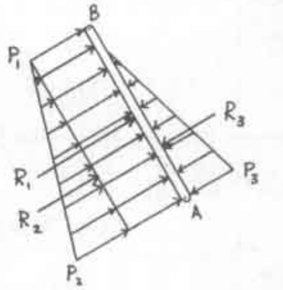
9.48 (cont.)

Pressure distributions and resultants on the gate.



$P_1 = \gamma(d_2 - 2\sin\theta) = 41.74\ kN/m^2$

$P_2 = \gamma d_2 = 58.68\ kN/m^2$

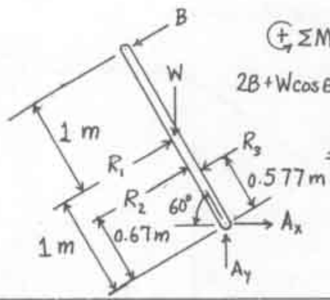
$P_3 = \gamma d_1 = 14.67\ kN/m^2$

$R_1 = P_1(2)(4) = 333.9\ kN$ acting at 1.0m from A.

$R_2 = \frac{1}{2}(P_2 - P_1)(2)(4) = 67.76$ kN acting at 0.67m from A.

$R_3 = \frac{1}{2}P_3\left(\frac{d_1}{\sin\theta}\right)(4) = 50.82\ kN$ acting at $\frac{d_1}{\sin\theta} = 0.577$ m from A.

Equilibrium of the gate:



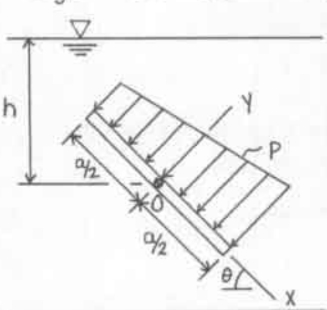
$\circlearrowleft^{+} \Sigma M_A = 0$

$2B + W\cos\theta + R_3(0.577) - R_1 - 0.67R_2 = 0$

$\underline{B = 172.6\ kN}$

9.49

A rectangular plate is submerged in a *liquid* at an angle θ as shown. Derive a formula for the distance between the center of the plate and the center of pressure for the pressure force on one face of the plate.



γ = Specific weight of the fluid.

b = Width of the plate.

$p = \gamma(h + x\sin\theta)$

Resultant pressure force:

$$F = \int p\,dA = \int_{-a/2}^{a/2} \gamma(h + x\sin\theta)b\,dx$$

$$F = \gamma b\left[hx + \frac{x^2}{2}\sin\theta\right]_{-a/2}^{a/2}$$

$$F = \gamma b\left[\frac{ah}{2} + \frac{a^2}{8}\sin\theta - \left(-\frac{ah}{2} + \frac{a^2}{8}\sin\theta\right)\right]$$

$\underline{F = \gamma abh}$

Moment of the pressure force about O.

$$M_o = \int p\,x\,dA = \int_{-a/2}^{a/2} \gamma(hx + x^2\sin\theta)b\,dx$$

$$M_o = \gamma b\left[\frac{hx^2}{2} + \frac{x^3}{3}\sin\theta\right]_{-a/2}^{a/2}$$

$$M_o = \gamma b\left[\frac{ha^2}{8} + \frac{a^3}{24}\sin\theta - \left(\frac{ha^2}{8} - \frac{a^3}{24}\sin\theta\right)\right]$$

$\underline{M_o = \frac{\gamma b a^3}{12}\sin\theta}$

Location of center of pressure

$$x_p = \frac{M_o}{F} = \frac{\frac{\gamma b a^3 \sin\theta}{12}}{\gamma abh}$$

$$\underline{\underline{x_p = \frac{a^2}{12h}\sin\theta}}$$

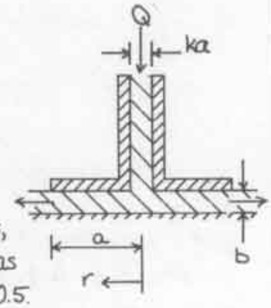
9.50

Oil is pumped through a tube and flows out between a circular disk and a flat plate. The flow rate is Q.

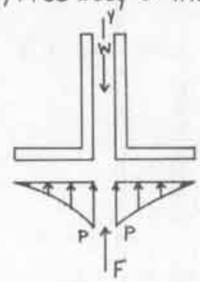
$p = (6\mu_v Q/\pi b^3)\ln(a/r)$

a.) Derive a formula for gap b in terms of μ_v, k, Q, a, and W.

b.) For $\mu_v = 0.007\ \frac{N\cdot s}{m^2}$, $Q = 0.1\ m^3/s$, $a = 300\ mm$, $W = 20$ N, plot b as a function of k for $0.1 < k < 0.5$.



a.) Free Body of the tube and disk



Note that the pressure on the disk begins at $r = \frac{ka}{2}$ and ends at $r = a$.

Let the resultant pressure force be F.

For equilibrium of the tube and disk

$\Sigma F_y = 0:\quad F - W = 0\qquad \underline{F = W}$

Find F:

$$F = \int p\,dA\ ,\quad p = \frac{6\mu_v Q}{\pi b^3}\ln\left(\frac{a}{r}\right),\quad dA = 2\pi r\,dr$$

$$F = \frac{12\mu_v Q}{b^3}\int_{\frac{ka}{2}}^{a} r\ln\left(\frac{a}{r}\right)dr = \frac{12\mu_v Q}{b^3}\int_{\frac{ka}{2}}^{a} r(\ln a - \ln r)\,dr$$

$$F = \frac{12\mu_v Q}{b^3}\left[\frac{r^2}{2}\ln a - \left(\frac{r^2}{2}\ln r - \frac{r^2}{4}\right)\right]_{\frac{ka}{2}}^{a}$$

$$F = \frac{12\mu_v Q}{b^3}\left[\frac{a^2}{4} - \frac{k^2a^2}{8}\left(\ln\left(\frac{2}{k}\right) + \frac{1}{2}\right)\right]$$

Substitute F=W and solve for b:

$$\underline{\underline{b = \left\{\frac{12\mu_v Q}{W}\left[\frac{a^2}{4} - \frac{k^2a^2}{8}\left(\ln\left(\frac{2}{k}\right) + \frac{1}{2}\right)\right]\right\}^{1/3}}}$$

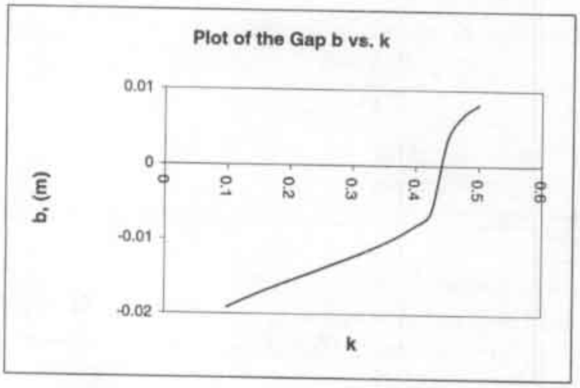
(Continued)

9.50 (cont.)

b.) $\mu_v = 0.007\ \frac{N\cdot s}{m^2}$, $Q = 0.1\ m^3/s$, $a = 0.3m$, $W = 20N$

$$\therefore\ b = \left\{ \frac{12(.007)(0.1)}{20}\left[\frac{(0.3)^2}{4} - \frac{k^2(0.3)^2}{8}\left(\ln\left(\frac{2}{k}\right)+\frac{1}{2}\right)\right]\right\}^{1/3}$$

$$\underline{\underline{b = \left\{ 9.45(10^{-6}) - 4.725(10^{-6})\left[\ln\left(\frac{2}{k}\right)+\frac{1}{2}\right]\right\}^{1/3}}}$$

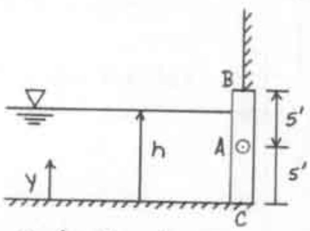


9.51

a.) For the gate in Example 9.8 (shown below) plot the force P as a function of h for $0 \le h \le 15 ft$.

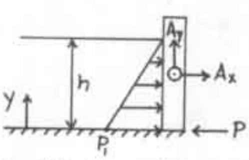
b.) Find the maximum value of P and the corresponding value(s) of h.

c.) Check your results for $h = 5ft$, $10ft.$, and $15ft$.



First break up the function into 2 parts.
$0 \le h \le 10ft$ and $10ft \le h \le 15ft$

For $0 \le h \le 10ft.$, the pressure distribution on the gate is triangular.



Resultant Pressure Force

$P_1 = \gamma h$, $F = \frac{1}{2}(15)P_1 h = \left(\frac{15}{2}\right)\gamma h^2$

Center of Pressure

$y_p = h/3$

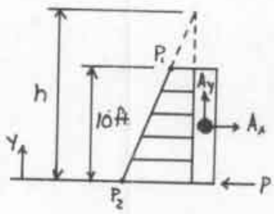
Equilibrium of the gate.

$\Sigma M_A = 0 \quad F(5-y_p) - 5P = 0$

$P = F(1 - \frac{y_p}{5}) = \left(\frac{15}{2}\right)\gamma h^2\left(1 - \frac{h}{15}\right)$

$\underline{\underline{P = 468h^2 - 31.2h^3}}$

Now for $10ft \le h \le 15ft$, the pressure distribution on the gate is trapezoidal.



Resultant Pressure Force

$P_1 = \gamma(h-10)$, $P_2 = \gamma h$

$F = 15\left(\frac{P_1+P_2}{2}\right)(10) = 150\gamma(h-5)$

Center of Pressure (From Table D.2)

$y_p = 10 - \frac{10(2P_2+P_1)}{3(P_2+P_1)} = \frac{15h-100}{3h-15}$

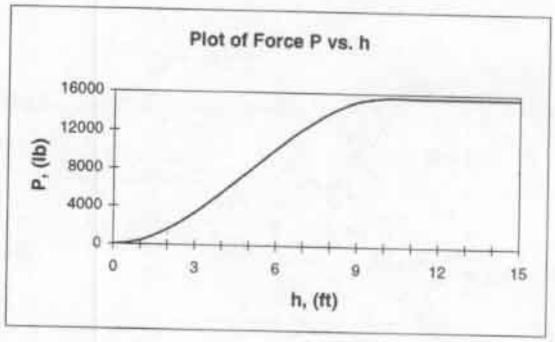
(After Simplifying)

Equilibrium of the gate.

$\Sigma M_A = 0 \quad F(5-y_p) - 5P = 0$

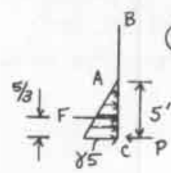
$150\gamma(h-5)\left[5 - \left(\frac{15h-100}{3h-15}\right)\right] = 5P$

$\underline{\underline{P = 250\gamma = 15,600\ lb}}$



b.) $\underline{\underline{P_{max} = 15,600\ lb \text{ for } 10ft \le h \le 15ft.}}$

C.) For $h = 5ft$.

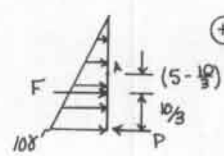


$\Sigma M_A = -P(5) + \frac{1}{2}(5\gamma)(5)(15)(5 - \frac{5}{3}) = 0$

$5P = 39000\ lb\,ft$

$\underline{\underline{P = 7800\ lb}}$ ✓

For $h = 10ft$.

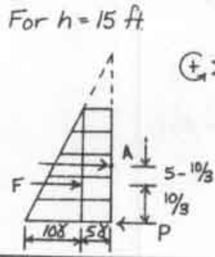


$\Sigma M_A = -5P + \frac{1}{2}(10\gamma)(10)(15)(5 - \frac{10}{3}) = 0$

$5P = 78,000\ lbft$

$\underline{\underline{P = 15,600\ lb}}$ ✓

For $h = 15ft$



$\Sigma M_A = -5P + \frac{1}{2}(10\gamma)(10)(15)(5 - \frac{10}{3}) = 0$

$5P = 78,000\ lbft$

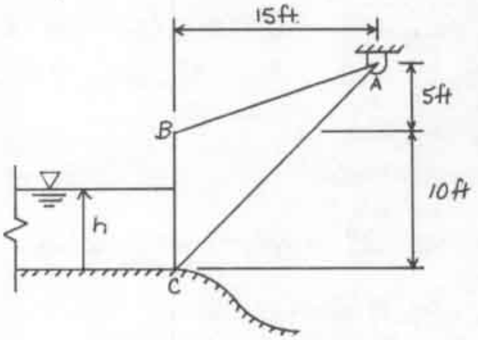
$\underline{\underline{P = 15,600\ lb}}$ ✓

9.52

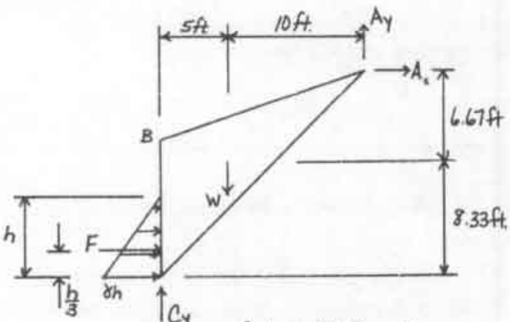
Water is slowly filled from point C to point B. Neglect friction

a) Plot the support reactions at A and C as functions of the height h of water for $0 \le h \le 10$ ft in increments of 0.5 ft.

b) Discuss results with respect to requirements for the supports at A and C.



FBD of Gate



Gate is 20 ft wide.

$W = 150\ \text{lb/ft}^3\left(\frac{10\text{ft}\cdot 15\text{ft}}{2}\right)(20\text{ft}) = 225{,}000$ lb.

$\Sigma F_x = A_x + F = 0$

$A_x = -\frac{1}{2}(\gamma h)(h)(20)$, $\gamma = 62.4\ \text{lb/ft}^3$

$\underline{\underline{A_x = -624h^2}}$ (lb)

$\Sigma M_A = W(10) - C_y(15) + F(15 - \frac{1}{3}h) = 0$

$15C_y = 2{,}250{,}000 + 624h^2(15 - \frac{h}{3})$

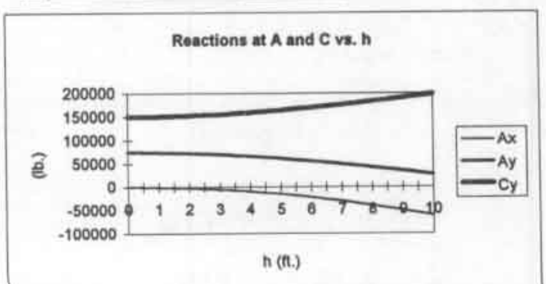
$\underline{\underline{C_y = 150{,}000 + 624h^2 - 13.87h^3}}$ (lb)

$\Sigma F_y = -W + C_y + A_y = 0$

$A_y = W - C_y$

$A_y = 225{,}000 - 150{,}000 - 624h^2 + 13.87h^3$

$\underline{\underline{A_y = 13.87h^3 - 624h^2 + 75{,}000}}$ (lb)

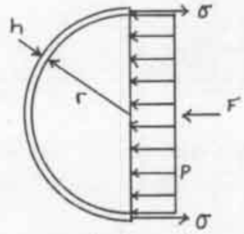


b) C increases with added height. So, the gate will tend to wedge itself tight at C as the reservior is filled. The vertical reaction at A is maximum when the reservior is empty but the horizontal reaction is maximum when the reservior is full.

9.53

A thin-walled spherical shell with an internal radius of $r = 10$ in. and a wall thickness of $h = 0.2$ in. is subjected to an internal pressure p. The fracture stress is 60,000 psi. Find the internal pressure that causes fracture.

FBD of a hemisphere



$F = \pi r^2 p = 100\pi p$ lb.

Find the cross-sectional area of shell

$A = (10.1)(2\pi)(0.2) = 4.04\pi\ \text{in}^2$

For equilibrium of shell:

$\Sigma F_x = \sigma A - F = 0$ $\qquad \sigma(4.04\pi) = 100\pi p$

$p = \frac{60{,}000(4.04\pi)}{100\pi}$ $\qquad \underline{\underline{p = 2424\ \text{psi}}}$

9.54

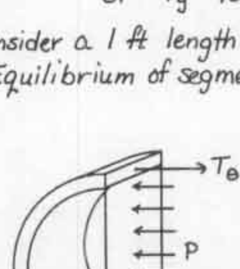
An airplane fuselage of circular cross section contains air at 12 lb/in² absolute pressure. Atmospheric pressure is 9 lb/in². The radius of the fuselage is 4.5 ft. Find the hoop tension.

Gage pressure is $P_g = 12 - 9 = 3\ \text{lb/in}^2$

or $P_g = 432\ \text{lb/ft}^2$

Consider a 1 ft length of fuselage:

Equilibrium of segment gives:



$\Sigma F = 0$: $2T_\theta = 2r(1)p = 0$

$T_\theta = pr$

$T_\theta = 432(4.5)$

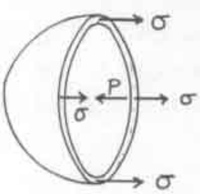
$\underline{\underline{T_\theta = 1944\ \text{lb/ft}}}$

(For each foot of length of fuselage)

9.55

A thin-wall spherical shell with radius r and thickness h ($h \ll r$) has internal pressure p. Find the average tensile stress in the wall in terms of p, h, and r. Neglect weight.

FBD of a hemispherical shell:



For equilibrium

$\Sigma F = 0:$

$\sigma A_1 - p A_2 = 0$

$\sigma = p\left(\frac{A_2}{A_1}\right)$

$A_1 = (2\pi r)h$

$A_2 = \pi r^2$

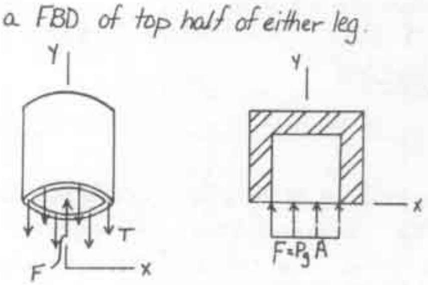
$\therefore\ \sigma = p\,\frac{\pi r^2}{2\pi r h}$

$$\underline{\underline{\sigma = \frac{pr}{2h}}}$$

9.56

A 6 in. diameter pipe with closed ends is cast into a U-shape and pressurized to $P_g = 150$ psi. Find the tensile force in a leg of the pipe.

Take a FBD of top half of either leg.



Resultant pressure force on the closed end is

$F = 150\,(\pi r^2)$ $\quad F = 4241$ lb

For equilibrium

$\Sigma F_y = 0: \quad F - T = 0$

$\underline{T = 4241 \text{ lb.}}$

9.57

A hollow conical shell weighs 450 N and rests on the bottom of a tank of water. Pressure is 87.3 kPa inside the shell. Find the force required to pull the shell from the bottom of the tank.

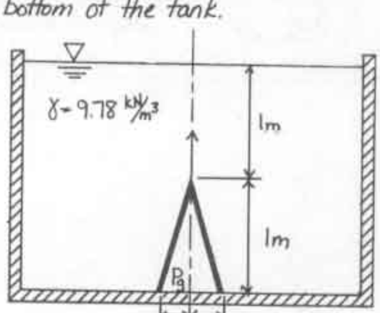
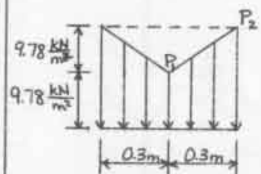


Figure a.

The shell is held down by the forces due to atmospheric pressure (F_{atm}), the weight (W_w) of water above, and by its own weight (W_s). It is pushed up by the force (F_{in}) due to the pressure inside. Therefore, the force P required to pull the shell loose is:

$$P = W_w + W_s + F_{atm} - F_{in} \qquad (a)$$

The water pressure on the shell is shown in Fig. b.



$P_1 = 9.78\,\frac{kN}{m^3}\,(1m) = 9.78\ kN/m^2$

$P_2 = 9.78\,\frac{kN}{m^3}\,(2m) = 19.56\ \frac{kN}{m^2}$

Figure b → Water Pressure

By Figure b and Table D4,

$$W_w = (19.56)\pi(0.30)^2 - (9.78)\left(\tfrac{1}{3}\right)\pi(0.30)^2 = 4.609\ kN \qquad (b)$$

By Figure a,

$$F_{atm} = \pi(0.30)^2(101.3) = 28.64\ kN \quad (c)$$

$$F_{in} = \pi(0.25)^2(87.3) = 17.14\ kN \quad (d)$$

Hence, with $W_s = 0.450$ kN, Equations (a),(b),(c), and (d) yield

$P = 28.64 + 0.450 + 28.64 - 17.14$

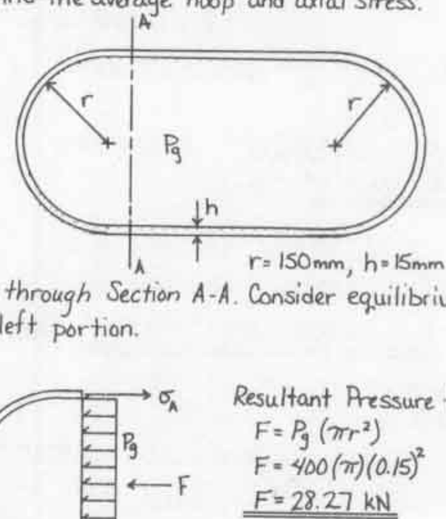
or $\quad \underline{P = 16.56\ kN}$

9.58

A cylindrical tank with hemispherical end caps has internal pressure $P_g = 400$ kPa.

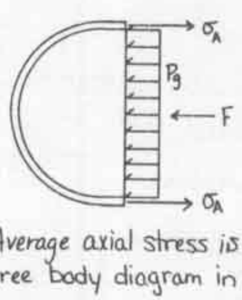
a) Find the pressure force on an end cap.

b) Find the average hoop and axial stress.



$r = 150$mm, $h = 15$mm

a) Cut through Section A-A. Consider equilibrium of the left portion.



Resultant Pressure force is:

$F = P_g\,(\pi r^2)$

$F = 400(\pi)(0.15)^2$

$\underline{\underline{F = 28.27\ kN}}$

b) Average axial stress is found from equilibrium of the free body diagram in part (a).

$\Sigma F = 0: \quad \sigma_A A_c - F = 0$

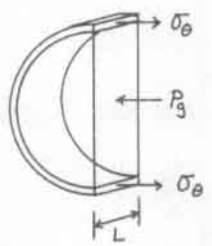
$\sigma_A\left[(2\pi)(r + h/2)h\right] - F = 0$

$$\underline{\underline{\sigma_A = 1905\ kPa = 1.905\ MPa}}$$

(Continued)

9.58 (cont.)

For average hoop stress: Cut free body as shown.



$\Sigma F = 0: \ \sigma_\theta(2Lh) - P_g(2rL) = 0$

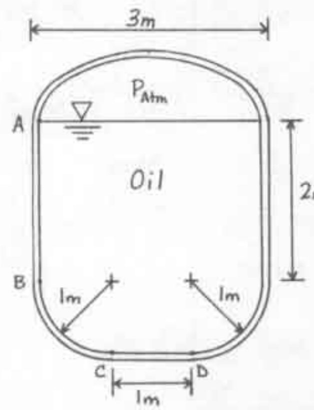
$\sigma_\theta = P_g \frac{r}{h}$

$\sigma_\theta = 10 P_g$

$\underline{\underline{\sigma_\theta = 4000 \text{ kPa} = 4.00 \text{ MPa}}}$

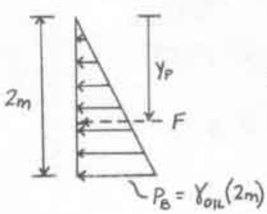
9.59

Oil with s.g. = 0.84 fills a bay of a tanker. The free surface of the oil is subjected to atmospheric pressure. Find the magnitudes and orientations of the pressure forces on AB, BC, and CD.



$\gamma_{OIL} = 0.84(\gamma_{H_2O}) = 0.84(9.78) = 8.22 \text{ kN/m}^3$

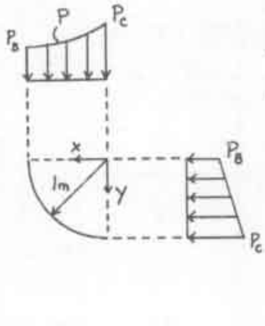
Segment AB



$F = \frac{1}{2}(8.22)(2)(2)$

$\underline{\underline{F = 16.43 \text{ kN/m} \ @ \ \theta = 0^\circ}}$ from horizontal

Segment BC



$P_B = 2\gamma_{OIL} = 16.43 \text{ kN/m}^2$

$P_C = 3\gamma_{OIL} = 24.65 \text{ kN/m}^2$

$P = \gamma_{OIL}\left(2 + \sqrt{R^2 - x^2}\right)$

X projection of resultant:

$\underline{F_x = \frac{1}{2}(P_B + P_C)(1) = 20.54 \text{ kN/m}}$

Y projection of resultant:

$F_y = \int_0^1 p\,dx = \int_0^1 \gamma_{OIL}\left[2 + \sqrt{R^2 - x^2}\right] dx$

$F_y = 8.22\left[2x\Big|_0^1 + \frac{1}{2}\left(x\sqrt{R^2 - x^2} + R^2 \sin^{-1}\left(\frac{x}{R}\right)\right)\Big|_0^1\right]$ with $R = 1$

$F_y = 8.22\left[2 + \pi/4\right]$

$\underline{F_y = 22.90 \text{ kN/m}}$

Magnitude:

$F = \sqrt{F_x^2 + F_y^2}$

$F = \sqrt{20.54^2 + 22.90^2}$

$\underline{F = 30.76 \text{ kN/m}}$

Orientation:

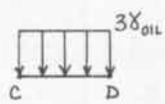
$\tan\theta = \frac{F_y}{F_x}$

$\tan\theta = 1.15$

$\underline{\theta = 48.11^\circ}$

$\underline{\underline{F = 30.8 \text{ kN/m} \ @ \ \theta = 48.1^\circ}}$

Segment CD



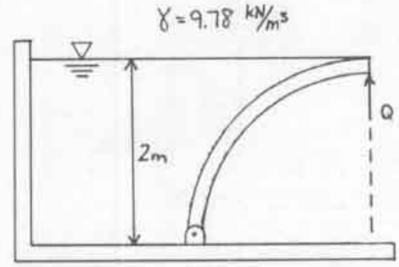
$F = 3(8.22) = 24.7 \frac{\text{kN}}{\text{m}}$

$\underline{\underline{F = 24.7 \frac{\text{kN}}{\text{m}} \ @ \ \theta = 90^\circ}}$

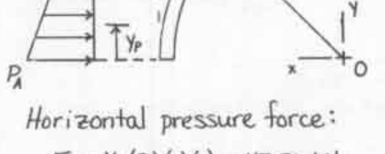
9.60

The quarter-circle water gate is 6m long and is hinged at A. Find Q required for equilibrium.

$\gamma = 9.78 \text{ kN/m}^3$



Pressure force projections



$P_A = 2\gamma = 19.56 \text{ kN/m}^2$

$d = R - \sqrt{R^2 - x^2}$

$P = \gamma d = \gamma\left(R - \sqrt{R^2 - x^2}\right)$

$R = 2m$

(Note: Left-Hand Coordinates)

Horizontal pressure force:

$F_x = \frac{1}{2}(P_A)(2)(6) = 117.36 \text{ kN}$ $\quad y_P = 0.67m$

Vertical pressure force:

$F_y = \int_0^R 6p\,dx = 6\gamma\int_0^2 \left(2 - \sqrt{2^2 - x^2}\right) dx$

$F_y = 6\gamma\left[2x\Big|_0^2 - \frac{1}{2}\left(x\sqrt{4 - x^2} + 4\sin^{-1}\left(\frac{x}{2}\right)\right)\Big|_0^2\right]$

$F_y = 6\gamma\left[4 - \pi\right] = 50.37 \text{kN}$

(Continued)

9.60 (cont.)

Resultant axis for F_y:

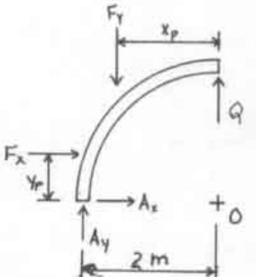
$$M_o = \int_0^R x(6p)\,dx = 6\gamma\int_0^2 \left(2x - x\sqrt{2^2-x^2}\right)dx$$

$$M_o = 6\gamma\left[x^2\Big|_0^2 - \left(-\tfrac{1}{3}\sqrt{(2^2-x^2)^3}\right)\Big|_0^2\right]$$

$$M_o = 6\gamma\left[4 - \tfrac{8}{3}\right] = 78.24\ \text{kN}\cdot\text{m}$$

$$x_P = \frac{M_o}{F_y} = \frac{78.24}{50.37} = 1.553\ \text{m}.$$

Equilibrium of the gate:



$\circlearrowleft^{+} \Sigma M_A = 0$:

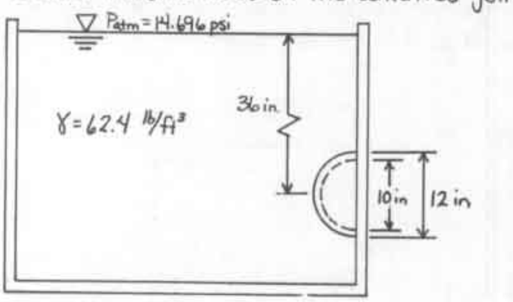
$$2Q - (2-x_p)F_y - y_pF_x = 0$$

$$Q = \frac{(2-x_p)F_y + y_pF_x}{2}$$

$\underline{\underline{Q = 50.38\ \text{kN}}}$

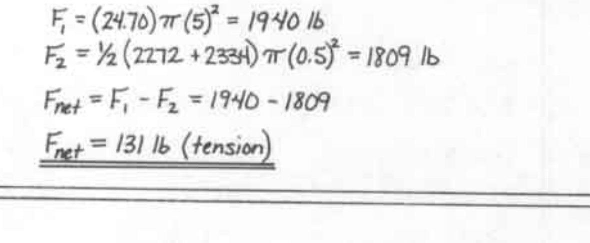
9.61

A hollow hemispherical shell is cemented to a vertical wall. The shell contains gas at $P_g = 10$ psi. Find the net tensile force on the cemented joint.



In order to find the tensile force, the only forces required are horizontal. Vertical forces apply shear to the joint.

FBD of hemisphere:



$$F_1 = (24.70)\pi(5)^2 = 1940\ \text{lb}$$

$$F_2 = \tfrac{1}{2}(2272 + 2334)\pi(0.5)^2 = 1809\ \text{lb}$$

$$F_{net} = F_1 - F_2 = 1940 - 1809$$

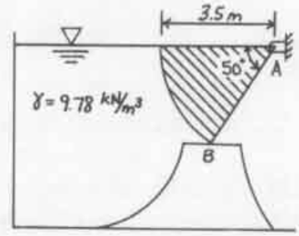
$\underline{\underline{F_{net} = 131\ \text{lb (tension)}}}$

9.62

Water is at the top of the gate shown. The gate is 3 m long, weighs 256 kN, and has a circular sector cross section.

a) Find the magnitude and line of action of the pressure force on the gate.

b) Find the moment required to raise the gate.



a) Projections of pressure force

$d_B = 3.5\sin 50^\circ$

$d_B = 2.68$ m

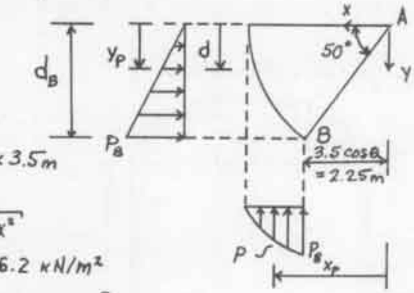
$d = \sqrt{3.5^2 - x^2}$

for $2.25 < x < 3.5$ m

$\therefore\ p = \gamma d$

$p = \gamma\sqrt{3.5^2 - x^2}$

$P_B = \gamma d_B = 26.2$ kN/m²



Horizontal pressure force:

$$F_x = \tfrac{1}{2}(P_B)(d_B)(3)$$

$F_x = 105.4$ kN at depth $y_P = \tfrac{2}{3}(2.68)$

$\underline{\underline{F_x = 105.4\ \text{kN} \ @\ y_P = 1.79\ \text{m}}}$

Vertical pressure force:

$$F_y = -\int_{2.25}^{3.5} 3p\,dx = -3\gamma\int_{2.25}^{3.5}\sqrt{3.5^2-x^2}\,dx$$

$$F_y = -3\gamma\left[\tfrac{1}{2}\left(x\sqrt{3.5^2-x^2} + 3.5^2\sin^{-1}\left(\tfrac{x}{3.5}\right)\right)\right]_{2.25}^{3.5}$$

$\underline{F_y = -3\gamma\left[2.329\right] = -68.32\ \text{kN}}$ (upward)

Center of vertical pressure:

$$\circlearrowleft^{+} M_A = -\int_{2.25}^{3.5} 3px\,dx = -3\gamma\int_{2.25}^{3.5} x\sqrt{3.5^2-x^2}\,dx$$

$$M_A = -3\gamma\left[-\tfrac{1}{3}\sqrt{(3.5^2-x^2)^3}\right]_{2.25}^{3.5}$$

$$M_A = -3\gamma\left[6.423\right] = -188.45\ \text{kN}\cdot\text{m}$$

$\underline{x_p = \frac{M_A}{F_y} = 2.76\ \text{m}}$

Resultant force and center of pressure

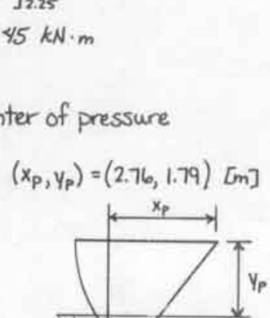
$F = \sqrt{F_x^2 + F_y^2}$

$\underline{\underline{F = 125.6\ \text{kN}}}$

$(x_P, y_P) = (2.76, 1.79)$ [m]

$\tan\theta = \frac{-F_y}{F_x}$

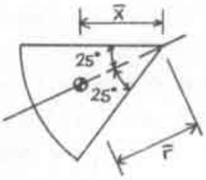
$\underline{\underline{\theta = 33.0^\circ}}$



(Continued)

9.62 (cont.)

b) Find the center of gravity of the gate.



From Table D.2

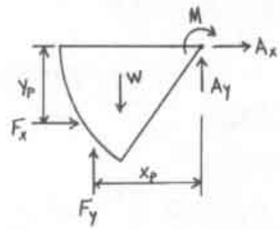
$\bar{r} = \frac{2r\sin\theta}{3\theta}$, $r = 3.5\text{ m}$

$\theta = 25° = 0.436$ rad

$\therefore \bar{r} = 2.26\text{m}$

$\bar{x} = \bar{r}\cos\theta = 2.05\text{m}$

FBD of gate with moment at A required to lift the gate -- no reaction at B.



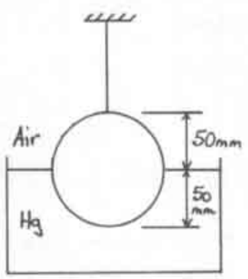
$\circlearrowleft\!+\ \Sigma M_A = 0: \ W\bar{x} + F_x y_p - F_y x_p - M = 0$

$M = 256(2.05) + 105.4(1.79) - 68.3(2.76)$

$\underline{\underline{M = 525.0 \text{ kN}\cdot\text{m}}}$

9.63

A steel ball of radius $r = 50\text{mm}$ and specific weight of 76.8 kN/m^3 is suspended in air and mercury as shown. Find the tension in the cord.



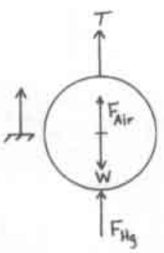
$\gamma_s = 76.8 \text{ kN/m}^3$

$\gamma_{Hg} = 133 \text{ kN/m}^3$

$\gamma_{Air} = 12.018 \text{ N/m}^3$

(See Sec. 9.2)

FBD of ball:



$V = \frac{4\pi(.05)^3}{3} = 0.000524 \text{ m}^3$

$W = \gamma_s V = 40.2 \text{ N}$

The buoyant force of air is

$F_{Air} = \frac{0.000524(12.018)}{2} = 0.00315 \text{ N}$

The buoyant force of mercury is

$F_{Hg} = \frac{.000524(133\times10^3)}{2} = 34.82 \text{ N}$

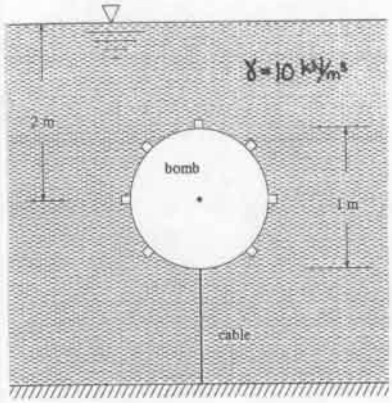
$\Sigma F_y = -40.21 + .00315 + 34.82 + T = 0$

$\underline{\underline{T = 5.39 \text{ N}}}$

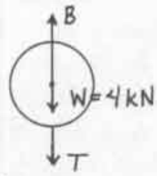
The buoyant force of air is negligible due to its low specific weight.

9.64

A submerged bomb weighs 4kN. Find the cable tension when it is held 2m below the surface in sea water.



FBD of Bomb:



$B = \frac{4}{3}\pi(0.5)^3(10) = 5.236 \text{ kN}$

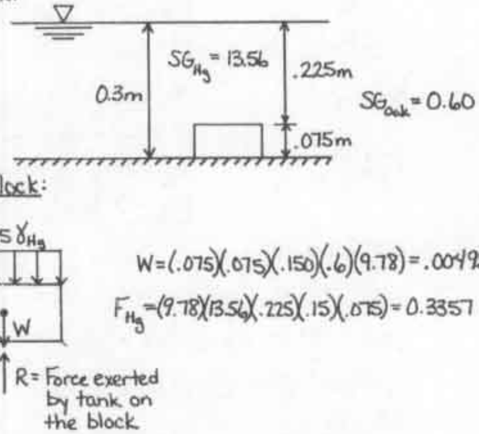
For Equilibrium:

$\Sigma F_y = -T - 4 + 5.236 = 0$

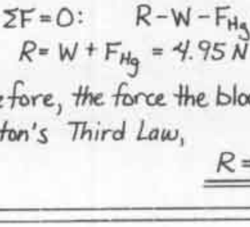
$\underline{\underline{T = 1.236 \text{ kN}}}$

9.65

An oak block (75mm × 75mm × 150mm) is pressed tightly on the bottom of a tank containing 300mm of Mercury. Find the force R that the block exerts on the tank.



FBD of the Block:



$W = (.075)(.075)(.150)(.6)(9.78) = .00495 \text{ kN}$

$F_{Hg} = (9.78)(13.56)(.225)(.15)(.075) = 0.3357 \text{ kN}$

Equilibrium of the block:

$\Sigma F = 0: \quad R - W - F_{Hg} = 0$

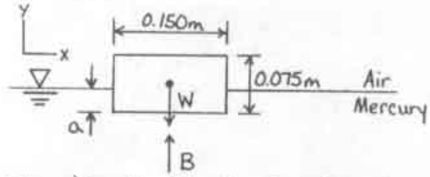
$R = W + F_{Hg} = 4.95 \text{ N} + 335.7 \text{ N} = 340.6 \text{ N} \uparrow$

Therefore, the force the block exerts on the tank is, by Newton's Third Law,

$\underline{\underline{R = 341 \text{ N}\downarrow}}$

9.66

An oak block (75mm × 75mm × 150mm) with SG = 0.60 floats as shown in mercury. Find the depth a to which it sinks.



$W = (0.075)(0.075)(0.150)(0.6)(9.78) = 0.00495\ kN$

$B = (0.075)(a)(0.150)(13.56)(9.78) = 1.4919\,a$

$\Sigma F_y = -W + B = 0 \qquad W = B$

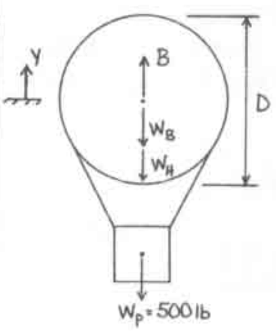
$1.4919\,a = 0.00495$

$\underline{a = 0.00332\ m = 3.32\ mm}$

9.67

A spherical balloon weighs 0.10 lb/ft² per square foot of surface area. It is to be filled with hydrogen, and the internal pressure is to be the same as the external atmospheric pressure. The specific weight of hydrogen (γ_H) is equal to 6.9% of that for air at the same temperature and pressure. Find the diameter of the balloon required to lift a 500 lb. payload to 40,000 ft, if the specific weight of air at 40,000 ft is $\gamma_{Air} = 0.020$ lb/ft³.

<u>FBD of Balloon</u>:



For a balloon diameter D:

$SA = \pi D^2$, $V = \frac{1}{6}\pi D^3$ (See Tables D.3 and D.4)

$B = 0.020\left(\frac{\pi}{6}\right)D^3$

$\underline{B = 0.01047 D^3}$

$W_H = 0.069B$

$\underline{W_H = 7.226(10^{-4})D^3}$

$W_B = 0.10\,\pi D^2$

$\underline{W_B = 0.3142 D^2}$

For Equilibrium:

$\Sigma F_y = 0: \quad B - W_B - W_H - W_p = 0$

$.01047D^3 - 7.226(10^{-4})D^3 - 0.3142D^2 - 500 = 0$

$9.749(10^{-3})D^3 - 0.3142D^2 - 500 = 0$

$D^3 - 32.23D^2 - 51{,}287 = 0$

$\underline{D = 51.54\ ft}$ is the only real root.

(the two imaginary roots are $D \approx -9.66 \pm 30.05i$; $i = \sqrt{-1}$)

9.68

A ship that weighs 30,000 kips and has an average specific weight of $\gamma_s = 360$ lb/ft³ is sunk in sea water $\gamma_w = 64$ lb/ft³. Air tanks are used to raise the ship. One tank weighs 20 kips and has a volume of 5000 ft³.

a) What lift force does one tank exert on the ship?

b) How many tanks are needed to bring the deck to the surface? The super structure weighs 5000 kip

a) Buoyant force on one submerged tank:

$B_T = \gamma_w(5000) = 320{,}000\ lb$

$B_T = 320\,k$

Net lift force: $F_T = B_T - W_T = 320 - 20$

$\underline{F_T = 300\ kips}$

b) Volume of the ship

$V_s = \frac{30{,}000}{0.360} = 83{,}333\ ft^3$ (Total)

$V_{ss} = \frac{5{,}000}{0.360} = 13{,}889\ ft^3$ (Superstructure)

$V_H = V_s - V_{ss} = 69{,}444\ ft^3$ (Hull)

Buoyant force on the hull

$B_H = \gamma_w V_H = 4{,}444\ kips$

For equilibrium of the ship:

Total Weight = Total Buoyant Force

$30{,}000 + n(20) = 4{,}444 + n(320)$

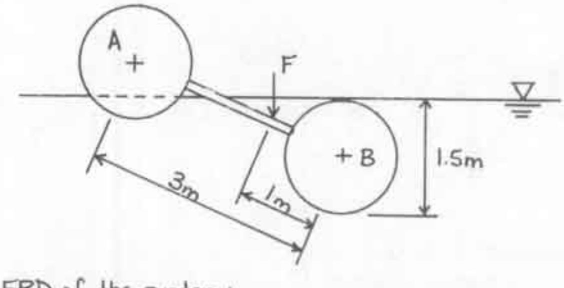
where n = number of tanks

$\therefore n = \frac{30{,}000 - 4{,}444}{300} = 85.19$

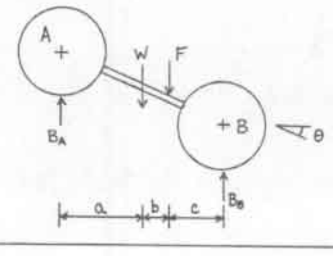
<u>Use 86 tanks</u>

9.69

Two spherical floats are held in position by force F. The beam and floats weigh 8900 N. Find F. Ignore the buoyant force on the beam.



FBD of the system:



$W = 8.9\ kN$

$a = 1.5\cos\theta$

$b = 0.5\cos\theta$

$c = 1.0\cos\theta$

$B_B = \gamma V_B$

$V_B = \frac{\pi}{6}D^3$ (Table D.3)

$B_B = 9.78\left(\frac{\pi}{6}\right)(1.5)^3 = 17.28\ kN$

(continued)

9.69 (cont.)

For equilibrium:

$\circlearrowleft \Sigma M_A = 0$

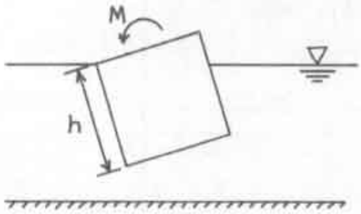
$(3\cos\theta)B_B - (1.5\cos\theta)W - (2\cos\theta)F = 0$

$F = \dfrac{3B_B - 1.5W}{2}$

$\underline{F = 19.25\ \text{kN}}$

9.70

A cube of ice floats as shown. The ice has sg = 0.90. What couple M is required to hold the cube.



Unit weight of water : γ_w

Unit weight of ice : $\gamma_I = 0.9\gamma_w$

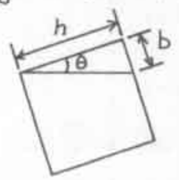
Weight of the cube : $W = \gamma_I h^3 = 0.9\gamma_w h^3$

Buoyant Force on cube : $B = W$

Volume of water displaced : $V_w = \dfrac{B}{\gamma_w} = 0.9h^3$

$\therefore$ The triangular wedge of ice above the water surface has volume $V_T = 0.1h^3$

Find the angle of rotation θ:



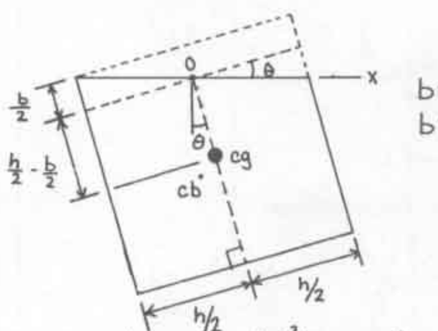
$V_T = (\tfrac{1}{2}hb)h^2$

$b = h\tan\theta$

$V_T = \tfrac{1}{2}h^3\tan\theta = 0.1h^3$

$\therefore\ \underline{\theta = 11.31^\circ}$

Find the center of buoyancy for the submerged part of the cube. See equations (b), Example 9.11 or Table D.2.



$b = h\tan\theta$

$b = 0.2h$

$$x_{cb} = \frac{(h - b/2)\sin\theta}{2} - \frac{(h/2)^2(2 + \tan^2\theta)\sin\theta}{6(h - b/2)}$$

$$y_{cb} = \frac{(h - b/2)\cos\theta}{2} + \frac{(h/2)^2\sin\theta\tan\theta}{6(h - b/2)}$$

Substitute for b and θ:

$x_{cb} = 0.06973h\ ,\quad y_{cb} = 0.4431h$

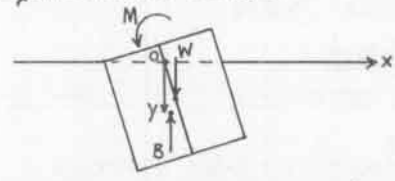
Find the center of gravity for the cube with respect to the origin O.

$h/2 - b/2 = 0.40h$

$x_{cG} = (0.4h)\sin\theta = 0.07845h$

$y_{cG} = (0.4h)\cos\theta = 0.3922h$

Equilibrium of the cube



$\Sigma M_0 = 0:\quad M + B(x_{cb}) - W(x_{cG}) = 0$

$M = W(x_{cG} - x_{cb})$

$M = 0.9\gamma_w h^3(0.07845h - 0.06973h)$

$\underline{\underline{M = 0.007848\,\gamma_w h^4}}$

In U.S. Customary units, $\gamma_w = 62.4\ \text{lb/ft}^3$. Then,

$\underline{\underline{M = 0.490h^4}}$

9.71

A frustum of a cone is cemented to the bottom of a 10 ft tank of water. Find the net pressure force on the cone.

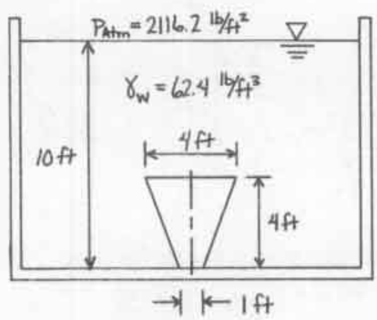


Figure (a)

By symmetry, the net horizontal force due to pressure is zero. Therefore, only vertical pressure forces need be considered. (Fig. (b))

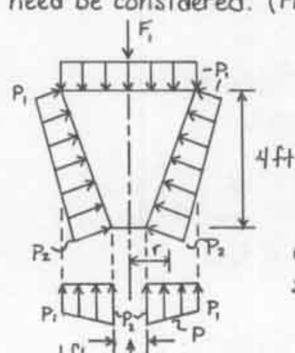


Figure (b)

By Figures (a) and (b),

$p_1 = 2116.2 + 6(62.4) = 2491\ \text{lb/ft}^2$

$p_2 = 2116.2 + 10(62.4) = 2740\ \text{lb/ft}^2$

Therefore,

$F_1 = p_1\pi(2)^2 = 31,303\ \text{lb}$

On the projected horizontal surface (Fig. (b)),

$p = p_2 - c(r - 0.5)\quad$ (a)

(continued)

9.71 (cont.)

For $r=2$, $p=2491$. Hence, $c=166$, and by Eqn. (a),

$$p = 2823 - 166r \quad (b)$$

The element of area of the ring (projected horizontal surface area) is

$$dA = 2\pi r\,dr$$

Therefore, with Eq. (b),

$$F_2 = \int p\,dA = 2\pi \int_{0.5}^{2} (2823 - 166r)\,r\,dr = 30{,}520 \text{ lb}$$

The net vertical force acting on the body is

$$F = F_1 - F_2 = 31{,}303 - 30{,}520$$

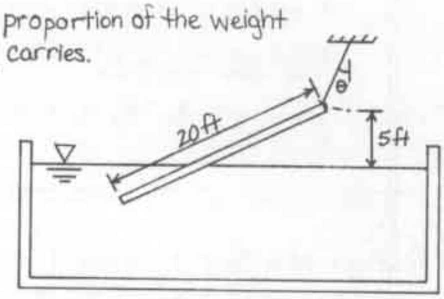
$$\underline{F = 783 \text{ lb} \downarrow}$$

9.72

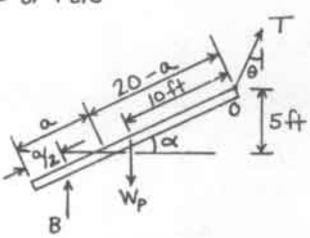
A slender, uniform pole has one end submerged in water and the other end hung from a string. The pole has specific weight $\gamma_p = 24$ lb/ft³.

a) Find θ.

b) Find the proportion of the weight the string carries.



a) FBD of Pole



$\Sigma F_x = 0$:

$T\sin\theta = 0$

$\underline{\therefore \theta = 0°}$

b) Let A = cross sectional area of the pole.

Weight of the pole: $W_p = \gamma_p A(20)$

$W_p = 480A$

Buoyant Force: $B = \gamma_w A a$

$B = 62.4Aa$

Equilibrium: $\circlearrowleft^{+} \Sigma M_o = 0$

$$(10\cos\alpha)W_p - (20 - a/2)\cos\alpha\, B = 0$$

$$4800A = (20 - a/2)62.4Aa$$

$$a^2 - 40a + 153.8 = 0$$

$a = 4.311$ ft or ($a = 35.69$ ft ← meaningless)

$\therefore B = 62.4A(4.311) \qquad B = 269.0A$

$\Sigma F_y = 0: \quad T + B - W_p = 0$

$T = 480A - 269A \qquad T = 211A$

$$\frac{T}{W} = \frac{211A}{480A} = 0.44$$

The string carries 44% of the pole weight.

9.73

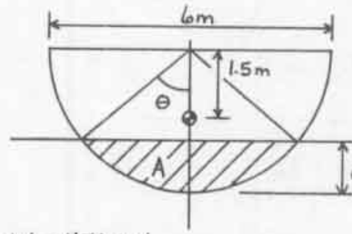
A barge with a hull in the shape of a half cylinder. $D = 6$m and $L = 15$m. The center of gravity is 1.5m below the axis of the cylinder. The barge weighs 450 kN.

a) What is its draft?

b) What is the angle of roll if one edge of the deck is at the water level?

c) What moment is required to cause this roll?

a)



d = draft

A = Area of barge cross section that is submerged.

$\gamma_w = 9.78$ kN/m³

$B = W = 450$ kN

$B = \gamma_w L A \quad \therefore A = \frac{450}{15(9.78)}$

$$A = 3.067 \text{ m}^2$$

From Table D.2, for a circular segment

$$A = \frac{r^2(2\theta - \sin 2\theta)}{2}$$

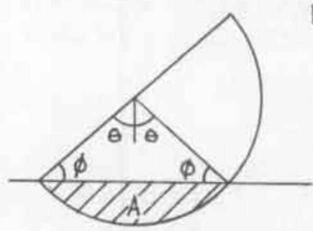
$$3.067 = \frac{3^2}{2}(2\theta - \sin 2\theta)$$

By trial and error

$$\theta = 0.8381 \text{ rad}$$

$d = r - r\cos\theta \qquad \underline{d = 0.993 \text{ m}}$

b) Angle of roll ϕ:



By Geometry:

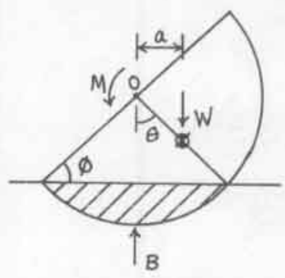
$\phi + \theta = \pi/2$

$\underline{\phi = 0.7328 \text{ rad}}$

$\underline{\phi = 42.0°}$

c) Required Moment M

FBD of barge in rolled position:



$a = 1.5\sin\theta$

$a = 1.115$ m

For Equilibrium: $\circlearrowleft^{+} \Sigma M_o = 0$

$M - Wa = 0$

$M = Wa$

$\underline{M = 501.8 \text{ kN}}$

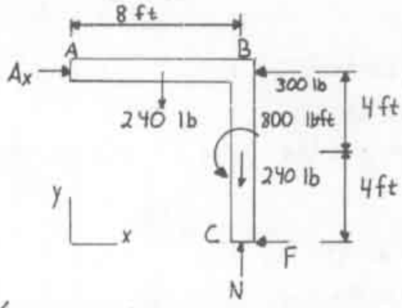
10.1

The bar ABC weighs 480 lb. The weight is distributed evenly.

a) Draw the FBD

b) Find the forces on the bar at A and C.

a/ FBD



b/ Equilibrium Equations

$\Sigma F_y = -480 + N = 0$

$\underline{N = 480 \text{ lb}}$

$\circlearrowleft^{+} \Sigma M_C = 800 + 300(8) - A_x(8) + 240(4) = 0$

$\underline{A_x = 520 \text{ lb}}$

$\Sigma F_x = A_x - 300 - F = 0$

$\underline{F = 220 \text{ lb}}$

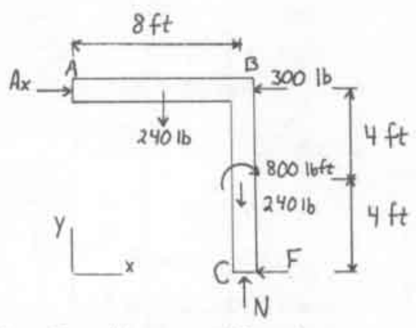
10.2

Bar ABC weighs 480 lb. The couple acts clockwise.

a) Draw the FBD

b) Find the forces on the bar at A and C

a/ FBD



b/ Equilibrium Equations

$\Sigma F_y = -480 + N = 0$

$\underline{N = 480 \text{ lb}}$

$\circlearrowleft^{+} M_C = 300(8) + 240(4) - 800 - A_x(8) = 0$

$\underline{A_x = 320 \text{ lb}}$

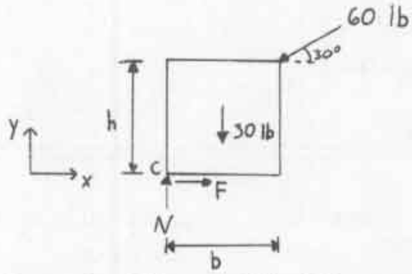
$\Sigma F_x = A_x - 300 - F = 0$

$\underline{F = 20 \text{ lb}}$

10.3

Find the maximum ratio $\frac{h}{b}$ that prevents tipping.

FBD for impending tipping



$\circlearrowleft^{+} \Sigma M_C = 0: (60 \cos 30°)h - (60 \sin 30°)b - 30\left(\frac{b}{2}\right) = 0$

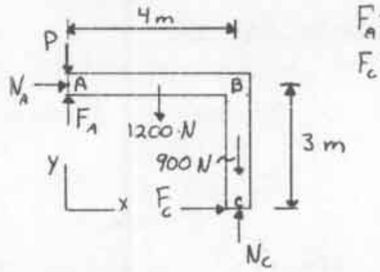
$51.96\, h = 45\, b$

$\underline{\underline{\frac{h}{b} = 0.866}}$

10.4

The bar ABC weighs $300 \frac{N}{m}$. $\mu_s = 0.5$ at A and C. Find the force P that causes impending motion.

FBD



$F_A = 0.5\, N_A$

$F_c = 0.5\, N_c$

Equilibrium Equations

$\circlearrowleft^{+} \Sigma M_A = N_c(4) - (0.5 N_c)(3) - 900(4) - 1200(2) = 0$

$N_c = 2400 \text{ N}$

$\Sigma F_x = N_A - F_c = 0 \rightarrow N_A = 0.5\, N_c$

$N_A = 1200 \text{ N}$

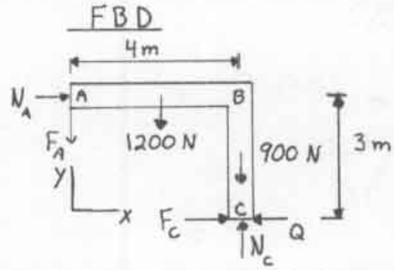
$\Sigma F_y = F_A + N_c - 2100 - P = 0 \rightarrow (0.5 N_A) + N_c = 2100 + P$

$\underline{P = 900 \text{ N}}$

10.5

The bar weighs $300 \frac{N}{m}$. A horizontal force Q is applied at C that acts to the left. $\mu_s = 0.5$ at A and C. Find Q for impending slip.

FBD



$F_A = 0.5\, N_A$

$F_c = 0.5\, N_c$

(continued)

10.5 Cont.

Equilibrium Equations

$\circlearrowleft M_C = N_A(3) - (0.5 N_A)(4) - 1200(2) = 0$

$N_A = 2400$ N

$\Sigma F_y = N_C - 2100 - 0.5 N_A = 0$

$N_C = 3300$ N

$\Sigma F_x = N_A + 0.5 N_C - Q = 0$

$\underline{\underline{Q = 4050 \text{ N}}}$

10.6

Block A weighs 100 lb. Block B weighs 40 lb. $\mu_s = 0.15$

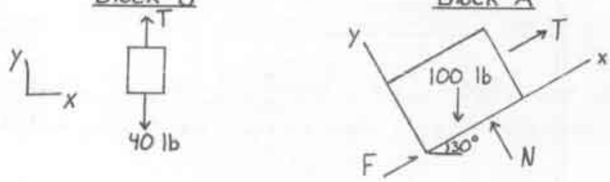
a) Determine if the system is in equilibrium.

b) Find the minimum μ_s to maintain equilibrium.

a/

Free Body Diagrams

Block B Block A



Equilibrium Equations

Block B

$\Sigma F_x = 0$

$\Sigma F_y = -40 + T = 0$

$T = 40$ lb

Block A

$\Sigma F_x = -100 \sin 30° + F + T = 0$

$\underline{F = 10 \text{ lb}}$

$\Sigma F_y = -100 \cos 30° + N = 0$

$\underline{N = 86.6 \text{ lb}}$

$F_{max} = \mu_s N$

$F_{max} = 12.99$ lb

Since $F < F_{max}$, the system is in equilibrium.

b/ If the blocks are on the verge of motion,

$F = \mu_s N$

$10 = \mu_s (86.6)$ $\therefore \underline{\underline{\mu_s = 0.1155}}$

10.7

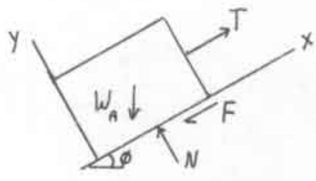
Block A weighs 250 N and is attached to hoist. B which moves it on a ramp with angle ϕ

a) Find a formula for the tension as a function of ϕ for which the block will not move. $\mu_s = 0.12$.

b) If $\phi = 30°$, find Tmax and Tmin.

a/

FBD



Equilibrium Equations (for block sliding up ramp impending)

$\Sigma F_y = N - W_A \cos \phi = 0$

$N = 250 \cos \phi$ (1)

$\Sigma F_x = T - F - W_A \sin \phi = 0$

$T = F + 250 \sin \phi$ (2)

$F = \mu_s N = 0.12 N$ (3)

solving (1), (2), and (3) for T,

$T = 30 \cos \phi + 250 \sin \phi$ (a)

If the block is on the verge of sliding down the ramp,

$T = -F + 250 \sin \phi$ (4)

solving (1), (3), and (4) for T,

$T = -30 \cos \phi + 250 \sin \phi$ (b)

by (a) and (b) the range of T for static equilibrium:

$\underline{\underline{250 \sin \phi - 30 \cos \phi \leq T \leq 250 \sin \phi + 30 \cos \phi \text{ [N]}}}$

b/ For $\phi = 30°$,

$\underline{\underline{99.0 \leq T \leq 151.0 \text{ [N]}}}$; $\underline{\underline{T_{max} = 151 \text{ N}}}$; $\underline{\underline{T_{min} = 99.0 \text{ N}}}$.

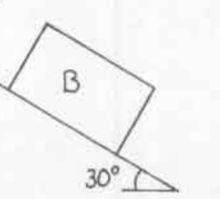
10.8

The cord is cut in example 10.6. Force T is removed and $\mu_k = 0.2$. $W_A = 600$ N, $W_B = 900$ N, $\mu_s = 0.5$

a) Find if block A moves.

b) Find if block B moves.

a/



$\mu_s = 0.5 = \tan \phi_s$

$\tan \phi = \tan 45° = 1.0$

$\mu_s < \tan \phi$; $\phi_s < \phi$

$\therefore$ Block A will slide

b/

$\mu_s = 0.5 = \tan \phi_s$

$\tan \phi = \tan 30° = 0.577$

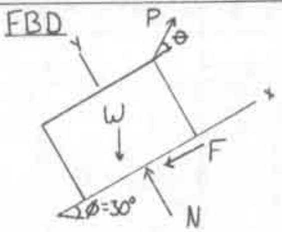
$\mu_s < \tan \phi$; $\phi_s < \phi$

$\therefore$ Block B will slide

10.9

The block rests on a plane of angle ϕ and is pulled by a force P which acts at an angle θ from the plane. $\mu_s = 0.2$, $\phi = 30°$ Find the angle θ for which $\frac{W}{P}$ is a maximum.

FBD



by example 10.4 (b),

$$\frac{W}{P} = \frac{\cos\theta + \mu_s \sin\theta}{\sin\phi + \mu_s\cos\phi} = \frac{\cos\theta + 0.2\sin\theta}{0.5 + 0.1732}$$

at $\frac{W}{P_{max}}$ $\left(\frac{W}{P}\right)' = 0$: $\frac{d(W/P)}{d\theta} = 0$

$$\frac{d(W/P)}{d\theta} = 0 = 1.485(-\sin\theta + 0.2\cos\theta)$$

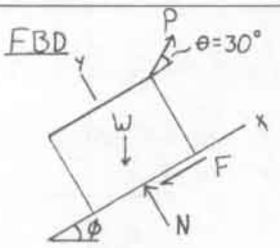
$\tan\theta = 0.2$ $\quad\therefore\ \underline{\underline{\theta = 11.31°}}$

Note: at $\theta = 11.31°$, $\frac{W}{P} = 1.515$

10.10

The block rests on a plane of angle ϕ and is pulled by a force P which acts at an angle θ from the plane. $\mu_s = 0.2$, $\theta = 30°$. Find the angle ϕ for which $\frac{W}{P}$ is a maximum

FBD



From example 10.4 (b)

$$\frac{W}{P} = \frac{\cos\theta + \mu_s\sin\theta}{\sin\phi + \mu_s\cos\phi} = \frac{0.966}{\sin\phi + 0.2\cos\phi}$$

at $\frac{W}{P}$ max, $\left(\frac{W}{P}\right)' = \frac{d(W/P)}{d\phi} = 0$

$$\frac{d(W/P)}{d\phi} = 0 = \frac{-0.966(\cos\phi - 0.2\sin\phi)}{(\sin\phi + 0.2\cos\phi)^2}$$

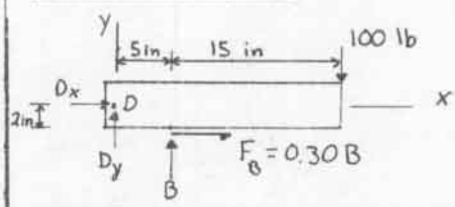
$\tan\phi = 5$

$\underline{\underline{\phi = 78.7°}}$ Note: at $\phi = 78.7°$, $W/P = 0.947$

10.11

$\mu_s = 0.30$ between wheel B and bar AD. Friction of bearings is negligible. The weight of bar AD is negligible. Find the minimum force F needed to turn B.

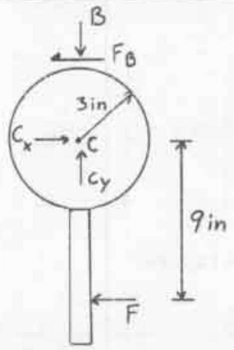
F.B.D. of bar AD:



$\circlearrowleft \Sigma M_D = 0$: $5B + 2(0.30B) - 20(100) = 0$

$B = 357.1$ lb, $F_B = 107.1$ lb

F.B.D. of wheel B & Arm:



$\circlearrowleft \Sigma M_C = 0$:

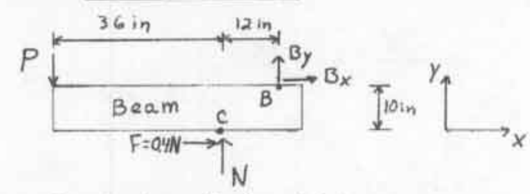
$(107.1)3 - F(9) = 0$

$\underline{\underline{F = 35.7 \text{ lb}}}$

10.12

Force P = 65 lb Keeps the shaft from moving due to torque M. $\mu_s = 0.40$. Find the magnitude of M. The shaft and beam are in contact at point **C**.

FBD of bar AB

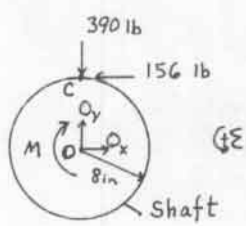


$\circlearrowleft M_B = P(48) + (0.4N)(10) - N(12) = 0$

$P = 0.1667 N = 65$ lb

$\therefore N = 390$ lb

FBD of shaft

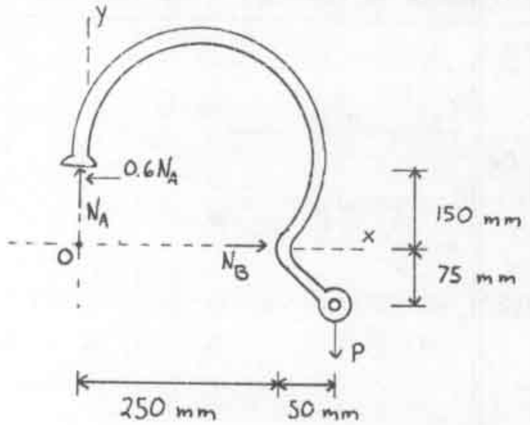


$\circlearrowleft \Sigma M_O = 156(8) - M = 0$

$\underline{\underline{M = 1248 \text{ lb}\cdot\text{in} = 104.0 \text{ lb}\cdot\text{ft}}}$

10.13

For surface A, $\mu_s = 0.60$. There is no friction for surface B, $\mu_s = 0$. Determine if the hook can maintain load P. The hook's weight is negligible.



$\Sigma F_y = N_A - P = 0$

$N_A = P \qquad (a)$

$\circlearrowleft \Sigma M_o = 0 = 0.6\,N_A\,(150) - P(300) = 0 \qquad (b)$

sub from (a) into (b)

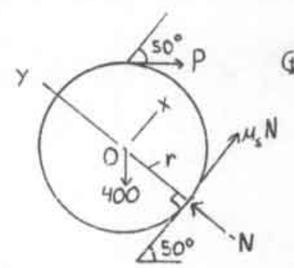
$\underline{90P - 300\,P < 0}$

Equilibrium cannot be maintained.

$\therefore$ The hook is not safe.

10.14

The cylinder weighs 400 N and is on the verge of sliding. Find force P and the friction coefficient μ_s required to hold the cylinder in equilibrium



$\circlearrowleft \Sigma M_o = 0$:

$\mu_s N r - P r = 0$

$\underline{P = \mu_s N} \qquad (a)$

$\Sigma F_x = \mu_s N + P\cos 50° - 400\sin 50° = 0 \qquad (b)$

sub from (a) into (b)

$P(1 + \cos 50°) = 400\sin 50°$

$P = \dfrac{400\sin 50°}{1+\cos 50°} = \underline{186.5\ N}$

$\Sigma F_y = N - 400\cos 50° - P\sin 50° = 0$

$N = 400\cos 50° + P\sin 50° = 400\ N$

From (a): $\underline{\mu_s = P/N = 0.467}$

Summary: $P = 186.5\ N$

$\mu_s = 0.467$

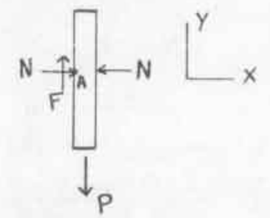
10.15

The coefficient of static friction is $\mu_s = 0.42$. Cam C is designed to prevent bar B from slipping due to force P. Neglect the weight of the cam and bearing friction. Find if the cam will operate properly. B and C make contact at A.

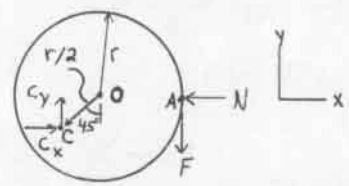
$\Sigma F_y = F - P = 0$

$F = P$

FBD of bar B:



FBD of cam C:



$\circlearrowleft \Sigma M_C = N\left(\frac{r}{2}\cos 45°\right) - F\left(r + \frac{r}{2}\sin 45°\right) = 0$

$F = 0.2612\ N$

For equilibrium, μ_s only needs to equal 0.2612, since $\mu_s = 0.42 > 0.2612$, the cam will hold the bar in place.

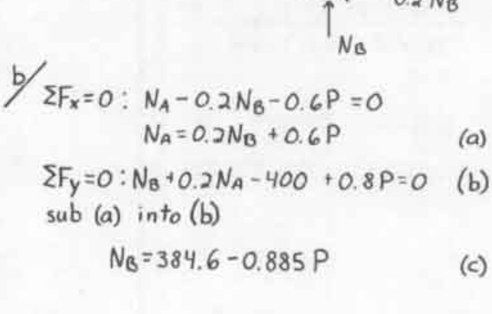
10.16

The cylinder weighs 400 N. $\mu_s = 0.2$. Motion is impending.

a) Draw the FBD; Label all forces.

b) Find the magnitude of P.

a/



b/ $\Sigma F_x = 0$: $N_A - 0.2N_B - 0.6P = 0$

$N_A = 0.2N_B + 0.6P \qquad (a)$

$\Sigma F_y = 0: N_B + 0.2N_A - 400 + 0.8P = 0 \qquad (b)$

sub (a) into (b)

$N_B = 384.6 - 0.885\,P \qquad (c)$

(Continued)

10.16 Cont.

$\circlearrowleft M_c=0: \; 0.15P-0.15(0.2N_A)-0.15(0.2N_B)=0$

$P=0.2\,(N_A+N_B) \qquad (d)$

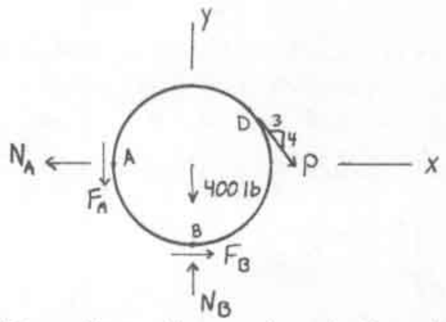
sub from (a) and (c) into (d)

$\underline{\underline{P=84.5\ N}}$

10.17

The force P in Figure 10.16 is reversed. Find whether or not the cylinder moves.

FBD

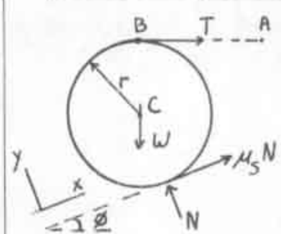


Since the wall cannot pull the cylinder to the left, $N_A=0$. So $F_A=0$ as well. $\Sigma M_B=0$ cannot be satisfied if $P \neq 0$. Hence, the cylinder will roll to the right.

10.18

Cylinder C rests on an inclined plane and is held in place by cord AB. Find the minimum μ_s to maintain equilibrium.

FBD of cylinder



$\circlearrowleft \Sigma M_C=0=r(\mu_s N)-r(T)=0$

$T=\mu_s N \qquad (a)$

$\Sigma F_x=0:$

$\mu_s N+T\cos\phi-W\sin\phi=0 \quad (b)$

Sub from (a) into (b):

$T+T\cos\phi-W\sin\phi=0$

$T=W\left(\dfrac{\sin\phi}{1+\cos\phi}\right) \qquad (c)$

$\Sigma F_y=0: \; N-W\cos\phi-T\sin\phi=0 \qquad (d)$

Sub from (a) and (c) into (d) for N and T:

$\dfrac{W}{\mu_s}\left(\dfrac{\sin\phi}{1+\cos\phi}\right)-W\cos\phi-W\left(\dfrac{\sin^2\phi}{1+\cos\phi}\right)=0$

$\dfrac{1}{\mu_s}\left(\dfrac{\sin\phi}{1+\cos\phi}\right)=\dfrac{\cos\phi+\cos^2\phi+\sin^2\phi}{1+\cos\phi}$

$\dfrac{\sin\phi}{\mu_s}=1+\cos\phi \qquad \therefore\ \underline{\underline{\mu_s=\dfrac{\sin\phi}{1+\cos\phi}}}$

10.19

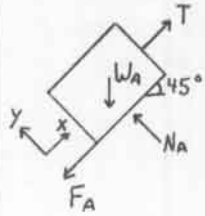
Two boxes are attached by a light cord and placed on two inclined planes.

$W_A=600\ N$, $W_B=900\ N$, $\mu_s=0.5$, $\mu_K=0.2$

Determine if the boxes move.

Approach: Assume that motion of the boxes to the right is impending. Find the cord tension required to move A. Then check the friction force on B.

FBD of A



$\Sigma F_y=0: \; N_A-W_A\cos 45°=0$

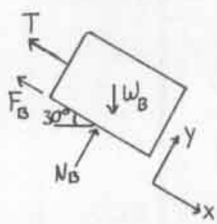
$N_A=424.3\ N$

$\therefore F_A=\mu_s N_A=212.1\ N$

$\Sigma F_x=0: \; T-F_A-W_A\sin 45°=0$

$\underline{T=636.4\ N}$

FBD of B



$\Sigma F_y=0: \; N_B-W_B\cos 30°=0$

$N_B=779.4\ N$

$\Sigma F_x=0: \; W_B\sin 30°-T-F_B=0$

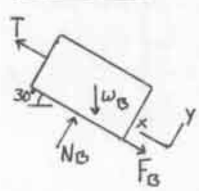
$F_B=450-636.4\ N$

$\underline{F_B=-186.4\ N}$

The friction force at B would have to push the block B *down* the plane in order for the boxes to move. This is not possible. <u>So, the boxes *do not* move to the right.</u>

Approach: Assume that motion of the boxes to the left is impending. Find the cord tension required to move B. Then check the friction force on A.

FBD of B



$\Sigma F_y=0: \; N_B-W_B\cos 30°=0$

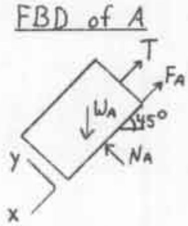
$N_B=779.4\ N$

$\therefore F_B=\mu_s N_B=389.7\ N$

$\Sigma F_x=0: \; T-F_B-W_B\sin 30°=0$

$\underline{T=839.7\ N}$

FBD of A



$\Sigma F_y=0: \; -W_A\cos 45°+N_A=0$

$N_A=424.3\ N$

$\Sigma F_x=0: \; -T-F_A+W_A\sin 45°=0$

$F_A=-839.7+W_A\sin 45°$

$\underline{F_A=-415.4\ N}$

(continued)

10.19 Cont.

The friction force at A would have to push block A down the plane in order for the boxes to move. This is not possible.

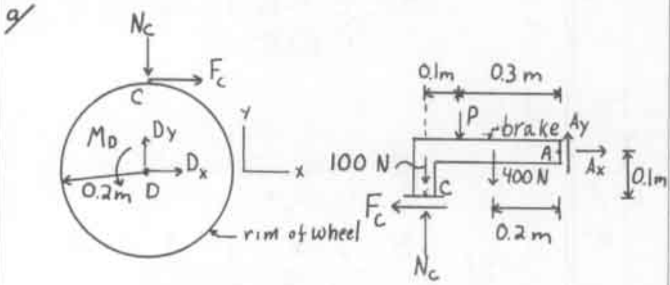
So, the boxes do not move to the left.

Thus, the boxes do not move.

10.20

The wheel D is supported by an axle D at its center. The brake weighs 500 N. The wheel is at impending motion due to the moment $M_D = 20$ N·m. The rim and brake are in contact at C.
a) Draw the FBD of the wheel and bar.
b) Write equilibrium equations.
c) Solve for P if $\mu_s = 0.20$.

a/



b/ $\circlearrowleft \Sigma M_D = M_D - F_c(0.2) = 0$

$F_c = 100$ N

$F_c = \mu_s N_c$

$N_c = \dfrac{F_c}{\mu_s} = \dfrac{100}{\mu_s}$

$\circlearrowleft \Sigma M_A = 400(0.2) + 100(0.4) - N_c(0.4) - F_c(0.1) + P(0.3) = 0$

$P = -366.67 + \frac{4}{3} N_c$

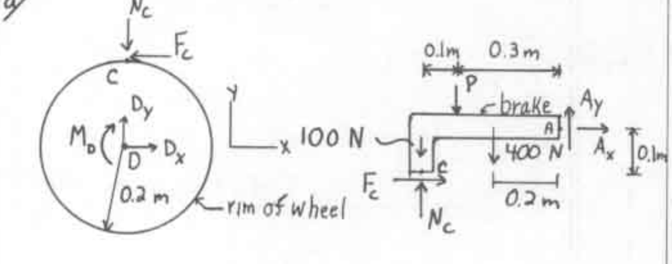
c/ $P = -366.67 + \frac{4}{3}\left(\frac{100}{0.2}\right)$

$P = 300$ N

10.21

Couple M_O acts clockwise with a magnitude of 20 N·m. The bar weighs 500 N. The wheel is on the verge of motion. The rim contacts the brake at C.
a) Draw the FBD's for the wheel and bar.
b) Write the equilibrium equations.
c) Solve for P if $\mu_s = 0.20$.

a/



b/ wheel

$\circlearrowleft \Sigma M_D = -M_D + F_c(0.2) = 0$

$F_c = 100$ N

$F_c = \mu_s N_c$

$N_c = \dfrac{F_c}{\mu_s} = \dfrac{100}{\mu_s}$

brake

$\circlearrowleft \Sigma M_A = F_c(0.1) + 100(0.4) + P(0.3) + 400(0.2) - N_c(0.4) = 0$

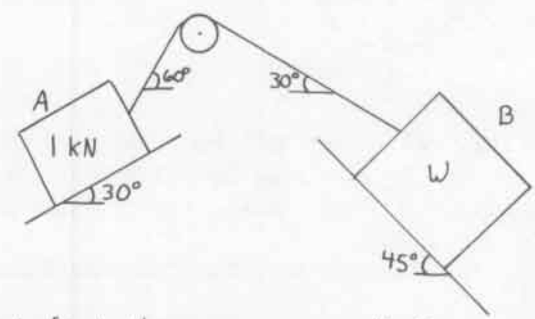
$P = \dfrac{\frac{40}{\mu_s} - 130}{0.3}$

c/ $\mu_s = 0.2$

$P = 233$ N

10.22

Two blocks rest on inclined planes and are connected by a flexible cord that passes over a pulley. $\mu_s = 0.6$ and $\mu_k = 0.4$. Find W to cause impending sliding.



First, find the minimum weight W.

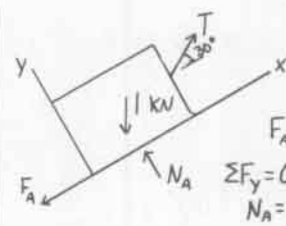
$\mu_s = 0.60$, $\tan 30° = 0.577 \Rightarrow \mu_s > \tan 30°$

∴ Block A will not slide down the ramp if $W = 0$

∴ There is no minimum weight.

Next, find the maximum weight W for impending motion to the right.

FBD of block A:



$F_A = \mu_s N_A$

$\Sigma F_y = 0: N_A + T(\sin 30°) - 1(\cos 30°) = 0$

$N_A = \cos 30° - T \sin 30°$ (a)

$\Sigma F_x = 0: T\cos 30° - 1(\sin 30°) - \mu_s N_A = 0$ (b)

Sub from (a) into (b) for N_A

$T\cos 30° - \sin 30° - 0.6(\cos 30° - T\sin 30°) = 0$

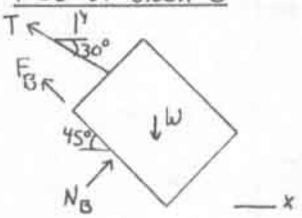
$T(\cos 30° + 0.6 \sin 30°) = \sin 30° + 0.6\cos 30°$

$T = 0.874$ KN

(continued)

10.22 Cont.

FBD of block B



$\Sigma F_x = 0 = N_B \sin 45° - F_B \cos 45° - T\cos 30°$

$N_B(\sin 45° - 0.6\cos 45°) = T\cos 30°$

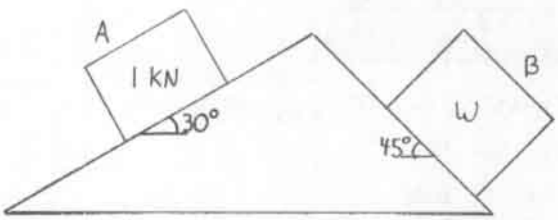
$N_B = 2.68$ KN

$\Sigma F_y = 0 = T\sin 30° + F_B \sin 45° + N_B \cos 45° - W = 0$

$\underline{\underline{W = 3.47 \text{ KN}}}$

10.23

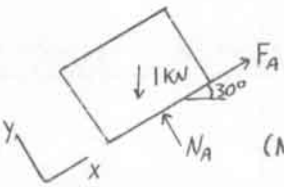
Two blocks are placed on inclined planes.
$\mu_s = 0.6$, $\mu_k = 0.4$
a) Determine if the blocks move.
b) Determine the frictional forces that act on each block.



a/ For block A, $\mu_s > \tan 30°$ ∴ The block does not move.

For block B, $\mu_s < \tan 45°$ ∴ Block B slides.

b/ FBD of block A:

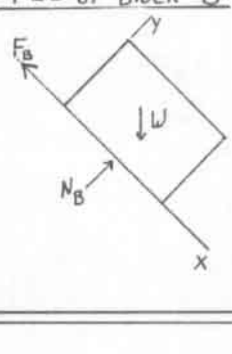


$\Sigma F_y = 0$: $N_A - 1(\cos 30°) = 0$
$N_A = 0.866$ KN

$\Sigma F_x = 0$: $F_A - 1(\sin 30°) = 0$
$\underline{F_A = 0.5 \text{ KN}}$

(Note: $F_A < \mu_s N_A$)

FBD of block B:



$\Sigma F_y = 0$: $N_B - W\cos 45° = 0$
$N_B = W/\sqrt{2}$

Since the block slides,

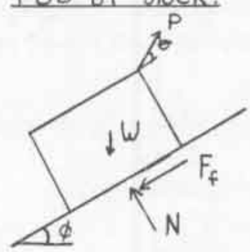
$F_B = \mu_k N_B = \dfrac{0.4}{\sqrt{2}} W$

$\underline{\underline{F_B = 0.283 W}}$

10.24

A block rests on a plane of angle ϕ and a coefficient of static friction μ_s. A force P pulls up the plane at an angle of θ relative to the plane.
a) Find the angle θ for which P is a minimum for impending motion up the plane.
b) Find the relative values of P and N.

FBD of block:



$\mu_s = \tan \phi_s$

a/ From Example 10.4, equation (c),

$$P = W\left(\frac{\sin(\phi + \phi_s)}{\cos(\phi_s - \theta)}\right) \qquad (a)$$

Regardless of W, ϕ, and ϕ_s, P_{min} occurs at the maximum value of $\cos(\phi_s - \theta)$

∴ $\underline{\underline{\theta = \phi_s = \tan^{-1}\mu_s}}$

b/ From equation (a)

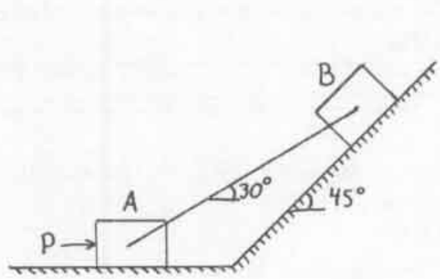
$\underline{\underline{P = W(\sin(\phi + \phi_s))}}$

From Example 10.4 equation (c),

$\underline{\underline{N = W\cos\phi_s \cos(\phi_s + \phi)}}$

10.25

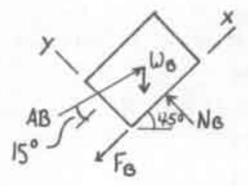
Weightless rod AB connects A and B.
$\mu_s = 0.40$, $W_A = 400$ N, $W_B = 600$ N.
a) Find P required to initiate sliding up the plane.
b) Find minimum P to hold the blocks in the position shown.



(continued)

10.25 Cont.

a/ FBD of block B: with impending sliding up the plane



$F_B = \mu_s N_B$

$\Sigma F_y = 0: N_B - W_B \cos 45° - AB \sin 15° = 0$

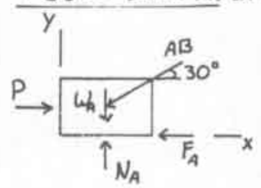
$N_B = 424.3 + 0.259\, AB$

$\Sigma F_x = 0: AB \cos 15° - W_B \sin 45° - \mu_s N_B = 0$

$0.966 AB - 424.3 - 0.4(424.3 + 0.259\, AB) = 0$

$AB = 689 \text{ N}$

FBD of block A: with impending sliding to the right



$\Sigma F_y = 0: N_A - W_A - AB \sin 30° = 0$

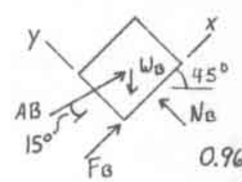
$N_A = 745 \text{ N}$

$F_A = \mu_s N_A = 298 \text{ N}$

$\Sigma F_x = 0: P - F_A - AB \cos 30° = 0$

$P = 895 \text{ N}$

b/ FBD of block B: with impending sliding down the plane.



$F_B = \mu_s N_B$

$\Sigma F_y = 0$: - same as part a) -

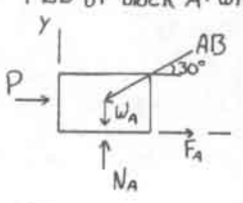
$N_B = 424.3 + 0.259\, AB$

$\Sigma F_x = 0: AB \cos 15° - W_B \sin 45° + \mu_s N_B = 0$

$0.966 AB - 424.3 + 0.4(424.3 + 0.259\, AB) = 0$

$AB = 238 \text{ N}$

FBD of block A: with impending sliding to the left.



$\Sigma F_y = 0: N_A - W_A - AB \sin 30° = 0$

$N_A = 519. \text{ N}$

$F_A = \mu_s N_A = 208 \text{ N}$

$\Sigma F_x = 0: P + F_A - AB \cos 30° = 0$

$P = -1.49 \text{ N}$

The force P is not needed to prevent sliding down the plane.

10.26

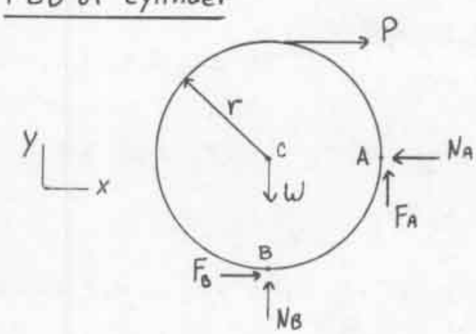
Force P causes impending slip of the cylinder
Cylinder has weight W.

a) Draw the FBD

b) Find a formula for P in terms of W, μ_{sA}, μ_{sB}, and r.

c) Find P for $W = 400$ N, $\mu_{sA} = 0.1$, $\mu_{sB} = 0.2$, $r = 300$ mm

a/ FBD of cylinder



b/ $\Sigma F_y = N_B + F_A - W = 0$ (1)

$\Sigma F_x = P + F_B - N_A = 0$ (2)

$\circlearrowleft \Sigma M_D = -P(r) + F_B(r) + F_A(r) = 0$ (3)

$F_A = \mu_{sA} N_A \; ; \; F_B = \mu_{sB} N_B$ (4)

Solving (1), (2), (3), (4) for P yields:

$$P = \frac{W(\mu_{sB})(1 + \mu_{sA})}{(1 + 2\mu_{sB}\mu_{sA} - \mu_{sA})}$$

c/ $W = 400$ N, $\mu_{sA} = 0.1$, $\mu_{sB} = 0.2$, $r = 0.3$ m

solve for P:

$P = 93.6 \text{ N}$

10.27

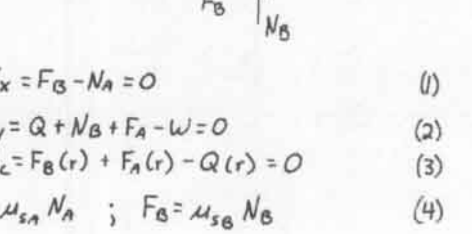
A force Q acts vertically upward at D as shown. The cylinder has weight W.

a) Draw the FBD.

b) Find a formula for Q in terms of W, μ_{sA}, μ_{sB}, and r

c) Find Q for $W = 400$ N, $\mu_{sA} = 0.1$, $\mu_{sB} = 0.2$, $r = 300$ mm

a/ FBD of cylinder



b/ $\Sigma F_x = F_B - N_A = 0$ (1)

$\Sigma F_y = Q + N_B + F_A - W = 0$ (2)

$\circlearrowleft \Sigma M_C = F_B(r) + F_A(r) - Q(r) = 0$ (3)

$F_A = \mu_{sA} N_A \; ; \; F_B = \mu_{sB} N_B$ (4)

(continued)

10.27 Cont.

Solving (1), (2), (3), and (4) for Q:

$$Q = \frac{W(1+\mu_{sA})\mu_{sB}}{(1+2\mu_{sA}\mu_{sB}+\mu_{sB})}$$

c/ $W = 400\ N$, $\mu_{sA} = 0.1$, $\mu_{sB} = 0.2$, $r = 0.3\ m$

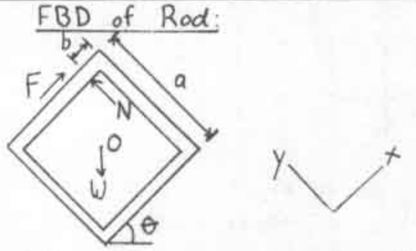
Solving Q:

$Q = 71.0\ N$

10.28

A rod bent into a square is hung on a peg. The rod will not slip, regardless of the position of the peg. Find the smallest value of μ_s required to keep static equilibrium in terms of a and b.

FBD of Rod:



For sliding to be impending, $F = \mu_s N$. Enforce moment equilibrium about O.

$\circlearrowleft \Sigma M_O = 0$: $F(\frac{a}{2}) - N(\frac{a}{2} - b) = 0$

$\mu_s N(\frac{a}{2}) = N(\frac{a}{2} - b)$; $\mu_s = 1 - \frac{2b}{a}$

as $b \to 0$, $\mu_s \to 1$ $\therefore \mu_s \geq 1$ if rod is not to slip regardless of position of peg.

10.29

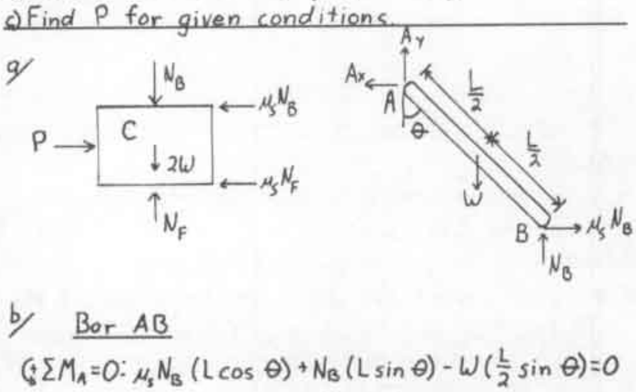
The machine part weighs 2W. The bar weighs W. Force p causes impending motion. The coefficient of static friction is μ_s. Pin A is frictionless.

a) Draw the FBDs.

b) Find P in terms of W, L, θ, and μ_s.

c) Find P for given conditions.

a/



b/ Bar AB

$\circlearrowleft \Sigma M_A = 0$: $\mu_s N_B(L\cos\theta) + N_B(L\sin\theta) - W(\frac{L}{2}\sin\theta) = 0$

$N_B L(\mu_s \cos\theta + \sin\theta) = W\frac{L}{2}\sin\theta$

$$N_B = \frac{W\sin\theta}{2(\mu_s\cos\theta + \sin\theta)} \qquad (a)$$

Block c

$\Sigma F_y = 0$: $N_F - N_B - 2W = 0$

$N_F = 2W + N_B \qquad (b)$

$\Sigma F_x = 0$: $P - \mu_s N_B - \mu_s N_F = 0$

$P = \mu_s(N_B + N_F) \qquad (c)$

Sub from (b) into (c)

$P = \mu_s(2W + 2N_B) \qquad (d)$

Sub from (a) into (d)

$$P = \mu_s\left(2W + \frac{W\sin\theta}{(\mu_s\cos\theta + \sin\theta)}\right)$$

$$P_s = \mu_s W\left(2 + \frac{\sin\theta}{\mu_s\cos\theta + \sin\theta}\right) \quad (e)$$

c/ For $W = 200\ lb$, $L = 10\ ft$, $\theta = 45°$, $\mu_s = 0.2$

Eq. (e) yields

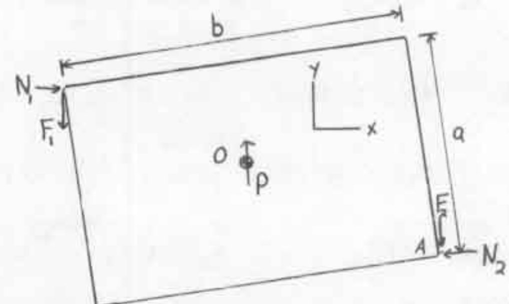
$P = 40\left[2 + \frac{0.7071}{0.8485}\right]$

$P = 113.3\ lb$

10.30

The drawer slides between parallel walls on guides. The depth of the drawer is a and the width is b. Show that $a/b > \mu_s$ to prevent jamming.

FBD of drawer:



The horizontal guides are frictionless. Therefore the forces in the z-direction can be disregarded because they have no effect on the problem. Since the angle the drawer is turned is very small, it can be disregarded also.

$\Sigma F_x = N_1 - N_2 = 0 \Rightarrow N_1 = N_2$

At impending slip, $F_1 = \mu_s N_1$ and $F_2 = \mu_s N_2$

$\circlearrowleft \Sigma M_O = -N_1(\frac{a}{2}) - N_2(\frac{a}{2}) + F_1(\frac{b}{2}) + F_2(\frac{b}{2}) = 0$

$N_1 a = \mu_s N_1 b$

$\therefore \mu_s = \frac{a}{b}$

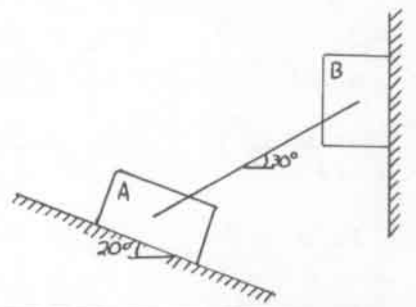
$\therefore \frac{a}{b} > \mu_s$ to avoid jamming

10.31

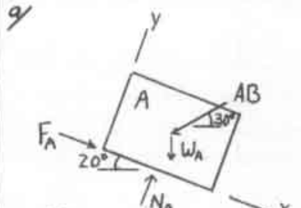
Two blocks are connected by a rod as shown. $W_A = 50$ N, $W_{rod} = 0$, $\mu_{sA} = 0.35$, $\mu_{sB} = 0.20$

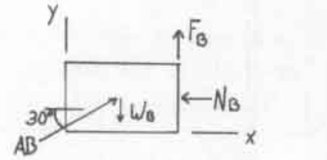
a) Draw FBDs and write equilibrium equations for the blocks that determine the maximum weight of B.

b) Determine the maximum weight of B.



a/





Block A:

$\Sigma F_x = 0$:

$F_A + W_A \sin 20° - AB \cos 50° = 0$ (1)

$\Sigma F_y = 0$:

$N_A - W_A \cos 20° - AB \sin 50° = 0$ (2)

Slip is impending:

$F_A = 0.35 N_A$ (3)

Block B:

$\Sigma F_x = 0$:

$AB \cos 30° - N_B = 0$ (4)

$\Sigma F_y = 0$:

$AB \sin 30° + F_B - W_B = 0$ (5)

Slip is impending:

$F_B = 0.20 N_B$ (6)

b/ Solve (2) for N_A and sub into (3).

Sub (3) into (1) to get:

$0.35(47.0 + AB \sin 50°) + 17.1 - AB \cos 50° = 0$

$\therefore AB = 89.5$ N

Sub AB into (4) $\longrightarrow$ $N_B = 77.5$ N

Sub N_B into (6) $\longrightarrow$ $F_B = 15.5$ N

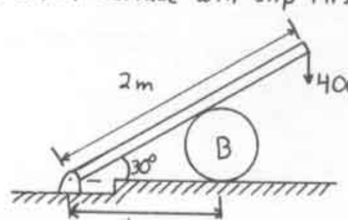
Sub AB and F_B into (5) $\longrightarrow$ $\underline{\underline{W_B = 60.3 \text{ N is maximum } W}}$

10.32

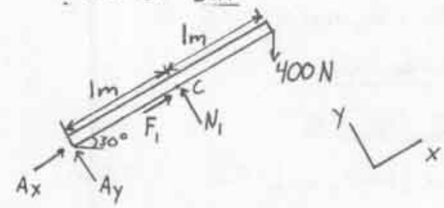
Roller B weighs 200 N. The coefficient of static friction for all contacting surfaces is μ_s. The bar is weightless.

a) Find smallest μ_s that prevents slip.

b) At which surface will slip first occur.



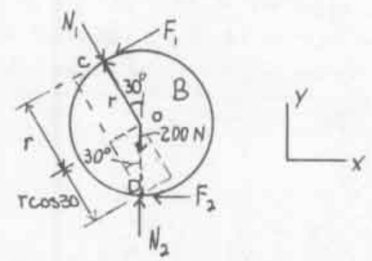
FBD of bar



$\circlearrowleft \Sigma M_A = N_1(1) - 400(2)(\cos 30°) = 0$

$N_1 = 692.8$ N

FBD of roller



By FBD of the roller,

$\circlearrowleft \Sigma M_O = F_1(r + r\cos 30°) - N_1(r \sin 30°) = 0$

$F_1 = 0.268 N_1 = \mu_s N_1$

Therefore, the minimum coefficient of static friction needed to prevent slipping at C is,

$\underline{\mu_s = 0.268}$

By the FBD of the roller,

$\circlearrowleft \Sigma M_O = F_1 r - F_2 r = 0$

$\therefore F_1 = F_2 = 0.268 N_1 = 0.268(692.8) = 185.67$ N

at impending slipping at C.

Also, by the FBD of the roller,

$\Sigma F_y = N_2 - 200 - N_1 \cos 30° - F_1 \sin 30° = 0$

or $N_2 = 892.8$ N

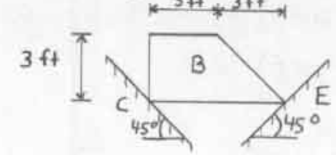
Hence, since $N_2 > N_1$, $\underline{\underline{\text{slipping is more apt to occur at C.}}}$

10.33

A homogeneous wedge B rests on walls of a sluice. $\mu_{sE} = 0$. $W_B = 3W$

a) Find minimum μ_{sC} to prevent slipping

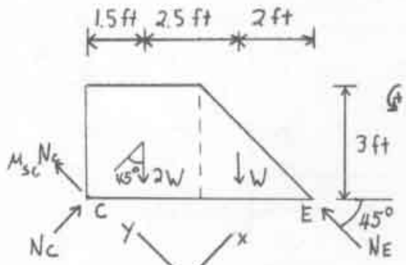
b) A boy of weight 2W stands at the center of the 3 ft face. $\mu_{sC} = 0.4$. Determine if the wedge slips.



(continued)

10.33 Cont.

a) FBD of the block - Slip is impending



$\circlearrowleft \Sigma M_c = 0$

$N_E(\sin 45°)6 - 2W(1.5) - W(4) = 0$

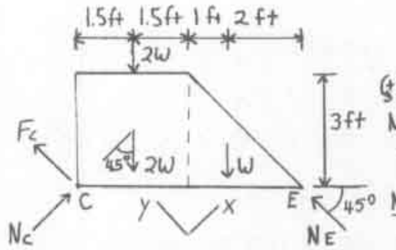
$\underline{N_E = 1.65\,W}$

$\Sigma F_x = 0: N_c - 3W(\cos 45°) = 0 \rightarrow \underline{N_c = 2.121W}$

$\Sigma F_y = 0: \mu_{sc} N_c + 1.65W - 3W(\sin 45°) = 0 \rightarrow \mu_{sc} N_c = 0.471\,W$

$\therefore \mu_{sc} = 0.222$ = minimum μ_{sc} to prevent slip.

b) FBD of the block



$\circlearrowleft \Sigma M_c = 0:$

$N_E(\sin 45°)6 - 4W(1.5) - W(4) = 0$

$\underline{N_E = 2.36\,W}$

$\Sigma F_x = 0: N_c - 5W(\cos 45°) = 0 \rightarrow \underline{N_c = 3.54\,W}$

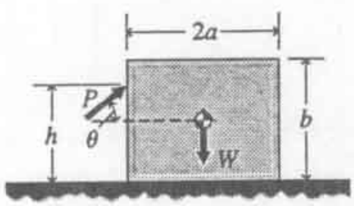
$\Sigma F_y = 0: F_c + 2.36W - 5W(\sin 45°) = 0 \rightarrow \underline{F_c = 1.176\,W}$

$\mu_{sc\,min} = \frac{F_c}{N_c} = \frac{1.176\,W}{3.54\,W} = \underline{0.332}$

$\mu_{sc} > \mu_{sc\,min}$ $\therefore$ The block will not slip

10.34

The block of weight W is motionless. It is pushed by a force P.



a) If no sliding occurs, show that $\frac{P}{W} < \frac{\sin \phi_s}{\cos(\theta - \phi_s)}$ for $\tan \phi_s = \mu_s$.

b) If no tipping occurs, show that $\frac{P}{W} < \frac{a}{h\cos\theta + 2a\sin\theta}$

a/ FBD for impending slip:

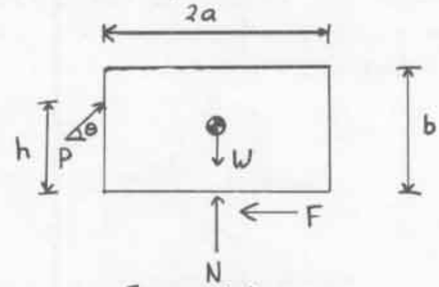


Figure (a)

$F = \mu_s N$ (a)

$\Sigma F_y = N + P\sin\theta - W = 0$

$N = -P\sin\theta + W$ (b)

$\Sigma F_x = P\cos\theta - F = 0$

$F = P\cos\theta = \mu_s N = (\tan \phi_s) N$ (c)

$N = P\cos\theta \left(\frac{\cos \phi_s}{\sin \phi_s}\right)$ by (c) & (a)

$\frac{P\cos\theta \cos\phi_s}{\sin\phi_s} + P\sin\theta = W$ by (b)

solve for $\frac{P}{W}$ $\frac{P}{W} = \frac{\sin \phi_s}{\cos(\theta - \phi_s)}$ for impending motion

$\therefore \frac{P}{W} < \frac{\sin \phi_s}{\cos(\theta - \phi_s)}$ for no slip

b/ FBD for impending tip:

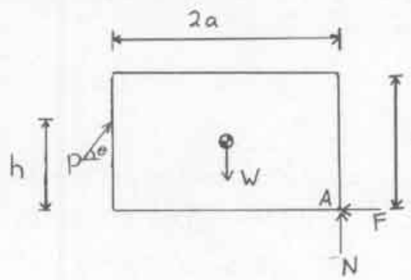


Figure b

$\circlearrowleft \Sigma M_A = W(a) - P\cos\theta\,(h) - P\sin\theta\,(2a) = 0$

$Wa = P(h\cos\theta + 2a\sin\theta)$

$\therefore \frac{P}{W} = \frac{a}{h\cos\theta + 2a\sin\theta}$ at impending tip

For no tip,

$\frac{P}{W} < \frac{a}{h\cos\theta + 2a\sin\theta}$

10.35

For Example 10.5 let $\mu_1 = k$, $\mu_2 = 2k$, $\mu_3 = 3k$. Find the minimum k for which the system is self-locking.

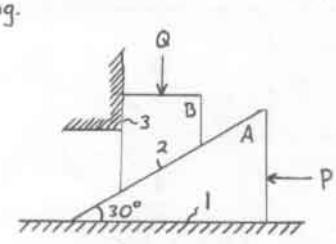
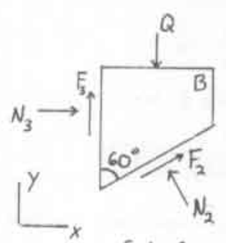


Figure (a)

For the system to be self-locking, motion will not occur for any value of Q, with $P = 0$

FBD of Block B for impending motion



$F_3 = 3kN_3$, $F_2 = 2kN_2$

$\Sigma F_x = 0: N_3 + (2kN_2)\sin 60° - N_2 \cos 60° = 0$

$N_3 = N_2(0.5 - 1.732k)$ (a)

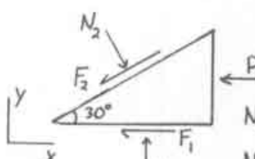
$\Sigma F_y = 0: 3kN_3 - Q + N_2 \sin 60° + 2kN_2 \cos 60° = 0$ (b)

Sub from (a) into (b) for N_3 and solve for N_2

$3kN_2(0.5 - 1.732k) - Q + 0.866N_2 + kN_2 = 0$

$$N_2 = \frac{Q}{[0.866 + 2.5k - 5.196k^2]} \quad (c)$$

FBD of wedge A for impending motion



$F_1 = kN_1$, $F_2 = 2kN_2$

$\Sigma F_x = 0:$

$N_2 \sin 30° - 2kN_2 \cos 30° - kN_1 - P = 0$

$N_2 = (P + kN_1)/(0.5 - 1.732k)$ (d)

$\Sigma F_y = 0: N_1 - N_2 \cos 30° - 2kN_2 \sin 30° = 0$

$N_1 = N_2(0.866 + k)$ (e)

Sub from (e) into (d) for N_1 and solve for N_2

$$N_2 = \frac{P}{(0.5 - 2.6k - k^2)} \quad (f)$$

Equate (f) and (c), solve for P, set $P = 0$ and solve for k.

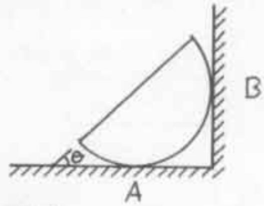
$$P = \frac{Q(0.5 - 2.6k - k^2)}{(0.866 + 2.5k - 5.196k^2)} = 0$$

For $Q \neq 0$ then $(0.5 - 2.6k - k^2) = 0$

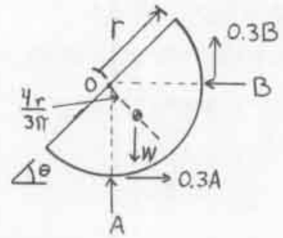
$\therefore k = 0.180$

10.36

A uniform semicircular disk has weight W. $\mu_s = 0.3$. Find the maximum angle θ for equilibrium.



FBD of the disk with slip impending



$\Sigma F_x = 0: \ 0.3A - B = 0$

$B = 0.3A$ (a)

$\Sigma F_y = 0: A + 0.3B - W = 0$

Sub from (a) for B:

$A = 0.9174\,W$

$B = 0.2752\,W$ (b)

$\Sigma M_O = 0: \ (0.3A)r + (0.3B)r - W\left(\frac{4r}{3\pi}\sin\theta\right) = 0$

Sub from (b) for A and B:

$0.3r(0.9174W + 0.2752W) - W\left(\frac{4r}{3\pi}\sin\theta\right) = 0$

$W\left(0.3578r - \frac{4r}{3\pi}\sin\theta\right) = 0$

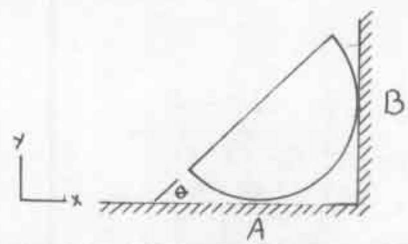
$\therefore \theta = 57.46°$ or $\theta = 1.003$ rad

10.37

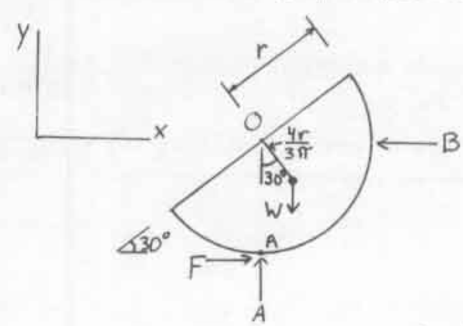
The concrete half-disk leans against a smooth wall at an angle $\theta = 30°$, μ_s between the concrete and floor is 0.30.

a) Find the friction force required for equilibrium.

b) Determine whether or not it can exist.



FBD of the Disk with slip impending



(continued)

10.37 cont.

a/ $\Sigma F_y = A - W = 0$

$A = W \qquad (a)$

$\circlearrowleft \Sigma M_A = Fr - W\left(\frac{4r}{3\pi}\sin 30^\circ\right) = 0 \qquad (b)$

Solving (a) and (b) for F,

$F = \frac{2A}{3\pi} = 0.212\,A$

b/ for impending slip,

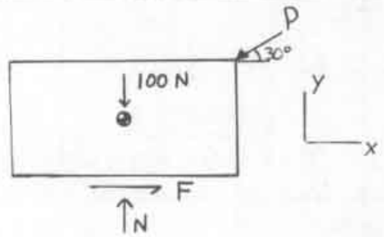
$F_s = \mu_s A = 0.3\,A$

Since $F_s > F$, this friction force can exist.

10.38

A block of weight $W = 100$ N sets on a horizontal plane with $\mu_s = 0.60$ and $\mu_k = 0.20$. Force P acts down and to the left at an angle of 30°. Plot the friction force F for $0 \le P \le 200$ N.

FBD:



At impending slip, $F = 0.6N$

$\Sigma F_x = 0: 0.6N - P\cos 30^\circ = 0$

$N = 1.443P \qquad (a)$

$\Sigma F_y = 0: N - P\sin 30 - 100 = 0$

Sub from (a) for N:

$P = 106.0$ N

Prior to slip:

$\Sigma F_x = 0: \quad F - P\cos 30^\circ = 0$

$F = 0.866P \qquad (b)$

After slip:

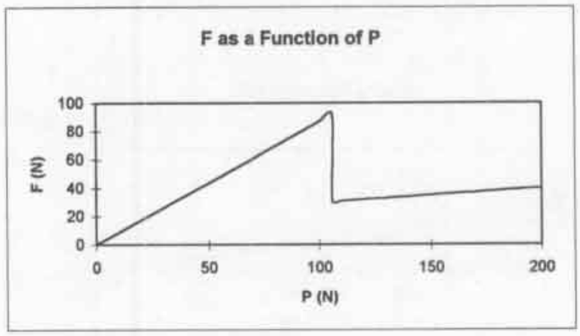
$\Sigma F_y = 0: \quad N - 100 - P\sin 30 = 0$

$N = 100 + 0.5P$

$F = 0.2N = 20 + 0.1P \qquad (c)$

Plot equation (b) for $0 \le P \le 106$ N

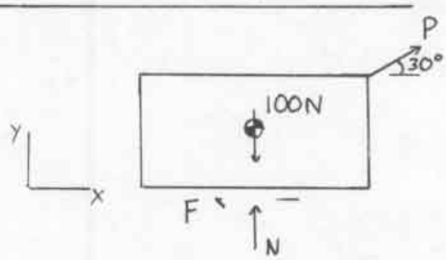
Plot equation (c) for $106 < P \le 200$



10.39

A block of weight $W = 100$ N sets on a horizontal plane with $\mu_s = 0.60$ and $\mu_k = 0.20$. Force P acts up and to the right at an angle of 30°. Plot the friction force F for $0 \le P \le 200$ N.

FBD



At impending slip, $F = 0.6N$

$\Sigma F_x = 0: \; P\cos 30^\circ - 0.6N = 0$

$N = 1.443P \qquad (a)$

$\Sigma F_y = 0: \; N - 100 + P\sin 30^\circ = 0$

Sub from (a) for N

$P = 51.47$ N

Prior to slip:

$\Sigma F_x = 0: \; P\cos 30^\circ - F = 0$

$F = 0.866P \qquad (b)$

After slip:

$\Sigma F_y = 0: \; N - 100 + P\sin 30^\circ = 0$

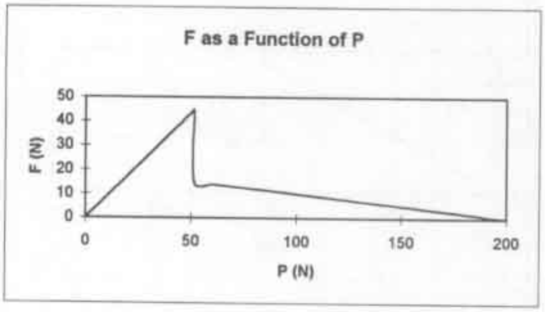
$N = 100 - 0.5P$

$F = 0.2N = 20 - 0.1P \qquad (c)$

(continued)

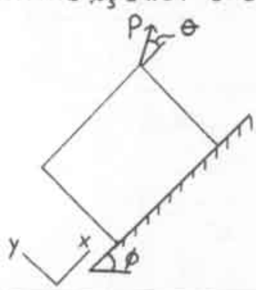
10.39 Cont.

Plot equation (b) for $0 \le P \le 51.47$ N
Plot equation (c) for $51.47 < P \le 200$ N



10.40

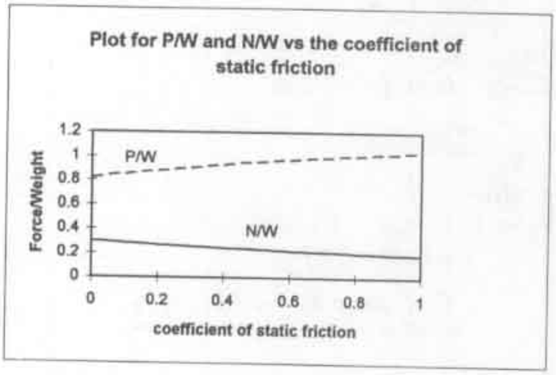
A block of weight W rests on an inclined plane of angle ϕ and coefficient of static friction μ_s. A force P pulls on a block at an angle θ from the plane. Plot P/W and N/W as functions of μ_s for $0 \le \mu_s \le 1.0$. $\theta = 30°$ and $\phi = 45°$



From example 10.4, equations (b) and (c):

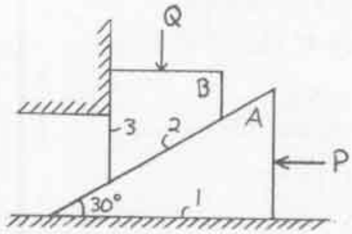
$$\frac{P}{W} = \frac{\sin\phi + \mu_s \cos\phi}{\cos\theta + \mu_s \sin\theta}$$

$$\frac{N}{W} = \frac{\cos(\theta + \phi)}{\cos\theta + \mu_s \sin\theta}$$

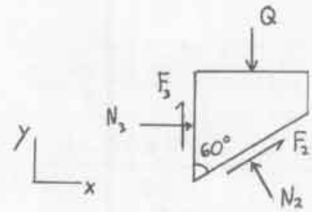


10.41

Wedge A and block B carry load Q and force P. $\mu_1 = 0.1$, $\mu_2 = 0.2$, $\mu_3 = 0.3$. Show that $P = 0.2163\,Q$ for motion of block B downward impending.



FBD of block B for impending motion downward.



$F_2 = 0.2\,N_2$
$F_3 = 0.3\,N_3$

$\Sigma F_x = 0$: $N_3 + (0.2N_2)\sin 60° - N_2 \cos 60° = 0$

$\underline{N_3 = 0.3268\,N_2}$ (a)

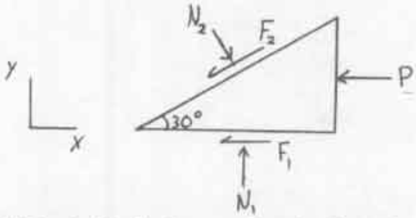
$\Sigma F_y = 0$: $(0.3N_3) - Q + N_2 \sin 60° + (0.2N_2 \cos 60°) = 0$ (b)

Sub from (a) into (b) for N_3 and solve for N_2

$0.3(0.3268N_2) - Q + 0.866N_2 + 0.1N_2 = 0$

$\underline{N_2 = 0.9398\,Q}$ (c)

FBD of wedge A for impending motion



$\Sigma F_y = 0$: $N_1 - N_2(\cos 30°) - (0.2)N_2(\sin 30°) = 0$

$N_1 = 0.9660\,N_2$ (d)

Sub from (c) into (d) for N_2:

$\underline{N_1 = 0.9079\,Q}$ (e)

$\Sigma F_x = N_2 \sin 30° - (0.2N_2)\cos 30° - (0.1N_1) - P = 0$ (f)

Sub from (c) and (e) into (f) for N_1 and N_2:

$0.4699Q - 0.1628Q - 0.0908Q = P$

$\underline{\underline{P = 0.2163\,Q}}$

10.42

The conveyor system shown in fig. a. Both drums A and B are on the verge of slipping. The friction of pulley P is negligible.

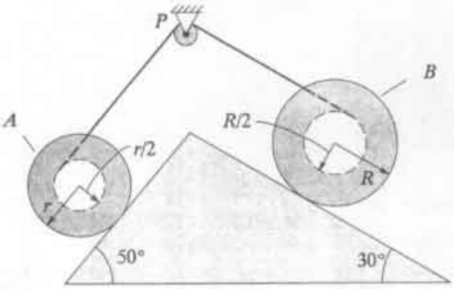


Figure a

a) Find the coefficients of static friction μ_{sA} and μ_{sB} between the inclined surfaces and drums A and B.

b) Determine the ratio W_B/W_A of the weights of the drums.

FBD of drum A

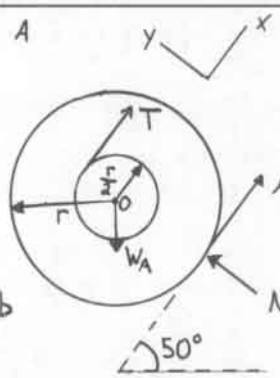


Figure b

$\circlearrowleft \Sigma M_O = 0: \mu_{sA} N_A r - T \frac{r}{2} = 0$

or $T = 2\mu_{sA} N_A$ (a)

$\Sigma F_x = T + \mu_{sA} N_A - W_A \sin 50° = 0$

or $3\mu_{sA} N_A = W_A \sin 50°$ (b)

$\Sigma F_y = N_A - W_A \cos 50° = 0$

or $N_A = W_A \cos 50°$ (c)

Hence, by Eqs. (b) and (c),

$$\mu_{sA} = \frac{1}{3} \tan 50° = 0.3972$$

Similarly, by equilibrium of drum B (Fig. c)

FBD of drum B

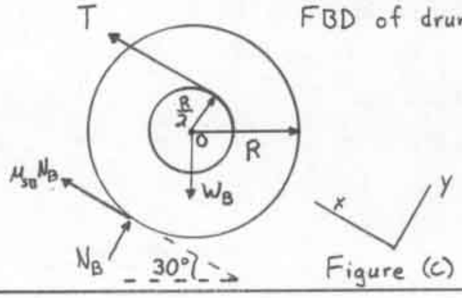


Figure (c)

$\circlearrowleft \Sigma M_O = 0: T \frac{R}{2} - \mu_{sB} N_B R = 0$

or $T = 2\mu_{sB} N_B$ (d)

$\Sigma F_x = T + \mu_{sB} N_B - W_B \sin 30° = 0$

or $3\mu_{sB} N_B = W_B \sin 30°$ (e)

$\Sigma F_y = N_B - W_B \cos 30° = 0$

or $N_B = W_B \cos 30°$ (f)

Hence by Eqs. (e) and (f)

$$\mu_{sB} = \frac{1}{3} \tan 30° = 0.1924$$

b/ By Eqs. (a) and (d)

$2\mu_{sA} N_A = 2\mu_{sB} N_B$ (g)

Then, by Eqs. (c), (f), and (g), we find

$$\frac{W_B}{W_A} = \frac{\mu_{sA} \cos 50°}{\mu_{sB} \cos 30°} = 1.532$$

10.43

A sailor applies a force of 120 lb to the handle of a windlass (Fig. a). One end of the 1-in diameter anchor rope is fixed to the shaft and the other end is attached to the anchor. The coefficient of sliding friction of the bearing is $\mu_k = 0.30$.

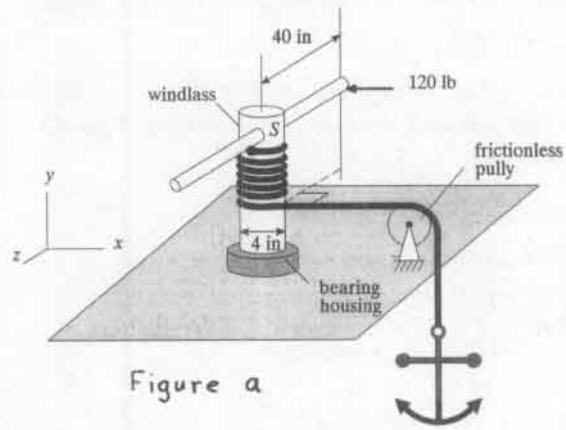


Figure a

a) Determine the weight of the anchor the sailor can raise

b) The ratio W/W', where W' is the weight of an anchor the sailor can raise if friction is negligible.

(continued)

10.43 Cont.

a/ Consider the top view of the shaft and handle (Fig. b). Assume that the bearing reactions on the shaft are equivalent to a concentrated force N (normal to the shaft) and the friction force $\mu_k N$ (tangent to the shaft) that act at some unknown point on the shaft circumference. Neglect the weight of the rope.

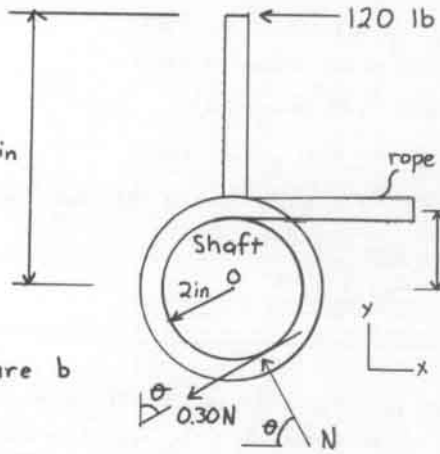


Figure b

By fig. b,

$\Sigma F_x = W - N\cos\theta - 0.30N\sin\theta - 120 = 0$ (a)

$\Sigma F_y = N\sin\theta - 0.30N\cos\theta = 0$ (b)

$\circlearrowleft \Sigma M_O = 120(40) - 2.5W - 0.30N(2) = 0$ (c)

By Eq. (b),

$\tan\theta = 0.30;\ \theta = 16.7°$ (d)

By Eqs. (a), (c), and (d), eliminating N and solving for W, we find,

$\underline{W = 1584 \text{ lb}}$

b/ If friction is negligible, the term involving N does not appear in Eq. (c). Then,

$2.5W' = 120(40)$

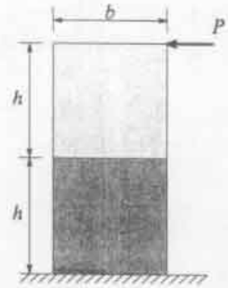
or $W' = 1920$ lb (f)

So, by Eqs. (e) and (f),

$$\underline{\underline{\frac{W}{W'} = 0.825 = 82.5\%}}$$

10.44

Two identical crates are placed on a horizontal surface, one on top of the other (Fig. a). The force P acts horizontally at the top of the upper crate and increases gradually. The coefficient of static friction is 0.20 between the lower crate and the floor, and 0.35 between the two crates.



Determine where the first movement occurs in the system.

Figure a

The analysis of Example 10.3 holds for the two crates taken as a single crate of height 2h and weight 2W, where W is the weight of each crate. Then, for impending sliding as a unit, by Eq. (c) of Example 10.3,

$P = P_{sliding} = \mu_{sc(Floor)}(2W)$

or $P_{sliding} = (0.20)(2W) = 0.40W$ (a)

For impending tipping as a unit, by Eq. (d) of Example 10.3,

$P = P_{tipping} = \dfrac{2Wb}{2(2h)} = \dfrac{Wb}{2h}$

or, since b = h = 600 mm,

$P_{tipping} = \dfrac{W}{2} = 0.50W$ (b)

Consider next the free-body diagram of the upper crate Fig. b)

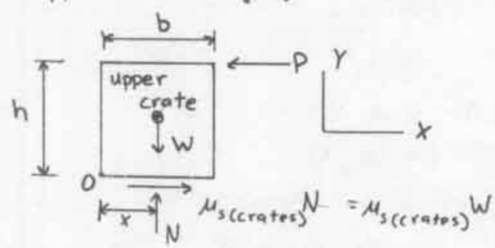


Figure b

For impending sliding of the upper crate,

$\Sigma F_x = \mu_{s(crates)}W - P = 0$

or

$P = P_{sliding} = \mu_{s(crates)}W$

or $P_{sliding} = 0.35W$ (c)

For impending tipping of the upper crate x = 0 (see Fig. b). Then, by Fig. b,

$\circlearrowleft \Sigma M_O = Ph - W\,b/2 = 0$ or $P = P_{tipping} = \dfrac{Wb}{2h}$

since b = h = 600 mm,

$P_{tipping} = 0.5W$ (d)

(Continued)

10.44 Cont.

Comparing Eqs. (a), (b), (c), and (d) for the various impending motions, we see that the minimum value of P occurs for impending sliding of the upper crate [see Eq. (c)]. Hence, the first movement in the system occurs by sliding of the upper crate.

10.45

A wooden block of weight W is set on a wooden plank (Fig. a). The coefficients of static and kinetic friction are $\mu_s = 0.50$ and $\mu_k = 0.30$.

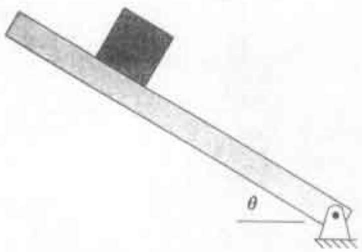


Figure a

Plot the ratios F/W, where F is the frictional force that acts on the block, as a function of the angle θ for $0° \le \theta \le 90°$

Consider the free-body diagram of the block (Fig. b).

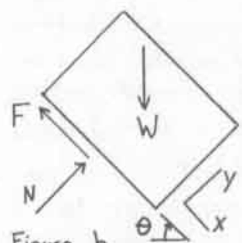


Figure b

For $F \le \mu_s N$, by Fig. b

$$\Sigma F_x = W \sin\theta - F = 0 \qquad (a)$$

$$\Sigma F_y = N - W\cos\theta = 0 \qquad (b)$$

By Eqs. (a) and (b),

$$F = W\sin\theta \qquad (c)$$

$$N = W\cos\theta \qquad (d)$$

Equation (c) remains valid for

$$F \le \mu_s N = \mu_s W\cos\theta = 0.5\,W\cos\theta \qquad (e)$$

when

$$F = 0.5W\cos\theta \qquad (f)$$

sliding of the block is impending. Once sliding occurs,

$$F = \mu_k N = \mu_k W\cos\theta = 0.3\,W\cos\theta \qquad (g)$$

By Eqs. (c) and (e), sliding is impending when

$$F = W\sin\theta = 0.5\,W\cos\theta$$

or when

$$\tan\theta = 0.5\ ;\ \theta = 26.565°$$

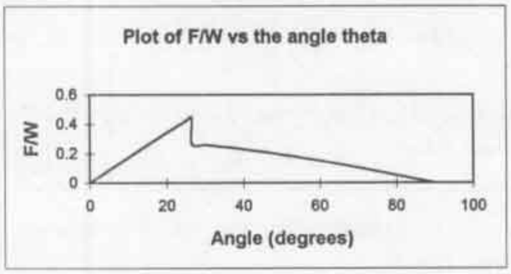
Thus, for $0 \le \theta \le 26.565°$, by Eq. (c)

$$\frac{F}{W} = \sin\theta \qquad (h)$$

For $26.565° \le \theta \le 90°$, by Eq. (g)

$$\frac{F}{W} = 0.30\cos\theta \qquad (i)$$

Equations (h) and (i) give F/W as a function of θ. See the plot below.



10.46

Figure a represents the jaws of a tension-testing machine. The bar to be tested is clamped by wedges B and is subjected to a tension force P (Fig. a)

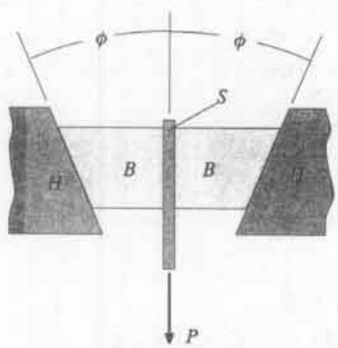


Figure a

The free-body diagram of the left-side wedge is shown in Fig. b.

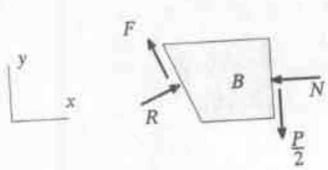


Figure b

(Continued)

10.46 Cont.

The bar should not slip in the wedges for any force P. As P increases the wedges slide slightly on the retainers (Fig a). The coefficient of static friction between B and the bar is μ_s and the coefficient of sliding friction between B and H is μ_K.

a) Write the equations of equilibrium for the wedge B (Fig. b). Neglect the weight of the wedge.

b) For $P/2 < \mu_s N$ and $F = \mu_K R$, show that for the jaws to operate properly

$$\tan\phi < \frac{\mu_s - \mu_K}{1 + \mu_s \mu_K} \qquad (a)$$

c) Noting that $\mu_s = \tan\phi_s$ and $\mu_K = \tan\phi_K$, show that $\phi < \phi_s - \phi_K$ (b)

a/ By the free-body diagram of Fig. b and with Fig. a,

$$\Sigma F_x = R\cos\phi - N - F\sin\phi = 0 \qquad (a)$$

$$\Sigma F_y = R\sin\phi - \frac{P}{2} + F\cos\phi = 0 \qquad (b)$$

b/ For no impending slipping between B and the bar, and for sliding between B and H,

$$P/2 < \mu_s N, \quad F = \mu_K R \qquad (c)$$

By Eqs. (a) and (b),

$$F = P/2 \cos\phi - N\sin\phi \qquad (d)$$

$$R = P/2 \sin\phi + N\cos\phi \qquad (e)$$

By Eqs. (c), (d), and (e)

$$\frac{F}{R} = \mu_K = \frac{P/2\cos\phi - N\sin\phi}{P/2\sin\phi + N\cos\phi}$$

Dividing numerator and denominator by $\cos\phi$ we find

$$\frac{F}{R} = \mu_K = \frac{P/2 - N\tan\phi}{P/2\tan\phi + N} \qquad (f)$$

and solving Eq. (f) for N we find,

$$N = \frac{P}{2}\left(\frac{1 - \mu_K\tan\phi}{\mu_K + \tan\phi}\right) \qquad (g)$$

With $\frac{P}{2} < \mu_s N$ [see Eq. (c)] and Eq. (g) we have

$$\frac{P}{2} < \mu_s \frac{P}{2}\left(\frac{1 - \mu_K\tan\phi}{\mu_K + \tan\phi}\right)$$

or rearranging, we obtain

$$\tan\phi < \frac{\mu_s - \mu_K}{1 + \mu_s\mu_K}$$

c/ Now let

$$\mu_s = \tan\phi_s, \quad \mu_K = \tan\phi_K \qquad (i)$$

Substituting Eq. (i) into (h) yields

$$\tan\phi < \frac{\tan\phi_s - \tan\phi_K}{1 + \tan\phi_s\tan\phi_K} \qquad (j)$$

By Eq. (B.19), Appendix B,

$$\frac{\tan\phi_s - \tan\phi_K}{1 + \tan\phi_s\tan\phi_K} = \tan(\phi_s - \phi_K) \qquad (k)$$

Hence, by Eqs (j) and (k)

$$\phi < \phi_s - \phi_K$$

10.47

A sheet of aluminum is passed through a roll-finishing machine (Fig. a).

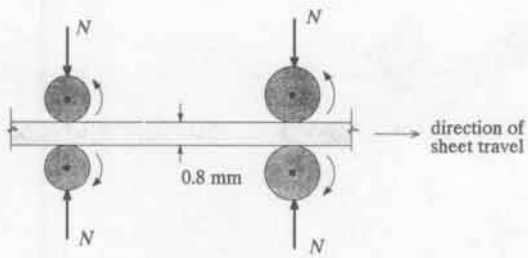


Figure a

The thickness of the sheet between the pairs of rollers is 0.8 mm. The coefficient of static friction between a roller and the sheet is $\mu_s = 0.80$. Since the rollers are drivers at the same angular speed, the larger rollers pull the sheet through faster than the smaller rollers, stretching the aluminum sheet between the larger and smaller rollers so that the tensile stress in the sheet is 430 N/mm².

Find the normal force N per unit width (direction perpendicular to the plane of Fig. a) with which the rollers must be pressed against the sheet to prevent slipping.

Consider the free-body diagram of the sheet for a length extending from a cut between the larger and smaller rollers to a cut in front of the larger rollers (Fig. b).

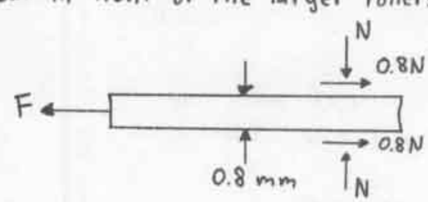


Figure b

(Continued)

10.47 Cont.

Let the width of the strip be 1mm wide. Then, $F = (430 \frac{N}{mm^2})(0.8\ mm)(1\ mm)$

By Fig. b,

$$\Sigma F_x = 2(0.8)N - (430)(0.8) = 0$$

Hence,

$$\underline{\underline{N = 215\ N \text{ per unit width}}}$$

10.48

In Example 10.5, let the weight W_A of the wedge and W_B of the block be $W_A = W_B = Q/20$ (Fig. a). The coefficients of static friction for surfaces 1, 2, and 3 are 0.1, 0.2, and 0.3 respectively.

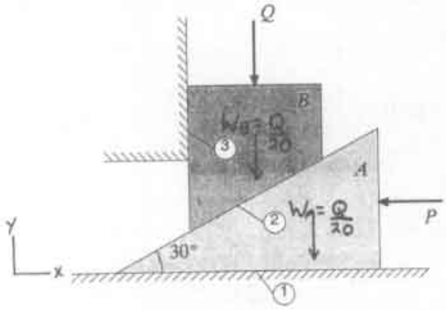


Figure a

Find the range of the force P for which there is no motion.

First consider the case where the block B tends to move upward (on the verge of slipping). Figure b is the free-body diagram of the block and wedge as a unit.

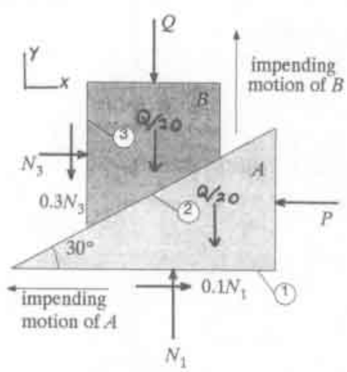


Figure b

By Fig. b,

$$\Sigma F_x = N_3 + 0.1N_1 - P = 0 \qquad (a)$$

$$\Sigma F_y = N_1 - 0.3N_3 - Q - Q/20 - Q/20 = 0$$

or

$$\Sigma F_y = N_1 - 0.3N_3 - 1.1Q = 0 \qquad (b)$$

Figure C is the free-body diagram of the wedge.

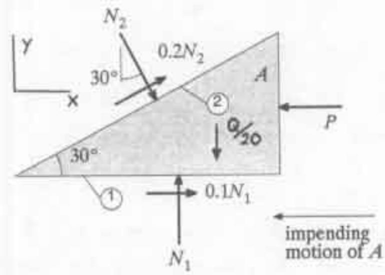


Figure c

By Fig. C,

$$\Sigma F_x = 0.1N_1 + 0.2N_2 \cos 30° + N_2 \sin 30° - P = 0 \quad (c)$$

$$\Sigma F_y = N_1 + 0.2N_2 \sin 30° - N_2 \cos 30° - Q/20 = 0 \quad (d)$$

Thus, by Eqs. (a), (b), (c), and (d)

$$\begin{aligned} &0.1N_1 + N_3 - P = 0 \\ &N_1 - 0.3N_3 - 1.1Q = 0 \\ &0.1N_1 + 0.6732N_2 - P = 0 \\ &N_1 - 0.766N_2 - 0.05Q = 0 \end{aligned} \qquad (e)$$

Solving Eqs. (e) for P, we find

$$\underline{\underline{P = 1.401Q}} \qquad (f)$$

If the block B is on the verge of moving down, the frictional forces in Figs. b and c are reversed in sense. Then Eqs. a, b, c, and d yield (with friction forces reversed)

$$\begin{aligned} &N_3 - 0.1N_1 - P = 0 \\ &N_1 + 0.3N_3 - 1.1Q = 0 \\ &-0.1N_1 + 0.3268N_2 - P = 0 \\ &N_1 - 0.966N_2 - 0.05Q = 0 \end{aligned} \qquad (g)$$

Solving Eqs. (g) for P, we find

$$\underline{\underline{P = 0.2222Q}} \qquad (h)$$

Hence, by Eqs. (f) and (h), the range of force P for no motion is

$$\underline{\underline{0.2222Q \le P \le 1.401Q}}$$

10.49

In Fig. a, the coefficient of friction between the automobile tires and the pavement is 0.75.

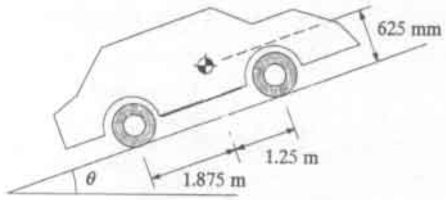


Figure a

Find the angle θ of the steepest hill the car can climb at constant speed for the following cases.

a) The car has four-wheel drive
b) The car has only rear-wheel drive
c) The car has only front-wheel drive

In such a problem with several different cases, it is efficient to consider the general case and specialize it for individual cases. Thus, consider the free-body diagram of the car with friction acting at all the tires (Fig. b)

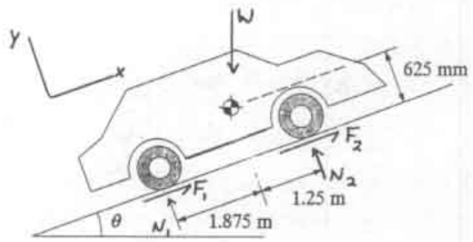


Figure b

General case:

$$\Sigma F_x = F_1 + F_2 - W\sin\theta = 0$$

$$\Sigma F_y = N_1 + N_2 - W\cos\theta = 0$$

$$\circlearrowleft \Sigma M_G = (F_1+F_2)(0.625) + N_2(1.25) - N_1(1.875) = 0$$

or

$$F_1 + F_2 = W\sin\theta \qquad (a)$$

$$N_2 + N_1 = W\cos\theta \qquad (b)$$

$$F_1 + F_2 = 3N_1 - 2N_2 \qquad (c)$$

By Eqs. (a) and (b)

$$\tan\theta = \frac{F_1+F_2}{N_1+N_2} \qquad (d)$$

Also,

$$F_1 = \mu N_1 \ , \quad F_2 = \mu N_2 \qquad (e)$$

a/ The car has four-wheel drive.
Then, by Eq. (e)

$$F_1 = 0.75N_1 \ , \quad F_2 = 0.75N_2$$

and by Eq. (d)

$$\underline{\tan\theta = 0.75 \ ; \quad \theta = 36.87°}$$

b/ The car has only rear-wheel drive.
Then,

$$F_1 = 0.75N_1 \ , \quad F_2 = 0$$

By Eq. (c),

$$0.75N_1 = 3N_1 - 2N_2 \quad \text{or} \quad N_2 = 1.125N_1$$

Then, by Eq. (d)

$$\tan\theta = \frac{0.75N_1}{2.125N_1} = 0.3529$$

Hence,

$$\underline{\theta = 19.44°}$$

c/ The car has only front-wheel drive.
Then,

$$F_1 = 0 \ , \quad F_2 = 0.75N_2$$

By Eq. (c),

$$0.75N_2 = 3N_1 - 2N_2$$

or $N_1 = 0.9166\ N_2$

Then, by Eq. (d),

$$\tan\theta = \frac{0.75N_2}{1.9166N_2} = 0.3913$$

Therefore,

$$\underline{\underline{\theta = 21.37°}}$$

Thus, four-wheel drive allows the car to climb a steeper incline, followed by the front-wheel drive and the rear-wheel drive.

10.50

a) Draw a free-body diagram of the block of example 10.4 for sliding impending down the plane (Fig. a)

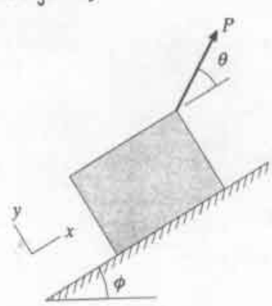


Figure a

b) Derive equations for P and N analogous to Eqs. (d) and (e) of example 10.4.

(Continued)

10.50 Cont.

a/

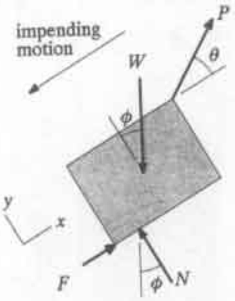


Fig. b - free-body diagram

b/ By Fig. b,

$$\Sigma F_x = P\cos\theta - W\sin\phi + F = 0$$
$$\Sigma F_y = P\sin\theta - W\cos\phi + N = 0 \qquad (a)$$

When slipping is impending, the frictional force is,

$$F = F_s = \mu_s N \qquad (b)$$

Where μ_s is the coefficient of static friction. Substituting Eq. (b) into Eqs. (a), we obtain

$$P\cos\theta + \mu_s N = W\sin\phi$$
$$P\sin\theta + N = W\cos\phi$$

By Cramer's rule (Appendix A·2)

$$P = \frac{\begin{vmatrix} W\sin\phi & \mu_s \\ W\cos\phi & 1 \end{vmatrix}}{\begin{vmatrix} \cos\theta & \mu_s \\ \sin\theta & 1 \end{vmatrix}} = W\left(\frac{\sin\phi - \mu_s\cos\phi}{\cos\theta - \mu_s\sin\theta}\right) \qquad (c)$$

$$N = \frac{\begin{vmatrix} \cos\theta & W\sin\phi \\ \sin\theta & W\cos\phi \end{vmatrix}}{\begin{vmatrix} \cos\theta & \mu_s \\ \sin\theta & 1 \end{vmatrix}} = W\left(\frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\cos\theta - \mu_s\sin\theta}\right)$$

or

$$N = W\left[\frac{\cos(\theta+\phi)}{\cos\theta - \mu_s\sin\theta}\right] \qquad (d)$$

By definition,

$$\tan\phi_s = \mu_s = \frac{\sin\phi_s}{\cos\phi_s} \qquad (e)$$

Substituting Eq. (e) into Eqs. (c) and (d) yields after simplification

$$\underline{\underline{P = W\left[\frac{\sin(\phi - \phi_s)}{\cos(\phi_s + \theta)}\right]}}$$

$$\underline{\underline{N = W\left[\frac{\cos\phi_s\cos(\theta+\phi)}{\cos(\phi_s+\theta)}\right]}}$$

10.51

A uniform bar of weight W and square cross section lies on a horizontal floor (Fig. a)

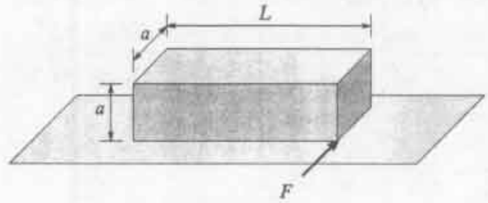


Figure a

The horizontal force F causes impending motion. The coefficient of static friction is μ_s.

a) Show that the bar is on the verge of rotating about the vertical axis at the distance $L/\sqrt{2}$ from the end 1 at which the force F is applied.

b) Then show that $F = \mu_s W(\sqrt{2} - 1)$

a/ Consider a top view of the bar which shows the horizontal forces acting on the bar (Fig. b). Point O is the axis of rotation.

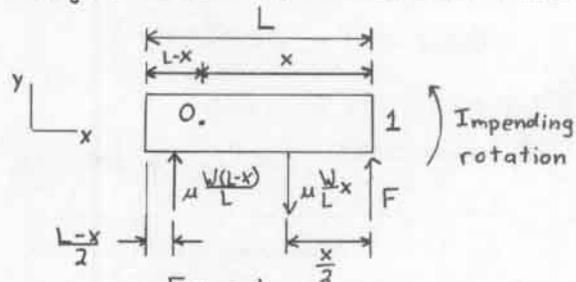


Figure b

The frictional forces oppose the rotation about O, and since the pressure exerted on the bar is uniform act at the midpoints of L-x and x (Fig. b)

By Fig. a,

$$\circlearrowleft \Sigma M_1 = \mu\frac{W}{L}x\left(\frac{x}{2}\right) - \mu\frac{W}{L}(L-x)\left(x + \frac{L-x}{2}\right) = 0$$

Simplification yields,

$$\mu\frac{W}{L}\frac{x^2}{2} - \mu\frac{W}{L}\left(\frac{L^2}{2} - \frac{x^2}{2}\right) = 0$$

or $\underline{\underline{x = \frac{L}{\sqrt{2}}}}$

b/ By Fig. b,

$$\Sigma F_y = F - \mu\frac{W}{L}x + \mu\frac{W}{L}(L-x) = 0$$

With $x = L/\sqrt{2}$, this equation yields

$$\underline{\underline{F = \mu W(\sqrt{2} - 1)}}$$

10.52

Let $\mu_s = \tan\phi_s = 0.2$ ($\phi_s = 11.31°$) in Example 10.4.
a) Plot P/W for $0 \le \theta < 90°$, with $\phi = 30°$.
b) Plot P/W for $0 \le \phi \le 90°$, with $\theta = 30°$
c) Check parts a and b for $\theta = \phi = 30°$

By Eq. (d) of Example 10.4, with $\phi_s = 11.31°$, we have in general

$$\frac{P}{W} = \frac{\sin(\phi + 11.31)}{\cos(11.31 - \theta)} \qquad (a)$$

a/ For $\phi = 30°$, Eq. (a) yields

$$\frac{P}{W} = \frac{\sin 41.31°}{\cos(11.31 - \theta)} = \frac{0.6601}{\cos(11.31 - \theta)} \qquad (b)$$

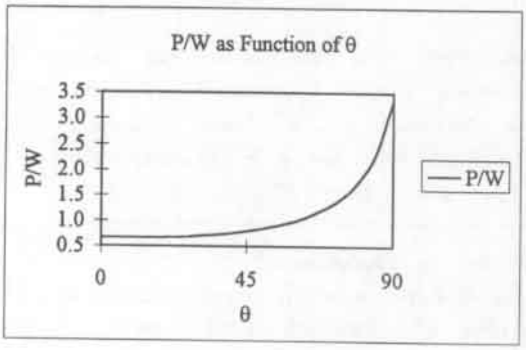


Figure a: for $0 \le \theta < 90°$

b/ For $\theta = 30°$, Eq. (a) yields

$$\frac{P}{W} = \frac{\sin(\phi + 11.31°)}{\cos(-18.69°)} = 1.013 \sin(\phi + 11.31°) \quad (c)$$

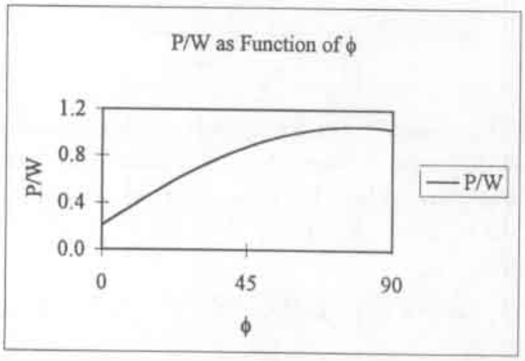


Figure b: for $0 \le \phi \le 90°$

c/ By Eq. (a) for $\phi = \theta = 30°$,

$$\frac{P}{W} = 0.6970$$

This value corresponds to the plot values at $\theta = 30°$ and at $\phi = 30°$.

10.53

The block-wedge system of example 10.5 is shown in Fig. a.

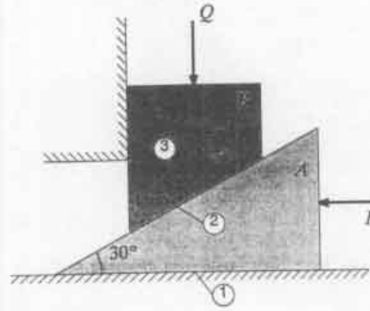


Figure a

Find the smallest coefficient of static friction μ_s between block A and the horizontal plane that will make the system self-locking; that is, the system does not move for P=Q

Consider the free-body diagram of the wedge A (Fig. b). For surface ②, $\mu = 0.2$ and for surface ①, $\mu = \mu_s$.

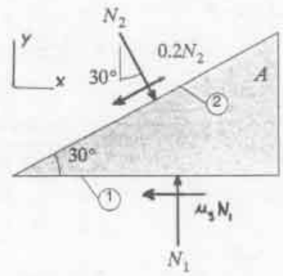


Figure b

By Fig. b,

$$\Sigma F_x = N_2 \sin 30° - 0.2 N_2 \cos 30° - \mu_s N_1 = 0 \qquad (a)$$

$$\Sigma F_y = N_1 - N_2 \cos 30° - 0.2 N_2 \sin 30° = 0 \qquad (b)$$

By Eq. (b),

$$N_1 = 0.966\, N_2 \qquad (c)$$

Substitution of Eq. (c) into Eq. (a) yields

$$0.966 \mu_s = \sin 30° - 0.2 \cos 30° = 0.3268$$

or

$$\underline{\underline{\mu_s = 0.338}}$$

10.54

A Roman contractor attempted to move a stone column by mounting it as an axle between two large wheels (of radii $r_1 < r_2$ and rigidly attached to the column and rolling it (Fig a).

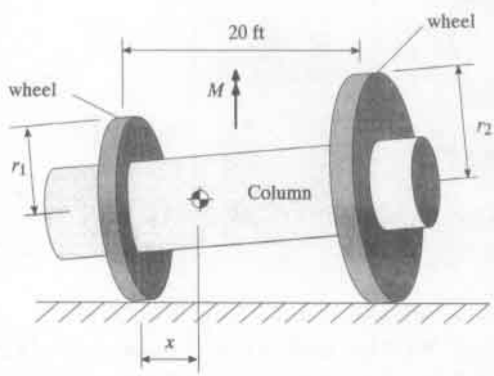


Figure a

The contractor's reputation was ruined because he could not keep the column rolling straight, since $r_1 < r_2$. The system (column and wheels) weighed 30,000 lb, and the center of gravity was a distance $x < 10$ ft from the smaller wheel. The coefficient of kinetic friction was 0.60. The wheel with the lighter load skidded and the other wheel rolled.

a) Find which wheel skidded.

b) Find the couple M that must be applied to keep the column rolling straight on level ground.

c) Find whether or not the contractor could have succeeded by shifting the column axially so that more load was carried by one wheel.

Consider the front view of the column and wheels (Fig. b)

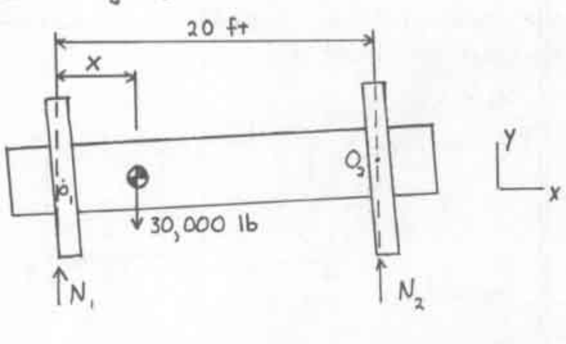


Figure b - (Front view)

By Fig. b,

$\circlearrowleft\!+ \Sigma M_{O_1} = 20 N_2 - 30{,}000\, x = 0 ; \quad N_2 = 1500x \text{ (lb)}$

$\circlearrowleft\!+ \Sigma M_{O_2} = -20 N_1 + 30{,}000(20 - x) = 0 ; \; N_1 = 30{,}000 - 1500x$

Since $x < 10$ ft, $N_2 < N_1$. Therefore, wheel 2 skidded.

b) Consider the top view of the column and wheel (Fig. c)

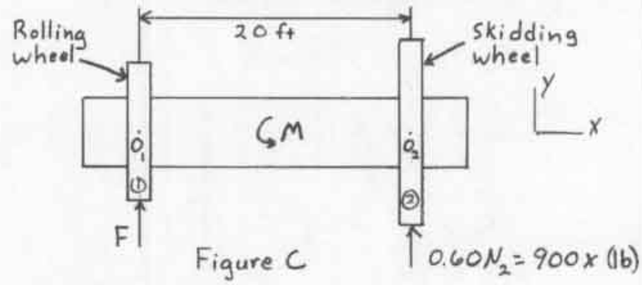


Figure C

By Fig. C,

$\circlearrowleft\!+ \Sigma M_{O_1} = 20(900x) + M = 0$

or $\underline{\underline{M = -18000x \text{ lb·ft}}}$ (a)

Thus, as viewed from the top a clockwise couple of magnitude 18000x must be applied to keep the column rolling straight.

c) By Eq. (a) note that $M \to 0$ as $x \to 0$.

Therefore, the contractor could have succeeded by moving the column axially so that most of the load was carried on one wheel.

10.55

A uniform stick of length L and weight W leans at an angle θ in a vertical plane against a ledge of height h (Fig. a).

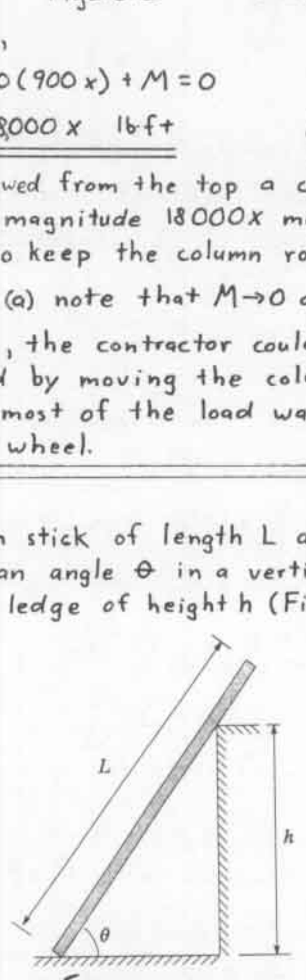


Figure a

a) Show that, if the stick is on the verge of slipping,

$$\cos\theta - \cos^3\theta = \frac{2\mu_s h}{(1+\mu_s^2)L} \qquad (I)$$

where μ_s is the coefficient of static friction

(Continued)

10.55 Cont.

b) Show that, if

$$\frac{\mu_s h}{(1+\mu_s^2)L} > \frac{1}{\sqrt{27}} \qquad (\mathrm{II})$$

the stick will not slip at any angle.

a/ Consider the free-body diagram of the stick (Fig. b) for impending slipping

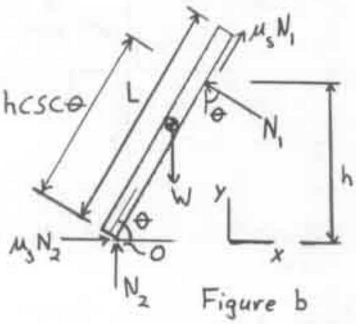


Figure b

By Fig. b,

$$\Sigma F_x = \mu_s N_2 - N_1 \sin\theta + \mu_s N_1 \cos\theta = 0 \qquad (a)$$

$$\Sigma F_y = N_2 + N_1 \cos\theta + \mu_s N_1 \sin\theta - W = 0 \qquad (b)$$

$$\circlearrowleft \Sigma M_O = N_1 h \csc\theta - W(L/2)\cos\theta = 0 \qquad (c)$$

By Eq. (c),

$$N_1 = \frac{WL}{2h} \sin\theta \cos\theta \qquad (d)$$

By Eqs. (b) and (d),

$$N_2 = W\left[1 - \frac{L}{2h}\sin\theta\cos\theta(\cos\theta + \mu_s \sin\theta)\right] \qquad (e)$$

Also, by Eqs. (a) and (d),

$$N_2 = \frac{WL}{2\mu_s h}(\sin\theta)(\cos\theta)(\sin\theta - \mu_s\cos\theta) \qquad (f)$$

Equating Eqs. (e) and (f) and simplifying, we obtain

$$\sin^2\theta\cos\theta = \frac{2h}{L}\cdot\frac{\mu_s}{1+\mu_s^2}$$

or, since $\sin^2\theta = 1 - \cos^2\theta$

$$\cos\theta - \cos^3\theta = \frac{2\mu_s h}{(1+\mu_s^2)L} \qquad (g)$$

Equation (g) verifies Eq. (I)

b/ Let $f(\theta) = \cos\theta - \cos^3\theta$

For $f(\theta)$ to be a maximum, $df/d\theta = 0$. Hence

$$\frac{df}{d\theta} = -\sin\theta + 3\sin\theta\cos^2\theta = 0$$

Hence, either $\sin\theta = 0$ $(\theta = 0)$ or $3\cos^2\theta - 1 = 0$ $(\cos\theta = 1/\sqrt{3})$. The case $\theta = 0$ is the trivial case (the stick stands vertically). Therefore, $\cos\theta = 1/\sqrt{3}$ is the desired result.

Hence, the maximum value of $f(\theta)$ is

$$[f(\theta)]_{max} = [\cos\theta - \cos^3\theta]_{max} = \frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}$$

or

$$(\cos\theta - \cos^3\theta)_{max} = \frac{2\sqrt{3}}{9} \qquad (h)$$

Therefore, by Eqs. (g) and (h), there is no slipping if

$$\frac{2\sqrt{3}}{9} < \frac{2\mu_s h}{(1+\mu_s^2)L}$$

or,

$$\frac{\mu_s h}{(1+\mu_s^2)L} > \frac{\sqrt{3}}{9} = \frac{1}{\sqrt{27}} \qquad (i)$$

Equation (i) verifies Eq. (II)

10.56

The sense of the force P in Fig. P 10.29 is directed to the left (see Fig. a below). Part C weighs 2W and bar AB weighs W. The coefficent of static friction is μ_s.

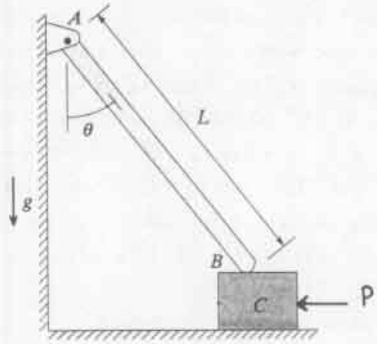


Figure a

a) Rework Problem 10.29 for this case.

b) For $\mu_s = 0.2$, plot P/W as a function of θ and determine a value for θ for which the machine part cannot be moved to the left

a/ The free-body diagrams of bar AB and part C are shown in Figs. b and c for impending motion of part C.

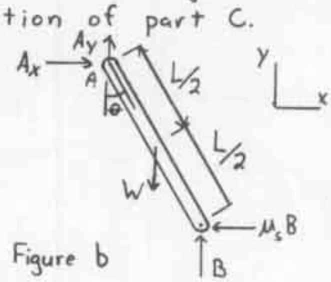


Figure b

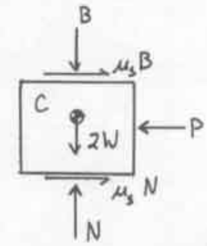


Figure c

(Continued)

10.56 Cont.

By Fig. b,

$$\circlearrowleft \Sigma M_A = B(L\sin\theta) - \mu_s B(L\cos\theta) - W\left(\frac{L}{2}\sin\theta\right) = 0$$

or

$$B = \frac{W\sin\theta}{2(\sin\theta - \mu_s\cos\theta)} \qquad (a)$$

By Fig. c,

$$\Sigma F_x = \mu_s B + \mu_s N - P = 0 \qquad (b)$$

$$\Sigma F_y = N - 2W - B = 0 \qquad (c)$$

By Eq. (c), $N = 2W + B$ (d)

Substituting Eq. (d) into Eq. (b) and solving for B, we obtain

$$B = \frac{P - 2\mu_s W}{2\mu_s} \qquad (e)$$

Equating Eqs (a) and (e) and solving for P, we find

$$P = \mu_s W\left(\frac{3\sin\theta - 2\mu_s\cos\theta}{\sin\theta - \mu_s\cos\theta}\right) \qquad (f)$$

For $W = 200\text{ lb}$, $L = 10\text{ ft}$, $\theta = 45°$, and $\mu_s = 0.2$, Eq. (f) yields

$\underline{P = 130\text{ lb}}$

b/ For $\mu_s = 0.2$, Eq. (f) yields

$$\frac{P}{W} = \frac{3\sin\theta - 0.4\cos\theta}{5\sin\theta - \cos\theta} \qquad (g)$$

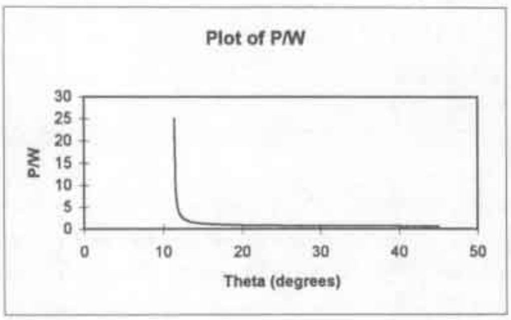


Figure d

For part C not to move to the left (system self-locking), it must not move no matter how large the force P. Hence, by Eq. (g), as $P \to \infty$,

$$5\sin\theta - \cos\theta \to 0$$

or

$$\tan\theta = 0.2 \ (=\mu_s); \quad \theta = 11.31°$$

Thus, if $\theta \le 11.31°$, the system is self-locking.

10.57

For a complete statement of the problem refer to the text. Briefly stated you are required to design a door stop that will keep a door of various sizes ($2.5\text{ ft} \le a \le 4\text{ ft}$, Fig. a) from closing for various floor surface materials ($0.1 \le \mu_s \le 0.9$, Fig. b). The door's spring hinges exerts a closing moment Mc ($10\text{ lbft} \le M_c \le 40\text{ lbft}$, Fig. a)

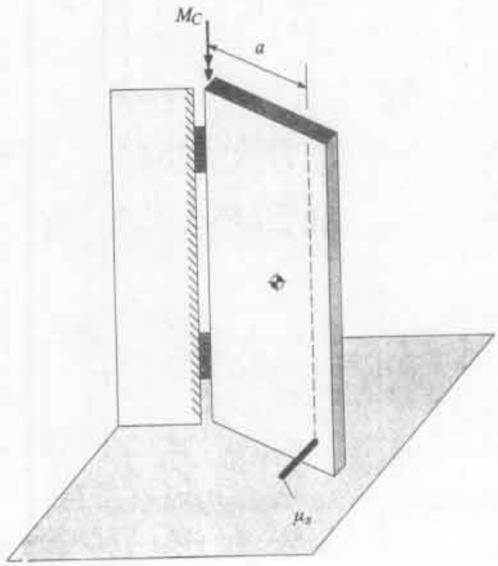


Figure a

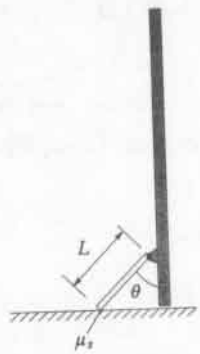


Figure b

Design Objectives: Examine the various parameters (L, a, θ, μ_s) that might affect the operation, and design a door stop that will keep the door from closing.

Consider the free-body diagrams of the door (Fig. c) and the stop (Fig. d). (Note that the stop is a two force member.)

(Continued)

10.57 Cont.

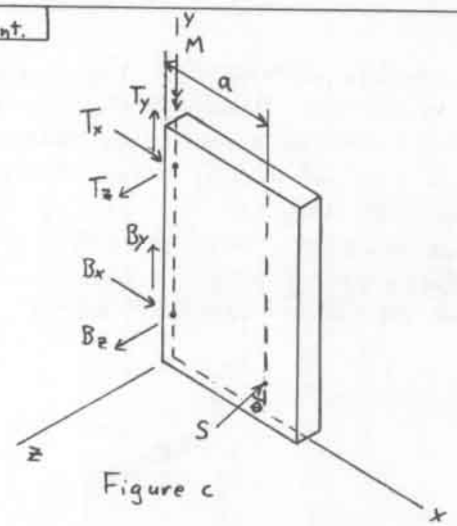


Figure c

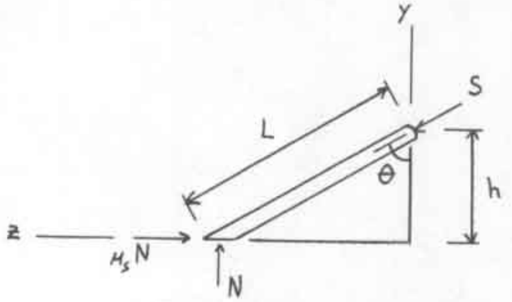


Figure d

By Fig. c,

$$\circlearrowleft\!+\ \Sigma M_y = S(\sin\theta)a - M = 0$$

or $\quad S = \dfrac{M}{a\sin\theta}$ $\qquad$ (a)

By Fig. d, at impending motion,

$$\circlearrowleft\!+\ \Sigma M_A = \mu_s N(L\cos\theta) - N(L\sin\theta) = 0 \qquad (b)$$

$$\Sigma F_y = N - S\cos\theta = 0 \qquad (c)$$

$$\Sigma F_z = S\sin\theta - \mu_s N = 0 \qquad (d)$$

By Eq. (b)

$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta \qquad (e)$$

By Eqs. (a), (c), and (e)

$$N = \frac{M}{\mu_s a} \qquad (f)$$

Hence, $\quad \mu_s N = \dfrac{M}{a}$ $\qquad$ (g)

Note that in the relevant equations for the design, the length L of the stop doesn't enter. However, the angle θ does have an effect, as do M, a, and μ_s.

In order to design a door stop to hold the door open for all coefficients of static friction ($0.1 \le \mu_s \le 0.9$), you should choose the minimum value for μ_s ($\mu_s = 0.1$).

Then by Eq. (e)

$$\underline{\underline{\tan\theta = 0.1\ ;\quad \theta = 5.71^\circ}} \qquad (h)$$

For this value of θ and with the observation that the typical height above the floor for the door-stop connection is approximately $h = 4$ in, we find from Fig. d.

$$\underline{\underline{L = \frac{h}{\cos\theta} = \frac{4\text{ in}}{\cos 5.71^\circ} = 4.02\text{ in}}}$$

The worse-case scenario for any door would be to have

$$M_C = 40\text{ lb}\cdot\text{ft}$$
$$a = 2.5\text{ ft}$$
$$\mu_s = 0.1$$

Then, by Eq. (f) and (a)

$$N = \frac{M}{\mu_s a} = \frac{40}{(0.1)(2.5)} = 160\text{ lb}$$

$$S = \frac{M}{a\sin\theta} = \frac{40}{(2.5)\sin 5.71^\circ} = 160.8\text{ lb}$$

The door door-stop connection must be capable of sustaining the load $S = 160.8$ lb

Summary: The following parameters meet the worse-case scenario.

$\mu_s = 0.1$

$\theta = 5.71^\circ$

$M_c = 40$ lb·ft

$a = 2.5$ ft

$L = 4.02$ in

10.58

The rock crusher of Problem 6.9 as shown in Fig. a. Modify the crusher to exert a force of 20,000 lb on the rock. The coefficient of static friction between the rock and horizontal surface is $\mu_s = 0.20$, F has a maximum value of 1 ton. Rods AC and BC must maintain the angles of 15° and the weight of block B is negligible.

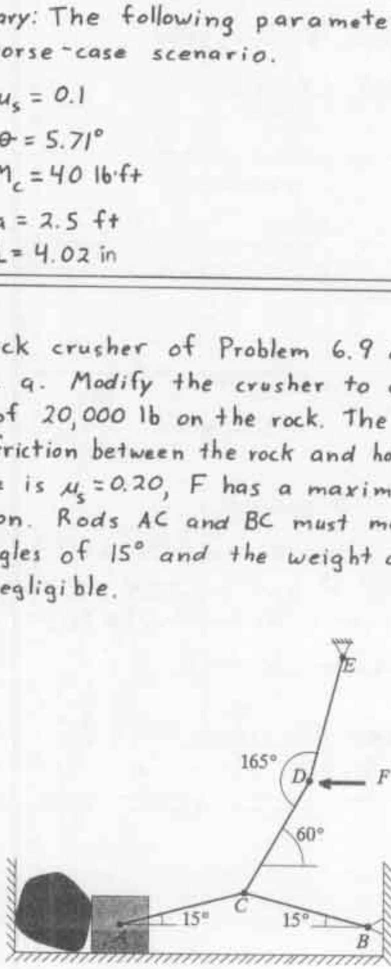


Figure a

(Continued)

10.58 Cont.

Find a design to meet the requirements.

To analyze the rock crusher as shown in Fig. a, consider first the free-body diagram of block A (Fig. b), when motion impends.

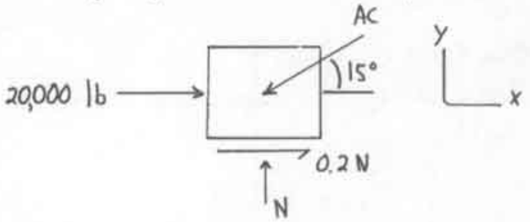


Figure b

By Fig. b,

$$\Sigma F_x = 20{,}000 + 0.2N - AC\cos 15° = 0 \qquad (a)$$

$$\Sigma F_y = N - AC\sin 15° = 0 \qquad (b)$$

The solution of Eqs. (a) and (b) is

$$AC = 21{,}878 \text{ lb} \qquad (c)$$

$$N = 5662 \text{ lb} \qquad (d)$$

Next, consider the free-body diagram of pin C (Fig. c)

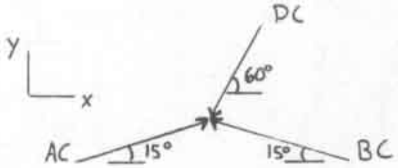


Figure c

By Fig. c,

$$\left.\begin{aligned}\Sigma F_x &= AC\cos 15° - BC\cos 15° - DC\cos 60° = 0\\ \Sigma F_y &= AC\sin 15° + BC\sin 15° - DC\sin 60° = 0\end{aligned}\right\} \qquad (e)$$

The solution of Eqs. (e)

$$BC = 16{,}016 \text{ lb} \qquad (f)$$

$$DC = 11{,}325 \text{ lb} \qquad (g)$$

By the free-body diagram of pin D (Fig. d),

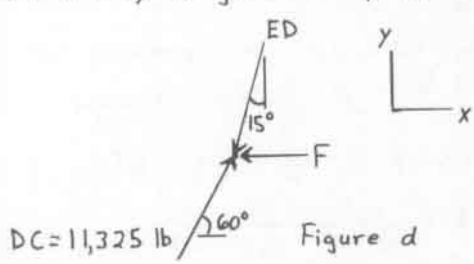


Figure d

$$\left.\begin{aligned}\Sigma F_x &= (11{,}325)\cos 60° - ED\sin 15° - F = 0\\ \Sigma F_y &= (11{,}325)\sin 60° - ED\cos 15° = 0\end{aligned}\right\} \qquad (h)$$

The solution of Eqs. (h) is

$$ED = 10{,}154 \text{ lb}$$

$$F = 3035 \text{ lb}$$

Since $F = 3035$ lb > 2000 lb, the crusher rods DC and ED must be modified. There are several possibilities. One possibility is shown in Fig. e (ED = DC)

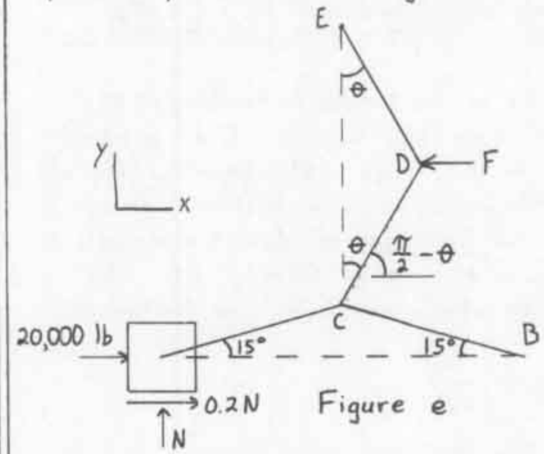


Figure e

As before (see Fig. b and Eqs. (a) and (b), $AC = 21{,}878$ lb, and by the free-body diagram of pin C (see Figs. e and f).

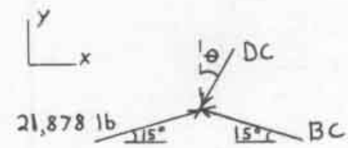


Figure f

$$\left.\begin{aligned}\Sigma F_x &= (21{,}878)\cos 15° - BC\cos 15° - DC\sin\theta = 0\\ \Sigma F_y &= (21{,}878\sin 15° + BC\sin 15° - DC\cos\theta = 0\end{aligned}\right\} \qquad (i)$$

By Eqs. (i), we find

$$DC\,(1.0353\sin\theta + 3.8637\cos\theta) = 43756 \qquad (j)$$

By the free-body diagram of pin D (Fig. g), with $F = 2000$ lb,

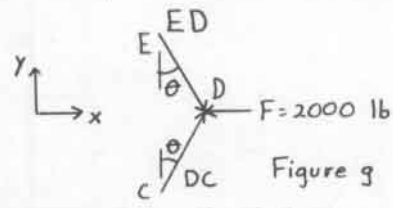


Figure g

$$\Sigma F_x = DC\sin\theta + ED\sin\theta - 2000 = 0$$

$$\Sigma F_y = DC\cos\theta - ED\cos\theta = 0$$

Therefore, $\quad 2DC\sin\theta = 2000 \qquad (k)$

Hence, by Eqs. (j) and (k)

$$DC = \frac{1000}{\sin\theta} = \frac{43756}{1.0353\sin\theta + 3.8637\cos\theta}$$

(Continued)

10.58 Cont.

or after simplification,

$$\tan\theta = 0.09044 \;;\; \theta = 5.167°$$

Thus, if $0 < \theta \le 5.167°$, the force $F = 2000$ lb will crush the rocks. For a little safety, take $\theta = 5°$. Then, the required force F is slightly less than 2000 lb (1936 lb).

10.59

The weight of the bar AB in Problem 10.29 is W (kN), and the weight W_c of part C may lie in the range $0.5W \le W_c \le 2W$ (Fig. a). The coefficient μ_s of static friction between the bar and part C of different materials may lie in the range $0.1 \le \mu_s \le 0.6$. The surface on which part C rests is frictonless.

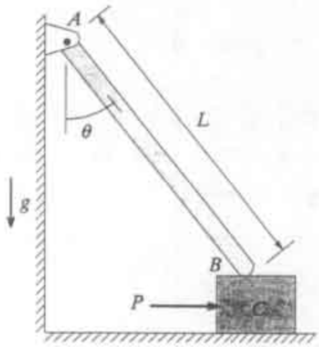


Figure a

Design the system so that the part C may move to the right when force P pushes to the right, but cannot be moved to the left when P pushes to the left.

First assume that P pushes to the right. The free-body diagrams of the bar AB and part C are shown in Figs. b and c, for motion impending.

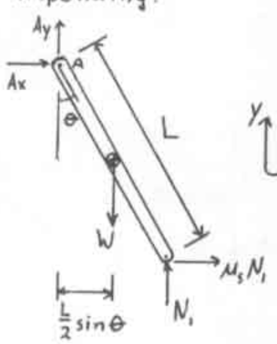


Figure b

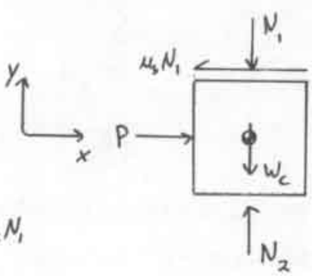


Figure c

By Fig. b,

$$\circlearrowleft \Sigma M_A = N_1 (L\sin\theta) + \mu_s N_1 (L\cos\theta) - W\left(\frac{L}{2}\sin\theta\right) = 0$$

or

$$N_1 = \frac{W\sin\theta}{2(\sin\theta + \mu_s\cos\theta)} \qquad (a)$$

By Fig. c,

$$\Sigma F_x = P - \mu_s N_1 = 0 \;; \quad P = \mu_s N_1 \qquad (b)$$

By Eqs. (a) and (b), for impending motion to the right,

$$P = \frac{\mu_s W\sin\theta}{2(\sin\theta + \mu_s\cos\theta)}$$

independent of W_c. Therefore, for sliding to the right,

$$P > \frac{\mu_s W\sin\theta}{2(\sin\theta + \mu_s\cos\theta)} \qquad (c)$$

regardless of W_c and for all angles $\theta \ge 0$.

Next assume that P pushes to the left. The free-body diagrams of bar AB and part C are shown in Figs. d and e.

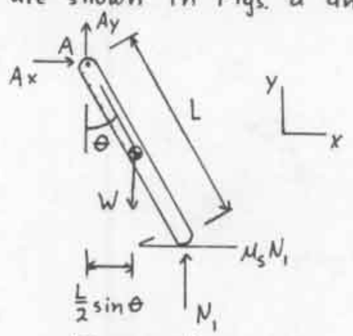


Figure d

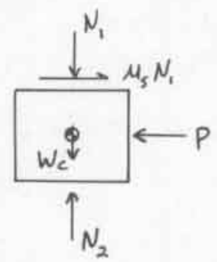


Figure e

By Fig. d,

$$\circlearrowleft \Sigma M_A = N_1 (L\sin\theta) - \mu_s N_1 (L\cos\theta) - W\left[\frac{L}{2}\sin\theta\right] = 0$$

or

$$N_1 = \frac{W\sin\theta}{2(\sin\theta - \mu_s\cos\theta)} \qquad (d)$$

By Fig. e,

$$\Sigma F_x = \mu_s N_1 - P = 0 \;; \quad P = \mu_s N_1 \qquad (e)$$

By Eqs. (d) and (e), for impending motion to the left,

$$P = \frac{\mu_s W\sin\theta}{2(\sin\theta - \mu_s\cos\theta)} \qquad (f)$$

To prevent sliding to the left,

$$P \le \frac{\mu_s W\sin\theta}{2(\sin\theta - \mu_s\cos\theta)} \qquad (g)$$

The largest magnitude of the right-hand side of Eq. (g) occurs as $\sin\theta - \mu_s\cos\theta \to 0$ or as

$$\tan\theta \to \mu_s$$

Then

$$P \le \infty$$

(Continued)

10.59 Cont.

Thus, if $\theta = \tan^{-1}\mu_s$, the part C will never move to the left.

For $\mu_s = 0.6$, $\theta = 30.96°$. For $\mu_s = 0.1$, $\theta = 5.71°$. Note however, if we take $\theta = 30.96°$ and $\mu_s = 0.1$, Eq. (g) yields $P \le 6W$. Hence, for $\theta = 30.96°$ and $\mu_s = 0.1$, if $P > 6W$ sliding to the left will occur. Thus, we must have $\theta = 5.71°$ to ensure sliding is prevented for all μ_s where $0.1 \le \mu_s \le 0.60$.

Note also for sliding to the right, with $\theta = 5.71°$, Eq. (c) yields

$P > 0.043W$ for $\mu_s = 0.6$

and

$P > 0.091W$ for $\mu_s = 0.1$

Finally, the length L of bar AB does not affect P. The length L may be chosen to fit the space requirements of the machine.

10.60

The coefficient of static friction between the block C in Problem 10.59 and the surface is μ_s (see Problem 10.59 for design requirements). Determine whether a design can meet the requirement and evaluate its feasibility.

We proceed as in Problem 10.59 and assume first that force P pushes to the right. The free-body diagrams of the bar and part C are shown in Figs. a and b for impending motion

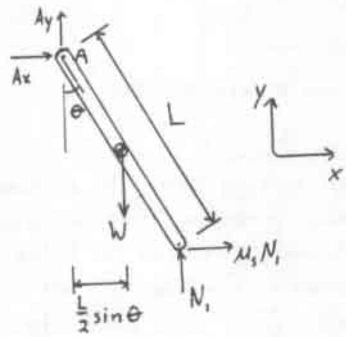


Figure a

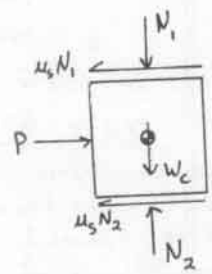


Figure b

By Fig. a,

$$\circlearrowleft \Sigma M_A = N_1(L\sin\theta) + \mu_s N_1 (L\cos\theta) - W\left(\frac{L}{2}\sin\theta\right) = 0$$

or,

$$N_1 = \frac{W\sin\theta}{2(\sin\theta + \mu_s\cos\theta)} \qquad (a)$$

By Fig. b,

$$\Sigma F_x = P - \mu_s N_1 - \mu_s N_2 = 0$$

or

$$P = \mu_s (N_1 + N_2) \qquad (b)$$

$$\Sigma F_y = N_2 - N_1 - W_c = 0 \quad \text{or} \quad N_2 = N_1 + W_c \qquad (c)$$

By Eqs. (a), (b), and (c), we find for impending motion,

$$P = \frac{\mu_s W\sin\theta}{(\sin\theta + \mu_s\cos\theta)} + \mu_s W_c$$

Therefore, to move part C to the right

$$P > \frac{\mu_s W\sin\theta}{(\sin\theta + \mu_s\cos\theta)} + \mu_s W_c \qquad (d)$$

If P pushes to the left, the free-body diagrams of bar AB and part C are shown in Figs. c and d, for impending motion.

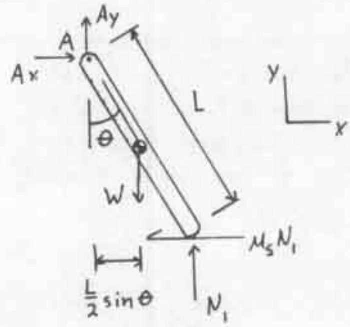


Figure c

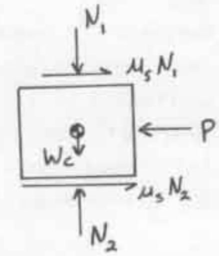


Figure d

By Fig. c,

$$\circlearrowleft \Sigma M_A = N_1(L\sin\theta) - \mu_s N_1 (L\cos\theta) - W\left(\frac{L}{2}\sin\theta\right) = 0$$

or

$$N_1 = \frac{W\sin\theta}{2(\sin\theta - \mu_s\cos\theta)} \qquad (e)$$

By Fig. d,

$$\Sigma F_x = \mu_s N_1 + \mu_s N_2 - P = 0$$

or

$$P = \mu_s (N_1 + N_2) \qquad (f)$$

$$\Sigma F_y = N_2 - N_1 - W_c = 0$$

or

$$N_2 = N_1 + W_c \qquad (g)$$

By Eqs. (e), (f), and (g), we find for impending motion,

$$P = \frac{\mu_s W\sin\theta}{(\sin\theta - \mu_s\cos\theta)} + \mu_s W_c$$

Therefore, if part C is not to move to the left,

$$P \le \frac{\mu_s W\sin\theta}{(\sin\theta - \mu_s\cos\theta)} + \mu_s W_c \qquad (h)$$

As in the results of Problem 10.59, the largest value of the right-hand side of Eq. (h) occurs as $\sin\theta - \mu_s\cos\theta \to 0$.

or as

$$\tan\theta \to \mu_s$$

(Continued)

10.60 Cont.

Then, $P \leq \infty$. Thus if $\theta = \tan^{-1}\mu_s$, the part C will never move to the right, irrespective of the value of W_C [see Eq. (h)]. Hence, as in Problem 10.59, with $\theta = \tan^{-1}(0.1) = 5.71°$, part C will not move to the left for any P. Again as in Problem 10.59, the value of L may be chosen to fit space requirements.

Thus, you may assure your supervisor that the design is feasible.

10.61

Baggage is conveyed to airline passengers on a ramp conveyor-belt system (Fig. a). The coefficient of static friction between various types of baggage and a proposed conveyor belt range from 0.2 to 0.9. The angle θ must be greater than 20°

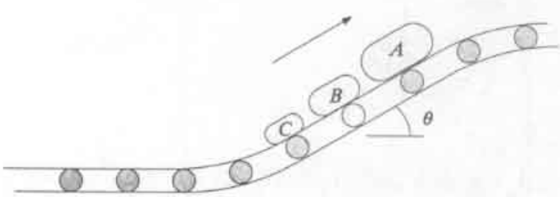


Figure a

a/ Design a conveyor system so that the suitcases $W_A = 75$ lb, $\mu_A = 0.2$; $W_B = 50$ lb, $\mu_B = 0.6$; $W_C = 30$ lb, $\mu_C = 0.9$ can be conveyed in the order shown.

b/ Discuss your design, considering the possibility that the order of the suitcases is changed. Based upon your study, what might you recommend for the surface material of the belt.

a/ Consider a free-body diagram of the system of suitcases in which contact occurs among the suitcases (Fig. a)

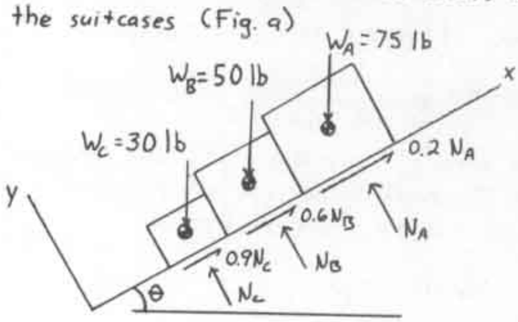


Figure b

By Fig. b, for suitcases A, B, and C, respectively,

$\Sigma F_{y_A} = N_A - W_A \cos\theta = 0; \quad N_A = 75\cos\theta \quad (a)$

$\Sigma F_{y_B} = N_B - W_B \cos\theta = 0; \quad N_B = 50\cos\theta \quad (b)$

$\Sigma F_{y_C} = N_C - W_C \cos\theta = 0; \quad N_C = 30\cos\theta \quad (c)$

Also by Fig b and Eqs. (a), (b), and (c),

$$\Sigma F_x = (0.2)(75\cos\theta) + (0.6)(50\cos\theta) + (0.9)(30\cos\theta) - 75\sin\theta - 50\sin\theta - 30\sin\theta = 0$$

or

$$72\cos\theta = 155\sin\theta$$

Hence,

$$\tan\theta = 0.4645; \quad \theta = 24.92°$$

As a bit of a safety factor take

$$\theta = 24°$$

to ensure no slipping due to jerks and starts

b/ The design of part a is highly dependent on the sequence in which the suitcases are placed on the conveyor belt. It is unlikely that baggage personnel would be able to place the suitcases on the belt in a given order. For example, if suitcase A is placed behind suitcases B and C, it would slide down the belt. This is seen by Fig. (c) and the equilibrium conditions; that is, for A not to slide

$$\theta \leq 11.3°$$

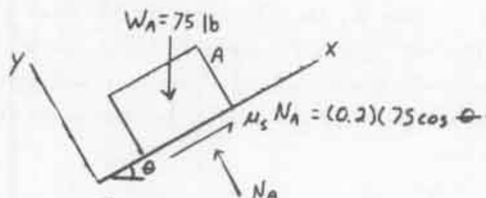


Figure C

$\Sigma F_x = (0.2)(75\cos\theta) - 75\sin\theta = 0$

or $\tan\theta = 0.2, \quad \theta = 11.31°$

Consequently, to ensure that the suitcases do not slip for any sequence, the conveyor belt material should produce a large coefficient of static friction for all baggage materials. It should probably be a soft rubber-like material. (see Table 10.1)

10.62

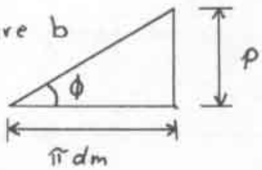
A push screwdriver is shown in Fig. a. The pitch of the thread is $p = 1$ in and the mean diameter is $d_m = 3/8$ in. The coefficent of kinetic friction is $\mu_k = 0.20$.

Figure a

Find the torque exerted on the screw head when you push on the handle with a force $P = 30$ lb

By Fig. E10.7 (see Fig. b below)

Figure b



$\tan\phi = \frac{p}{\pi d_m} = \frac{1}{\pi(3/8)} = 0.8488$

or $\phi = 40.33°$

By Eq. (10.6), $\tan\phi_k = \mu_k = 0.20$

or $\phi_k = 11.31°$

As reviewed from above, the torque exerted on the screw head by the screwdriver is clockwise (Fig. c)

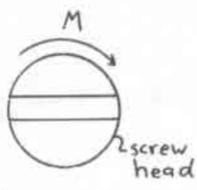


Figure c

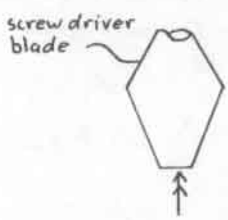


Figure d

Hence, the torque exerted by the screw head on the screwdriver blade is counterclockwise (Fig. d). This torque tends to raise the load P (see Fig. 10.10), and by Eq. (10.7), where $r = d_m/2$,

$$M = Pr\tan(\phi_k + \phi)$$

Therefore,

$$M = (30)\left(\frac{3}{16}\right)\tan(11.31° + 40.33°)$$

$$\underline{M = 7.107 \text{ lb}\cdot\text{in}}$$

10.63

A press is used for bending or straightening steel rods (Fig. a). The screw threads have a mean diameter $d_m = 50$ mm, and a pitch $p = 5$ mm; the coefficient of kinetic friction is $\mu_k = 0.20$

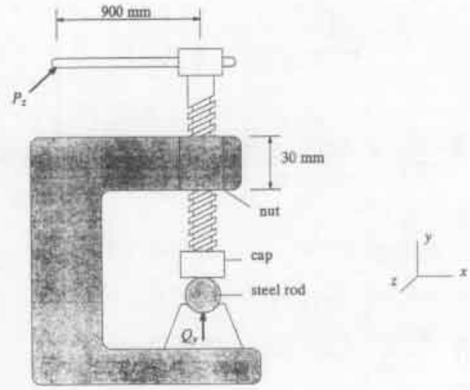
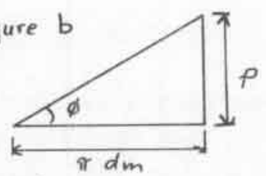


Figure a

Find the force P_z required to produce a vertical $Q_y = 100$ kN on the rod.

By Fig. b (see Fig. E 10.7),

Figure b



$\tan\phi = \frac{p}{\pi d_m} = \frac{5}{\pi(50)} = 0.0318$ or $\phi = 1.823°$

By Eq. (10.6). $\tan\phi_k = \mu_k = 0.20$

or $\phi_k = 11.31°$

Since the cap moves toward the load Q_y (see Fig. 10.10), Eq. (10.7) applies. So,

$$M = (900 \text{ mm})P_z = Q_y\ r\tan(\phi_k + \phi)$$

where $r = d_m/2 = 25$ mm Hence,

$$P_z = \frac{(100 \text{ kN})(25 \text{ mm})}{(900 \text{ mm})}\tan(1.823° + 11.31°)$$

or $\underline{P_z = 648 \text{ N}}$

10.64

The mean radius of a single-threaded, square-thread jackscrew is $r_m = 1.00$ in. The pitch is $p = 0.40$ in, and the coefficient of kinetic friction between the screw and nut is $\mu_k = 0.15$.

a) Find the force, with lever arm of 18 in, required to raise a load of 6000 lb at a constant rate.

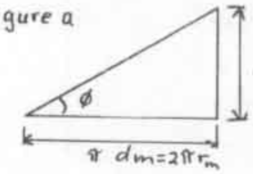
b) Find the force required to lower the load at a constant rate.

(Continued)

10.64 Cont.

a/ By Fig. a,

Figure a



$\tan\phi = \frac{P}{2\pi r_m} = \frac{0.40}{2\pi(1.00)} = 0.06366$; $\phi = 3.643°$

By Eq. (10.6),

$\tan\phi_k = \mu_k = 0.15$; $\phi_k = 8.531°$

By Eq. (10.7), $M = (18\text{ in})R = Pr\tan(\phi_k + \phi)$

or

$$R = \frac{(6000)(1.00)}{(18)}\tan(8.531° + 3.643°)$$

Hence, $\underline{\underline{R = 71.91\text{ lb}}}$

b/ To lower the load, by Eq. (10.9),

$M = 18R = Pr\tan(\phi_k - \phi)$

or

$$R = \frac{(6000)(1.00)}{(18)}\tan(8.531° - 3.643°)$$

Hence, $\underline{\underline{R = 28.51\text{ lb}}}$

10.65

The screw of a book press has 5 threads per inch, a mean diameter $d_m = 1.2$ in, and a coefficient of kinetic friction $\mu_k = 0.12$. Two 20 lb forces are applied as shown (Fig. a).

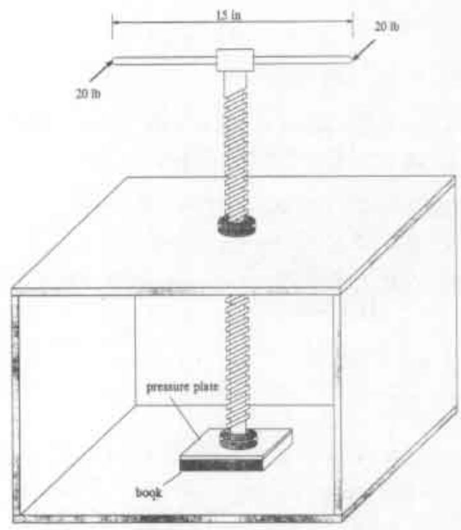
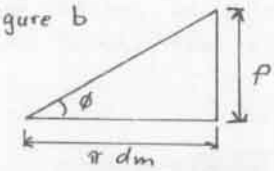


Figure a

Find the fore that the press exerts on the book.

By Fig. b,

Figure b



$\tan\phi = \frac{P}{\pi d_m} = \frac{1/5}{\pi(1.2)} = 0.05305$; $\phi = 3.037°$

By Eq. (10.6),

$\tan\phi_k = \mu_k = 0.12$; $\phi_k = 6.843°$

Since the cap moves toward the plate (load), (see Fig. 10.10), Eq. (10.7) applies. So,

$M = (20\text{ lb})(15\text{ in}) = P(d_m/2)\tan(\phi_k + \phi)$

or

$$P = \frac{(20)(15)}{(0.60)\tan(6.843° + 3.037°)}$$

Hence,

$\underline{\underline{P = 2871\text{ lb}}}$

10.66

A jackscrew that is to support large loads must be designed to be self-locking. The coefficient of kinetic friction may vary from 0.10 to 0.40. The mean radius of the threads is $r_m = 50$ mm.

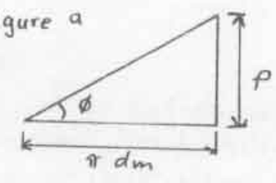
Find the required pitch of the jackscrew.

If the jackscrew is to be self-locking, it must be so for the lowest coefficient of friction. Therefore let $\mu_k = 0.10$. Also for self locking,

$$\phi_k > \phi \qquad (a)$$

By Fig. a, $\tan\phi = \frac{P}{\pi d_m} = \frac{P}{\pi(2)(50)} = \frac{P}{100\pi}$ (b)

Figure a



By Eq. (10.6), $\tan\phi_k = \mu_k = 0.10$ (c)

By Eqs. (a), (b), and (c),

$0.10 > P/100\pi$

or the critical pitch is

$\underline{\underline{P_{critical} = 10\pi = 31.416\text{ mm}}}$

10.67

A single-thread jackscrew with square threads is used to raise the front end of a car that weighs 15 kN (Fig. a). The screw has a mean diameter $d_m = 38$ mm and pitch $p = 10$ mm, and coefficient of kinetic friction $\mu_k = 0.1$. Collar friction is negligible.

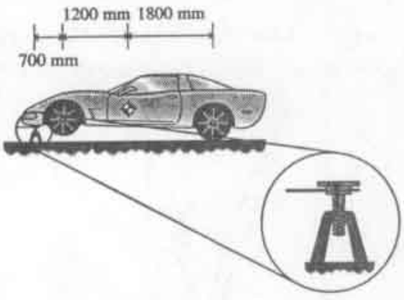


Figure a

Find the turning moment required to raise the front wheels off the ground.

The free-body diagram of the car with the front wheels off the ground is shown in Fig. b.

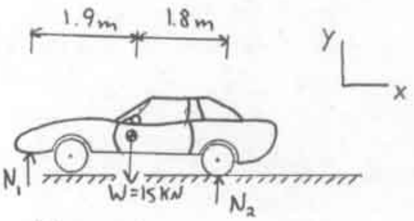


Figure b

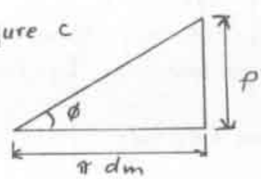
By Fig. b,

$\circlearrowleft \Sigma M_{N_2} = 15(1.8) - N_1(3.7) = 0$

$N_1 = 7297$ N

For the jackscrew, angle ϕ is determined by (see fig. c),

Figure c



$\tan\phi = \frac{p}{\pi d_m} = 10/\pi(38) = 0.08377$ or $\phi = 4.788°$

By Eq. (10.6),

$\tan\phi_k = \mu_k = 0.1$ or $\phi_k = 5.711°$

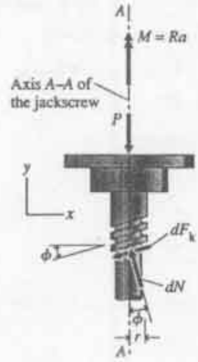
Then, by Eq. (10.7)

$M = P r \tan(\phi_k + \phi) = (7297)(19)\tan(5.711° + 4.788°)/1000$

or $\underline{\underline{M = 25.69 \text{ N·m}}}$

10.68

A square-thread screw, with outer and inner diameter of 1.5 in and 1.25 in, respectively, and 4 threads per inch, is used as a jack. The coefficient of kinetic friction is $\mu_k = 0.1$. A force $R = 60$ lb is applied to the lever at a distance $a = 16$ in (see Fig. a).



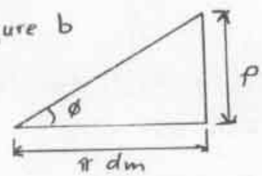
a) Find the load P that the 60 lb force can raise.

b) Find the force R required to lower the load P of part a.

Figure a

a/ By Fig. b

Figure b



$\tan\phi = \frac{p}{\pi d_m} = \frac{1/4}{\pi\left(\frac{1.5+1.25}{2}\right)} = 0.05787$

$\phi = 3.312°$ (a)

By Eq. (10.6),

$\tan\phi_k = \mu_k = 0.1$; $\phi_k = 5.711°$ (b)

Then, by Eq. (10.7),

$M = Ra = P r \tan(\phi_k + \phi)$

$(60)(16) = P(1.375/2)\tan(5.711° + 3.312°)$

or $\underline{\underline{P = 8793 \text{ lb}}}$ (c)

b/ To lower the lower P, by Eq (10.9),

$M = Ra = P r \tan(\phi_k - \phi)$ (d)

By Eqs. (a), (b), (c), and (d)

$R(16) = (8793)\left(\frac{1.375}{2}\right)\tan(5.711° - 3.312°)$

or $\underline{\underline{R = 15.83 \text{ lb}}}$

10.69

A 5° wedge is used to split a block of wood (Fig. a). The coefficient of friction between the wedge and block is 0.25. The wedge is driven with a force of 240 lb.

a) Find the splitting force N normal to the wedge (neglect friction).

b) Find the splitting force N including friction.

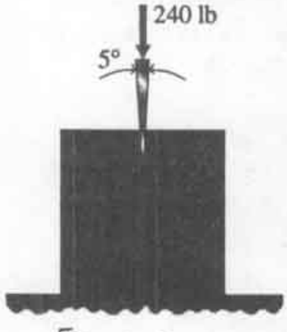


Figure a

a/ The free-body diagram of the wedge is shown in Fig. b.

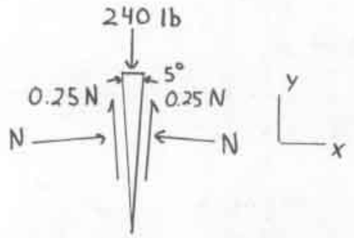


Figure b

By Fig. b, ignoring the friction forces 0.25 N,

$$\Sigma F_y = 2N \sin(2.5°) - 240 = 0$$

or

$$\underline{\underline{N = 2751 \text{ lb}}}$$

b/ By Fig. b, including the friction forces 0.25 N,

$$\Sigma F_y = 2N \sin(2.5°) + 2N(0.25)\cos(2.5°) - 240 = 0$$

or

$$\underline{\underline{N = 408.7 \text{ lb}}}$$

Thus, friction greatly reduces the splitting force. Hence, it is a good idea to lubricate the wedge when splitting wood.

10.70

The compound jackscrew (Fig. a) has a pitch $p = 1$ in, a mean screw diameter $d_m = 1.5$ in, and coefficient of kinetic friction $\mu_k = 0.1$

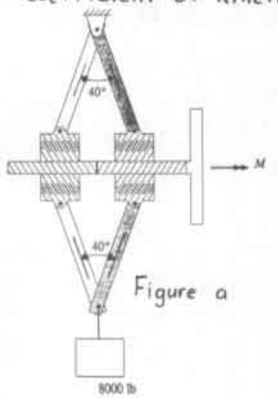


Figure a

Find the moment M required to raise the 800 lb weight.

The angles ϕ_k and ϕ are given by

$$\tan\phi_k = \mu_k = 0.1 \quad \text{or} \quad \phi_k = 5.711° \qquad (a)$$

and,

$$\tan\phi = \frac{p}{\pi d_m} = \frac{1}{\pi(1.5)} = 0.2122$$

$$\text{or} \quad \phi = 11.981° \qquad (b)$$

also,

$$r = \frac{d_m}{2} = 0.75 \text{ in} \qquad (c)$$

Consider next the free-body diagram of the weight and the two lower rods (Fig. b).

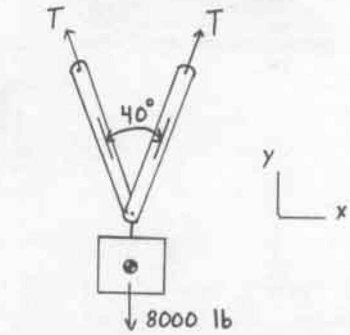


Figure b

By Fig. b, neglecting the weight of the rods,

$$\Sigma F_y = 2T\cos 20° - 8000 = 0$$

or

$$T = 4256.7 \text{ lb}$$

Then, by the free-body diagram of a nut (say, the left nut), Fig. c,

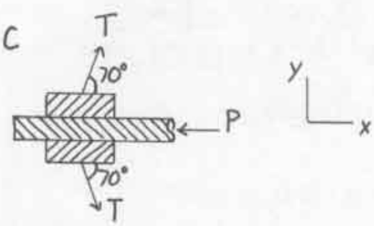


Figure C

$$\Sigma F_x = 2T\cos 70° - P = 0$$

or

$$P = 2T\cos 70 = 2(4256.7)\cos 70°$$

Therefore,

$$P = 2911.8 \text{ lb} \qquad (d)$$

The moment applied to each nut is M/2 (see Fig. a). Therefore, by Eq. (10.7),

$$\frac{M}{2} = Pr\tan(\phi_k + \phi) \qquad (e)$$

Hence, by Eqs. (a), (b), (c), (d), and (e)

$$M = 2(2911.8)(0.75)\tan(5.711° + 11.981°)$$

or

$$\underline{\underline{M = 1393 \text{ lb}\cdot\text{in}}}$$

10.71

A single-thread jackscrew with a square thread has 0.2 threads per millimeter, outer and inner diameters of 18.75 mm and 14.375 mm, and a coefficient of kinetic friction $\mu_k = 0.12$.

a) Find the efficiency η of the jackscrew.

b) Find whether or not the jackscrew is self-locking.

a/ By the given data,

$p = 1/0.2 = 5$ mm $\qquad d_m = \dfrac{(18.75+14.375)}{2} = 16.5625$ mm

$\tan\phi_k = \mu_k = 0.12$ or $\phi_k = 6.8428°$ $\qquad$ (a)

$\tan\phi = \dfrac{p}{\pi d_m} = \dfrac{5}{\pi(16.5625)} = 0.09609$

$\phi = 5.4889°$ $\qquad$ (b)

Then, by Eqs. (10.12), (a), and (b)

$$\eta = \frac{\tan\phi}{\tan(\phi_k+\phi)} = \frac{0.09609}{0.2186}$$

or $\quad \underline{\underline{\eta = 0.440 = 44\,\%}}$

b/ The jackscrew is self-locking when $\phi_k > \phi$ (se discussion near the end of sec. 10.4).

By Eqs. (a) and (b),

$$\phi_k (= 6.8428°) > \phi (= 5.4889°)$$

Hence, the jackscrew is self-locking.

10.72

The physical properties of a jack screw are (Fig. a)

$d_m = 72$ mm

$p = 16$ mm

Under the action of load P alone (K=0), the screw is on the verge of moving downward (Fig. b).

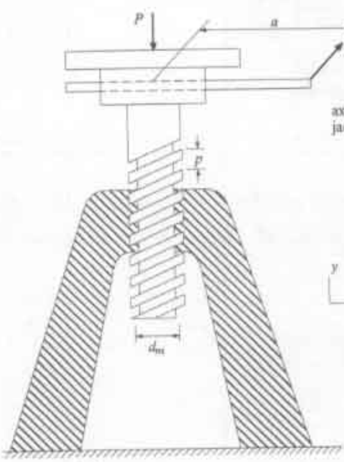


Figure a

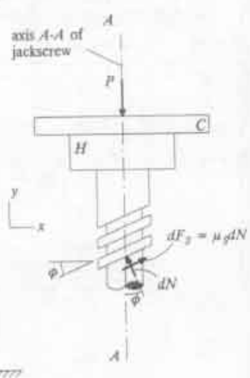


Figure b

a) Find the coefficient of static friction μ_s (see Fig. b).

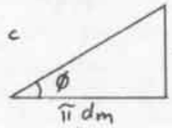
b) Find the efficiency of the jackscrew.

a/ By Fig. b,

$$\Sigma F_x = \mu_s \cos\phi \int dN - \sin\phi \int dN = 0$$

Hence, $\quad \mu_s = \tan\phi \qquad$ (a)

Figure c



By Fig. c,

$$\tan\phi = \frac{p}{\pi d_m} = \frac{16}{\pi(72)} = 0.07074 \qquad \text{(b)}$$

By Eqs. (a) and (b),

$$\underline{\underline{\mu_s = 0.07074}} \qquad \text{(c)}$$

b/ By Eq. (10.9), with $M = Ra = 0$,

$P\tan(\phi_k - \phi) = 0$

so, $\quad \phi_k = \phi$

$\tan\phi_k = \tan\phi = 0.07074 \qquad$ (d)

Hence, $\quad \phi_k = \phi = 4.046° \qquad$ (e)

The efficiency η is by Eqs. (10.12), and (e)

$$\eta = \frac{\tan\phi}{\tan(\phi_k+\phi)} = \frac{0.07074}{0.1422}$$

or, $\quad \underline{\underline{\eta = 0.4975 = 49.75\,\%}}$

10.73

Let the coefficient of static friction be $\mu_s = 0.30$ and of kinetic friction be $\mu_k = 0.10$ for the jackscrew in problem 10.72. Let a load P=50 kN be applied - (see Figs. a and b)

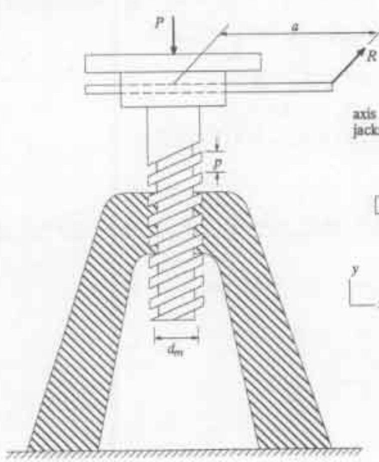


Figure a

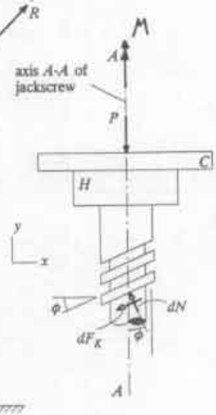


Figure b

(Continued)

10.73 cont.

a) Find the moment $M_s = Ra$ required to cause motion to be pending upward.

b) Find the moment $M_k = Ra$ to maintain motion upward at a constant speed.

c) Determine the efficiency of the jackscrew.

d) Find the mechanical advantage of the jackscrew for lowering the load if $a = 400$ mm.

a/ For motion pending upward, μ_k in Eq. 10.7 is replaced by μ_s. Then,

$$M_s = Ra = Pr\tan(\phi_s + \phi) \qquad (a)$$

Where, with the given data above and in problem 10.72,

$$\tan\phi_s = \mu_s = 0.30\,;\ \phi_s = 16.699° \qquad (b)$$

$$\tan\phi_k = \mu_k = 0.10\,;\ \phi_k = 5.711° \qquad (c)$$

and by Eq. (a), with $\phi = 4.046°$ from problem 10.72,

$$M_s = (50\text{ kN})(0.036\text{m})\tan(16.699° + 4.046°)$$

or

$$\underline{M_s = 681.8\ \text{N}\cdot\text{m}} \quad \text{counter clockwise}$$

b/ By Eqs. (10.7) and (c), with $\phi = 4.046°$,

$$M_k = Pr\tan(\phi_k + \phi)$$

$$M_k = (50\text{kN})(0.036\text{ m})\tan(5.711° + 4.046°)$$

so $\underline{M_k = 309.5\ \text{N}\cdot\text{m}}$ counterclockwise

c/ By Eq. (10.12) and Eq. (c), with $\phi = 4.046°$, the efficiency is

$$\eta = \frac{\tan\phi}{\tan(\phi_k + \phi)} = \frac{0.07074}{0.17196}$$

or $\underline{\eta = 0.411 = 41.1\%}$

d/ By Eq. (10.10), with $a = 400$ mm, the mechanical advantage for lowering the load is

$$m = \frac{a}{r}\cot(\phi_k - \phi)$$

or

$$m = \frac{(0.40)}{(0.036)}\cot(5.711° - 4.046°)$$

so,

$$\underline{m = 382}$$

10.74

A jackscrew shown in Fig. a.

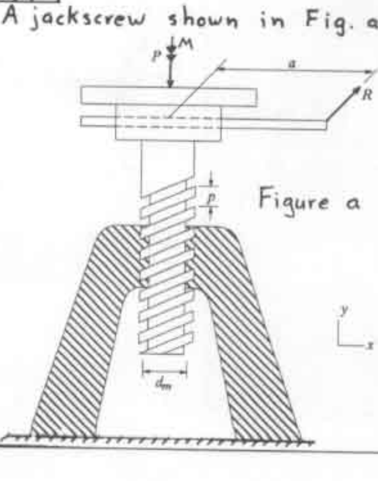


Figure a

a) Draw a diagram showing the normal and tangential forces that act on an infinitesimal length of thread of the jackscrew for the case where P is lowered by a clockwise moment $M = Ra$.

b) Derive Eq. (10.9).

a/ The diagram is shown in Fig. b

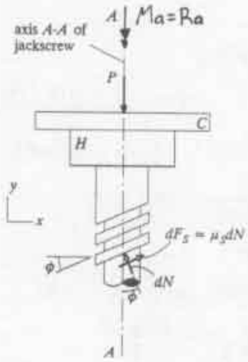


Figure b

b/ To derive Eq. (10.9), by Fig. b,

$$\Sigma F_y = \mu_k \sin\phi \int dN + \cos\phi \int dN - P = 0$$

$$\Sigma M_{A\text{-}A} = r(\mu_k\cos\phi \int dN) - r(\sin\phi \int dN) - M = 0$$

Therefore,

$$P = \mu_k \sin\phi \int dN + \cos\phi \int dN$$
$$\frac{M}{r} = \mu_k\cos\phi \int dN - \sin\phi \int dN \qquad (a)$$

Elimination of $\int dN$ from Eqs. (a) yields

$$M = \frac{Pr(\mu_k\cos\phi - \sin\phi)}{\mu_k\sin\phi + \cos\phi} \qquad (b)$$

Let $\tan\phi_k = \mu_k$. Thus, Eq. (b) may be written as,

$$M = Pr\left(\frac{\tan\phi_k - \tan\phi}{1 + \tan\phi_k\tan\phi}\right)$$

or

$$\underline{M = Pr\tan(\phi_k - \phi)}$$

where $\phi_k = \tan^{-1}\mu_k$

10.75

A jackscrew has a pitch $p = 16$mm, a mean diameter $d_m = 64$mm (see Fig. a), and a coefficient of sliding friction $\mu_k = 0.40$.

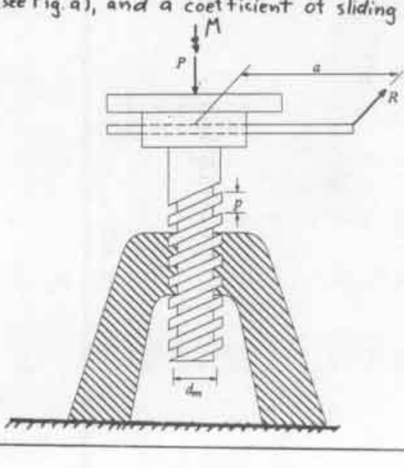


Figure a

(Continued)

10.75 Cont.

a) Find the moment required to lower a load $P = 45\ kN$.

b) Find the moment required to raise a load $P = 45\ kN$.

c) Determine the mechanical advantage for raising the load if $a = 300\ mm$

a/ By Eq. (10.6),

$$\tan \phi_K = \mu_K = 0.40 \quad \text{or} \quad \phi_K = 21.80°$$

also, $$\tan \phi = \frac{p}{\pi d_m} = \frac{16}{\pi(64)} = 0.07958$$

$$\phi = 4.5499°$$

Hence, to lower the load $P = 45\ kN$, Eq. (10.9) yields

$$M = Pr \tan(\phi_K - \phi)$$

or $$M = (45\ kN)(0.032\ m)\tan(21.80° - 4.5499°)$$

so, $$\underline{M = 447.1\ N \cdot m} \quad \text{clockwise}$$

b/ To raise the load $P = 45\ kN$, Eq. (10.7) yields

$$M = Pr \tan(\phi_K + \phi)$$

or $$M = (45\ kN)(0.032\ m)\tan(21.80° + 4.5499°)$$

so, $$\underline{M = 713.3\ N \cdot m} \quad \text{counter clockwise}$$

c/ The mechanical advantage for raising the load, if $a = 300\ mm$, is by Eq. (10.8)

$$m = \frac{a}{r}\cot(\phi_K + \phi)$$

or $$m = \frac{(0.300\ m)}{(0.032\ m)}\cot(21.80° + 4.5499°)$$

so, $$\underline{\underline{m = 18.93}}$$

10.76

The coefficient of kinetic friction in Problem 10.75 is reduced to $\mu_K = 0.05$

a) Find the mechanical advantage if the load $P = 45 kN$ is raised

b) Determine the moment required to lower the load $P = 45\ kN$.

The data is the same as in Problem 10.75, except that now $\mu_K = 0.05$.

a/ When the load $P = 45 kN$ is raised, the mechanical advantage is, by Eq. (10.8),

$$m = \frac{a}{r}\cot(\phi_K + \phi) \qquad (a)$$

where now

$$\tan \phi_K = 0.05\ ; \quad \phi_K = 2.862° \qquad (b)$$

Hence, by Eqs. (a), and (b), with $a = 300 mm$, $r = 32\ mm$, $\phi = 4.5499°$ from Problem 10.75,

$$m = \left(\frac{0.300\ m}{0.032\ m}\right)\cot(2.862° + 4.5499°)$$

or $$\underline{\underline{m = 72.1}}$$

b/ The clockwise moment required to lower $P = 45\ kN$ is, by Eq. (10.9),

$$M = Pr \tan(\phi_K - \phi)$$

or $$M = (45\ kN)(0.032\ m)\tan(2.862° - 4.5499°)$$

so, $$M = -42.4\ N \cdot m = \underline{\underline{42.4\ N\,m \text{ counter clockwise}}}$$

Hence, a counter clockwise moment is required to lower the load at constant speed. If no moment is applied, the load will accelerate downward under its own weight.

10.77

To level a floor, a carpenter needs to drive a wedge between a floor joist and a support wall (Fig. a). The wedge angle is 4°, and the coefficient of friction is 0.7. The vertical load on the wall is 1.5 kN. The carpenter can deliver a 1 kN blow to the wedge.

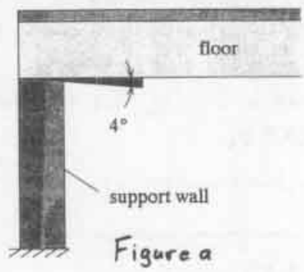


Figure a

a) Ignoring friction, find whether or not the carpenter can drive the wedge between the floor joist and wall.

b) Including friction, repeat part a

a/ The free-body diagram of the wedge is shown in Fig. b,

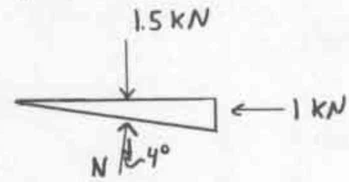


Figure b

By Fig. b,

$$\Sigma F_y = N \cos 4° - 1.5\ kN = 0\ ;\ N = 1.504\ kN$$

$$\Sigma F_x = N \sin 4° - 1\ kN = 0.105 - 1 = -0.895\ kN$$

Hence, the wedge can be driven between the joist and wall. But note that after the wedge is driven, without friction, a restraining force of 0.105 kN is required to keep the wedge in place.

(Continued)

10.77 Cont.

b/ With friction, the free-body diagram of the wedge is shown in Fig. c.

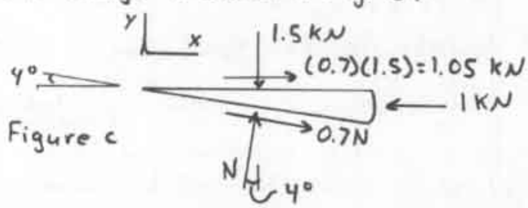


Figure c

By observation of Fig. c, the top friction force is 1.05 kN > 1 kN, without considering the contributions of N and 0.7N. Hence, the blow of 1kN is not sufficient to drive the wedge.

10.78

A jackscrew has a coefficient of kinetic friction $\mu_k = 0.10$.

a) Plot the efficiency η_a as a function of ϕ for the range $1.1\,\phi_k \le \phi \le 1.9\,\phi_k$, for raising a load.

b) Plot the efficiency η_b as a function of ϕ for the range $1.1\,\phi_k \le \phi \le 1.9\,\phi_k$, for lowering a load.

c) Plot the ratio η_b/η_a for the range $1.1\,\phi_k \le \phi \le 1.9\,\phi_k$.

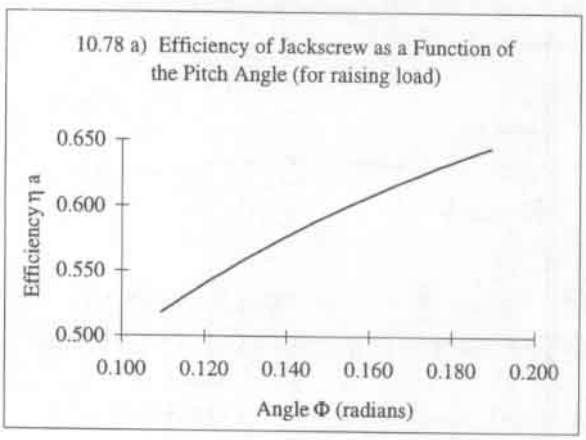
d) What conclusions can you draw from these results.

a/ By Eq. (10.6),

$$\tan\phi_k = \mu_k = 0.10 \quad ; \quad \phi_k = 5.711° \qquad (a)$$

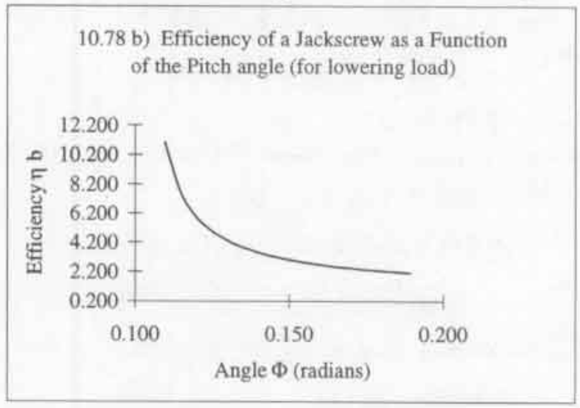
By Eq. (10.12), the efficiency η_a is

$$\eta_a = \frac{\tan\phi}{\tan(\phi_k+\phi)} = \frac{\tan\phi}{\tan(5.711°+\phi)} \qquad (b)$$



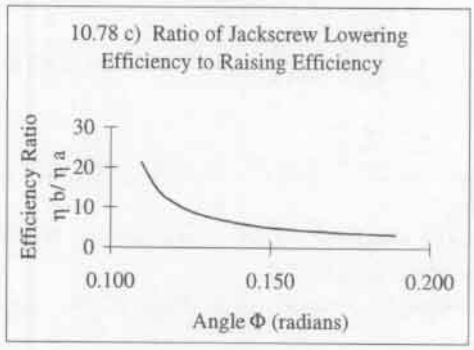
b/ The efficiency of a jackscrew is defined as the ratio of M (for a frictionless screw) to M (for a screw with friction). See the discussion preceding Eq. [10.11]. Hence, for lowering a load, Eq. (10.9) yields

$$\eta_b = \frac{\tan(-\phi)}{\tan(5.711°-\phi)} = \frac{\tan\phi}{\tan(\phi-5.711°)} \qquad (c)$$



c/ By Eqs. (b) and (c),

$$\frac{\eta_b}{\eta_a} = \frac{\tan(\phi+5.711°)}{\tan(\phi-5.711°)} \qquad (d)$$



d/ Since $\eta_b/\eta_a > 1$, for ϕ in the range $1.1\,\phi_k \le \phi \le 1.9\,\phi_k$, it is more efficient to lower the load than to raise it.

10.79

a) A building contractor asks you to develop specifications for a jack-screw which is capable of lifting 8 tons 10 inches.

b) You must write one paragraph explaining your recommended specifications.

a) Selection of parameters

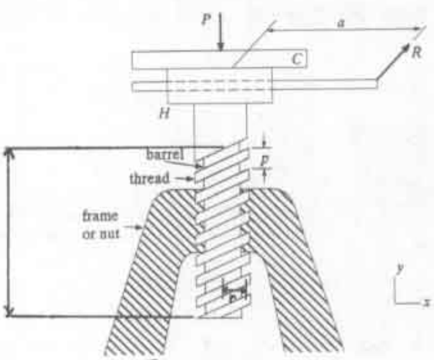


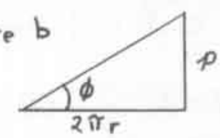
Figure a

To allow some tolerance in the design, assume the following specifications (see Fig. a):

- In order to lift the house 10 inches set the thread length to h = 13 inches, for a 3-inch working tolerance in the nut (Fig. a).
- Design for a load $P = 8$ tons $= 16{,}000$ lbs.
- Assume a reasonable force applied by a worker, say, $R = 50$ lbs
- A coefficient of kinetic friction of $\mu_k = 0.1$; requires a clean, lubricated screw.

Defining parameters and equations

Figure b



Thus, by Fig. b, the pitch angle is given by,

$$\text{Tan } \phi = \frac{p}{2\pi r} \qquad (a)$$

or

$$\phi = \text{Tan}^{-1} \frac{p}{2\pi r} \qquad (b)$$

By Eq. (b) in the derivation of Eq. 10.7 of the text,

$$M = Ra = Pr\left[\frac{\mu_k + \tan\phi}{1 - \mu_k \tan\phi}\right] \qquad (c)$$

Combining Eqs. (a) and (c), we get

$$\frac{Ra}{P} = r\left[\frac{\mu_k + p/2\pi r}{1 - \mu_k \frac{p}{2\pi r}}\right] \qquad (d)$$

Recasting Eq. (d) into the form $r^2 + c_1 r + c_2 = 0$, we obtain after some algebra,

$$r^2 + \left[\frac{p}{2\pi\mu_k} - \frac{Ra}{P\mu_k}\right] r + \frac{Ra\,p}{2\pi P} = 0 \qquad (e)$$

Dividing Eq. (e) by a^2, with $\mu_k = 0.1$, $R = 50$ lb, and $P = 16{,}000$ lb, we get

$$\left(\frac{r}{a}\right)^2 + \left(\frac{5p}{\pi a} - 0.03125\right)\left(\frac{r}{a}\right) + \frac{0.00156\,p}{\pi\ a} = 0$$

The Quadratic formula may be used to solve for r/a (see Table P10.79).

Table P10.79

a	p	p/a	r/a	r
16	0.1	0.0063	0.0212	0.3385
16	0.2	0.0125	0.0108	0.1725
16	0.25	0.0156	0.0047	0.076
24	0.1	0.0042	0.0245	0.5888
24	0.2	0.0083	0.0178	0.4261
24	0.25	0.0104	0.0143	0.3434
24	0.3	0.0125	0.0108	0.2587
36	0.1	0.0028	0.0268	0.964
36	0.2	0.0056	0.0223	0.8022
36	0.25	0.0069	0.02	0.7209
30	0.1	0.0033	0.0259	0.7764
30	0.2	0.0067	0.0205	0.6143
30	0.25	0.0083	0.0178	0.5326

For manufacturing purposes, let $r = 0.5$ in. Now check to see if the following inequality holds true,

$$Ra \geq Pr\left[\frac{\mu_k + \tan\phi}{1 - \mu_k \tan\phi}\right]$$

With $r = 0.5$ inches, $p = 0.25$ in ($\phi = \tan^{-1}\frac{p}{2\pi r} = 4.55°$), and $\mu_k = 0.10$

$Ra \geq 1448$ lb in Thus, with $a = 30$ in, a force $R = 48.3$ lb is sufficient to raise the load.

Summary of Design Specifications

$\mu_k = 0.1$ ($\phi_k = \tan^{-1} 0.1 = 5.711°$)	$a = 30''$
$h = 13''$	$p = 0.25''$
$\phi = 4.55°$	$r = 0.5''$

Note that since $\phi_k > \phi$ the jack screw is self-locking.

b) Although all of the parameters shown in Table P10.79 satisfy requirements, A 30 in bar fits reasonably into tight space requirements found at the sight. Also 4 threads per inch allows for adequate material strength necessary to raise 8 tons. A 1inch mean diameter screw will provide for ease in manufacturing.

10.80

A sailor who weighs 170 lb is lowered into a large ventilator shaft of a ship. (Fig. a). The coefficient of kinetic friction between the line and the ventilator is $\mu_K = 0.50$.

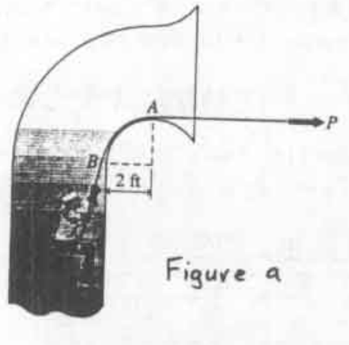


Figure a

Find the force P required to lower the sailor, neglecting the weight of the rope and the small frictional force between the sailor and shaft.

This is a problem of belt friction (Fig. b).

By Fig. b and Eq. (10.14),

$$\frac{T_1}{T_2} = e^{\mu_K \alpha}$$

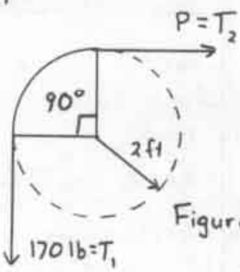


Figure b

Where $T_1 = 170$ lb,

$T_2 = P$, $\alpha = \pi/2$ rad,

and $\mu_K = 0.50$

Hence,

$$P = 170 \big/ \left[e^{(0.50)(\pi/2)} \right]$$

$$\underline{P = 77.51 \text{ lb}}$$

10.81

Given the data in Problem 10.80 Find the force P required to hoist the sailor up the ventilator shaft.

As in Problem 10.80, this is a problem of belt friction (Fig. a).

Now, however, $P = T_1$, $T_2 = 170$ lb

So, by Eq. (10.14),

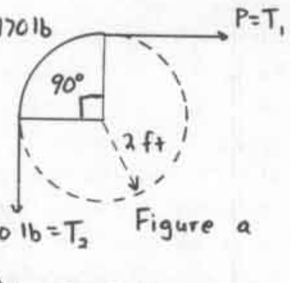


Figure a

$$\frac{T_1}{T_2} = e^{\mu_K \alpha} = \frac{P}{170}$$

Hence,

$$\underline{\underline{P = 170 \left[e^{(0.50)(\pi/2)} \right] = 372.9 \text{ lb}}}$$

10.82

A V-belt, with the cross section shown in Fig. a, is used to deliver power to the wheel-drive transmission of a tractor (Fig. b). The maximum torque delivered to the driven shaft is 240 lb·in, when the smaller belt tension of the driven shaft is 40 lb and slipping is impending between the belt and the driven sheave.

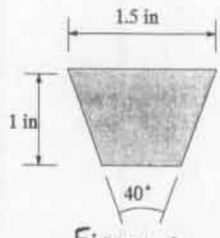


Figure a

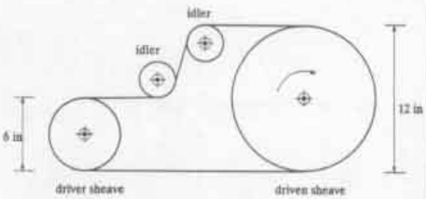


Figure b

Find the coefficient of friction between the belt and driven sheave.

Figure c is the free-body diagram of the driven sheave.

By Eq. (10.19),

$$\frac{T_1}{T_2} = e^{\mu_s \alpha / \sin\beta}$$

or

$$T_1 = 40\, e^{\mu_s (\pi) / \sin\beta}$$

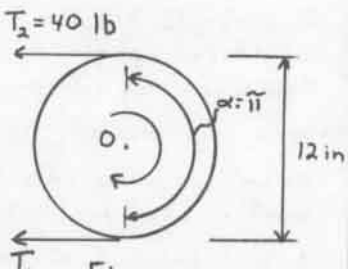


Figure c

By Figs. (10.17a) and (a)

$\beta = 20°$; Hence,

$$T_1 = 40\, e^{9.185 \mu_s} \qquad (a)$$

By Fig. c, and given that maximum torque delivered to the driven shaft is 240 lb·in,

$$\circlearrowleft\!+ \Sigma M_O = (T_1 - T_2)\, 6 = 240 \qquad (b)$$

By Eqs. (a) and (b),

$$40\left(e^{9.185 \mu_s} - 1\right) 6 = 240$$

$$\text{or} \quad e^{9.185 \mu_s} = 1.0 + 1.0 = 2.0$$

Hence,

$$\underline{\underline{\mu_s = 0.075}}$$

10.83

A V-belt with an angle of 40° has a maximum allowable tension of 400 N. It is strung around two identical pulleys of diameter 100 mm, which are 1 m apart. The coefficients of static and kinetic friction are $\mu_s = 0.20$ and $\mu_k = 0.10$.

Find the maximum torque that this system can transmit.

The maximum torque transmitted occurs at impending slipping. Consider the driven pulley (Fig. a).

By Eq. (10.19),

$$\frac{T_1}{T_2} = e^{\mu_s \alpha / \sin \beta} \qquad \text{(a)}$$

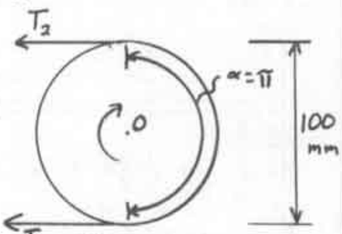


Figure a

By Fig. a, $\alpha = \pi$ rad.
Also $\beta = 40°/2 = 20°$.
Hence, by Eq. (a), with $\mu_s = 0.20$ and $T_1 = 400$ N,

$$T_2 = (400)/[e^{0.20\pi/\sin 20°}]$$

or

$$T_2 = 63.71 \text{ lb} \qquad \text{(b)}$$

By Fig. a, and Eq (b), with $T_1 = 400$ N,

$$M_{max} = \Sigma M_0 = (T_1 - T_2)(0.1\text{m})/2$$

$$M_{max} = (400 - 63.71)(0.05)$$

so

$$\underline{\underline{M_{max} = 16.81 \text{ N}\cdot\text{m}}}$$

10.84

The disk of the band brake system rotates clockwise (Fig. a). The coefficient of kinetic friction between the band and the disk is $\mu_k = 0.3$. A force of $P = 100$ lb is applied to the control lever.

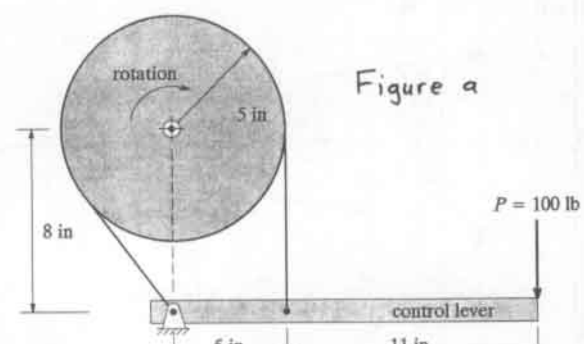


Figure a

Find the band tensions and the brake torque transmitted to the disk.

Consider first the free-body diagram of the disk (Fig. b)

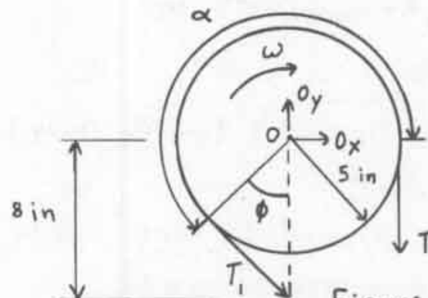


Figure b

By Fig. b,

$$\cos \phi = \frac{5}{8}\ ; \ \phi = 51.32°$$

Hence,

$$\alpha = 270° - 51.32° = 218.68° = 3.817 \text{ rad} \quad \text{(a)}$$

Also by Fig. b,

$$\circlearrowleft \Sigma M_0 = (T_1 - T_2)\,5 \qquad \text{(b)}$$

and by Eqs. (10.14) and (a),

$$T_1 = T_2\, e^{\mu_k \alpha} = T_2\, e^{(0.3)(3.817)} = 3.143\, T_2 \quad \text{(c)}$$

Next consider the free-body diagram of the control lever (Fig. c).

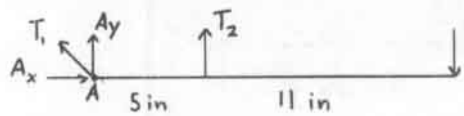


Figure c

By Fig. c,

$$\circlearrowleft \Sigma M_A = 5\, T_2 - (100)(16) = 0$$

or

$$\underline{\underline{T_2 = 320 \text{ lb}}} \qquad \text{(d)}$$

By Eqs. (c) and (d),

$$T_1 = 3.143\, T_2 = 3.143(320) = \underline{1005.8 \text{ lb}} \quad \text{(e)}$$

By Eqs. (b), (d), and (e), the braking torque is

$$M = (1005.8 - 320)(5) = 3428.8 \text{ lb}\cdot\text{in}$$

or $\underline{M = 285.7 \text{ lb}\cdot\text{ft}}$; counter clockwise

10.85

The rotation of the disk in Problem 10.84 is reversed (is counter clockwise). Repeat Problem 10.84 for this case.

For counter clockwise, T_1 and T_2 in Problem 10.84 are interchanged.

That is,

$$\underline{\underline{T_1 = 320 \text{ lb}}} \qquad \text{(a)}$$

(Continued)

10.85 Cont.

and by Eq. (10.14) and, with $\mu_k = 0.30$, $\alpha = 3.817$ rad,

$$T_2 = T_1 e^{-\mu_k \alpha} = 320 e^{(-0.3)(3.817)}$$

or

$$\underline{\underline{T_2 = 101.82 \text{ lb}}} \qquad (b)$$

The braking Torque is [see Eq. (10.15)]

$$M = (T_1 - T_2) r \qquad (c)$$

By Eqs. (a), (b), and (c), with $r = 5$ in,

$$M = (320 - 101.82) 5 = 1090.9 \text{ lb} \cdot \text{in}$$

or

$$\underline{\underline{M = 90.91 \text{ lb} \cdot \text{ft} \text{; clockwise}}}$$

10.86

A torque T is applied to pulley A (Fig. a) to drive pulley B. The pulleys have equal radii, with the tension in the slack side of the belt of 2kN. The coefficient of friction between the belt and pulley is 0.30.

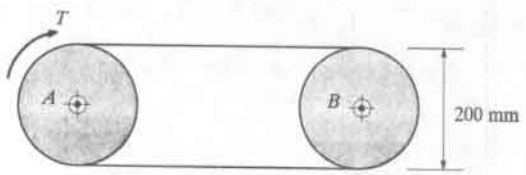


Figure a

a) Find the maximum possible torque that can be applied to pulley A.
b) Find the torque transmitted to pulley B.

a/ The free-body diagram of pulley A is shown in Fig. b.

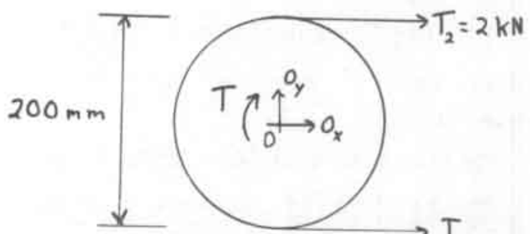


Figure b

By Eq. (10.14),

$$T_1 = T_2 e^{\mu \alpha} = 2 e^{0.30 \alpha} \qquad (a)$$

Since the pulleys have equal radii, $\alpha = \pi$ (see also Fig. a). Hence, Eq. (a) yields

$$\underline{\underline{T_1 = 2 e^{0.30 \pi} = 5.133 \text{ KN}}}$$

Then, by Eq. (10.15), the maximum possible torque that can be applied to pulley A is

$$M_{max} = (T_1 - T_2) r \qquad (a)$$

$$= (5.133 - 2)(0.10 \text{ m})$$

or

$$\underline{\underline{M_{max} = 313.3 \text{ N} \cdot \text{m}}}$$

b/ The tensions T_1 and T_2 in the belt also act on pulley B, and since its radius is also $r = 0.10$ m, Eq. (a) gives the torque transmitted to pulley B as

$$\underline{\underline{M_B = 313.3 \text{ N} \cdot \text{m}}}$$

10.87

The centers of the pulleys in Problem 10.86 are 750 mm apart and the *diameter* of the driver pulley A is increased to 400 mm (Fig. a). The coefficient of friction between the belt and pulleys is $\mu = 0.30$ and the slack tension in the belt remains at 2 kN.

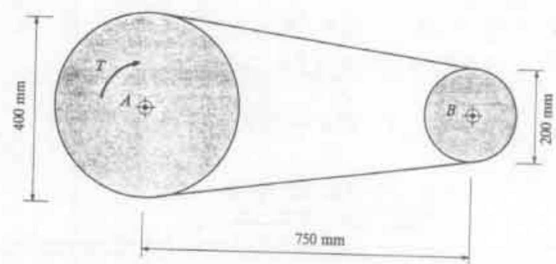


Figure a

Find the maximum possible torque that can be transmitted to pulley B.

A sketch of the system showing the contact angle α of the belt with pulley B is shown in Fig. b.

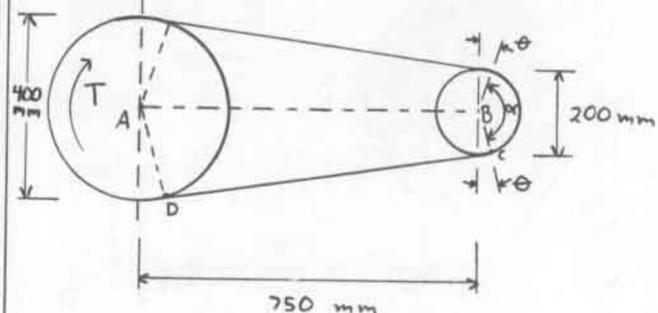


Figure b

(Continued)

10.87 Cont.

By Fig. b, we have the geometry shown in Fig. c,

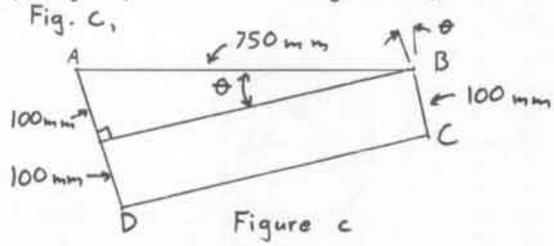


Figure c

By Fig. c,

$$\sin \theta = \frac{100}{750} = 0.1333$$

Therefore, $\theta = 7.662° = 0.1337$ rad.

Hence, by Fig. b,

$$\alpha = 180° - 2(7.662°) = 164.676°$$

or $\alpha = 2.874$ rad

Now by Eq. (10.14) and with $\mu = 0.30$, $\alpha = 2.874$ rad, and $T_2 = 2$ kN,

$$T_1 = T_2 e^{\mu\alpha} = 2e^{(0.30)(2.874)}$$

or

$$T_1 = 4.737 \text{ kN}$$

and by Eq. (10.15),

$$M_B = (T_1 - T_2) r = (4.737 - 2)(0.10)$$

or

$$\underline{\underline{M_B = 273.7 \text{ N}\cdot\text{m}}}$$

10.88

A idler pulley is installed to increase the angle of wrap of a belt-pulley system (Fig. a). A torque T is applied to the driver pulley A. The coefficient of friction is $\mu = 0.30$, and the slack tension in the belt is 2 kN.

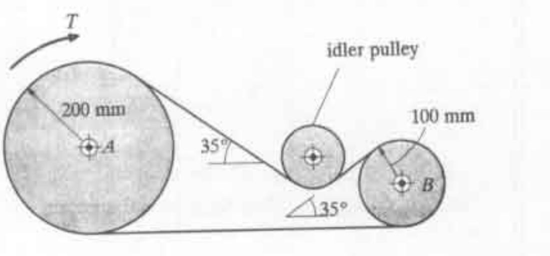


Figure a

a) Find the maximum torque T that can be transmitted to pulley B (and comparison with answer of Problem 10.87).

b) Determine the benefit of using an idler pulley; and the disadvantages of using an idler pulley.

By the sketch in Fig. b, we see that

$$\alpha = 180° + 35° = 215° = 3.752 \text{ rad} \qquad (a)$$

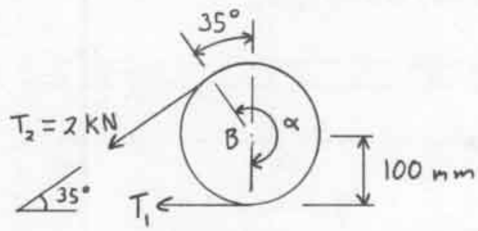


Figure b

Hence, by Eq. (10.14), with Eq. (a),

$$T_1 = T_2 e^{\mu\alpha} = 2e^{(0.30)(3.752)} = 6.165 \text{ kN}$$

Therefore, by Eq. (10.15) or by Fig. b,

$$M_B = (T_1 - T_2) r = (6.165 - 2)(0.10)$$

Therefore,

$$\underline{M_B = 416.5 \text{ N}\cdot\text{m}}$$

In Problem 10.87, we obtain $M_B = 273.7$ N·m. Hence, use of an idler pulley increased M_B by approximately 52%.

b) The use of the idler pulley increased the maximum transmitted torque to pulley B by about 52%, which is an advantage. A disadvantage is that a longer belt is required. A second disadvantage is that an additional pulley (the idler pulley) is required.

10.89

A worker can exert a 100 lb upward push on lever CD (Fig. a). The coefficients of static and kinetic friction between the belt and the fixed cylindrical drums or both $\mu = 0.40$.

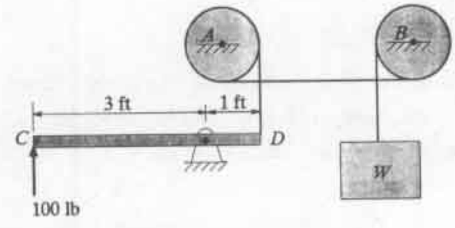


Figure a

a) Find the maximum weight W that the worker can lift.

b) Determine the maximum weight that the worker can hold.

(Continued)

10.89 Cont.

Consider first the free-body diagram of the lever CD (Fig. b).

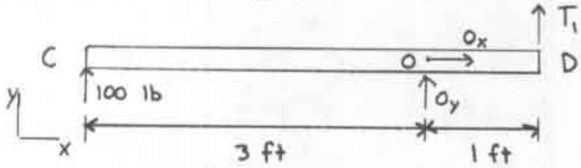


Figure b

By Fig. b,

$\circlearrowleft \Sigma M_0 = (1)T_1 - (100)(3) = 0$; $\therefore$ $\underline{\underline{T_1 = 300 \text{ lb}}}$

By the free body diagram of drum A (Fig. c), we have, with Eq. (10.14),

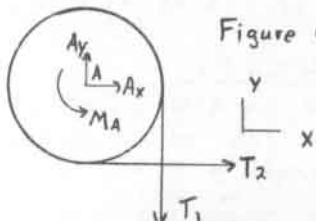


Figure C

$T_1 = T_2 e^{\mu\alpha} = T_2 e^{(0.40)(3\pi/2)} = 300 \text{ lb}$

or $T_2 = 45.55 \text{ lb}$

Then, by the free-body diagram of drum B (Fig. d), with Eq. (10.14), we have

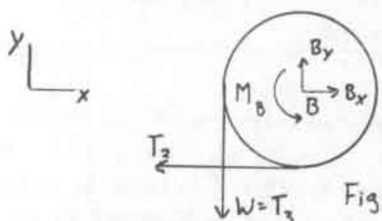


Figure d

$T_2 = T_3 e^{\mu\alpha} = W e^{(0.40)(3\pi/2)} = 45.55 \text{ lb}$

or $\underline{\underline{W = 6.916 \text{ lb}}}$

b/ To determine the maximum weight that the worker can hold, start with the free-body diagram of drum BC (Fig. e).

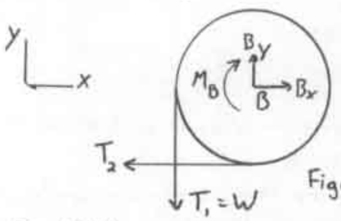


Figure e

By Fig. e and Eq. 10.14

$T_1 = W = T_2 e^{\mu\alpha} = T_2 e^{(0.40)(3\pi/2)}$

or $T_2 = 0.1518 W$ (a)

Then, by the free-body diagram of drum A (Fig. f), we have

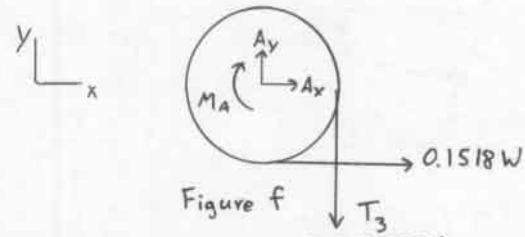
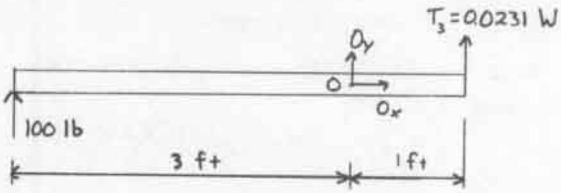


Figure f

$0.1518 W = T_3 e^{\mu\alpha} = T_3 e^{(0.40)(3\pi/2)}$

or $T_3 = 0.0231 W$

Finally, by the free-body diagram of lever CD (Fig. g), we obtain



$\circlearrowleft \Sigma M_o = (0.0231W)(1) - (100)(3) = 0$

or $\underline{\underline{W = 13,013 \text{ lb}}}$

10.90

A torque of 1 kN·m is transmitted to pulley D by its shaft (Fig. a). The coefficient of static friction between the belt and the pulley is $\mu_s = 0.60$. The shaft O and pin at A are frictionless.

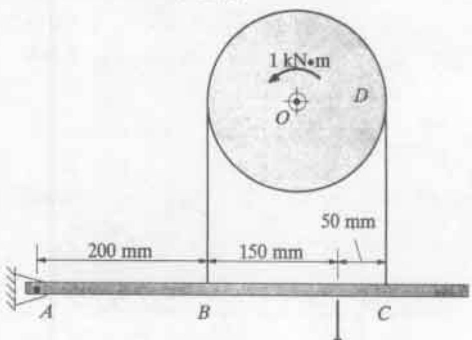


Figure a

a) Find the magnitude of F and the tensions in the belt at B and C.

b) Determine the location and magnitude of F so that the support reaction at A is zero

(Continued)

10.90 Cont.

a/ Consider first the free-body diagram of the bar ABC (Fig. b)

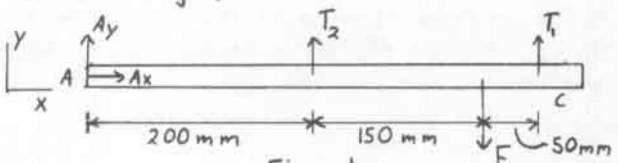


Figure b

By Fig. b,

$$\circlearrowleft \Sigma M_A = (400)T_1 + (200)T_2 - 350F = 0$$

or

$$T_1 + 0.5T_2 = 0.875F \quad (N) \qquad (a)$$

Next, consider the free-body diagram of the pulley (Fig. c).

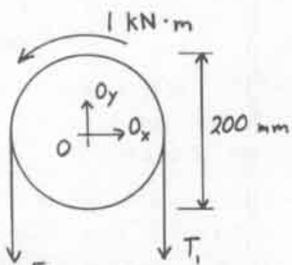


Figure c

By Fig. c and Eq.(10.14),

$$T_1 = T_2 e^{\mu_s \alpha} = T_2 e^{(0.60)(\pi)}$$

or

$$T_1 = 6.586T_2 \qquad (b)$$

Also, by Fig. c,

$$\circlearrowleft \Sigma M_o = (0.100)T_1 - (0.100)T_2 - 1000 = 0$$

or

$$T_1 - T_2 = 10{,}000 \text{ N} \qquad (c)$$

Then, Eqs. (b) and (c) yield

$$\underline{\underline{T_1 = 11{,}790.2 \text{ N} \,, \quad T_2 = 1{,}790.2 \text{ N}}} \qquad (d)$$

Hence, Eqs. (a) and (d) yield

$$\underline{\underline{F = 14{,}497 \text{ N} = 14.50 \text{ kN}}}$$

b/ The values of T_1 and T_2 determined in part a remain valid (they do not depend on F or its location)

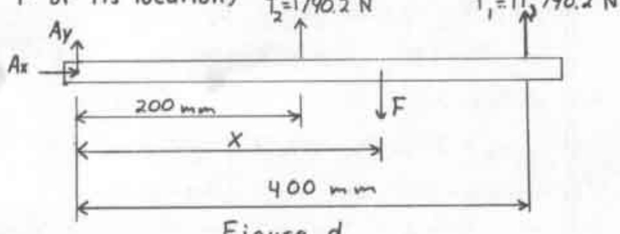


Figure d

By Fig. d,

$$\Sigma F_x = A_x = 0$$

$$\Sigma F_y = A_y + 1790.2 + 11{,}790.2 - F = 0 \qquad (e)$$

$$\circlearrowleft \Sigma M_A = (0.200)(1790.2) + 11{,}790.2(0.400) - F(x) = 0 \quad (f)$$

with $A_y = 0$, Eq. (e) yields

$$\underline{\underline{F = 13{,}580.4 \text{ N} = 13.58 \text{ kN}}} \qquad (g)$$

Equations (f) and (g) yield

$$\underline{\underline{x = 373.6 \text{ mm to the right of A}}}$$

10.91

To lower or raise herself in a tree, a tree surgeon rigs a rope to a limb (Fig. a). The coefficient of kinetic friction between the rope and limb and between the rope and her shoe is $\mu_k = 0.40$. She stands on the bight B of the rope and increases or reduces the pull on the strand S to go up or down. Assume the rope strands are parallel.

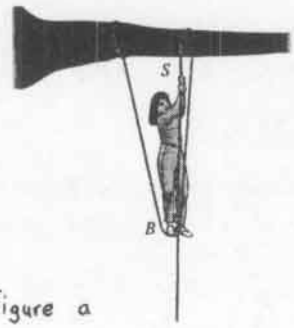


Figure a

a) Find the pull she must apply to S in order to go down slowly.

b) Determine the pull she must apply to S to go up slowly.

c) Comment on the results of a and b.

a/ To lower slowly, consider the free-body diagram of the tree branch and rope (Fig. b) where P is the pull exerted by the tree surgeon.

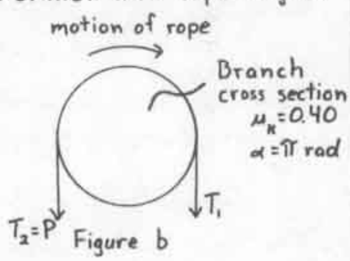


Figure b

By Fig. b and Eq. (10.14),

$$T_1 = T_2 e^{\mu \alpha} = P e^{(0.40)\pi} = 3.514\,P \qquad (a)$$

Since the woman pulls up with force P on the strand S, the free-body diagram of the bight B is shown in Fig. c

(Continued)

10.91 Cont.

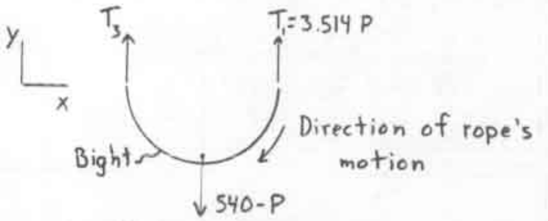


Figure c

Since the rope's motion is clockwise around the bight when the woman lowers herself, $T_3 > T_1$, and by Eqs. (10.14) and (a)

$$T_3 = T_1\ e^{(0.40)(\pi)} = T_1\,(3.514) = (3.514)^2\ P \qquad (b)$$

Also, by Fig. c,

$$\Sigma F_y = T_1 + T_3 - 540 + P = 0 \qquad (c)$$

Then, by Eqs. (a), (b), and (c),

$$(3.514)P + (3.514)^2 P + P = 540$$

or

$$\underline{P = 32.02\ \text{N}}$$

b/ To raise herself slowly, by Fig. b, the rope's motion is counter clockwise around the branch and $P > T_1$.
Hence, by Eq. (10.14)

$$P = T_1\, e^{(0.40)\pi} = 3.514\ T_1 \qquad (d)$$

Also, in Fig. c, the motion of the rope is counter clockwise around the bight and $T_1 > T_3$. Therefore, by Eq. (10.14)

$$T_1 = T_3\, e^{(0.40)\pi} = 3.514\ T_3 \qquad (e)$$

Therefore, by Eqs. (d) and (e)

$$T_1 = \frac{P}{3.514}\ ,\quad T_3 = \frac{P}{(3.514)^2} \qquad (f)$$

Again, by Fig. c,

$$\Sigma F_y = T_1 + T_3 - 540 + P = 0 \qquad (g)$$

Hence, by Eqs. (f) and (g),

$$0.2846\,P + 0.0810\,P + P = 540$$

or

$$\underline{P = 395.4\ \text{N}}$$

c/ The force (pull) required to raise herself is 12.35 times as large as the force (pull) required to lower herself. This is because her weight helps when she lowers herself, but when she raises herself, her weight acts as a deterrent.

10.92

In the belt-pulley system shown in Fig. a, a torque T is applied to the drive pulley A. The maximum tension in the belt is 6 kN. The coefficient of friction between the belt and the pulley is 0.25.

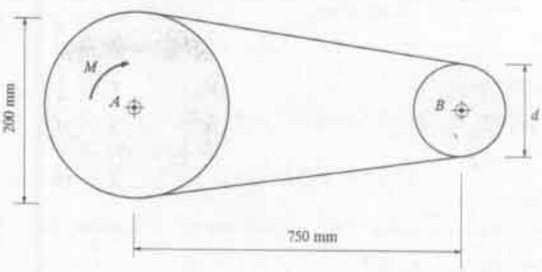


Figure a

Plot the maximum torque that can be transmitted to pulley B as a function of the diameter d for $100\ \text{mm} \le d \le 600\ \text{mm}$.

Two regions of d must be considered, namely $0 \le d \le 200$ mm and $200\ \text{mm} \le d \le 600$ mm.
First, consider $0 \le d \le 200$ mm (Fig. b).

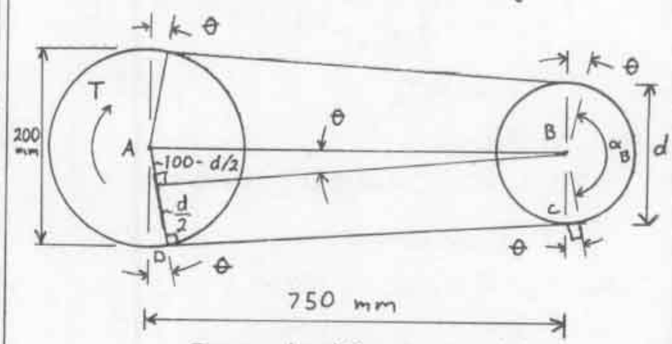


Figure b (Not to scale)

By Fig. b,

$$\sin\theta = \frac{100 - d/2}{750} = \frac{200 - d}{1500}$$

or,

$$\theta = \sin^{-1}\left(\frac{200-d}{1500}\right) \qquad (a)$$

Also, by Fig. b, for pulley B,

$$\alpha_B = \pi - 2\theta\ ;\quad 0 \le d \le 200\ \text{mm} \qquad (b)$$

For pulley A,

$$\alpha_A = \pi + 2\theta\ ;\quad 0 \le d \le 200\ \text{mm} \qquad (c)$$

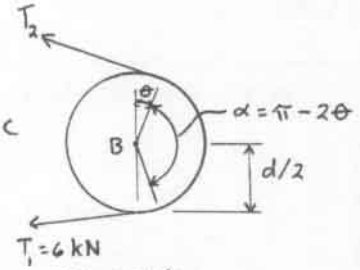
The smaller of α_A, α_B restricts the maximum moment transmitted to pulley B.
Thus, with $\alpha = \alpha_B = \pi - 2\theta$, $\mu = 0.25$,

(Continued)

10.92 Cont.

We obtain, by Eq. (10.14) and Fig. c

Figure c



$$T_1 = T_2\, e^{\mu\alpha} = T_2\, e^{(\pi - 2\theta)/4} = 6 \text{ kN}$$

or

$$T_2 = 6 / [e^{(\pi-2\theta)/4}] \qquad \text{(d)}$$

$$\theta = \sin^{-1}\left(\frac{200 - d}{1500}\right)$$

Hence, by Eqs. (10.15) and (d),

$$M_B = (T_1 - T_2)\frac{d}{2} = 3\left[1 - \frac{1}{e^{(\pi-2\theta)/4}}\right] d \quad (\text{N}\cdot\text{m})$$

$$0 \le d \le 200 \text{ mm} \qquad \text{(e)}$$

$$\theta = \sin^{-1}\left(\frac{200 - d}{1500}\right)$$

Next consider $200 \le d \le 600$ mm (Fig. d)

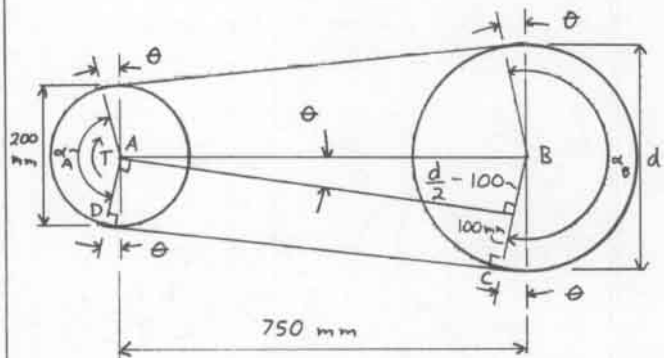


Figure d (Not to scale)

By Fig. d,

$$\sin\theta = \frac{d/2 - 100}{750} = \frac{d - 200}{1500}$$

or

$$\theta = \sin^{-1}\left(\frac{d - 200}{1500}\right) \qquad \text{(f)}$$

Also, by Fig. d, for pulley B

$$\alpha_B = \pi + 2\theta\ ;\ 200 \le d \le 600 \text{ mm} \quad \text{(g)}$$

For pulley A,

$$\alpha_A = \pi - 2\theta\ ;\ 200 \le d \le 600 \text{ mm} \quad \text{(h)}$$

The smaller of α_A, α_B restricts the maximum moment transmitted to pulley B.

Thus, with $\alpha = \alpha_A$, $\mu = 0.25$, we obtain by Eq. (10.14) and Fig. e,

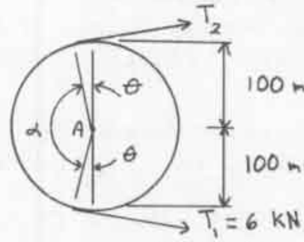


Figure e

$$T_1 = T_2\, e^{\mu\alpha} = T_2\, e^{(\pi - 2\theta)/4}$$

or

$$T_2 = 6 \text{ kN} / [e^{(\pi-2\theta)/4}] \qquad \text{(i)}$$

$$\theta = \sin^{-1}\left(\frac{d - 200}{1500}\right)$$

Hence, by Eqs. (10.15) and (i),

$$M_B = (T_1 - T_2)\frac{d}{2} = 3\left[1 - \frac{1}{e^{(\pi-2\theta)/4}}\right] d \quad (\text{N}\cdot\text{m})$$

$$200 \text{ mm} \le d \le 600 \text{ mm}$$

$$\theta = \sin^{-1}\left(\frac{d - 200}{1500}\right) \qquad \text{(j)}$$

Thus, Eqs. (e) and (j) define M_B as a function of d.

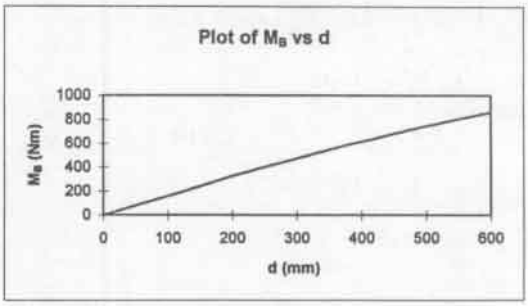


Figure f

10.93

To study the design of the belt-pulley system shown in Fig. a, an engineer applies a torque T to the driver pulley A and, by changing the length of the belt and position of the idler pulley, determines the maximum torque that can be transmitted to pully B

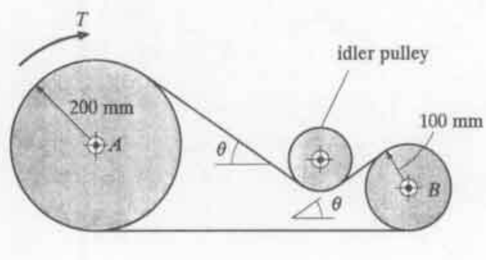


Figure a

(Continued)

10.93 Cont.

a) Plot the maximum torque that can be exerted by pulley A as a function of θ, for $15^\circ \le \theta \le 45^\circ$. The maximum tension in the belt is 5 kN, and the coefficient of friction is $\mu = 0.30$.

b) Determine the largest torque that can be transmitted to B in the range $15^\circ \le \theta \le 45^\circ$.

a/ Consider the sketch of pulley A (Fig. b).

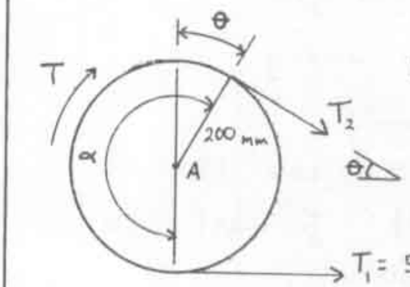


Figure b

By Eq. (10.14) and Fig. b, with $\mu = 0.30$ and $\alpha = \pi + \theta$

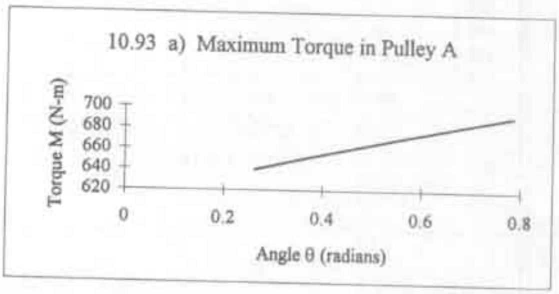
$$T_1 = T_2 e^{\mu\alpha} = T_2 e^{(0.30)(\pi+\theta)}$$

or $T_2 = 5 e^{-(0.30)(\pi+\theta)}$ (kN) (a)

Hence, by Eqs. (10.15) and (a),

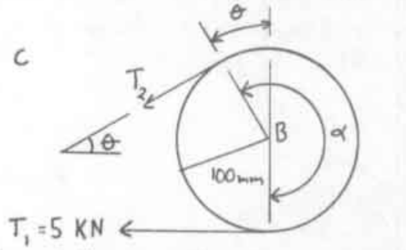
$$M_A = (T_1 - T_2) r$$

$$M_A = 1000\left[1 - e^{-0.30(\pi+\theta)}\right] \quad (b)$$



b/ Consider the sketch of pulley B. (Fig. c).

Figure C



By Eq. (10.14) and Fig. C, with $\mu = 0.30$ and $\alpha = \pi + \theta$,

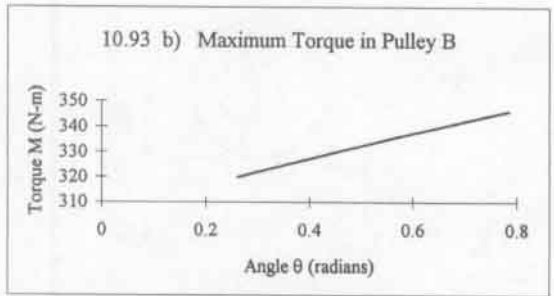
$$T_1 = T_2 e^{\mu\alpha} = T_2 e^{0.30(\pi+\theta)}$$

or $T_2 = 5 e^{-0.30(\pi+\theta)}$ (c)

Hence, by Eqs. (10.15) and (c),

$$M_B = (T_1 - T_2) r$$

$$M_B = 500\left[1 - e^{-0.30(\pi+\theta)}\right] \quad (d)$$



10.94

A manufacturing company wishes to use the belt-pulley system shown in Fig. a. However, the chief engineer wants to adjust the idler pulley so that the angle θ can be varied over the range $20^\circ \le \theta \le 40^\circ$, and wants to use belts of different materials for which the coefficient of static friction varies in the range $0.1 \le \mu_s \le 0.5$.

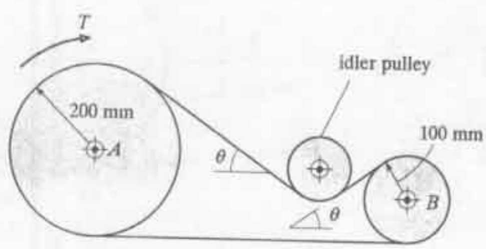


Figure a

The chief engineer asks you to design the belt-pulley system of Fig. a to meet the requirements and to determine the maximum tensions in the belt using a slack tension of 2 kN.

a) Plot design values of the maximum tension T_1 in the belt as a function of θ, and for $\mu_s = 0.1, 0.2, 0.3, 0.4$, and 0.5 (see Fig. 10.14).

b) Write a one-paragraph report for the chief engineer, explaining your design plot.

a/ As in Problem 10.93, the angle α of wrap for both pulleys is (see solution of P10.93)

$$\alpha = \pi + \theta \quad (a)$$ (Continued)

10.94 Cont.

Hence, to determine T_1, consider the diagram of pulley A (Fig. b)

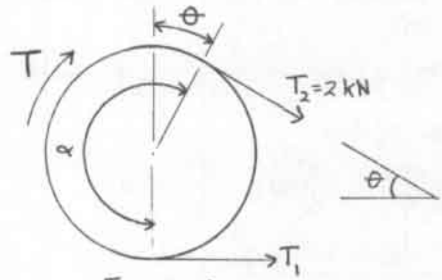
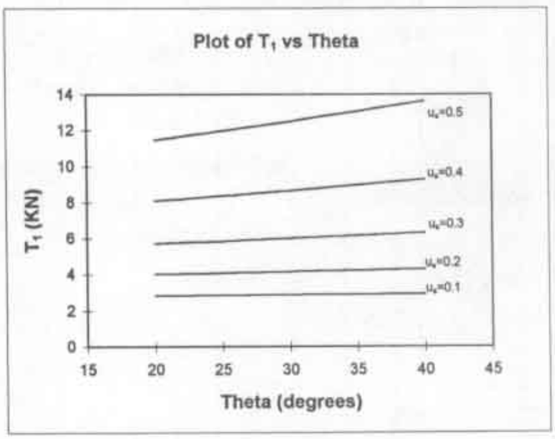


Figure b

By Eq. (10.14) and Fig. b, with $\alpha = \pi + \theta$,

$$T_1 = T_2 e^{\mu_s \alpha} = 2\, e^{\mu_s(\pi + \theta)} \quad (\text{kN}) \qquad (b)$$

Equation (b) expresses the maximum tension T_1 as a function of μ_s and θ. For a given value of μ_s, T_1 may be plotted as a function of θ for $20° \le \theta \le 40°$. See the plot of T_1 as a function of θ, for $\mu_s = 0.1, 0.2, 0.3, 0.4$, and 0.5.



b/ The plot of T_1 as a function of μ_s and θ shows that for a given value of μ_s, T_1 increases exponentially with θ. Also for a given angle θ, T_1 increases exponentially with μ_s. [see Eq. b and the plot of T_1 versus θ and μ_s.] For given values of μ_s and θ, the maximum tension T_1 in the belt can be read directly from the plot.

10.95

An airplane model is attached to a base (3 m in diameter) that rises on a concrete slab (Fig. a). The gravity axis of the model coincides with the geometric axis of the base. The coefficients of static and kinetic friction are $\mu_s = 0.60$ and $\mu_k = 0.25$. The pressure between the base and slab is uniform.

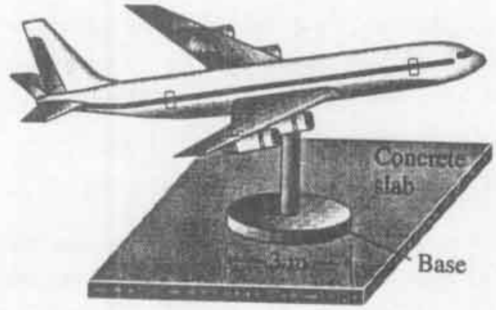


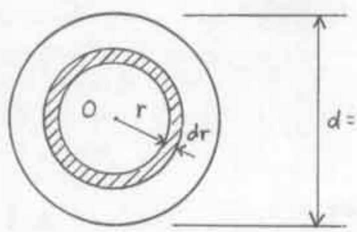
Figure a

a) Find the couple required to initiate rotation of the model about its gravity axis

b) Determine the couple required to maintain the rotation at a constant rate

a/ Consider a diagram of the bottom of the base showing an elemental ring (Fig. b) of the base

Figure b



The uniform pressure p on the base is

$$p = \frac{W}{\pi d^2/4} = \frac{360 \text{ kN}}{[\pi(3)^2/4]\,\text{m}^2} = 50.930 \frac{\text{kN}}{\text{m}^2} \qquad (a)$$

Hence, the element of normal force dN acting on the element of area $2\pi r\,dr$ (Fig. a) is

$$dN = p\,(2\pi r\,dr)$$

and the corresponding frictional force is, at impending rotation of the base,

$$dF = \mu_s\, p\,(2\pi r\,dr)$$

and the moment of dF about the gravity axis O is

$$dM = r\,dF = 2\pi r^2 \mu_s\, p\, dr \qquad (b)$$

The net moment is then [with Eq. (a) and $\mu_s = 0.60$]

$$M = 2\pi (0.60)(50.930) \int_0^{1.5} r^2\, dr \qquad (c)$$

or

$$\underline{\underline{M = 216 \text{ kN·m}}} \qquad (d)$$

(Continued)

10.95 Cont.

Hence, a couple of magnitude slightly larger than 216 kN·m and directed along the gravity axis of the model is required to initiate rotation.

b/ The couple required to maintain rotation has magnitude given by Eq. (c) with $\mu_s = 0.60$ replaced by $\mu_K = 0.25$.

Hence, to maintain the rotation,

$$\underline{M = 216(0.25/0.60) = 90\ \text{kN}\cdot\text{m}} \qquad (e)$$

Information Item: The moments given by Eqs. (d) and (e) could have been calculated directly by means of Eq. (10.21) and the appropriate value of the coefficient of friction.

10.96

A single-collar thrust bearing is illustrated in Fig. a. Assume that the pressure between the collar C and the base support B is uniformly distributed.

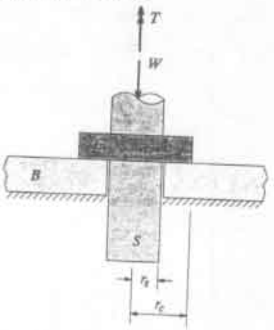


Figure a

a) Derive a formula for the pressure p between the collar and support in terms of W, r_s, and r_c.

b) Derive a formula for the torque T required to initiate rotation of the collar and shaft in terms of W, r_s, r_c, and μ_s (the coefficient of static friction).

The cross-sectional area of the collar is

$$A = \pi(r_c^2 - r_s^2) \qquad (a)$$

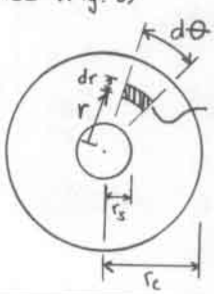
The pressure p is

$$\underline{\underline{p = \frac{W}{A} = \frac{W}{\pi(r_c^2 - r_s^2)}}} \qquad (b)$$

Where W is the weight of the collar and shaft.

b/ Consider a differential area $dA = r\,dr\,d\theta$ on the collar face (Fig. b)

Figure b



The tangential element of force on dA is

$$\mu_s\, dF = \mu_s\, p\, dA = \mu_s\, p\, r\,dr\,d\theta$$

and the moment of this force about O is, with Eq. (b),

$$dM_o = r(\mu_s dF) = \frac{\mu_s W}{\pi(r_c^2 - r_s^2)}\, r^2\, dr\, d\theta$$

Integration yields

$$M_o = \frac{\mu_s W}{\pi(r_c^2 - r_s^2)} \int_0^{2\pi} d\theta \int_{r_s}^{r_c} r^2\, dr$$

or

$$\underline{\underline{M_o = \frac{2\mu_s W (r_c^3 - r_s^3)}{3(r_c^2 - r_s^2)}}}$$

M_o is the torque at impending rotation. A torque slightly larger than M_o will initiate rotation.

10.97

The single-collar thrust bearing shown in Fig. a, is subjected to an axial load P=10,000 lb. The coefficients of static and kinetic friction are $\mu_s = 0.40$ and $\mu_k = 0.20$. Assume that the pressure is uniformly distributed on the collar C.

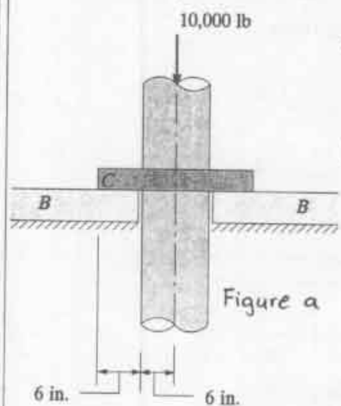


Figure a

a) Find the torque T_K required for constant rotation

b) Determine the torque T_s required to start rotation.

a/ By Eq. (10.21), with $r_i = r_c$ and $r_o = r_s$, and $R = P = 10{,}000$ lb,

$$T_K = \frac{2\mu_K P(r_c^2 - r_s^2)}{3(r_c^2 - r_s^2)} = \frac{2(0.20)(10{,}000)(12^3 - 6^3)}{3(12^2 - 6^2)}$$

or $\underline{\underline{T_K = 18666.\bar{6}\ \text{lb}\cdot\text{in} = 1555.\bar{5}\ \text{lb}\cdot\text{ft}}}$

b/ Also, by Eq. (10.21),

$$T_s = \frac{2\mu_s P(r_c^3 - r_s^3)}{3(r_c^2 - r_s^2)} = \frac{2(0.40)(10{,}000)(12^3 - 6^3)}{3(12^2 - 6^2)}$$

or $T_s = T_K(\mu_s/\mu_K) = 1555.5(0.40/0.20)$

so, $\underline{\underline{T_s = 3111.\bar{1}\ \text{lb}\cdot\text{ft}}}$

To start the rotation, the torque must be slightly larger than $3111.\bar{1}$ lb·ft

10.98

Two collars are used as shown in Fig. a to support a $P = 10{,}000$ lb load (see Problem 10.97), with $\mu_s = 0.40$ and $\mu_K = 0.20$.

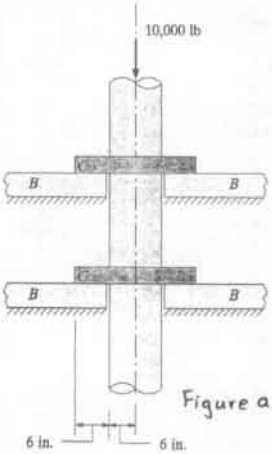


Figure a

a) Find the torque required to initiate rotation of the shaft. Assume that the load is divided equally between the collars and that the pressure is uniform on each collar.

b) Why are multiple collars used?

a/ Since each collar supports $\frac{1}{2}P = \frac{1}{2}(10{,}000) = 5000$ lb, the resistive torque T_R of each collar is, by Eq. (10.21),

$$T_R = \frac{2\mu_s (P/2)(r_c^3 - r_s^3)}{3(r_c^2 - r_s^2)} = \frac{2(0.40)(5000)(12^3 - 6^3)}{3(12^2 - 6^2)}$$

or

$$T_R = 777.\bar{7} \text{ lb·ft}$$

Hence the total resistive torque of two collars is $\underline{\underline{T_s = 2T_R = 1555.\bar{5} \text{ lb·ft}}}$

To initiate rotation, the applied torque must be slightly greater than $1555.\bar{5}$ lb·ft.

Alternatively, directly by Eq. (10.22) with $R = P/2$, $R = 5000$ lb and $n = 2$ (for two collars)

$$T_s = 2\frac{(2)(0.40)(5000)(12^3 - 6^3)}{3(12^2 - 6^2)} = 1555.5 \text{ lb·ft.}$$

b/ The same torque is required to initiate rotation of a single-collar bearing (Problem 10.97) as a double-collar bearing (Problem 10.98). However, in the two-collar system, each collar carries only 1/2 the load as in the single-collar bearing. Thus the two-collar system undergoes less wear because of the lower loads, and because, wear occurs more rapidly (non-linearly) with increasing load.

10.99

The weight that acts on the single-collar thrust bearing shown in Fig. a is $W = 30$ kN. The shaft radius is 200 mm, the collar radius is 350 mm and $\mu_s = 0.15$, $\mu_K = 0.10$.

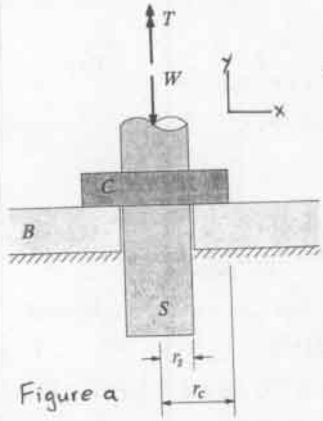


Figure a

a) Find the pressure p that exists between the collar C and the support B. Take p to be uniformly distributed.

b) Determine the torque T_s required to initiate rotation.

c) Find the torque T_K required to maintain rotation at a constant rate.

a/ By Fig. (a), with $W = 30$ kN, $r_c = 0.35$ m, $r_s = 0.20$ m, Eq. (10.20) yields

$$\underline{\underline{p = \frac{W}{\pi(r_c^2 - r_s^2)} = \frac{30000}{\pi(0.35^2 - 0.20^2)} = 115.75 \text{ kPa}}}$$

b/ By Eq. (10.21),

$$T_s = \frac{2\mu_s W(r_c^3 - r_s^3)}{3(r_c^2 - r_s^2)} = \frac{2(0.15)(30)(0.35^3 - 0.20^3)}{3(0.35^2 - 0.20^2)}$$

or

$$\underline{T_s = 1268.2 \text{ N·m}}$$

A torque slightly greater than 1268.2 N·m is required to initiate rotation.

c/ From Eq. (10.21), with $\mu = \mu_K$,

$$T_K = \frac{2\mu_K W(r_c^3 - r_s^3)}{3(r_c^2 - r_s^2)} = \frac{2(0.1)(30{,}000)(0.35^3 - 0.20^3)}{3(0.35^2 - 0.20^2)}$$

or

$$\underline{T_K = 845.\overline{45} \text{ N·m}}$$

10.100

The disk-clutch of Fig. a, with $r_0 = 2$ in and $r_1 = 5$ in, and coefficient of static friction $\mu_s = 0.60$. The pressure between the disks decreases linearly from 100 psi at $r = r_0$ to 40 psi at $r = r_1$.

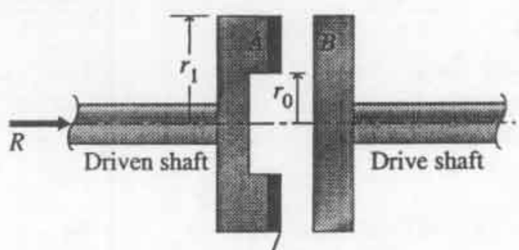


Figure a

(Continued)

10.100 Cont.

Find the maximum torque that can be transmitted by the clutch.

Since the pressure p varies linearly with r,

$$p = mr + b, \text{ (psi)} \qquad (a)$$

where

$$p = 100 \text{ psi}, \text{ for } r = r_o = 2 \text{ in}$$
$$p = 40 \text{ psi}, \text{ for } r = r_i = 5 \text{ in} \qquad (b)$$

Hence, by Eqs. (a) and (b),

$$100 = m(2) + b$$
$$40 = m(5) + b \qquad (c)$$

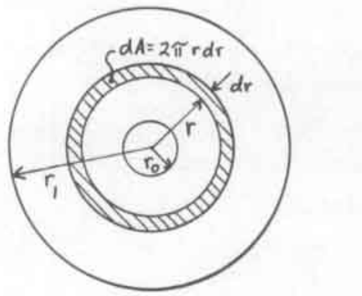
The solution of Eqs. (c) is

$$m = -20 \text{ psi/in}, \ b = 140 \text{ psi}$$

Therefore, the normal force on an element $dA = 2\pi r\,dr$ of the disk is (Fig. b)

$$dN = 2\pi(p)r\,dr = 2\pi(-20r + 140)r\,dr \qquad (d)$$

Figure b



Hence, the friction force on dA, at impending slipping of the disk, is with Eq. (d),

$$dF = \mu_s\,dN$$
$$= 2\pi\mu_s(-20r^2 + 140r)\,dr$$

and the resisting moment about O, due to dF, is

$$dM_o = [2\pi\mu_s(-20r^2 + 140r)\,dr]\,r$$

Integration yields the total resisting moment as

$$M_o = 2\pi\mu_s \int_{r_o}^{r_i} (-20r^3 + 140r^2)\,dr$$
$$= 2\pi\mu_s\left[-\frac{20}{4}(r_i^4 - r_o^4) + \frac{140}{3}(r_i^3 - r_o^3)\right]$$

or

$$M_o = 2\pi(0.60)[-5(5^4 - 2^4) + 46.\bar{6}(5^3 - 2^3)]$$

so,

$$\underline{\underline{T_s = M_o = 9104 \text{ lb·in} = 758.7 \text{ lb·ft}}}$$

10.101

The net force due to the pressure distribution on the disk face ($r_o \le r \le r_i$) in Problem 10.100 is assumed to be distributed uniformly on the disk face.

Find the maximum torque that can be transmitted by the clutch

By Eq. (d) of the solution of Problem 10.100, the elemental normal force acting on area $dA = 2\pi r\,dr$ is

$$dN = 2\pi(-20r^2 + 140r)\,dr \qquad (a)$$

Integration of Eq. (a) yields

$$N = 2\pi \int_{r_o=2}^{r_i=5} (-20r^2 + 140r)\,dr$$

$$N = 2\pi\left[-\frac{20}{3}[5^3 - 2^3] + \frac{140}{2}[5^2 - 2^2]\right]$$

or

$$N = 4335.40 \text{ lb} \qquad (b)$$

Assuming that N is distributed uniformly over the face of the disk, we find the pressure p = constant to be

$$p = \frac{N}{\pi(r_i^2 - r_o^2)} = \frac{4335.4}{\pi(5^2 - 2^2)} = 65.714 \text{ psi}$$

Hence, the element of moment due to the frictional force is

$$dM_o = (\mu_s\,p\,dA)r = \mu_s(65.714)(2\pi r\,dr)r$$

or

$$M_o = (2\pi)(0.60)(65.714)\int_{r_o=2}^{r_i=5} r^2\,dr$$

$$M_o = (2\pi)(0.60)(65.714)\left(\tfrac{1}{3}\right)(5^3 - 2^3)$$

or

$$\underline{\underline{M_o = T_s = 9661.7 \text{ lb·in}}}$$

Alternatively, by Eq. (10.21), for uniformly distributed load $N = R = 4335.40$ lb,

$$T_s = \frac{2\mu_s N(r_i^3 - r_o^3)}{3(r_i^2 - r_o^2)} \quad \frac{2(0.60)(4335.4)(5^3 - 2^3)}{3(5^2 - 2^2)}$$

or

$$\underline{\underline{T_s = 9661.7 \text{ lb·in}}}$$

10.102

Assume that the normal pressure p is distributed uniformly over the contacting surfaces of a cone clutch (Figs. a and b)

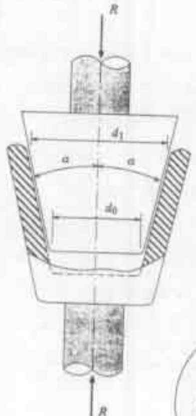


Figure a

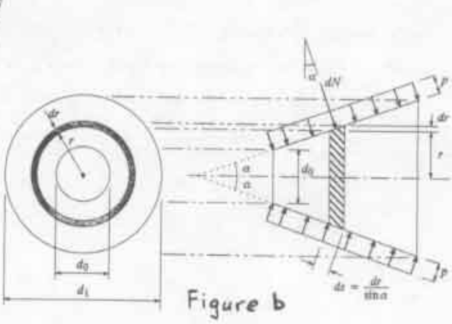


Figure b

(Continued)

10.102 Cont.

Show that the maximum torque T_{max} transmitted by the clutch is

$$T_{max}=\frac{2\mu_s R(r_1^3-r_0^3)}{3(\sin\alpha)(r_1^2-r_0^2)} \qquad (a)$$

where $2r_1=d_1$, $2r_0=d_0$, and

$$R=\pi p(r_1^2-r_0^2) \qquad (b)$$

From the hint and Fig. b,

$$dN=p(2\pi r)\frac{dr}{\sin\alpha} \qquad (c)$$

$$dR=dN(\sin\alpha) \qquad (d)$$

$$dT=(\mu_s dN)r \qquad (e)$$

Substitution of Eq. (c) into Eq. (e) yields

$$dT=\mu_s p(2\pi r)r\frac{dr}{\sin\alpha} \qquad (f)$$

and substitution of Eq. (c) into Eq. (d) gives

$$dR=p(2\pi r)dr \qquad (g)$$

Integration of Eq. (g) yields

$$R=2\pi p\int_{r_0}^{r_1} r\,dr \quad \text{or} \quad \underline{\underline{R=\pi p(r_1^2-r_0^2)}} \qquad (h)$$

Equation (h) verifies Eq. (b). Solving Eq. (h) for p and substituting p into Eq. (f) yields

$$dT=\frac{2\mu_s R}{(r_1^2-r_0^2)\sin\alpha}r^2 dr$$

Integration yields

$$T=\frac{2\mu_s R}{(r_1^2-r_0^2)\sin\alpha}\int_{r_0}^{r_1} r^2 dr$$

or

$$\underline{\underline{T=\frac{2\mu_s R(r_1^3-r_0^3)}{3\sin\alpha(r_1^2-r_0^2)}}} \qquad (i)$$

Equation (i) verifies Eq. (a)

10.103

In Fig. P10.102 a (see Fig. a below), $R=18$ kN, $d_0=100$ mm, $d_1=150$ mm, $\alpha=10°$, and $\mu_s=0.70$

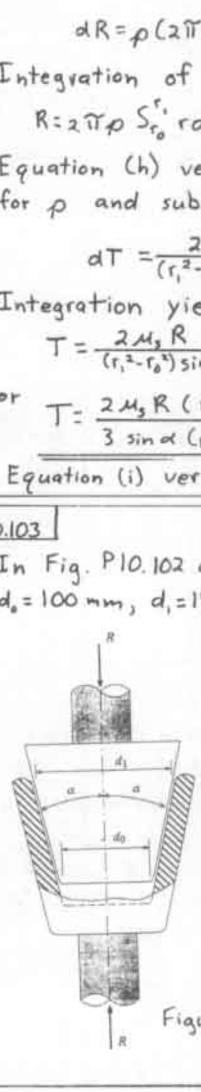


Figure a

Calculate the maximum torque that can be transmitted by the clutch.

By the formula given in Problem 10.102 (see also the solution of Problem 10.102),

$$T_{max}=\frac{2\mu_s R(r_1^3-r_0^3)}{3\sin\alpha(r_1^2-r_0^2)} \qquad (a)$$

With the given data and Eq. (a),

$$T_{max}=\frac{2(0.70)(18\text{ kN})\left[\left(\frac{0.150}{2}\right)^3-\left(\frac{0.100}{2}\right)^3\right]}{3(\sin 10°)\left[\left(\frac{0.150}{2}\right)^2-\left(\frac{0.100}{2}\right)^2\right]}$$

or

$$\underline{\underline{T_{max}=4595.5\ \text{N}\cdot\text{m}}}$$

10.104

The hollow flat pivot shown in Fig. a is subjected to a total load L, including its own weight. The coefficients of static and kinetic friction are μ_s and μ_k.

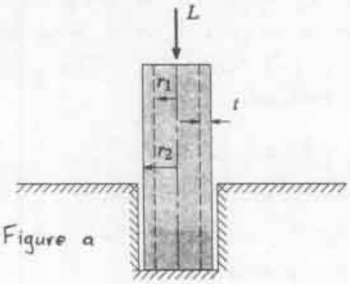


Figure a

a) Show that the torque T_s required to initiate rotation about the pivot's axis is

$$T_s=\frac{2\mu_s L}{3}\left(\frac{r_2^3-r_1^3}{r_2^2-r_1^2}\right) \qquad (a)$$

b) Find the torque T_k required to keep the pivot rotating at a constant rate.

c) Show that as $r_2\to r_1$, where $r_2-r_1=t$ (the thickness of the pivot wall) and for $t \ll r_1$, Eq. (a) is approximated by

$$T_s\approx\mu_s L r_1 \qquad (b)$$

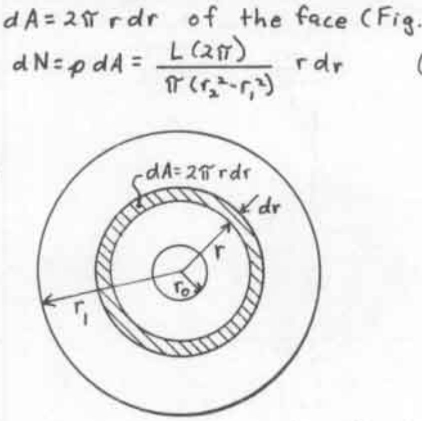
The pressure on the bottom face of the pivot is,

$$p=\frac{L}{\pi(r_2^2-r_1^2)} \qquad (c)$$

Hence, the normal force on an element of area $dA=2\pi r\,dr$ of the face (Fig. b) is

$$dN=p\,dA=\frac{L(2\pi)}{\pi(r_2^2-r_1^2)}r\,dr \qquad (d)$$

Figure b



(Continued)

10.104 Cont.

The corresponding friction force is, by Eq. (d)

$$dF = \mu_s dN = 2\mu_s L \frac{r\,dr}{(r_2^2 - r_1^2)}$$

and the resisting moment relative to O is

$$dM_o = r\,dF = \frac{2\mu_s L}{r_2^2 - r_1^2}\, r^2 dr$$

Integration yields

$$M_o = T_s = \frac{2\mu_s L}{3}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right) \qquad (e)$$

Equation (e) verifies Eq. (a)

Alternatively, by Eq. (10.21) with $R = L$, $r_i = r_2$, $r_o = r_1$,

$$T_s = \frac{2\mu_s L}{3}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)$$

b/ The torque T_K required to keep the pivot rotating at a constant rate may be obtained as was Eq. (e), with μ_K. Thus, by Eq. (a), with μ_s replaced by μ_K,

$$T_K = \frac{2\mu_K L}{3}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)$$

c/ Since $r_2 - r_1 = t$, $r_2 = r_1 + t$. Substitution for r_2 in Eq. (e) yields

$$T_s = \frac{2\mu_s L}{3}\left[\frac{(r_1+t)^3 - r_1^3}{(r_1+t)^2 - r_1^2}\right]$$

or

$$T_s = \frac{2\mu_s L}{3}\left[\frac{3r_1^2 t + 3r_1 t^2 + t^3}{2r_1 t + t^2}\right]$$

$$T_s = \frac{2\mu_s L}{3}\left[\frac{3r_1^2 + 3r_1 t + t^2}{2r_1 + t}\right]$$

Since $t \ll r_1$,

$$T_s \approx \frac{2\mu_s L}{3}\,\frac{3r_1^2}{2r_1} = \mu_s L r_1 \qquad (f)$$

Equation (f) verifies Eq. (b)

10.105

A metal ball is pressed against a thick flat plate by a force F (Fig. a). It flattens slightly so that the pressure over the small circle of contact of diameter 2a is

$$p = \frac{3F}{2\pi a^2}\sqrt{1 - \frac{r^2}{a^2}} \qquad (a)$$

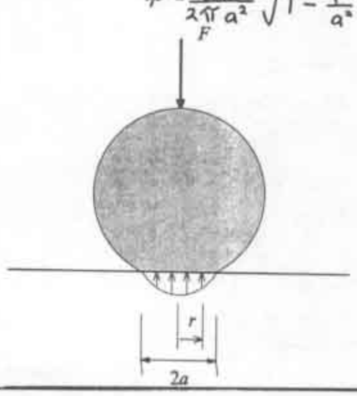


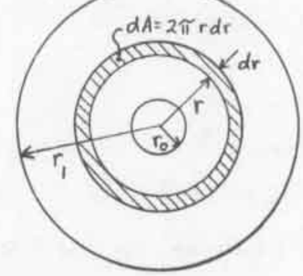
Figure a

Show that the torque required to rotate the ball slowly about the diametral axis perpendicular to the plate is

$$T = 3\pi\mu_K Fa/16 \qquad (b)$$

The normal force dN on an elemental area $dA = 2\pi r\,dr$ of the small circle of contact is (see Fig. b)

Figure b



$$dN = p\,dA = \frac{(2\pi)3F}{2\pi a^2}\sqrt{1 - r^2/a^2}\; r\,dr \qquad (c)$$

and the corresponding friction force is, with Eq. c,

$$dF = \mu_K dN = \frac{3\mu_K F}{a^2}\sqrt{1 - r^2/a^2}\; r\,dr \qquad (d)$$

Hence, the resistive moment about the diametral axis is, with Eq. (d),

$$dM = r\,dF = \frac{3\mu_K F}{a^2}\sqrt{1 - r^2/a^2}\; r^2 dr$$

Integration yields

$$M = T = \frac{3\mu_K F}{a^2}\int_0^a r^2\sqrt{1 - r^2/a^2}\; dr$$

$$= \frac{3\mu_K F}{a^3}\left\{-\frac{r}{4}\sqrt{(a^2-r^2)^3} + \frac{a^2}{8}\left[r\sqrt{a^2-r^2} + a^2 \sin^{-1}\frac{r}{a}\right]\right\}\Bigg|_0^a$$

$$= \frac{3\mu_K F}{a^3}\left(\frac{a^4}{8}\right)\left(\frac{\pi}{2}\right)$$

or

$$T = \frac{3\pi\mu_K Fa}{16} \qquad (e)$$

Equation (e) verifies Eq. (b)

10.106

An axial force P = 90,000 lb is required to press a locomotive wheel onto its axle, since the diameter of the wheel hole is slightly less than that of the axle (Fig. a). Once pressed on, the wheel is prevented from slipping around the axle by static friction due to pressure between the wheel and axle, whose diameter is 3.75 in.

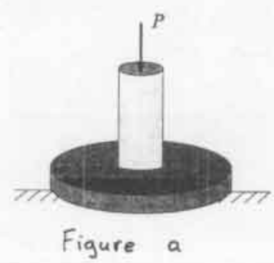


Figure a

(Continued)

10.106 Cont.

Find the maximum torque that the axle can transmit to the wheel; $\mu_s = \mu_k$ (Fig. b)

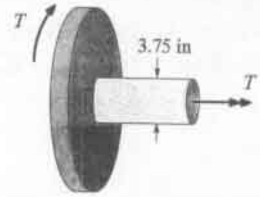


Figure b

Let A = area of contact between the wheel and axle, and p = pressure on A. Therefore, to press the wheel on the axle (Fig. a),

$$\mu_k p A = \mu_s p A = 90{,}000 \text{ lb} \qquad (a)$$

The normal force dN on an element dA of area of contact is

$$dN = p\,dA \qquad (b)$$

The corresponding friction force is, at impending sliding,

$$dF = \mu_s dN \qquad (c)$$

Hence, the element of torque about the axis of the axle (Fig. b) is, with Eqs. (b) and (c),

$$dT = \left(\frac{3.75}{2}\right) dF = \left(\frac{3.75}{2}\right) \mu_s p\, dA$$

Hence, $T = \left(\frac{3.75}{2}\right) \mu_s p \int dA$ or $T = \left(\frac{3.75}{2}\right) \mu_s p A$ (d)

So, by Eqs. (d) and (a)

$$T = \left(\frac{3.75}{2}\right)(90{,}000) = 168{,}750 \text{ lb}\cdot\text{in}$$

10.107

A plate sector of a circle (Fig. a) weighs 3.6 kN. It lies on a horizontal xy plane, and is constrained to rotate about the vertical axis through O. The pressure between the plate and the plane is uniform, and the coefficient of static friction is $\mu_s = 0.30$.

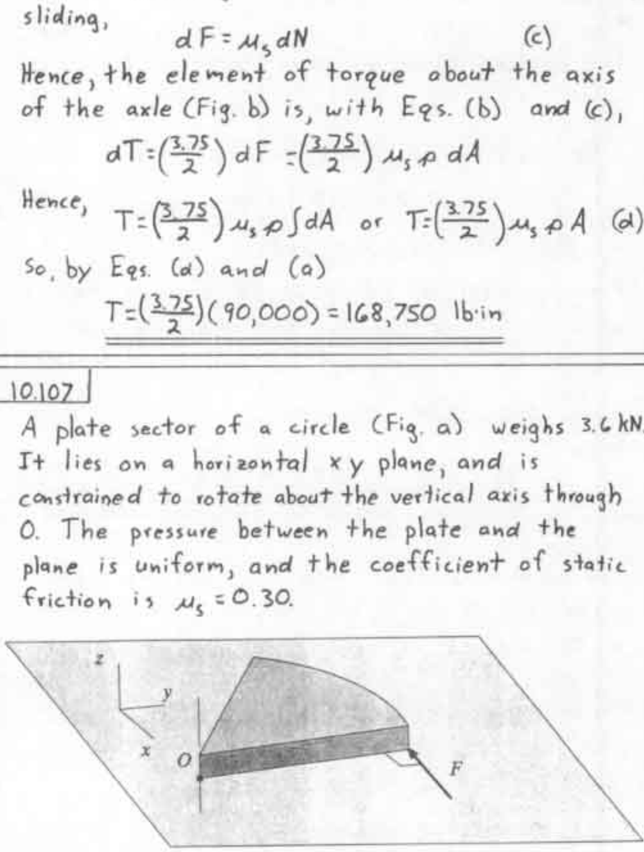


Figure a

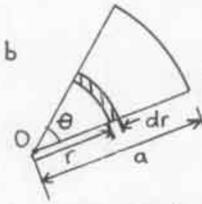
Find the magnitude of the tangential force F required to move the plate.

The pressure is

$$p = \frac{3.6 \text{ kN}}{\text{Area}} = \frac{3600 \text{ N}}{(1/2)\,\theta a^2} = \frac{7200}{\theta a^2} \qquad (a)$$

Where θ is the angle of the sector and a is the radius of the sector (Fig. b).

Figure b



By Fig. b, the normal force dN that acts on the element $dA = (r\theta)dr$ of the sector is

$$dN = p\,dA = p\,(r\theta)\,dr \qquad (b)$$

or with Eq. (a),

$$dN = \frac{7200}{a^2}\, r\, dr$$

and the corresponding friction force is, at impending motion,

$$dF_s = \mu_s dN = \frac{(0.30)\,7200}{a^2}\, r\, dr$$

Hence, the increment of moment about O (Fig. b) is

$$dM_o = r\,dF_s = \frac{2160}{a^2}\, r^2\, dr$$

Integration yields

$$M_o = \frac{2160}{a^2}\int_0^a r^2\, dr = 720\, a \qquad (c)$$

Thus, by Fig. a,

$$M_o = Fa = 720\, a$$

or $F = 720$ N

A force slightly larger than 720 N is required to move the plate.

10.108

From Fig. a, let the pressure p between the collar C and the support B vary linearly from $r = r_s$ to $r = r_c$, such that $p = p_s$ at $r = r_s$ and $p = 0$ at $r = r_c$.

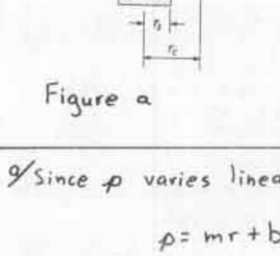


Figure a

a) Derive a formula for the pressure p in terms of W, r_c, and r_s.

b) Derive a formula for the torque T required to initiate rotation of the collar and shaft, in terms of W, r_s, r_c, and μ_s.

a) Since p varies linearly with r,

$$p = mr + b \qquad (a)$$

(Continued)

10.108 Cont.

For $r = r_s$, $p = p_s$ (b)

$r = r_c$, $p = 0$

By Eqs. (a) and (b)

$$m = -\frac{p_s}{r_c - r_s} \quad , \quad b = \frac{p_s r_c}{r_c - r_s} \qquad (c)$$

Equations (a) and (c) yield

$$p = \frac{p_s}{r_c - r_s}(-r + r_c) \qquad (d)$$

Now, with Eq. (d)

$$W = \int p\,dA = \int p\,2\pi r\,dr$$

or $$W = \frac{2\pi p_s}{r_c - r_s}\int_{r_s}^{r_c}(-r^2 + r_c r)\,dr$$

After integration and simplification,

$$W = \frac{\pi p_s}{3}(r_c - r_s)(r_c + 2r_s)$$

or $$p_s = \frac{3W}{\pi(r_c - r_s)(r_c + 2r_s)} \qquad (e)$$

Equations (d) and (e) yield

$$\underline{\underline{p = \frac{3W(r_c - r)}{\pi(r_c - r_s)^2(r_c + 2r_s)}}} \qquad (f)$$

b/ To derive a formula for torque T required to initiate rotation, note that the normal force dN on a contact area dA of the collar is, with Eq. (f),

$$dN = p\,dA = p\,(2\pi r\,dr)$$

or $$dN = \frac{6W(r_c - r)\,r\,dr}{(r_c - r_s)^2(r_c + 2r_s)}$$

and the corresponding friction force at impending rotation is

$$dF_s = \mu_s\,dN = \frac{6\mu_s W(r_c r - r^2)dr}{(r_c - r_s)^2(r_c + 2r_s)}$$

Hence, the increment of moment about the axis of the shaft is

$$dM = r\,dF_s = \frac{6\mu_s W(r_c r^2 - r^3)dr}{(r_c - r_s)^2(r_c + 2r_s)}$$

or $$M = \frac{6\mu_s W}{(r_c - r_s)^2(r_c + 2r_s)}\int_{r_s}^{r_c}(r_c r^2 - r^3)\,dr$$

Integration yields

$$M = T = \frac{6\mu_s W}{(r_c - r_s)^2(r_c + 2r_s)}\left[\frac{r_c}{3}(r_c^3 - r_s^3) - \frac{1}{4}(r_c^4 - r_s^4)\right]$$

or factoring out $(r_c - r_s)^2$ and simplifying,

$$\underline{\underline{T = \frac{\mu_s W(r_c^2 + 2r_c r_s + 3r_s^2)}{2(r_c + 2r_s)}}} \qquad (g)$$

To initiate rotation, the torque must be slightly larger than T as given by Eq. (g).

10.109

In the disk brake shown in Fig. a, the pads are pressed against the disk by forces N. The disk is rigidly attached to the shaft, and it does not slip when a torque T=2 kN·m is applied to the shaft. The coefficient of static friction is $\mu_s = 0.20$ and r = 175 mm.

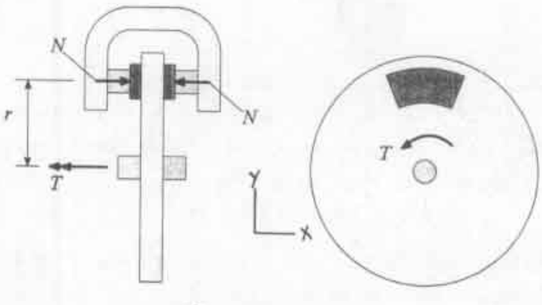


Figure a

Find the minimum required braking force N.

The braking force on each pad is N. Hence, the friction force on each pad is

$$F = \mu_s N = 0.20\,N$$

The resisting moment due to F is

$$M_R = r\,F = (0.175\text{ m})(0.20\text{ N})$$

or $$M_R = 0.035\text{ N} \quad \text{per pad}$$

For equilibrium of moments about the axis of the axle, since there are two pads,

$$\Sigma M = T - 2M_R = 0$$

or $$2(0.035\text{ N}) = 2000\text{ N}\cdot\text{m}$$

Hence, $$\underline{\underline{N = 28571\text{ N} = 28.571\text{ kN}}}$$

10.110

The pressure p that each shoe of the drum brake (Fig. a) exerts on the drum varies with $\sin\theta$, where θ is measured from line OB. At B, $p = 0$ and at A, $p = p_A$. The width of the lining in contact with the drum is b, and the inner radius of the drum is r. The coefficient of kinetic friction is μ_K.

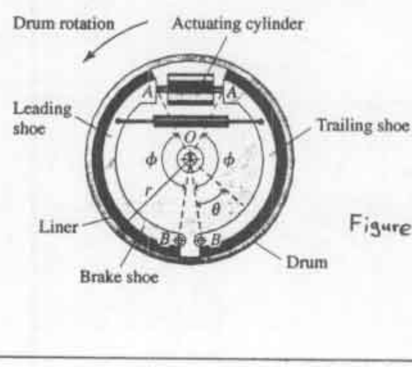


Figure a

(Continued)

10.110 Cont.

Derive a formula for the total moment M_d exerted on the drum by the shoes.

Each shoe exerts the pressure

$$p = k \sin\theta \qquad (a)$$

where k is a constant

at $\theta = 0$, $p = 0$ and at $\theta = \phi$, $p = p_A$.

at $\theta = 0$ Eq. (a) yields $p = 0$. At $\theta = \phi$, Eq. (a) yields $p_A = k \sin\phi$

Hence, $k = p_A / \sin\phi$. Hence, by Eq. (a),

$$p = p_A \frac{\sin\theta}{\sin\phi} \qquad (b)$$

Then, the increment of normal force dN exerted by each liner on an element dA of area of the drum is

$$dN = p\,dA = p\,br\,d\theta = p_A br \frac{\sin\theta}{\sin\phi}\,d\theta$$

and the corresponding frictional force is

$$dF = \mu_k dN = \mu_k p_A br \frac{\sin\theta}{\sin\phi}\,d\theta$$

The element of torque due to dF is

$$dM_d = r\,dF = \mu_k p_A b r^2 \frac{\sin\theta}{\sin\phi}\,d\theta$$

Thus, the torque due to the two liners is

$$M_d = \int 2\,dM_d = \frac{2\mu_k p_A b r^2}{\sin\phi}\int_0^{\phi} \sin\theta\,d\theta$$

or

$$M_d = 2\mu_k p_A b r^2 \left(\frac{1-\cos\phi}{\sin\phi}\right) \qquad (c)$$

By Eq. (B.21), Appendix B,

$$1 - \cos\phi = 2\sin^2(\phi/2) \qquad (d)$$

and by Eq. (B14),

$$\sin\phi = 2\sin(\phi/2)\cos(\phi/2) \qquad (e)$$

Then, by Eqs. (c), (d), and (e),

$$M_d = \frac{2\mu_k p_A b r^2 [2\sin^2(\phi/2)]}{2\sin(\phi/2)\cos(\phi/2)}$$

or

$$\underline{\underline{M_d = 2\mu_k p_A b r^2 \tan(\phi/2)}}$$

10.111

The actuating force that is transmitted to the cone clutch in Problem 10.102 is R = 20 kN (see Fig. a below)

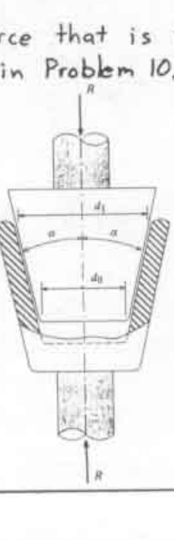


Figure a

a) For $d_o = 100$ mm and $d_1 = 150$ mm, plot the maximum torque transmitted by the clutch as a function of the angle α for $3° \le \alpha \le 15°$ (see Fig. b)

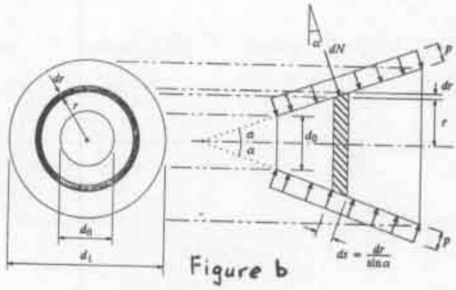


Figure b

b) Discuss the results

By the solution of Problem 10.102,

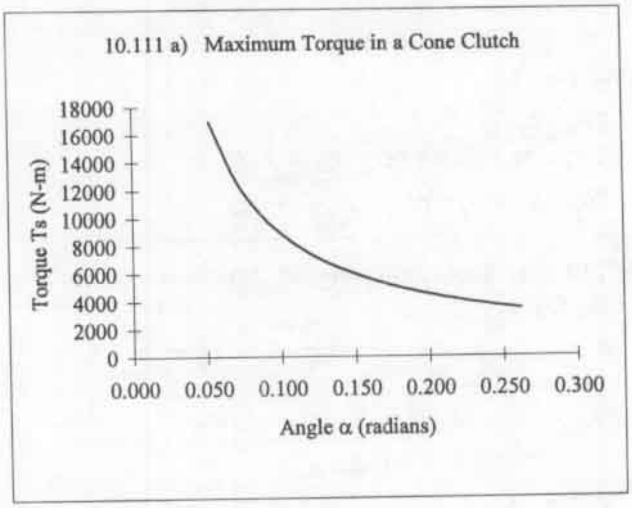
$$T_{max} = \frac{2\mu_s R (r_1^3 - r_o^3)}{3(\sin\alpha)(r_1^2 - r_o^2)} \qquad (a)$$

Hence, with the given values and noting that $r_o = d_o/2$, $r_1 = d_1/2$,

$$T_{max} = \frac{(2)(0.70)(20{,}000)[(0.075)^3 - (0.05)^3]}{(3)(\sin\alpha)[(0.075)^2 - (0.05)^2]}$$

or

$$T_{max} = \frac{886.6\overline{6}}{\sin\alpha} \quad N\cdot m$$



b) In the range $3° \le \alpha \le 15°$ (or $0.0523\,\text{rad} \le \alpha \le 0.2618\,\text{rad}$), the maximum torque decreased from $T_{max} \cong 16940$ N·m to $T_{max} \approx 3426$ N·m. In other words, the clutch is more efficient at $\alpha = 3°$ than at $\alpha = 15°$.

11.1

Given: The following beams (Figures a–g).

Find: The support reactions for each beam.

Solution:

a/ The free-body diagram of the beam is shown in Fig. a.

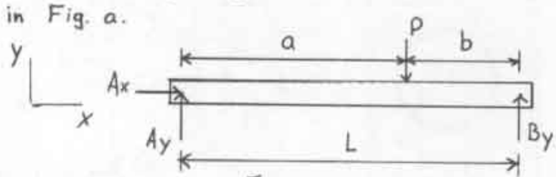


Figure a

By Fig. a,

$\Sigma F_x = \underline{A_x = 0}$

$\circlearrowleft \Sigma M_A = -P(a) + B_y(L) = 0\ ; \quad \underline{\underline{B_y = \frac{P(a)}{L}}}$

$\Sigma F_y = B_y - P + A_y = 0\ ; \quad \underline{\underline{A_y = \frac{P(L-a)}{L}}}$

b/ The free-body diagram of the beam is shown in Fig. b.

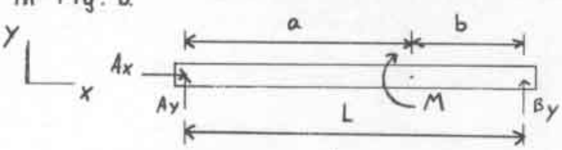


Figure b

By Fig. b,

$\Sigma F_x = \underline{\underline{A_x = 0}}$

$\circlearrowleft \Sigma M_A = -M + B_y(L) = 0\ ; \quad \underline{\underline{B_y = M/L}}$

$\Sigma F_y = A_y + B_y = 0\ ; \quad \underline{\underline{A_y = -M/L}}$

c/ The free-body diagram of the beam is shown in Fig. c.

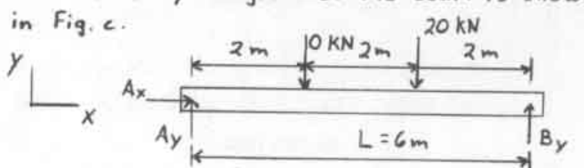


Figure c

By Fig. c,

$\Sigma F_x = \underline{\underline{A_x = 0}}$

$\circlearrowleft \Sigma M_A = -10\text{ KN}(2\text{ m}) - 20\text{ kN}(4\text{ m}) + B_y(6\text{ m}) = 0\ ; \underline{\underline{B_y = 16.67\text{ KN}}}$

$\Sigma F_y = A_y - 10\text{ kN} - 20\text{ KN} + B_y = 0\ ; \quad \underline{\underline{A_y = 13.33\text{ KN}}}$

d/ The free-body diagram of the beam is shown in Fig. d.

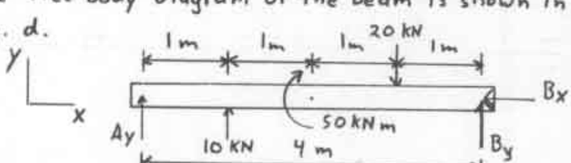


Figure d

By Fig. d,

$\Sigma F_x = \underline{\underline{B_x = 0}}$

$\circlearrowleft \Sigma M_B = -A_y(4\text{ m}) - 10\text{ kN}(3\text{ m}) - 50\text{ kN}\cdot\text{m} + 20\text{ kN}(1\text{ m}) = 0$

$\underline{\underline{A_y = -15\text{ kN}}}$

$\Sigma F_y = A_y + 10\text{ kN} - 20\text{ kN} + B_y = 0\ ; \quad \underline{\underline{B_y = 25\text{ kN}}}$

e/ The free-body diagram of the beam is shown in Fig. e.

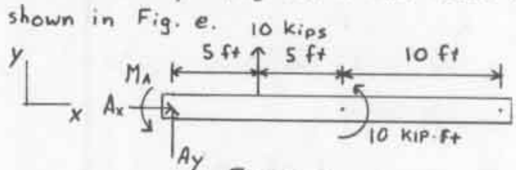


Figure e

By Fig. e,

$\Sigma F_x = \underline{\underline{A_x = 0}}$

$\Sigma F_y = A_y + 10\text{ kips} = 0\ ; \quad \underline{\underline{A_y = -10\text{ kips}}}$

$\circlearrowleft \Sigma M_A = M_A + 10\text{ kips}(5\text{ ft}) + 10\text{ kip ft} = 0$

$\underline{\underline{M_A = -60\text{ kip}\cdot\text{ft}}}$

f/ The free-body diagram of the beam is shown in Fig. f.

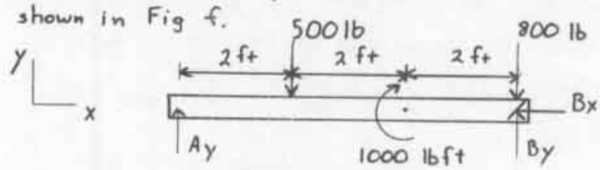


Figure f

By Fig. F,

$\Sigma F_x = \underline{\underline{B_x = 0}}$

$\circlearrowleft \Sigma M_B = -A_y(6\text{ ft}) + 500\text{ lb}(4\text{ ft}) - 1000\text{ lb}\cdot\text{ft} = 0$

$\underline{\underline{A_y = 166.7\text{ lb}}}$

$\Sigma F_y = A_y - 500 - 800 + B_y = 0\ ; \quad \underline{\underline{B_y = 1133.3\text{ lb}}}$

g/ The free-body diagram is shown in Fig. g.

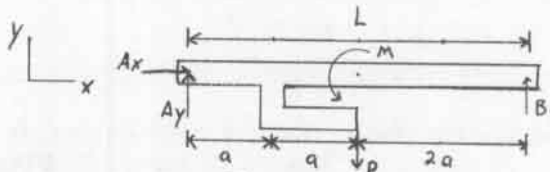


Figure g

By Fig. g,

$\Sigma F_x = \underline{\underline{A_x = 0}}$

$\circlearrowleft \Sigma M_A = M - P(2a) + B_y(L) = 0\ ; \quad \underline{\underline{B_y = \frac{2Pa - M}{L}}}$

$\Sigma F_y = A_y + B_y - P = 0$

$\underline{\underline{A_y = P - B_y = P - \frac{2Pa}{L} + \frac{M}{L} = \frac{P}{2} + \frac{M}{L}}}$

Alternatively,

$\underline{\underline{B_y = P - A_y = \frac{P}{2} - \frac{M}{L}}}$

11.2

Given:

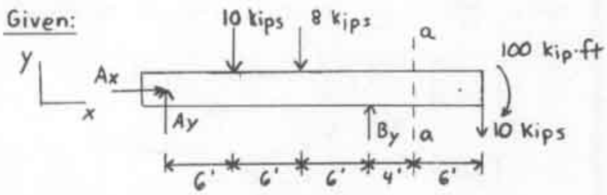


Figure a

Find: The shear and bending moment at section a-a.

Solution:

FBD (section to right of a-a)

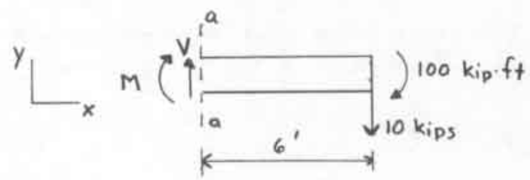


Figure b

By Fig. b,

$\Sigma F_y = -10 \text{ kips} + V = 0$; $\underline{V = 10 \text{ kips}}$

$\circlearrowleft \Sigma M_{a\text{-}a} = -M - 100 - 10(6) = 0$; $\underline{\underline{M = -160 \text{ kip·ft}}}$

11.3

Given: Figure a

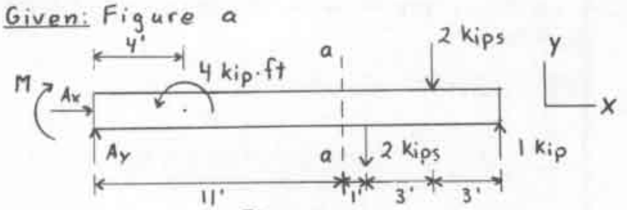


Figure a

Find: The shear force and bending moment at a-a.

Solution:

FBD (Section to right of a-a)

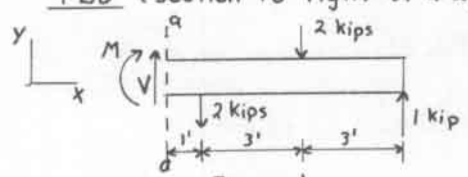


Figure b

By Fig. b,

$\Sigma F_y = -2 \text{ kips} - 2 \text{ kips} + 1 \text{ kip} + V = 0$

$\underline{\underline{V = 3 \text{ kips}}}$

$\circlearrowleft \Sigma M_{a\text{-}a} = -M - 2(1) - 2(4) + 1(7) = 0$

$\underline{\underline{M = -3 \text{ kip·ft}}}$

11.4

Given: Figure a

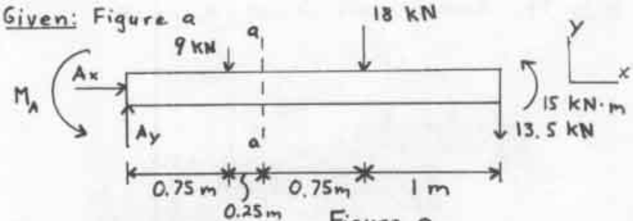


Figure a

Find: The shear force and bending moment at a-a.

Solution:

FBD (Section to right of a-a)

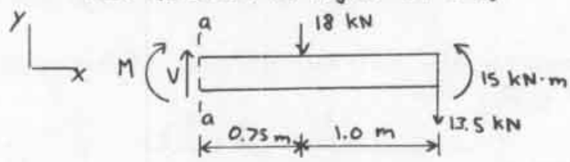


Figure b

By Fig. b,

$\Sigma F_y = V - 18 \text{ kN} - 13.5 \text{ kN} = 0$; $\underline{\underline{V = 31.5 \text{ kN}}}$

$\circlearrowleft \Sigma M_{a\text{-}a} = -M + 15 \text{ kN} - 13.5 \text{ kN}(1.75 \text{ m}) - 18 \text{ kN}(0.75 \text{ m}) = 0$

$\underline{\underline{M = -22.1 \text{ kN·m}}}$

11.5

Given:

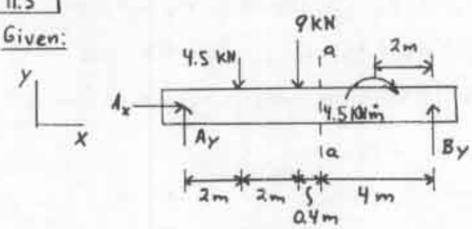


Figure b

Find: The shear force and bending moment at a-a.

Solution:

By Fig. a

$\circlearrowleft \Sigma M_a = -4.5(2) - 9(4) - 4.5 + B_y(8.4) = 0$

$B_y = 5.89 \text{ kN}$

FBD (section to the right of a-a)

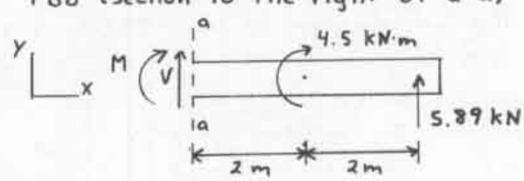


Figure b

By Fig. b,

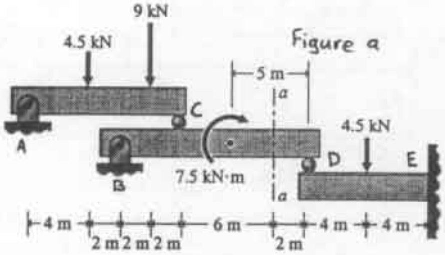
$\Sigma F_y = V + 5.89 = 0$; $\underline{\underline{V = -5.89 \text{ kN}; \text{ or, } V = 5.89 \text{ kN} \downarrow}}$

$\circlearrowleft \Sigma M_{a\text{-}a} = -4.5 + 5.89(4) - M = 0$

$\underline{\underline{M = 19.1 \text{ kN·m}}}$

11.6

Given: The beam system shown in Fig. a



Find: The shear force and the bending moment at a-a

Solution:

Consider the free-body diagram of beam AC (Fig. b).

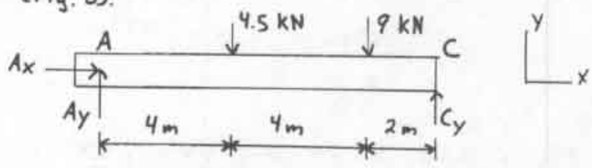


Figure b

By Fig. b,

$\Sigma F_x = A_x = 0$

$\circlearrowleft \Sigma M_A = -4.5(4) - 9(8) + C_y(10) = 0\,; \quad C_y = 9 \text{ kN}$

$\Sigma F_y = -4.5 - 9 + C_y + A_y = 0\,; \quad A_y = 4.5 \text{ kN}$

Next, consider the free-body diagram of beam BD (Fig. c)

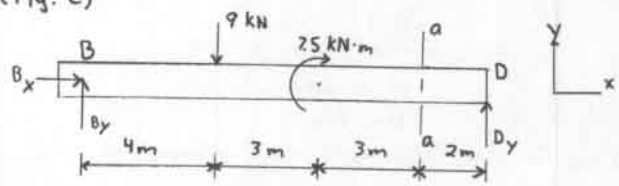


Figure c

$\Sigma F_x = B_x = 0$

$\circlearrowleft \Sigma M_B = -9(4) - 7.5 + D_y(12) = 0\,; \quad D_y = 3.625 \text{ kN}$

$\Sigma F_y = B_y - 9 + D_y = 0\,; \quad B_y = 5.375 \text{ kN}$

By the free-body diagram of beam BD from B to section a-a (Fig. d)

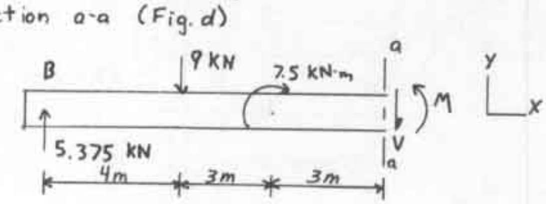


Figure d

$\Sigma F_y = 5.375 - 9 - V = 0\,; \quad \underline{\underline{V = -3.63 \text{ kN}; \text{ or, } V = 3.63 \text{ kN} \uparrow}}$

$\circlearrowleft \Sigma M_{a\text{-}a} = -5.375(10) + 9(6) - 7.5 + M = 0$

$\underline{\underline{M = 7.25 \text{ kN m}}}$

Alternatively, consider the free-body diagram of beam BD from section a-a to D (Fig. e)

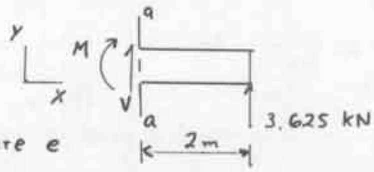


Figure e

$\Sigma F_y = V + 3.625 = 0\,; \quad \underline{\underline{V = -3.63 \text{ kN}; \text{ or, } V = 3.63 \text{ kN} \downarrow}}$

$\circlearrowleft \Sigma M_{a\text{-}a} = -M + 3.625(2) = 0\,; \quad \underline{\underline{M = 7.25 \text{ kN}\cdot\text{m}}}$

11.7

Given: The beams of Problem 11.1 (Figs. a–e).

Find: The shear force and bending moment as a function of x for each beam

Solution:

a)

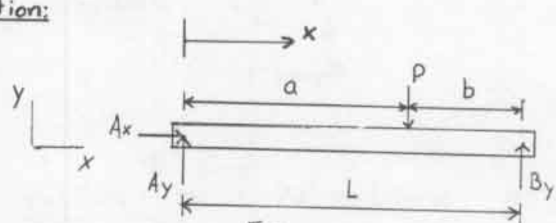


Figure a

For reactions see Problem 11.1 (a) ($A_x = 0$, $A_y = \frac{P(L-a)}{L}$, $B_y = Pa/L$)

FBD (section $0 \le x \le a$)

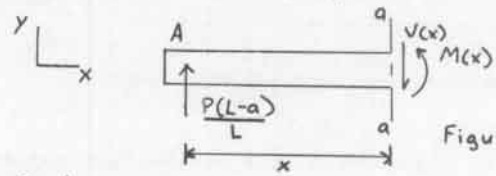


Figure b

By Fig. b,

$\Sigma F_y = -V(x) + \frac{P(L-a)}{L} = 0\,; \quad \underline{\underline{V(x) = \frac{P(L-a)}{L}}}$

$\circlearrowleft \Sigma M_{a\text{-}a} = -\frac{P(L-a)}{L}(x) + M(x)\,; \quad \underline{\underline{M(x) = \frac{P(L-a)}{L}(x)}}$

FBD (section $a \le x \le a+b$)

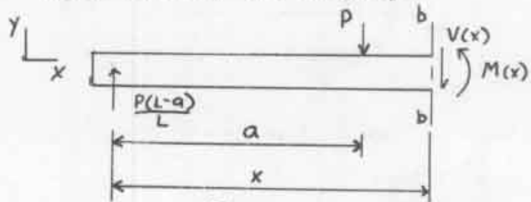


Figure c

$\Sigma F_y = \frac{P(L-a)}{L} - P + V(x) = 0\,; \quad V(x) = -P + \frac{PL}{L} - \frac{Pa}{L}$

$\underline{\underline{V(x) = -\frac{Pa}{L}}}$

(Continued)

11.7 Cont.

$\circlearrowleft\!+\ \Sigma M_{b\text{-}b} = -\frac{P}{L}(L-a)\,x + P(x-a) + M(x) = 0$

$M(x) = Px - \frac{P(a)x}{L} - Px + P(a)$; $M(x) = \frac{P(a)}{L}(-x+L)$

$\underline{\underline{M(x) = P(a)\left(1-\frac{x}{L}\right)}}$

b)

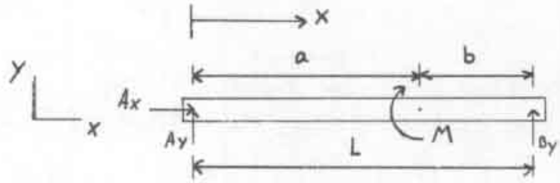


Figure d

For reactions see Problem 11.1 (b) ($A_x = 0$, $A_y = -\frac{M}{L}$, $B_y = \frac{M}{L}$)

FBD (section $0 \le x \le a$)

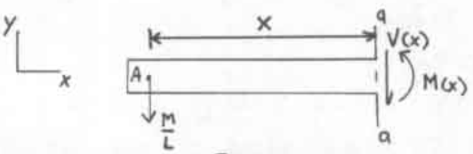


Figure e

By Fig. e,

$\circlearrowleft\!+\ \Sigma M_{a\text{-}a} = \frac{M}{L}x + M(x) = 0$

$\underline{\underline{M(x) = -\frac{Mx}{L}}}$ (For $0 \le x \le a$)

$\Sigma F_y = -V(x) - \frac{M}{L} = 0$; $\underline{\underline{V(x) = -\frac{M}{L}\text{ ; or, } V(x) = \frac{M}{L}\uparrow}}$

FBD (section $a \le x \le a+b$)

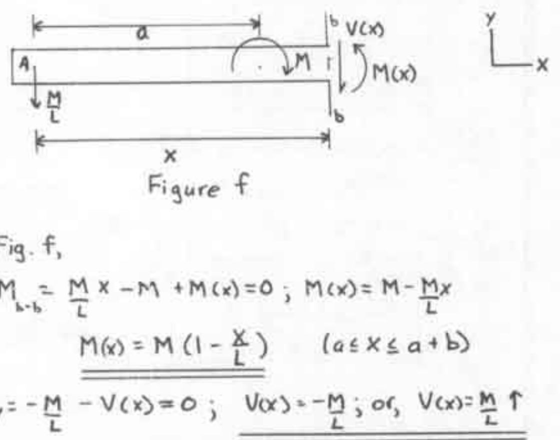


Figure f

By Fig. f,

$\circlearrowleft\!+\ \Sigma M_{b\text{-}b} = \frac{M}{L}x - M + M(x) = 0$; $M(x) = M - \frac{M}{L}x$

$\underline{\underline{M(x) = M\left(1-\frac{x}{L}\right)}}$ ($a \le x \le a+b$)

$\Sigma F_y = -\frac{M}{L} - V(x) = 0$; $\underline{\underline{V(x) = -\frac{M}{L}\text{ ; or, } V(x) = \frac{M}{L}\uparrow}}$

c)

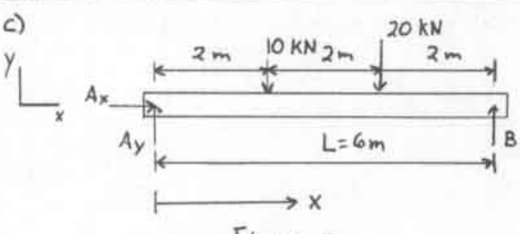


Figure g

For reactions see Problem 11.1 (c) ($A_x = 0$, $A_y = 13.33$ kN, $B_y = 16.67$ kN)

FBD (section $0 \le x \le 2$)

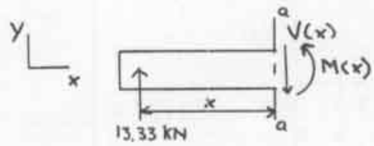


Figure h

By Fig. h,

$\Sigma F_y = 13.33 - V(x) = 0$; $\underline{\underline{V(x) = 13.3\bar{3}\text{ kN}}}$

$\circlearrowleft\!+\ \Sigma M_{a\text{-}a} = -13.33(x) + M(x) = 0$

$\underline{\underline{M(x) = 13.3\bar{3}\,x\ \ (\text{kN}\cdot\text{m})}}$

FBD (section $2 \le x \le 4$)

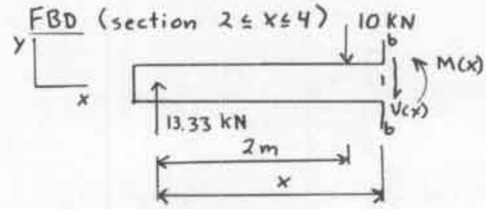


Figure i

By Fig. i,

$\Sigma F_y = 13.3\bar{3} - 10 - V(x) = 0$; $\underline{\underline{V(x) = 3.3\bar{3}\text{ kN}}}$

$\circlearrowleft\!+\ M_{b\text{-}b} = -13.3\bar{3}x + 10(x-2) + M(x) = 0$

$\underline{\underline{M(x) = 3.3\bar{3}x + 20\ \ (\text{kN}\cdot\text{m})}}$

FBD (section $4 \le x \le 6$)

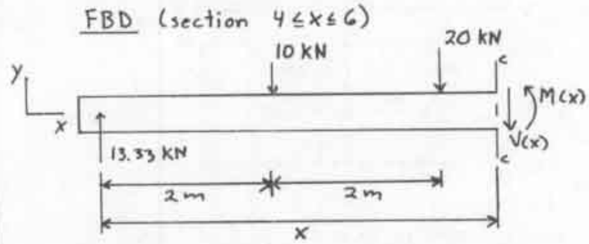


Figure j

By Fig. j,

$\Sigma F_y = 13.33 - 10 - 20 - V(x) = 0$; $\underline{\underline{V(x) = -16.67\text{ kN}}}$

$\circlearrowleft\!+\ \Sigma M_{c\text{-}c} = -13.33x + 10(x-2) + 20(x-4) + M(x) = 0$

$M(x) = 13.33x - 10x + 20 - 20x + 80$

$\underline{\underline{M(x) = -16.67x + 100}}$ (kN·m)

d)

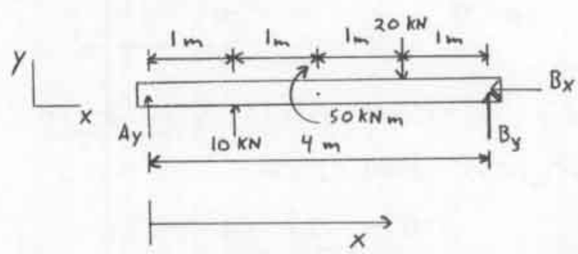


Figure k

(Continued)

11.7 Cont.

For reactions see Problem 11.1(d) ($A_y=-15$ kN, $B_x=0$, $B_y=25$ kN)

FBD (section $0 \le x \le 1$ m)

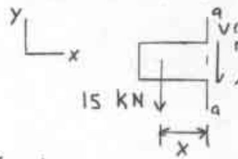


Figure L

By Fig. L,

$\Sigma F_y = -V(x) - 15 = 0$; $\underline{\underline{V(x) = -15 \text{ kN}}}$

$\circlearrowleft \Sigma M_{a\text{-}a} = 15(x) + M(x) = 0$; $\underline{\underline{M(x) = -15x}}$ (kN·m)

FBD (section $1 \le x \le 2$ m)

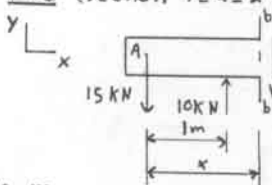


Figure m

By Fig. m,

$\Sigma F_y = -15 + 10 - V(x) = 0$; $\underline{\underline{V(x) = -5 \text{ kN}}}$

$\circlearrowleft \Sigma M_{b\text{-}b} = 15(x) - 10(x-1) + M(x) = 0$

$\underline{\underline{M(x) = -5x - 10 \quad (\text{kN·m})}}$

FBD (section $2 \le x \le 3$

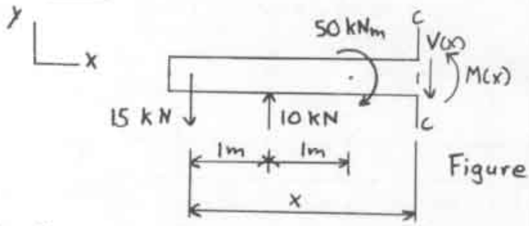


Figure n

By Fig. n,

$\Sigma F_y = -15 + 10 - V(x) = 0$; $\underline{\underline{V(x) = -5 \text{ kN}}}$

$\circlearrowleft \Sigma M_{c\text{-}c} = 15(x) - 10(x-1) - 50 + M(x) = 0$

$\underline{\underline{M(x) = -5x + 40 \quad (\text{kN·m})}}$

FBD (section $3 \le x \le 4$)

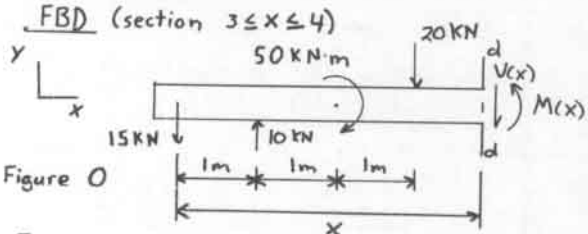


Figure O

By Fig. O,

$\Sigma F_y = -15 + 10 - 20 - V(x) = 0$; $\underline{\underline{V(x) = -25 \text{ kN}}}$

$\circlearrowleft \Sigma M_{d\text{-}d} = 15x - 10(x-1) + 20(x-3) - 50 + M(x) = 0$

$\underline{\underline{M(x) = -25x + 100 \quad (\text{kN·m})}}$

e)

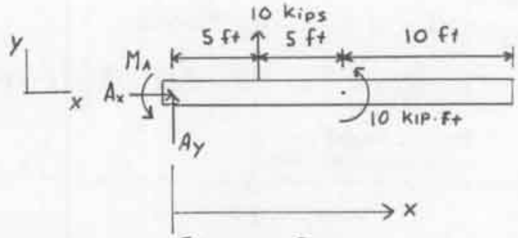


Figure P

For reactions see Problem 11.1 (e)
($A_x = 0$, $A_y = -10$ kips, $M_A = -60$ kip·ft)

FBD (section $0 \le x \le 5$)

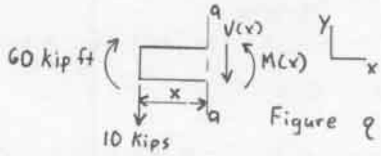


Figure q

By Fig. q,

$\Sigma F_y = -10 - V(x) = 0$; $\underline{\underline{V(x) = -10 \text{ kips}}}$

$\circlearrowleft \Sigma M_{a\text{-}a} = -60 + 10x + M(x) = 0$; $\underline{\underline{M(x) = -10x + 60 \ (\text{kip·ft})}}$

FBD (section $5 \le x \le 10$)

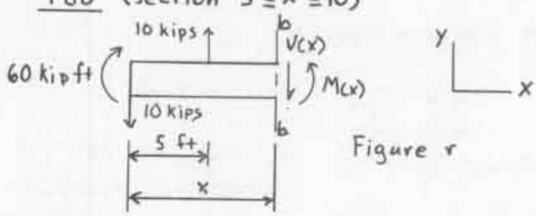


Figure r

By Fig. r,

$\Sigma F_y = -10 + 10 - V(x) = 0$; $\underline{\underline{V(x) = 0}}$

$\circlearrowleft \Sigma M_{b\text{-}b} = 10x - 10(x-5) + M(x) = 0$

$\underline{\underline{M(x) = 10 \text{ kip·ft}}}$

FBD (section $10 \le x \le 20$)

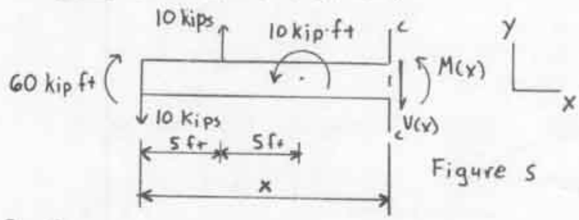


Figure s

By Fig. s,

$\Sigma F_y = -10 + 10 - V(x) = 0$; $\underline{\underline{V(x) = 0}}$

$\circlearrowleft \Sigma M_{c\text{-}c} = -60 + 10x - 10(x-5) + 10 + M(x) = 0$

$\underline{\underline{M(x) = 0}}$

(Continued)

11.7 Cont.

f)

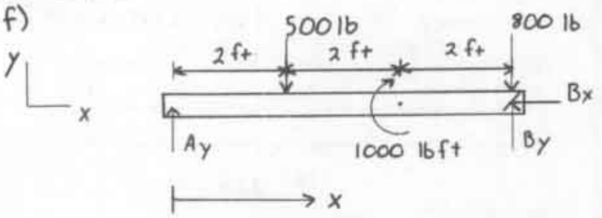


Figure t

For reactions see Problem 11.1 (f) (A_y=166.7 lb, B_x=0, B_y=1133 lb)

FBD (section $0 \le x \le 2$ ft)

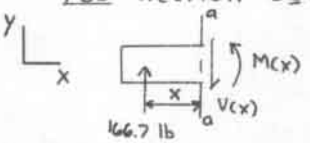


Figure u

By Fig. u,

$\Sigma F_y = 166.7 - V(x) = 0$; $\underline{\underline{V(x) = 166.7 \text{ lb}}}$

$\circlearrowleft \Sigma M_{a-a} = -166.7x + M(x) = 0$; $\underline{\underline{M(x) = 166.7x \text{ (lbft)}}}$

FBD (section $2 \le x \le 4$)

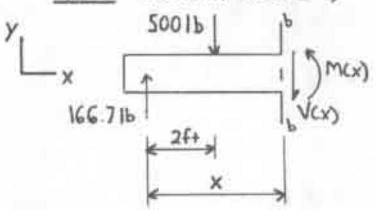


Figure v

By Fig. v,

$\Sigma F_y = 166.7 - 500 - V(x) = 0$; $\underline{\underline{V(x) = -333.3 \text{ lb}}}$

$\circlearrowleft \Sigma M_{b-b} = -166.7x + 500(x-2) + M(x) = 0$

$\underline{\underline{M(x) = -333.3x + 1000}}$ (lb·ft)

FBD (section $4 \le x \le 6$)

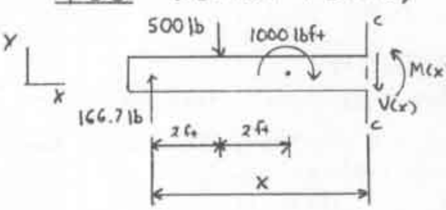


Figure w

By Fig. w,

$\Sigma F_y = 166.7 - 500 - V(x) = 0$; $\underline{\underline{V(x) = -333.3 \text{ lb}}}$

$\circlearrowleft \Sigma M_{c-c} = -166.7x + 500(x-2) - 1000 + M(x) = 0$

$\underline{\underline{M(x) = -333.3x + 2000}}$ (lbft)

g)

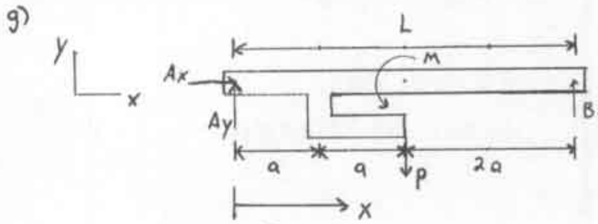


Figure x

For reactions see Problem 11.1 (g) ($A_x = 0$, $A_y = \frac{P}{2} + \frac{M}{L}$, $B_y = \frac{P}{2} - \frac{M}{L}$)

FBD (section $0 \le x \le a$)

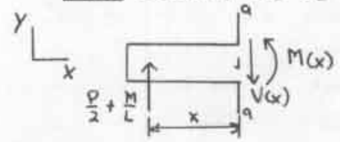


Figure y

By Fig. y,

$\Sigma F_y = \frac{P}{2} + \frac{M}{L} - V(x) = 0$; $\underline{\underline{V(x) = \frac{P}{2} + \frac{M}{L}}}$

$\circlearrowleft \Sigma M_{a-a} = -(\frac{P}{2} + \frac{M}{L})x + M(x) = 0$; $\underline{\underline{M(x) = (\frac{P}{2} + \frac{M}{L})x}}$

FBD (section $a \le x \le 2a$)

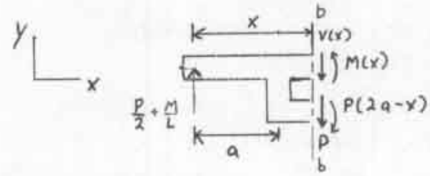


Figure z

By Fig. z,

$\Sigma F_y = \frac{P}{2} + \frac{M}{L} - V(x) - P = 0$; $\underline{\underline{V(x) = \frac{M}{L} - \frac{P}{2}}}$

$\circlearrowleft \Sigma M_{b-b} = -(\frac{P}{2} + \frac{M}{L})x - P(2a - x) + M(x) = 0$

$\underline{\underline{M(x) = (\frac{M}{L} - \frac{P}{2})x + 2Pa}}$

FBD (section $2a \le x \le 4a$)

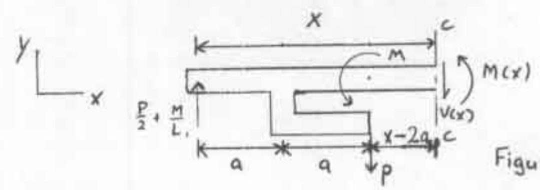


Figure a-a

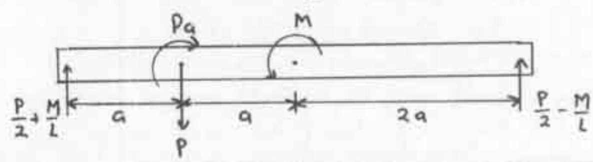
By Fig. a-a,

$\Sigma F_y = \frac{P}{2} + \frac{M}{L} - P - V(x) = 0$; $\underline{\underline{V(x) = \frac{M}{L} - \frac{P}{2}}}$

$\circlearrowleft \Sigma M_{c-c} = -(\frac{P}{2} + \frac{M}{L})x + M + P(x - 2a) + M(x) = 0$

$\underline{\underline{M(x) = (\frac{M}{L} - \frac{P}{2})x + 2Pa - M}}$

Information Note: The shear and moment in beam AB may also be determined with the loads illustrated in the following figure:



11.8

Given: To get a car that weighs 3000 lb over an 18 in Wall, a man plans to slowly drive the car across two boards until they tip, allowing him to drive down the other side (Fig. a). The boards will break if the bending moment in each board exceeds 40,000 lb·in.

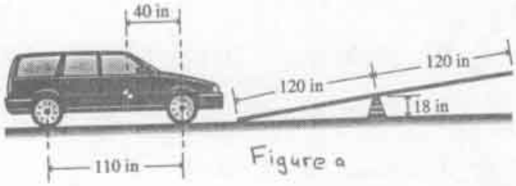


Figure a

Find: If the boards are strong enough.

Solution:

Note that the angle of inclination of the boards is $\theta = \sin^{-1}(18/120) = 8.63°$. Since θ is small, we assume that the boards are essentially horizontal.

To determine the forces that the car will exert on a board consider the free-body diagram of the car (Fig. b).

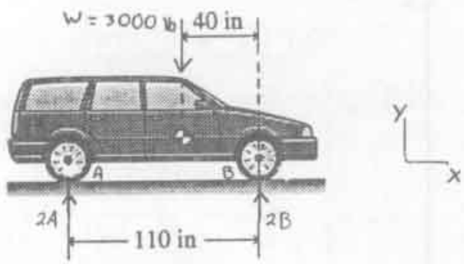


Figure b

By Fig. b,

$$\Sigma F_y = 2A + 2B - W = 0 \qquad (a)$$

$$\Sigma M_A = 2B(110) - W(70) = 0$$

The solution of Eqs. (a) is (with W = 3000 lb)

$$A = 545.455 \text{ lb} \qquad (b)$$
$$B = 954.545 \text{ lb}$$

The forces A and B change slightly as the car goes up the small incline, but we will neglect the small changes.

There are three cases to consider

a) Only the front wheels are on the boards.

b) The front and back wheels are on the boards but the front wheels have not passed over the wall.

c) The front and back wheels are on the boards, and the front wheels have passed over the wall.

Case a: Consider the free-body diagram of one board (Fig. c) $(0 < x < 110 \text{ in.})$

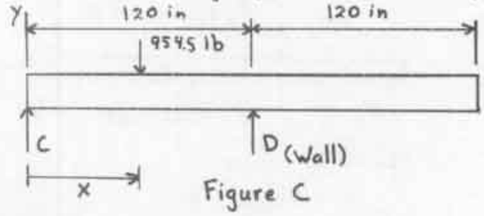


Figure C

By Fig. c,

$$\Sigma F_y = C + D - 954.545 \text{ lb} = 0$$

$$\Sigma M_A = 120D - 954.545x = 0$$

The solution of Eqs. (c) is

$$C = 954.545 - 7.9545x$$
$$D = 7.9545x \qquad (d)$$

For $0 < x < 110$, the moment at section x is given by Fig. d

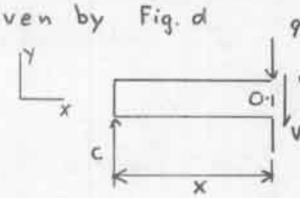


Figure d

$$\Sigma M_O = M - Cx = 0$$

or

$$M = (954.545 - 7.9545x)x$$
$$M = 954.545x - 7.9545x^2 \qquad (e)$$

The maximum moment occurs when $\frac{dM}{dx} = 0$

Hence from Eq. (e),

$$\frac{dM}{dx} = 954.545 - 15.909x = 0$$

or $x = 60$ ft $\qquad$ (f)

By Eqs. (e) and (f)

$$M_{(max)} = 28{,}635 \text{ lb·in}$$

Hence, the boards will not break for case a.

Case b: Consider the free-body diagram of one board (Fig. e) $(110 < x < 120 \text{ in})$

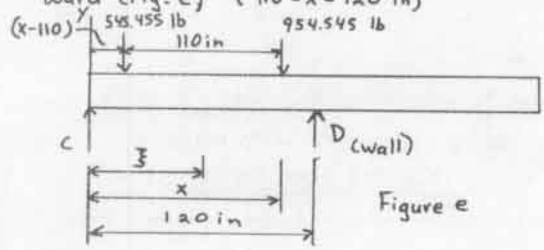


Figure e

By Fig. e,

$$\Sigma F_y = C + D - 1500 = 0 \qquad (g)$$

$$\Sigma M_c = -545.455(x - 110) - 954.545(x) + 120D = 0$$

(Continued)

11.8 Cont.

The solution of Eqs. (g) is

$$C = 2000 - 12.5x \qquad D = 12.5x - 500 \qquad \text{(h)}$$

Hence, the moment at ξ (Fig. e) is

$$M(\xi) = C\xi - 545.455[\xi - (x-110)]$$

or with the first of Eqs. (h),

$$M(\xi) = 1454.545\xi + 545.455x - 12.5x\xi - 60{,}000 \qquad \text{(i)}$$

Eq. (i) shows that $M(\xi)$ varies linearly with ξ for a given x. Also, by Fig. e,

$$x - 110 < \xi < x \qquad \text{(j)}$$

For example, for $x = 110$, $0 < \xi < 110$ in, Eq. (i) yields,

$$M(\xi) = 1454.545\xi - 1375\xi = 79.545\xi$$

at $\xi = 0$, $M(\xi) = 0$; at $\xi = 110$, $M(\xi) = 8750$ lb·in

8750 lb·in < 40,000 lb·in

For $x = 120$ in, $10 < \xi < 120$ in

$$M(\xi) = 5454.6 - 45.455\xi$$

at $\xi = 10$, $M(\xi) = 5000$ lb·in < 40,000 lb·in

at $\xi = 120$, $M(\xi) = 0$

Hence, in the range $110 < x < 120$ in the boards are safe.

Case C: Consider the free-body diagram of one board (Fig. f) $(120 < x < 160)$ in

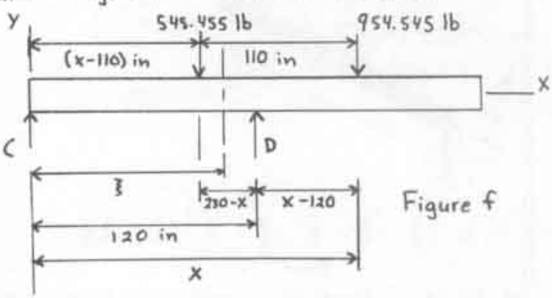


Figure f

By Fig. f,

$$\Sigma F_y = C + D - 1500 = 0$$
$$\circlearrowleft \Sigma M_D = 120C - 545.455(230 - x) + 954.545(x - 120) = 0 \qquad \text{(k)}$$

The solution of Eqs. (k) is

$$C = 2000 - 12.5x \qquad D = 12.5x - 500 \qquad (\ell)$$

Hence, the moment at ξ (Fig. f), for $x - 110 < \xi < 120$ in is,

$$M(\xi) = C(\xi) - 545.455[\xi - (x - 110)]$$

or with the first of Eqs. (ℓ)

$$M(\xi) = 1454.545\xi + 545.455x - 12.5x\xi - 60{,}000 \qquad \text{(m)}$$

Eq. (m) shows that for a given value of $x > 120$, $M(\xi)$ is a linear function of ξ. Noting that $x - 110 < \xi < 120$, Eq. (m) yields

$x = 120$ in, $\xi = 10$ in ; $M(\xi) = 5000$ lb·in < 40,000 lb·in

$x = 120$ in, $\xi = 120$ in, $M(\xi) = 0$

$x = 130$ in, $\xi = 20$ in, $M(\xi) = 7500$ lb·in < 40,000 lb·in

$x = 130$ in, $\xi = 120$ in, $|M(\xi)| = 9545.45$ lb·in < 40,000 lb·in

Continuing, the moment increases in magnitude until $x = 160$ in, at which distance the boards are tipping (Fig. g)

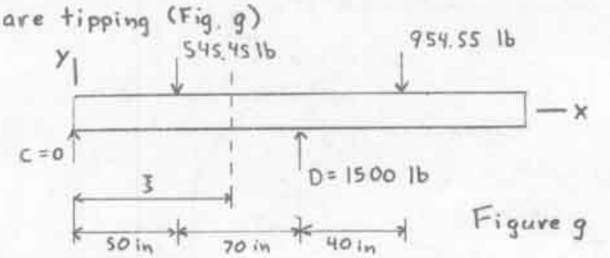


Figure g

Then, by Eq. (m)

$x = 160$ in, $\xi = 50$ in, $M(\xi) = 0$

$x = 160$ in, $\xi = 120$ in, $|M(\xi)| = 38{,}182$ lb·in < 40,000 lb·in

For $x > 160$, $M(\xi)$ decreases. Hence, the boards are strong enough.

Observation: Intuitively, one might surmise that in case a, the maximum moment occurs at $x = 60$ in (the boards act as a simple beam, with the load at the center), and in case c, the maximum moment occurs when the center of gravity of the car is centered on the wall (the boards act as a lever balanced on the wall). The above detailed computations verify intuition.

11.9

Given: The bow in Fig. a is bent by a horizontal pull of 35 lb.

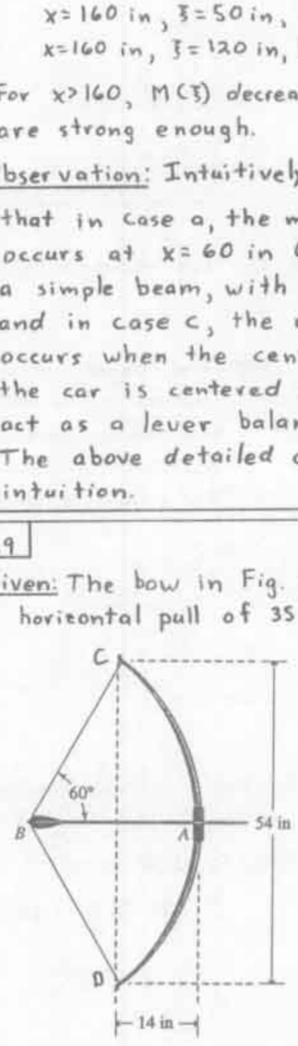


Figure a

Find: The shear force V, the axial force N, and the bending moment M at A.

(Continued)

11.9 Cont.

Solution:

FBD of Point B (Fig. b)

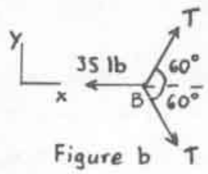


Figure b

By Fig. b,

$\Sigma F_x = -35 + 2T\cos 60° = 0$

$T = 35\ \text{lb}$

FBD of section CA (Fig. c)

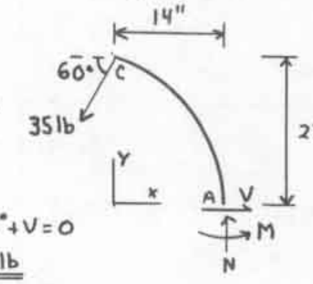


Figure C

By Fig. c,

$\Sigma F_x = -35\cos 60° + V = 0$

$\underline{\underline{V = 17.5\ \text{lb}}}$

$\Sigma F_y = -35\sin 60° + N = 0$

$\underline{\underline{N = 30.3\ \text{lb}}}$

$\circlearrowleft \Sigma M_A = 35\cos 60°(27) + 35\sin 60°(14) + M = 0$

$\underline{\underline{M = -897\ \text{lbin} = 897\ \text{lb·in}}}$

11.10

Given: The cam-follower system shown in Fig. a

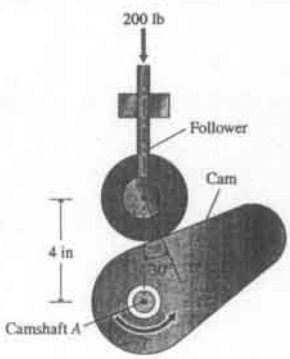


Figure a

Find: The torque T in the camshaft A to hold the follower in the position shown in Fig. a. Neglect friction.

Solution:

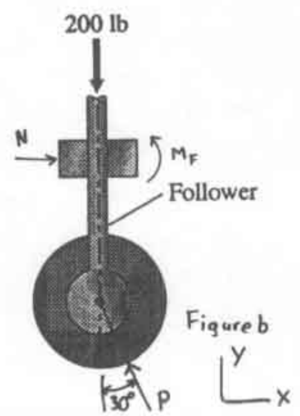


Figure b

By the free-body diagram of the follower (Fig. b)

$\Sigma F_y = -200 + P\cos 30° = 0$

$P = 231\ \text{lb} \qquad (a)$

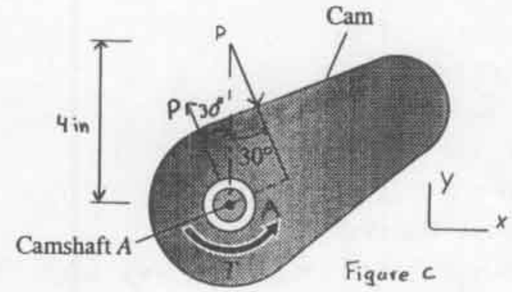


Figure c

By the free-body diagram of the cam (Fig. c) and Eq. (a),

$\circlearrowleft \Sigma M_A = T - P(4\sin 30°) = 0$

$\underline{\underline{T = 462\ \text{lb·in}}}$

11.11

Given: A magnetic bar hung horizontally from a quartz fiber and is subjected to a horizontal magnetic field H. (Fig. a)

When $\theta = 30°$, $H = H_1$

When $\theta = 45°$, $H = H_2$

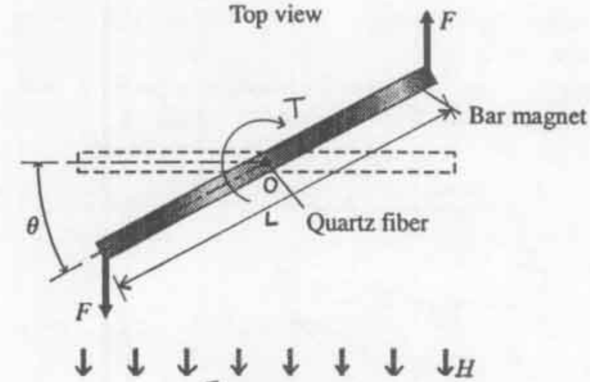


Figure a

The force F that the field exerts on each pole is proportional to H and the resisting torque in the fiber is proportional to θ.

Find: The ratio H_2/H_1

Solution:

The force F and torque T are

$$F = C_1 H \qquad (a)$$

$$T = C_2\theta \qquad (b)$$

Where C_1 and C_2 are constants of proportionality.

By Fig. a and Eqs. (a) and (b),

$\circlearrowleft \Sigma M_O = FL\cos\theta - T = 0$

or $C_1 HL\cos\theta = C_2\theta$

Therefore,

$$H = C\frac{\theta}{\cos\theta} \qquad (c)$$

Where $C = C_2/(C_1 L)$ is a constant.

(Continued)

11.11 Cont.

For $\theta = 30^\circ = \frac{\pi}{6}$ rad, $H = H_1$. So by Eq. (c),

$$H_1 = 0.6046C$$

For $\theta = 45^\circ = \frac{\pi}{4}$ rad, $H = H_2$. Hence, by Eq. (c),

$$H_2 = 1.1107C$$

Therefore,

$$\underline{\underline{\frac{H_2}{H_1} = 1.837}}$$

11.12

Given: Two pulleys C and D are fixed to a shaft (Fig. a)

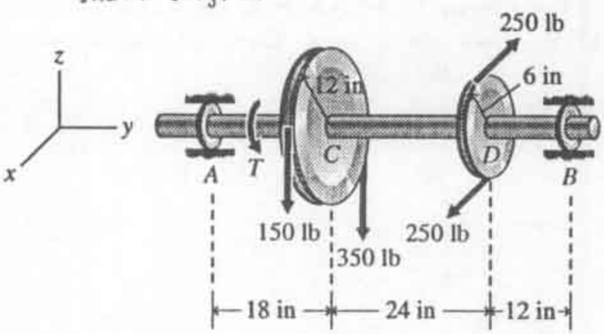


Figure a

Find:

a) The torque T required to maintain the shaft and pulleys in equilibrium

b) The reactions of the frictionless bearings A and B.

*Assume that no transverse loads are applied outside interval AB.

Solution:

a) By Fig. a,

$$\overset{+}{\rightarrow}\Sigma M = 0 = 150(12) - 350(12) - 250(6) - 250(6) + T$$

$$\underline{\underline{T = 5400 \text{ lb in}}}$$

b) Figure b is a diagram showing the transverse loads that act on the shaft.

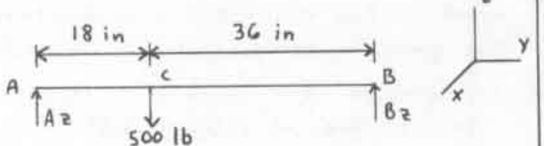


Figure b

By Fig. b,

$\circlearrowleft\Sigma M_A = -500(18) + B_z(54) = 0$; $\quad \underline{\underline{B_z = 166.7 \text{ lb}}}$

$\Sigma F_z = A_z + B_z - 500 = 0$; $\quad \underline{\underline{A_z = 333.3 \text{ lb}}}$

11.13

Given: A torque $T_1 = 100$ Nm is applied to shaft 1 of the gear system shown in Fig. a.

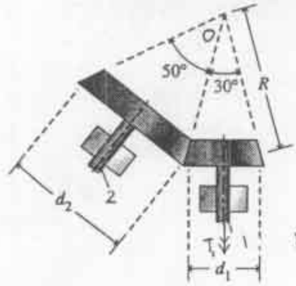


Figure a

Find: The torque T_2 transmitted to shaft 2.

Solution:

By Fig. a,

$d_1 = 2R\sin 30^\circ$; $\quad d_2 = 2R\sin 50^\circ$

or

$r_1 = R\sin 30^\circ$; $\quad r_2 = R\sin 50^\circ$

where r_1, r_2 are the radii of the pitch circles of gears 1, 2 respectively. Looking from point O along the axis of gear 1 (see Fig. b), we have, where C is the contact point of gears 1 and 2,

$\circlearrowleft\Sigma M_o = Fr_1 - T_1 = 0$

or, $\quad F = \dfrac{T_1}{r_1} \qquad$ (b)

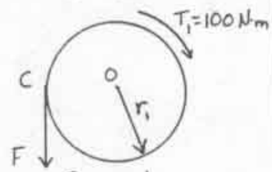


Figure b

Also looking from point O along axis 2 (Fig. c), we have, with Eq. (b),

$\circlearrowleft\Sigma M_o = Fr_2 = T_2 \qquad$ (c)

where T_2 is the torque transmitted to shaft 2. Hence, by Eqs. (a), (b), and (c),

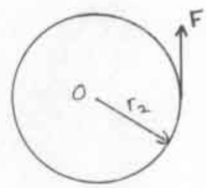


Figure c

$$T_2 = T_1\frac{r_2}{r_1} = T_1\frac{\sin 50^\circ}{\sin 30^\circ}$$

or, with $T_1 = 100$ N·m,

$$\underline{\underline{T_2 = 100\,\frac{\sin 50^\circ}{\sin 30^\circ} = 153.2 \text{ Nm}}}$$

11.14

Given: The quick-return mechanism is in equilibrium (Fig. a)

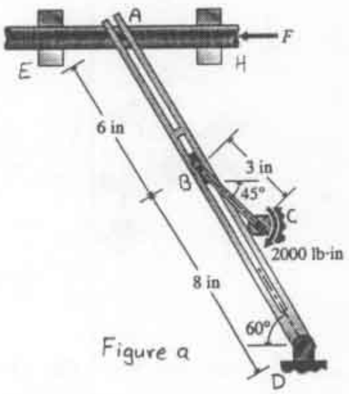


Figure a

Find: The thrust F developed by the quick-return mechanism. Neglect friction.

Solution:

Consider the free-body diagram of the crank BC (Fig. b).

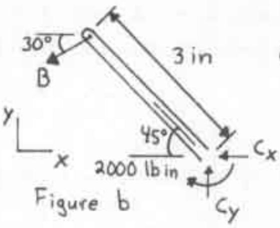


Figure b

By Fig. b,

$$\circlearrowleft \Sigma M_C = B(\cos 30°)(3\sin 45°) + B(\sin 30°)(3\cos 45°) - 2000 = 0$$

or

$$B = 690.2 \text{ lb} \qquad (a)$$

Next, consider the free-body diagram of the arm ABD (Fig. c)

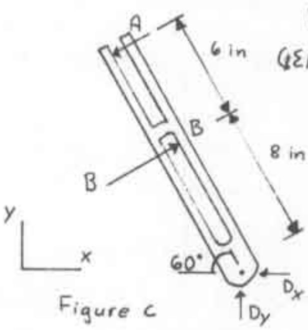


Figure c

By Fig. c,

$$\circlearrowleft \Sigma M_D = A(14) - B(8) = 0$$

or, with Eq. (a),

$$A = 394.4 \text{ lb}$$

Finally, consider the free-body diagram of the slider EF (Fig. d)

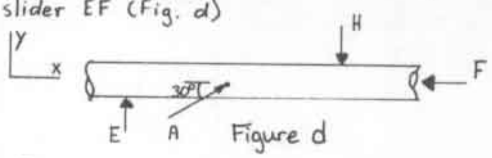


Figure d

$$\Sigma F_x = A\cos 30° - F = 0$$

$$F = 394.4(\cos 30°); \quad \underline{F = 341.6 \text{ lb}}$$

11.15

Given: A simplified representation of a train of gears (Fig. a). The torque in the drive shaft A is $T_A = 25$ N·m. Shaft B is rigidly attached to its two gears

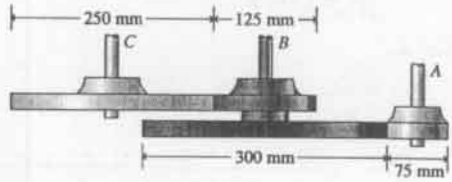


Figure a

Find:

a) The torque T_c transmitted to shaft c.

b) The force acting on a tooth of the smaller gear on shaft B.

Solution:

a) Consider the equilibrium of moments about the axes of shaft A and B (Fig. b).

Larger gear of shaft B

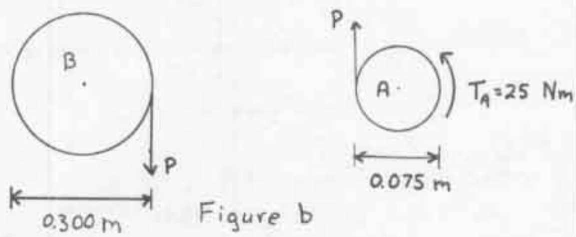


Figure b

For shaft A,

$$\circlearrowleft \Sigma M_A = T_A - P\left(\frac{0.075}{2}\right) = 0; \quad P = \frac{2T_A}{0.075} \text{ (N)} \qquad (a)$$

The torque transmitted to shaft B through the larger gear of shaft B is (Fig. b), with Eq. (a),

$$T_B = P\left(\frac{0.300}{2}\right) = 4T_A$$

or

$$\frac{T_B}{T_A} = 4 \qquad (b)$$

In words, Eq. (b) indicates that the ratio of torques between shafts B and A is equal to the ratio of the diameters of the gears in contact; that is, $0.300/0.075 = 4$

Next consider the equilibrium of moments about the axes of shafts B and C (Fig. C),

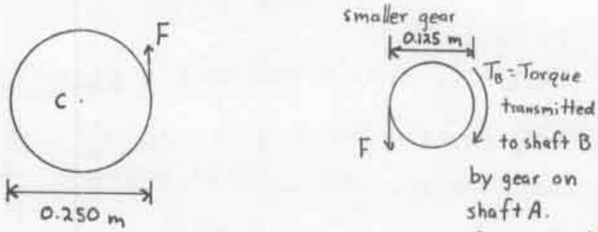


Figure C

(Continued)

11.15 Cont.

For shaft C, the torque transmitted by the smaller gear of shaft B is determined by (Fig. c),

$$T_c = F\left(\frac{0.250}{2}\right) \qquad (c)$$

where F is the force exerted on gear C by the smaller gear of B. For equilibrium of shaft B,

$$T_B = F\left(\frac{0.125}{2}\right) \qquad (d)$$

Equations (c) and (d) yield

$$\frac{T_c}{T_B} = 2 \qquad (e)$$

the ratio of the diameters of the gears in contact; that is $0.250/0.125 = 2$.

By Eqs. (b) and (e)

$$\frac{T_c}{T_A} = (4)(2) = 8$$

Hence, $\underline{T_c = 8T_A = 200 \text{ Nm}}$ (f)

b) By Eqs. (c) and (f), the force F exerted on the tooth of the smaller gear on shaft B is (Fig. c)

$$\underline{\underline{F = 1600 \text{ N}}}$$

11.16

Given: The linkage ABCD in Fig. a is in equilibrium

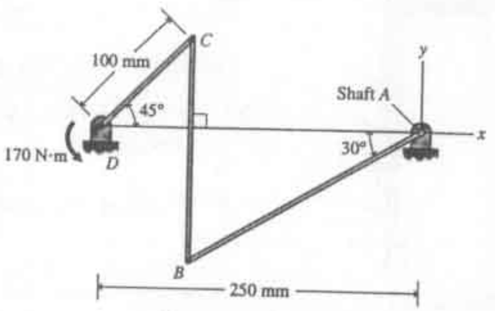


Figure a

Find: a) The torque transmitted to shaft A
b) The reactions R_x and R_y at A.

Solution:

Consider the free-body diagram of link CD (Fig. b).

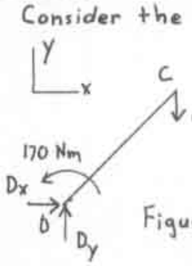


Figure b

$$\circlearrowleft + \Sigma M_D = -C(0.100 \cos 45°) + 170 = 0$$

$$C = 2404 \text{ N}$$

Next consider the free-body diagram of link AB (Fig. c)

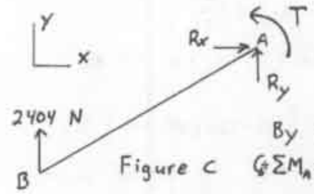


Figure c

By Fig. c,

$$\circlearrowleft + \Sigma M_A = -2404(0.250 - 0.100 \cos 45°) + T = 0$$

$$\underline{T = 431 \text{ N·m}}$$

b) Also by Fig. c,

$$\Sigma F_x = \underline{R_x = 0}$$

$$\Sigma F_y = 2404 + R_y = 0$$

$$\underline{\underline{R_y = -2404 \text{ N}}}$$

11.17

Given: The uniform ring in Fig. a is subjected to forces P. At the section a-a the moment due to these forces is $M_{a\text{-}a} = (\pi - 2)Pr/2\pi$.

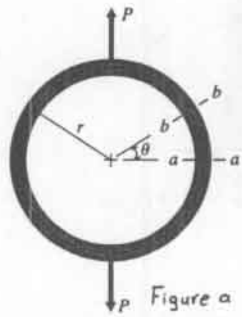


Figure a

Draw:
a) The free-body diagram of the upper half of the ring.

Write:
b) the expression for M_θ at the section b-b in terms of θ in the range $0 \le \theta \le \pi/2$.

Solution:

a) The free-body diagram of the upper half of the ring is shown in Fig. b, where symmetry has been used.

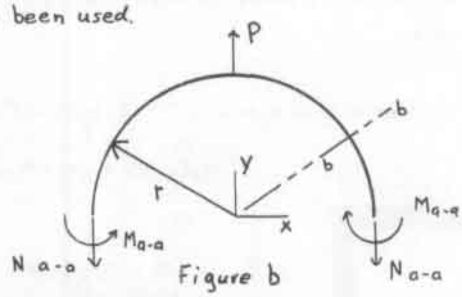


Figure b

b) Consider the free-body diagram of the section of the ring from $\theta = 0$ to θ (Fig. c)

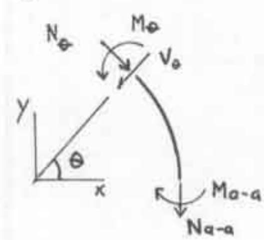


Figure c

(Continued)

11.17 Cont.

By Fig. b,

$$\Sigma F_y = 0 = -2N_{a\text{-}a} + P \ ; \quad N_{a\text{-}a} = \frac{P}{2}$$

Now summing Moments about the cut at θ (Fig. c) gives,

$$\circlearrowleft\Sigma M_\theta = -M_{a\text{-}a} + M_\theta + \frac{P}{2}(r - r\cos\theta) = 0 \quad (a)$$

Since $M_{a\text{-}a} = (\pi - 2)Pr/2\pi = \frac{Pr}{2} - \frac{Pr}{\pi}$, Eq. (a) yields,

$$M_\theta = \frac{Pr}{2} - \frac{Pr}{\pi} - \frac{Pr}{2}(1 - \cos\theta)$$

or

$$\underline{\underline{M_\theta = \frac{Pr}{2}\left(\cos\theta - \frac{2}{\pi}\right)}}$$

11.18

Given: The bent in Figure a

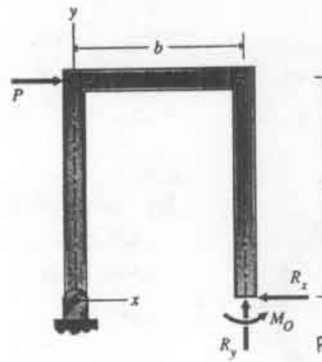


Figure a

Express:

a) M_o in terms of P, R_y, a, b

b) The shear V, bending moment M, and the axial force N in each vertical leg in terms of P, R_x, R_y, a, b, y.

c) The shear V, bending moment M, and axial force N in the top member in terms of P, R_x, R_y, a, b, and x.

Solution:

a) By the free-body diagram of the bent (Fig. b)

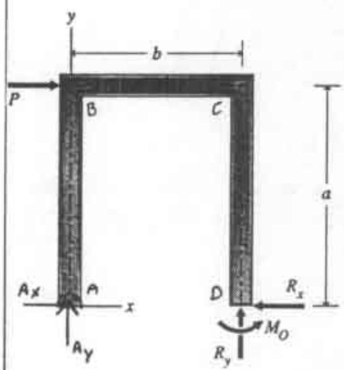


Figure b

$$\circlearrowleft\Sigma M_A = -Pa + (R_y)(b) + M_o = 0$$

or

$$\underline{\underline{M_o = P(a) - (R_y)b}}$$

b) By Fig. c, the free-body diagram of the right leg,

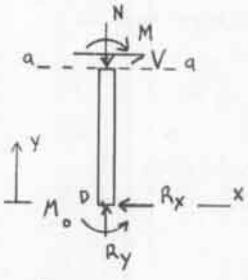


Figure C

$$\Sigma F_x = V - R_x = 0 \ ; \quad \underline{\underline{V = R_x}}$$

$$\Sigma F_y = R_y - N = 0 \ ; \quad \underline{\underline{N = R_y}}$$

$$\circlearrowleft\Sigma M_{a\text{-}a} = -M - (R_x)y + M_o = 0$$

or

$$\underline{\underline{M = P a - (R_y)b - (R_x)y}}$$

By the free-body diagram shown in Fig. c, for the left leg,

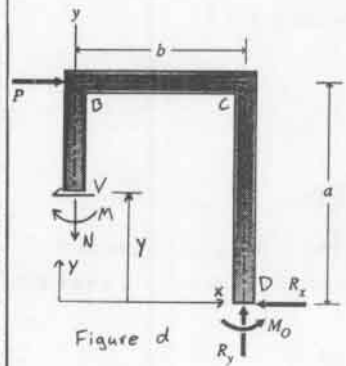


Figure d

$$\Sigma F_y = R_y - N = 0 ; \quad \underline{\underline{N = R_y}}$$

$$\Sigma F_x = P - R_x - V = 0 \ ; \quad \underline{\underline{V = P - R_x}}$$

$$\circlearrowleft\Sigma M_{cut} = -P(a - y) + (R_y)b - (R_x)y + M_o - M = 0$$

or

$$\underline{\underline{M = Py - (R_x)y = y(P - R_x)}}$$

c) By Fig. e, for the top member,

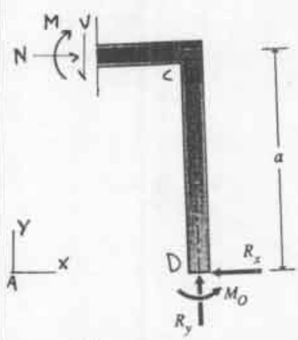


Figure e

$$\Sigma F_x = N - R_x = 0 ; \quad \underline{\underline{N = R_x}}$$

$$\Sigma F_y = -V + R_y = 0 ; \quad \underline{\underline{V = R_y}}$$

$$\circlearrowleft\Sigma M_{cut} = -M + M_o + (R_y)(b - x) - (R_x)a = 0$$

or

$$M = Pa - (R_y)b - (R_y)x + (R_y)b - (R_x)a$$

or

$$\underline{\underline{M = Pa - (R_y)x - (R_x)a}}$$

11.19

Given: Under the action of a torque applied to shaft C and force F, the mechanism in Fig. a is in equilibrium.

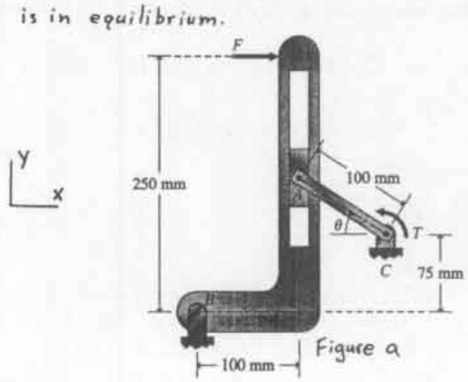


Figure a

(Continued)

11.19 Cont.

Express:

a) F in terms of T and θ. Neglect friction

b) The value of F for $T = 200$ N·m and $\theta = 30°$

Solution:

a) By the free-body diagram of crank AC,

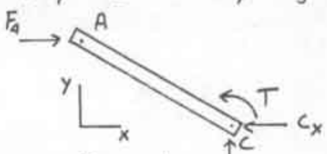


Figure b

$$\circlearrowleft \Sigma M_C = -F_A(0.100 \sin\theta) + T = 0$$

or $$F_A = \frac{T}{(0.100 \sin\theta)} = \frac{10T}{\sin\theta} \qquad (a)$$

By the free-body diagram of the L-shaped bar, (Fig. C), with Eq. (a),

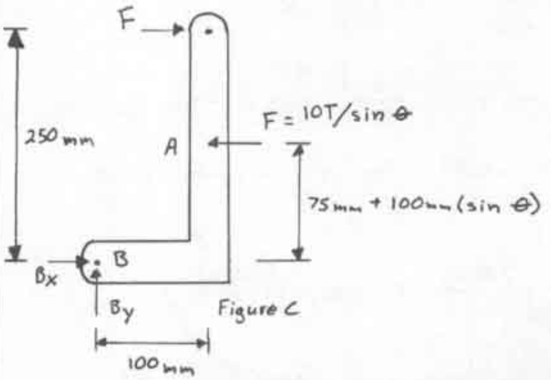


Figure C

$$\circlearrowleft \Sigma M_B = 0 = -F(0.25) + \frac{10T}{\sin\theta}(0.075 + 0.100 \sin\theta)$$

$$F(0.250) = \frac{10T}{\sin\theta}(0.075 + 0.100 \sin\theta)$$

$$\underline{\underline{F = \frac{3T}{\sin\theta} + 4T}} \qquad (b)$$

b) From Eq. (b), with $T = 200$ Nm and $\theta = 30°$

$$F = \frac{3(200)}{\sin 30°} + 4(200)$$

$$\underline{\underline{F = 2000 \text{ N}}}$$

11.20

Given: The torque in the drive shaft A is 50 N·m (Fig. a)

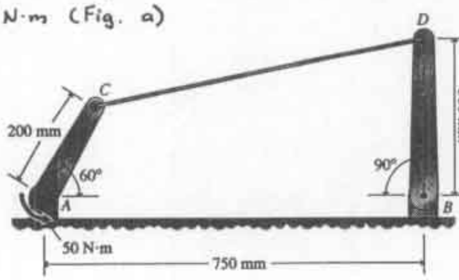


Figure a

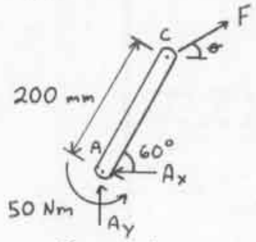
Find:

a) Construct free-body diagrams of the Members.

b) Calculate the torque transmitted to shaft B. Neglect friction.

Solution:

a) The free-body diagrams of the members are shown in Figs. b, C and d. Since CD is a two-force member, thus, the forces at C and D are equal in magnitude and opposite in sense (see Fig. c)



$$\tan\theta = \frac{126.80}{650} = 0.1951$$

$$\theta = 11.04°$$

Figure b

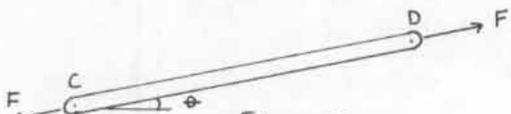


Figure c

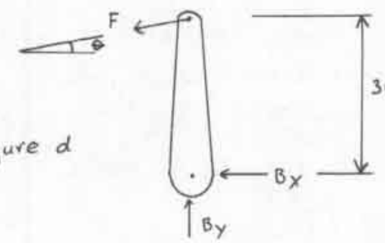


Figure d

b) By Fig. b,

$$\circlearrowleft \Sigma M_A = 50 + F \sin(11.04)(0.200 \cos 60°)$$
$$- F \cos(11.04)(0.200 \sin 60°) = 0$$

$$\therefore F = 331.4 \text{ N}$$

By Fig. d, the torque transmitted to axle B is

$$T_B = F(\cos 11.04°)(0.300)$$

or $$\underline{\underline{T_B = 97.58 \text{ N·m} \circlearrowright}}$$

11.21

Given: The applied magnetic field H is directed at 60° from the original axis of the magnet (Fig. a) The magnet of length L is suspended horizontally by a quartz fiber with resisting torque proportional to θ. When $\theta = 30°$, $H = H_1$, and when $\theta = 45°$, $H = H_2$. The force F is proportional to H.

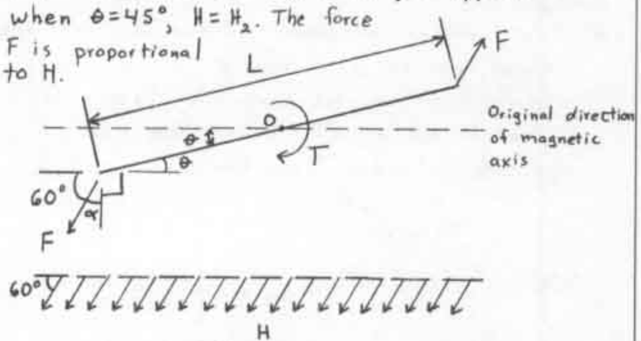


Figure a

Find: The ratio H_2/H_1.

Solution: By Fig. a, $60° + \alpha + 90° - \theta = 180°$; or

$\alpha = 30° + \theta$. Also by Fig. a,

$$\circlearrowleft \Sigma M_O = F(\cos\alpha)L - T = 0$$

or with $F = c_1 H$, $T = c_2\theta$, c_1, c_2 constants,

$$c_1 H \cos(30°+\theta)L = c_2\theta$$

$$\therefore H = \frac{c\theta}{\cos(30°+\theta)L} \qquad (a)$$

where $c = c_2/c_1 = \text{constant}$

For $\theta = 30° = \pi/6$ rad, Eq. (a) yields

$$H = H_1 = \frac{c(\pi/6)}{(\cos 60°)L} \qquad (b)$$

For $\theta = 45° = \pi/4$ rad, Eq. (a) yields

$$H = H_2 = \frac{c(\pi/4)}{(\cos 75°)L} \qquad (c),$$

Hence by Eqs. (b) and (c),

$$\frac{H_2}{H_1} = \frac{3}{2}\,\frac{\cos 60°}{\cos 75°} = 2.898$$

11.22

Given: The semicircular beam shown in Fig. a

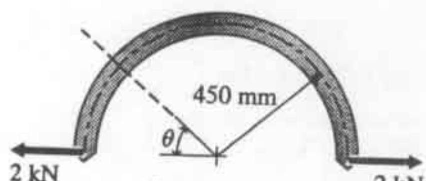


Figure a

Find: Formulas for the bending moment, the shear force, and the net tension in the semicircular beam at the section located at θ

Solution:

Figure b is the free-body diagram of the section of the beam subtended by angle θ.

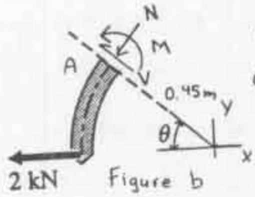


Figure b

By Fig. b,

$$\circlearrowleft \Sigma M_A = -2(0.45\sin\theta) + M = 0$$

$$M = 0.90\sin\theta \ [\text{kN}\cdot\text{m}]$$

$$\Sigma F_y = V\sin\theta - N\cos\theta = 0; \quad N = \frac{V\sin\theta}{\cos\theta} \qquad (a)$$

$$\Sigma F_x = -V\cos\theta - 2 - N\sin\theta = 0$$

$$-V\cos\theta - 2 - V\frac{\sin^2\theta}{\cos\theta} = 0$$

$$-V\left(\cos\theta + \frac{\sin^2\theta}{\cos\theta}\right) = 2 = -V\left(\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}\right)$$

$$V = -2\cos\theta \ [\text{kN}] \qquad (b)$$

By Eqs (a) and (b),

$$N = \frac{V\sin\theta}{\cos\theta} = \frac{-2\cos\theta\sin\theta}{\cos\theta}$$

$$N = -2\sin\theta \ [\text{kN}]$$

11.23

Given: The shaft-pulley system shown in Fig. a, where $P = 210\hat{\imath} - 75\hat{k}$ (N)

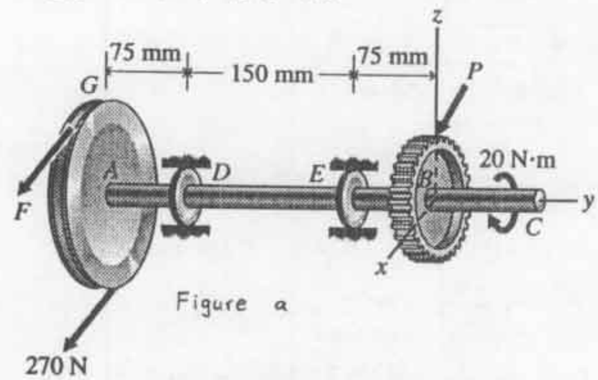


Figure a

Find:

a) The magnitude F of the tension in the belt at G and the reactions at D and E

b) The twisting moment in the shaft as a function of y.

(Continued)

11.23 Cont.

Solution:

a) By the free-body diagram of the system (Fig. b),

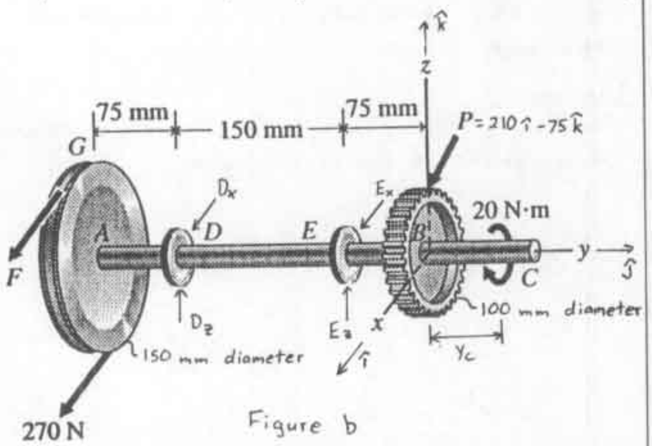


Figure b

$\xrightarrow{+} \Sigma M_y = F(0.075) - 270(0.075) + 210(0.05) - 20 = 0$

$\underline{F = 396.67 \text{ N}}$

$+\uparrow \Sigma M_z = F(0.300) + 270(0.300) + D_x(0.225) + E_x(0.075) = 0$ (a)

$\Sigma F_x = F + 270 + D_x + E_x + 210 = 0$

$D_x = -876.67 - E_x$ (b)

Substitution of Eq. (b) into Eq. (a) gives:

$396.67(0.300) + 270(0.300) + (-876.67 - E_x)(0.225) + 0.075 E_x = 0$

$\underline{E_x = 18.33 \text{ N}}$ (c)

By Eqs. (b) and (c),

$\underline{D_x = -876.67 - 18.33 = -895 \text{ N}}$

Also by Fig. b,

$\Sigma F_z = D_z + E_z - 75 = 0$; $D_z = 75 - E_z$ (d)

$\Sigma M_x = -D_z(0.225) - E_z(0.075) = 0$ (e)

The solution of Eqs. (d) and (e) is

$\underline{D_z = -37.5 \text{ N}}$; $\underline{E_z = 112.5 \text{ N}}$

b) Start from the left end of the shaft. Then,

$M_y = 0$

For $-0.300 \le y < 0$

$\xrightarrow{+} \Sigma M = F(0.075) - 270(0.075) + M_y = 0$

$M_y = -9.50$ N·m

For $0 \le y \le y_c$

$\xrightarrow{+} \Sigma M = F(0.075) - 270(0.075) + 210(0.050) + M_y = 0$

$M_y = -20$ N·m

For $y > y_c$

$\underline{M_y = 0}$

11.24

Given: The shaft O transmits a torque of 2 kip·in to arm OA. For equilibrium, the force F acts on piston B which is in a frictionless cylinder (Fig. a)

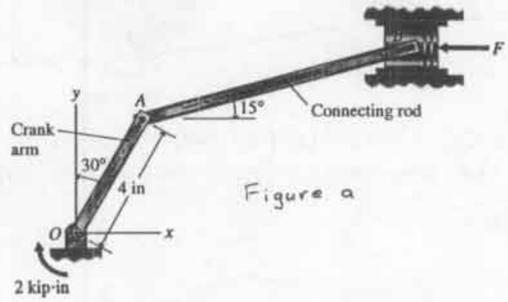


Figure a

Find:

a) The thrust force P in the connecting rod AB

b) The force F

c) The lateral force L that the cylinder exerts on B.

d) The support reactions R_x and R_y of the crank bearing O.

Solution:

a) By the free-body diagram of arm OA (Fig. b)

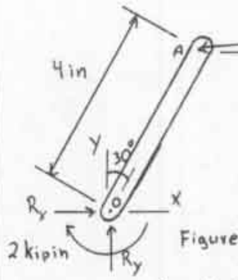
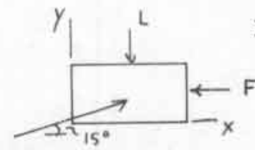


Figure b

$\circlearrowleft \Sigma M_o = -2 + (P\cos 15)(4\cos 30°) - (P\sin 15)(4\sin 30°) = 0$

$\underline{P = 0.7071 \text{ kip}}$

b) By the free-body diagram of piston B (Fig. c),



P = 0.7071 Kip Figure c

$\Sigma F_x = 0.7071 \cos 15° - F = 0$

$\underline{F = 0.683 \text{ kip}}$

c) Also by Fig. c

$\Sigma F_y = 0.7071 \sin 15° - L = 0$; $\underline{L = 0.183 \text{ kip}}$

d) By Fig. b,

$\Sigma F_x = R_x - P \cos 15° = 0$

or $\underline{R_x = 0.7071 \cos 15° = 0.683 \text{ kip}}$

$\Sigma F_y = R_y - P\sin 15° = 0$

or $\underline{R_y = 0.7071 \sin 15° = 0.183 \text{ kip}}$

11.25

Given: The torque of drive shaft A (Fig. a) is 90 N·m

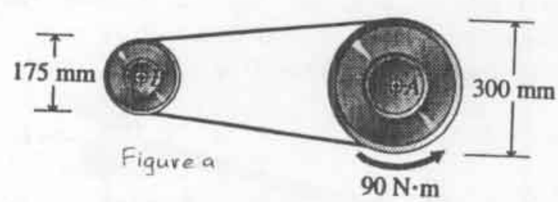


Figure a

Construct: appropriate free-body diagrams and determine the torque transmitted to shaft B.

Solution:

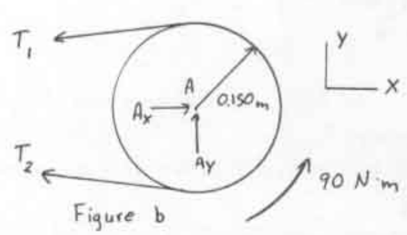


Figure b

By the free-body diagram of pulley A (Fig. b),

$\circlearrowleft \Sigma M_A = (T_1 - T_2)(0.15) + 90 = 0$; $T_1 - T_2 = -600$ N (a)

By the free-body diagram of pulley B (Fig. c),

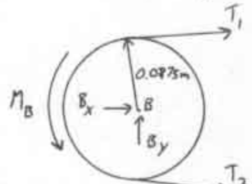


Figure c

$\circlearrowleft \Sigma M_B = (T_2 - T_1)(0.0875) + M_B = 0$

$$(T_1 - T_2) = 11.43\, M_B$$

By Eqs. (a) and (b),

$11.43\, M_B = -600$

or

$M_B = -52.5$ N·m

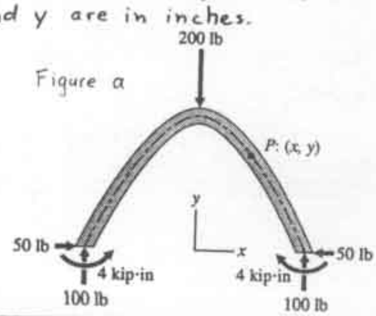
Therefore the torque transmitted is,

$$\underline{\underline{T = 52.5 \text{ N·m}}}$$

11.26

Given: The equation of the center line of the arch shown in Fig. a is $y = 120 - x^2/120$, x and y are in inches.

Figure a



Find:

Express the bending moment M, the shear force V, and the axial force N in terms of x for x>0. Neglect the weight of the arch.

Solution:

For y=0, x=120 in; then, by the free-body diagram of a segment of the arch (Fig. b),

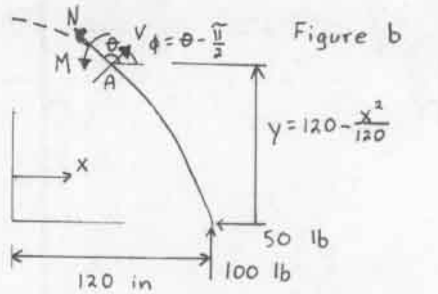


Figure b

$\circlearrowleft \Sigma M_A = 100(120 - x) - 50\left(120 - \frac{x^2}{120}\right) - 4000 + M = 0$

$$\therefore \underline{\underline{M = -2000 + 100x - \frac{5x^2}{12}}} \quad [\text{lb·in}]$$

The derivative of y with respect to x gives the slope of the arch at x; that is (see Fig. b),

$$\frac{dy}{dx} = \frac{d}{dx}\left(120 - \frac{x^2}{120}\right) = -\frac{x}{60} = \tan\theta$$

or by the triangle for θ (Fig. c)

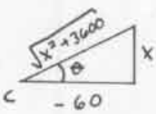


Figure c

$$\sin\theta = \frac{x}{\sqrt{x^2+3600}}, \quad \cos\theta = -\frac{60}{\sqrt{x^2+3600}} \qquad (a)$$

Now by Fig. b,

$\Sigma F_x = N\cos\theta + V\left(\cos\left(\theta - \frac{\pi}{2}\right)\right) - 50 = 0$

or $N\cos\theta + V\sin\theta = 50$ (b)

$\Sigma F_y = N\sin\theta + V\sin\left(\theta - \frac{\pi}{2}\right) + 100 = 0$

or $N\sin\theta - V\cos\theta = -100$ (c)

The solution of Eqs. (b) and (c) is

$N = 50\cos\theta - 100\sin\theta$

$V = 50\sin\theta + 100\cos\theta$ (d)

By Eqs. (a) and (d),

$$\underline{\underline{N = -\frac{(3000 + 100x)}{\sqrt{x^2 + 3600}}}} \quad [\text{lb}]$$

$$\underline{\underline{V = \frac{50x - 6000}{\sqrt{x^2 + 3600}}}} \quad [\text{lb}]$$

11.27

Given: The semicircular arch shown in Fig. a

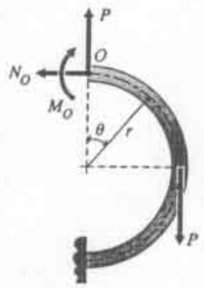


Figure a

Derive: formulas for the bending moment, the shear force, and the axial force at the section θ, for $\theta<\frac{\pi}{2}$ and for $\theta>\frac{\pi}{2}$, in terms of N_o, M_o, P, θ, and r.

Solution:

Consider first an element of the arch for $\theta<\frac{\pi}{2}$ (Fig. b).

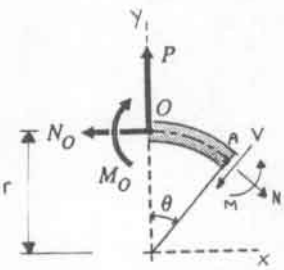


Figure b

By Fig. b,

$\circlearrowleft\Sigma M_A = M - Pr\sin\theta + N_o r(1-\cos\theta) - M_o = 0$

or

$\underline{\underline{M = M_o + Pr\sin\theta - Nr(1-\cos\theta);\ \theta<\frac{\pi}{2}}}$

$\Sigma F_x = -N_o - V\sin\theta + N\cos\theta = 0$

or

$N\cos\theta - V\sin\theta = N_o \qquad (a)$

$\Sigma F_y = P - V\cos\theta - N\sin\theta = 0$

or $N\sin\theta + V\cos\theta = P \qquad (b)$

The solution of Eqs. (a) and (b) is

$\underline{\underline{N = P\sin\theta + N_o\cos\theta;\ \theta<\frac{\pi}{2}}}$

$\underline{\underline{V = P\cos\theta - N_o\sin\theta;\ \theta<\frac{\pi}{2}}}$

Next, consider the element of the arch for $\theta>\pi/2$ (Fig. c). By Fig. C,

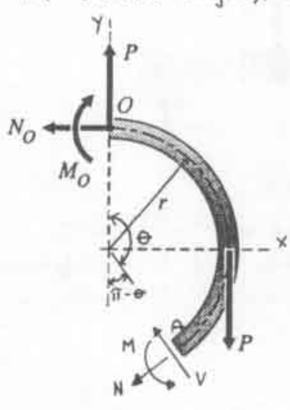


Figure C

$\circlearrowleft\Sigma M_A = M - P_r + N_o r[1+\cos(\pi-\theta)] - M_o = 0$

or

$\underline{\underline{M = M_o + P_r - N_o r(1-\cos\theta);\ \theta>\frac{\pi}{2}}}$

$\Sigma F_x = -N_o - V\sin(\pi-\theta) - N\cos(\pi-\theta) = 0$

or $N\cos\theta - V\sin = N_o \qquad (c)$

$\Sigma F_y = P + V\cos(\pi-\theta) - N\sin(\pi-\theta) - P = 0$

or $N\sin\theta + V\cos\theta = 0 \qquad (d)$

The solution of Eqs. (c) and (d) is

$\underline{\underline{N = N_o\cos\theta;\ \theta>\pi/2}}$

$\underline{\underline{V = -N_o\sin\theta;\ \theta>\pi/2}}$

11.28

Given: The semicircular arch shown in Fig. a rests on a frictionless surface.

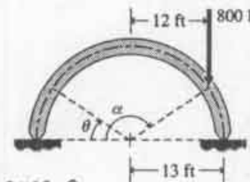


Figure a

Express: the bending moment, the shear force, and the axial force at section θ as functions of θ for $0<\theta<\alpha$ and for $\alpha<\theta<\pi$.

Solution:

The free-body diagram of the arch is shown in Fig. b.

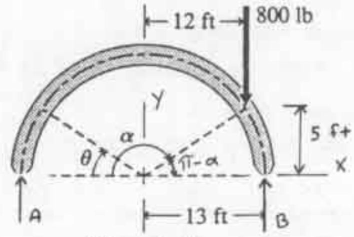


Figure b

By Fig. b,

$\Sigma F_y = A + B - 800 = 0 \quad \text{or} \quad A + B = 800 \qquad (a)$

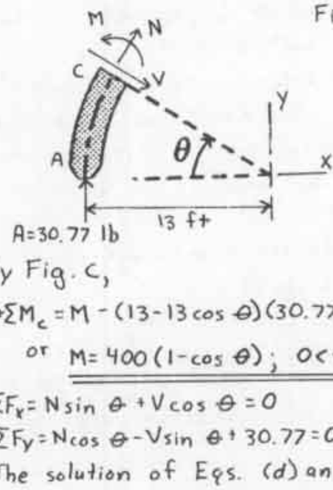
$\circlearrowleft\Sigma M_A = 26B - 25(800) = 0 \quad \text{or} \quad 26B = 20{,}000 \qquad (b)$

The solution of Eqs. (a) and (b) is

$A = 30.77\ \text{lb}$
$B = 769.23\ \text{lb} \qquad (c)$

The free-body diagram of the section of arch for $0<\theta<\alpha$ is shown in Fig. c.

Figure c



By Fig. C,

$\circlearrowleft\Sigma M_C = M - (13 - 13\cos\theta)(30.77) = 0$

or $\underline{\underline{M = 400(1-\cos\theta);\ 0<\theta<\alpha}}$ [lb·ft]

$\Sigma F_x = N\sin\theta + V\cos\theta = 0 \qquad (d)$

$\Sigma F_y = N\cos\theta - V\sin\theta + 30.77 = 0 \qquad (e)$

The solution of Eqs. (d) and (e) is

$\underline{\underline{N = -30.77\cos\theta;\ 0<\theta<\alpha}}$ [lb]

$\underline{\underline{V = 30.77\sin\theta;\ 0<\theta<\alpha}}$ [lb]

(Continued)

11.28 Cont

The free-body diagram of the section of the arch for $\alpha < \theta < \pi$ is shown in Fig. d.

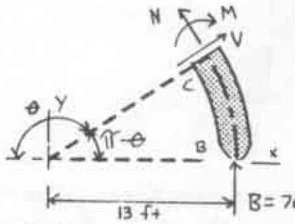


Figure d

By Fig. d,

$$\circlearrowleft \Sigma M_c = [13 - 13\cos(\pi - \theta)](769.23) - M = 0$$

$$\underline{\underline{M = 10{,}000(1 + \cos\theta)\ ; \quad \alpha < \theta < \pi}} \quad [\mathrm{lb \cdot ft}]$$

$$\Sigma F_x = V\cos(\pi - \theta) - N\sin(\pi - \theta) = 0$$

or $\quad N\sin\theta + V\cos\theta = 0 \qquad$ (f)

$$\Sigma F_y = N\cos(\pi - \theta) + V\sin(\pi - \theta) + 769.23 = 0$$

or $\quad N\cos\theta - V\sin\theta = 769.23 \qquad$ (g)

The solution of Eqs (f) and (g) is

$$\underline{\underline{N = 769.23\cos\theta\ ; \quad \alpha < \theta < \pi}} \quad [\mathrm{lb}]$$

$$\underline{\underline{V = -769.23\sin\theta\ ; \quad \alpha < \theta < \pi}} \quad [\mathrm{lb}]$$

Information Note: By Fig. b, $\tan(\pi - \alpha) = \frac{5}{12}$, or $\pi - \alpha = 22.62°$. Therefore, $\alpha = 180° - 22.62° = 157.38°$

11.29

Given: The spring at O produces a resisting torque $T = 500\,\theta$ N·m, where θ is in radians, that counteracts the moment about O due to force F (Fig. a)

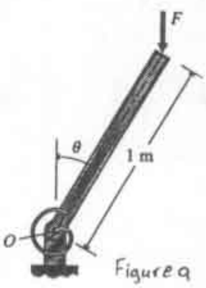


Figure a

Find:

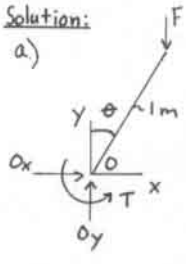
a) Derive a formula for the magnitude F in terms of θ, and Plot F as a function of θ for $0 < \theta < 180°$ and for $180° < \theta < 360°$.

b) Determine the magnitude of F when $\theta = 120°$ and compare to plot.

c) What does F do as θ approaches 180°? What happens as θ approaches 360°?

Solution:

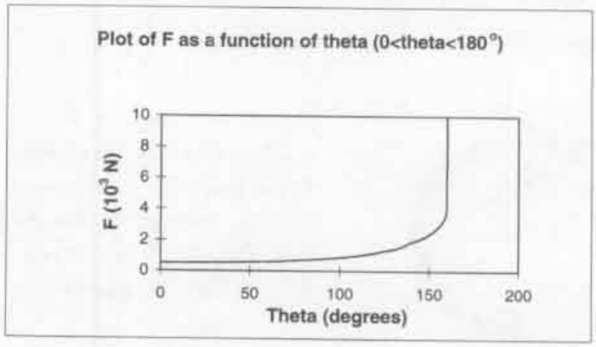
a)

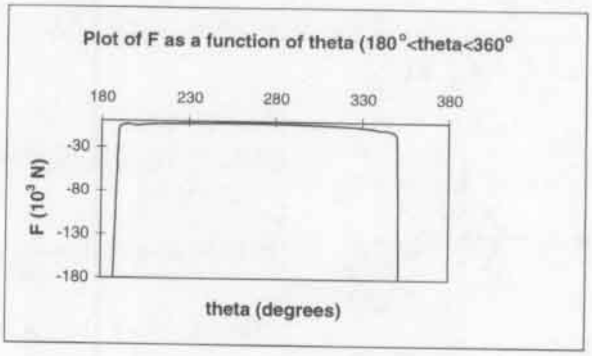


By the free-body diagram of the rod (Fig. b),

$$\circlearrowleft \Sigma M_o = -F(1\sin\theta) + T = 0$$

$$\underline{\underline{F = \frac{T}{\sin\theta} = \frac{500\,\theta}{\sin\theta}}}, \text{ for both } 0 < \theta < 180° \text{ and } 180° < \theta < 360° \qquad (a)$$





b) By Eq. a

$\underline{\underline{F = 1209\ \mathrm{N}}}$ This value agrees with the plot

c) As θ approaches 180°, F goes to ∞

As θ approaches 360°, F goes to $-\infty$

11.30

Given: A beam loaded as shown in Fig. a.

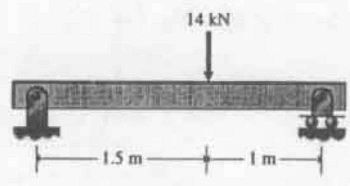


Figure a

Find: Determine the reactions of the supports and construct the shear and the bending-moment diagrams. Neglect the weight of the beam.

Solution:

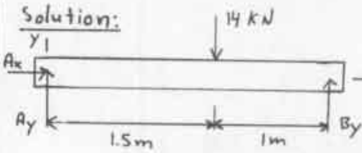


Figure b

By Fig. b, the free-body diagram of the beam,

$\Sigma F_x = \underline{\underline{A_x = 0}}$ $\qquad \circlearrowleft \Sigma M_A = -14(1.5) + B_y(2.5) = 0$

$\qquad \underline{\underline{B_y = 8.4\ \mathrm{kN}}}$

$\Sigma F_y = A_y - 14 + 8.4 = 0$

$\qquad \underline{\underline{A_y = 5.6\ \mathrm{kN}}}$

(Continued)

11.30 Cont.

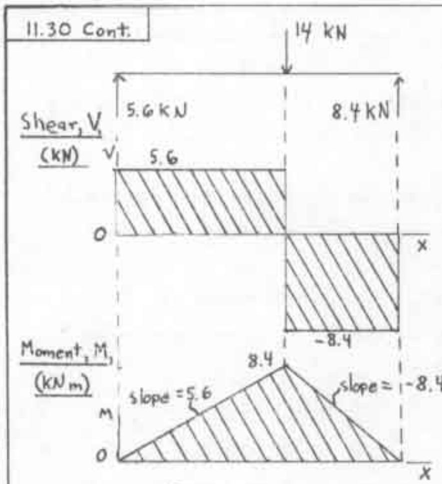


Figure c

Using the rules given in section 11.4, the shear and moment diagrams are constructed as shown in Fig. c

Also the formulas for shear and bending moment are as follows,

For $0 < x < 1.5$ m
$V = 5.6$ (kN)
$M = 5.6x$ (kN·m)

For $1.5 < x < 2.5$ m
$V = -8.4$ (kN)
$M = 21 - 8.4x$ (kN m)

11.31

Given: The results for the shaft of Problem 11.23 (see Fig. a, below) are as follows:

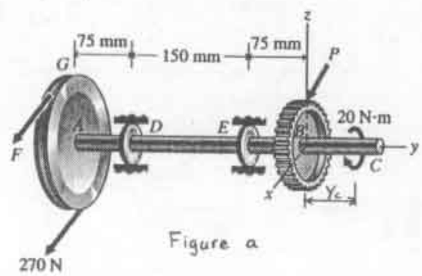


Figure a

For $y_c < y < -0.300$; $M = 0$
for $-0.300 = y < 0$; $M = -9.5$ Nm
for $0 = y \le y_c$; $M = -20$ Nm

Find: Construct the twisting-moment diagram for the shaft shown in Fig. b. (→→ = positive torque)

Solution:

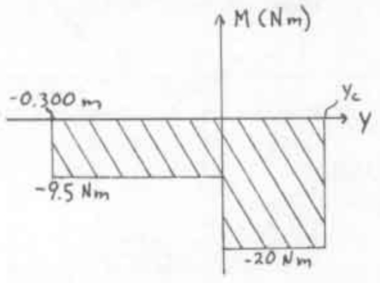


Figure b

11.32

Given: A beam loaded as shown in Fig. a. Neglect the weight of the beam.

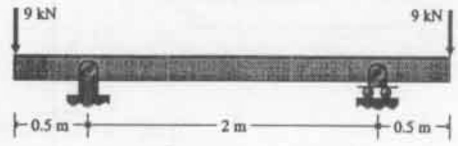


Figure a

Determine: The support reactions and construct the shear and the bending-moment diagrams.

Solution:

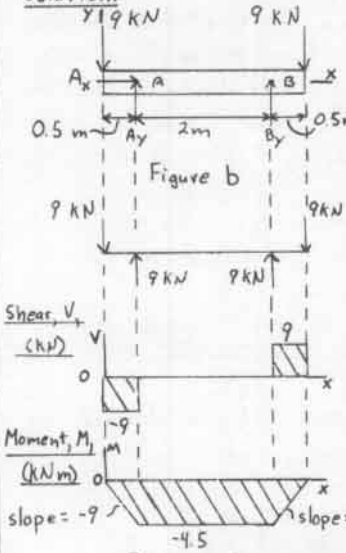


Figure c

By Fig. b, the free-body diagram of the beam,

$\Sigma F_x = A_x = 0$

$\circlearrowleft \Sigma M_A = 9(.5) + B_y(2) - 9(2.5) = 0$

$B_y = 9$ kN

$\Sigma F_y = -9 + A_y + B_y - 9 = 0$

$A_y = 9$ kN

Also note by symmetry, $A_y = B_y = 9$ kN.

Using the rules given in Section 11.4, the shear and moment diagrams are constructed as shown in Fig. c

The formulas for the shear and bending-moment are

For $0 < x < 0.5$ m,
$V = -9$ (kN)
$M = -9x$ (kN·m)

For $0.5 < x < 2.5$ m,
$V = 0$
$M = -9x + 9(x - 0.5)$
or $M = -4.5$ (kN m)

For $2.5 < x < 3$ m,
$V = 9$ (kN)
$M = -9x + 9(x - 0.5) + 9(x - 2.5)$
or $M = 9x - 27$ (kN·m)

11.33

Given: The cantilever beam loaded as shown in Fig. a. Neglect the weight of the beam.

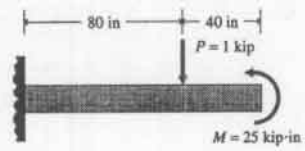


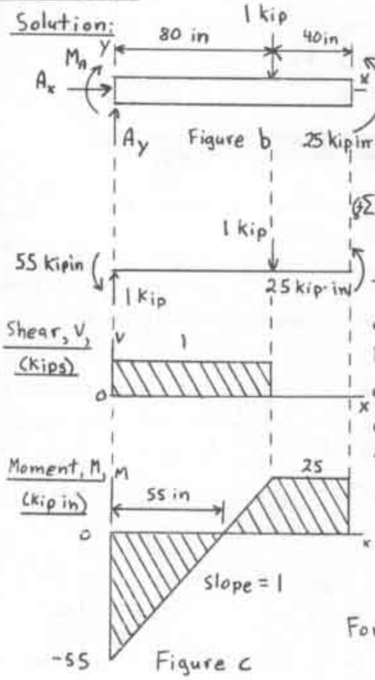
Figure a

Find: Determine the support reactions and construct the shear and the bending-moment diagrams.

(Continued)

11.33 Cont.

Solution:



By Fig. b,

$\underline{\Sigma F_x = A_x = 0}$

$\Sigma F_y = -1 + A_y = 0$

$\underline{A_y = 1 \text{ kip}}$

$\circlearrowleft \Sigma M_A = -1(80) + 25 - M_A = 0$

$\underline{M_A = -55 \text{ kip in}}$

The shear and moment diagrams are shown in Figure c. They may be constructed by the rules of section 11.4 or with the formulas for shear and moment.

For $0 < x < 80$ in,

$V = 1$ (kip)

$M = -55 + x$ (kip·in)

For $80 < x < 120$ in

$V = 0$

$M = 25$ (kip·in)

11.34

Given: The beam loaded as shown in Fig. a. Neglect the weight of the beam

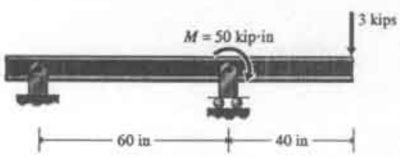
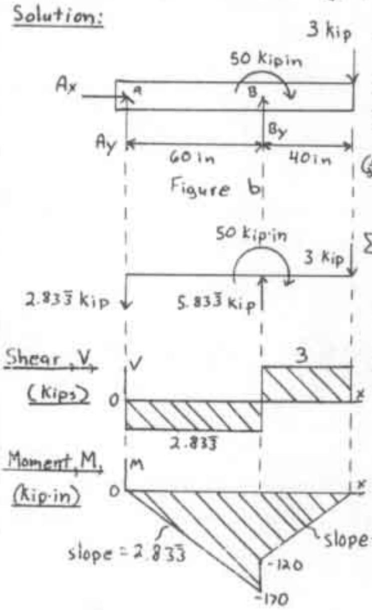


Figure a

Determine: The support reactions and construct the shear and the bending-moment diagrams.

Solution:



By the free-body diagram of the beam (Fig. b),

$\Sigma F_x = \underline{A_x = 0}$

$\circlearrowleft \Sigma M_A = B_y(60) - 50 - 3(100) = 0$

$\underline{B_y = 5.83\bar{3} \text{ kip}}$

$\Sigma F_y = A_y + B_y - 3 = 0$

$\underline{A_y = -2.83\bar{3} \text{ kip}}$

The shear and moment diagrams are shown in Figure c. They may be constructed by the rules of section 11.4 or with the formulas for shear and moment.

For $0 < x < 60$ in,

$V = -2.83\bar{3}$ (kip)

$M = -2.83\bar{3}x$ (kip·in)

For $60 < x < 100$ in,

$V = 3$ (kips)

$M = 3x - 300$ (kip·in)

11.35

Given: The following beams subjected to loads as shown (see also Figs. P11.1a - P11.1g and the solutions in P11.7)

The support reactions were found in P11.1

Construct: The shear and the bending-moment diagrams

Solution:

a)

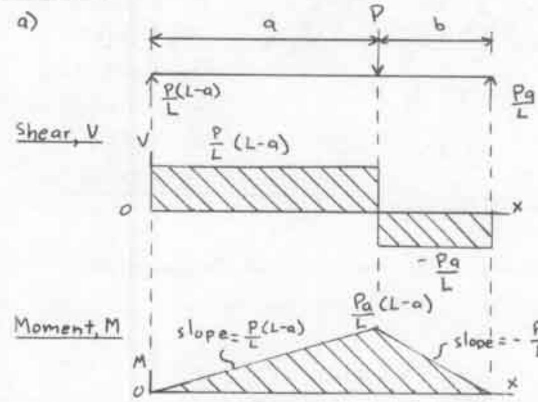


Figure a

b)

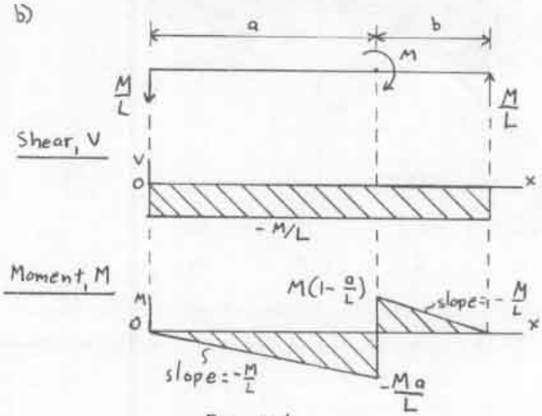


Figure b

c)

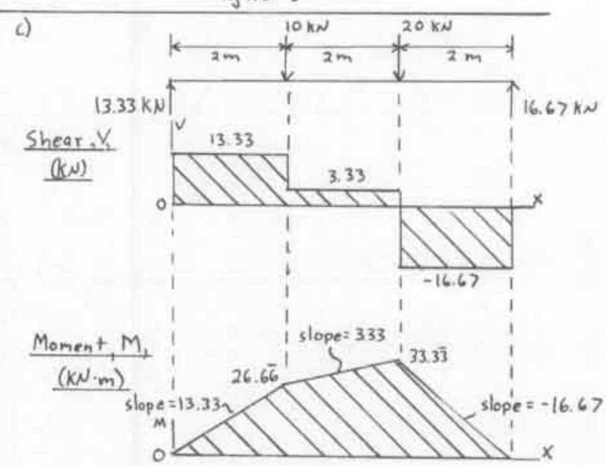


Figure c

(continued)

11.35 Cont.

d)

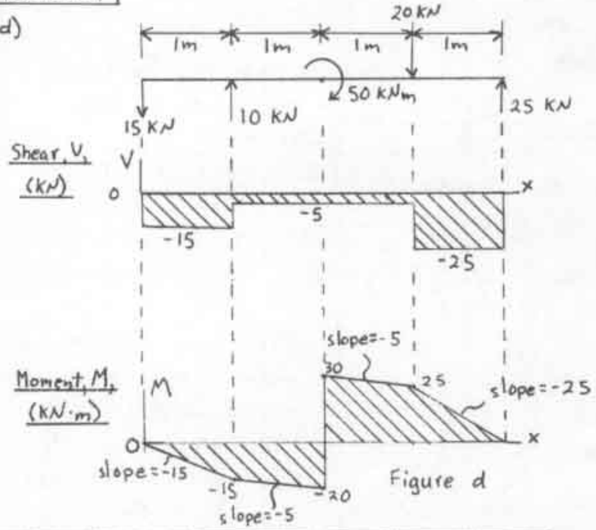


Figure d

e)

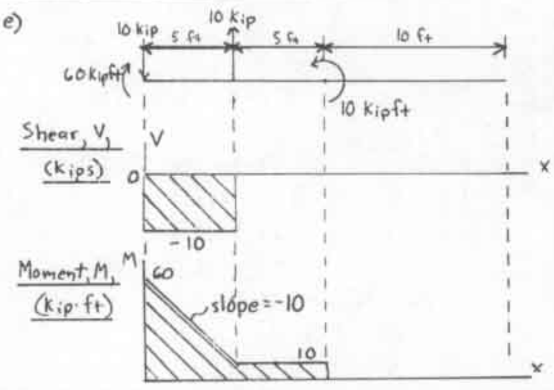


Figure e

f)

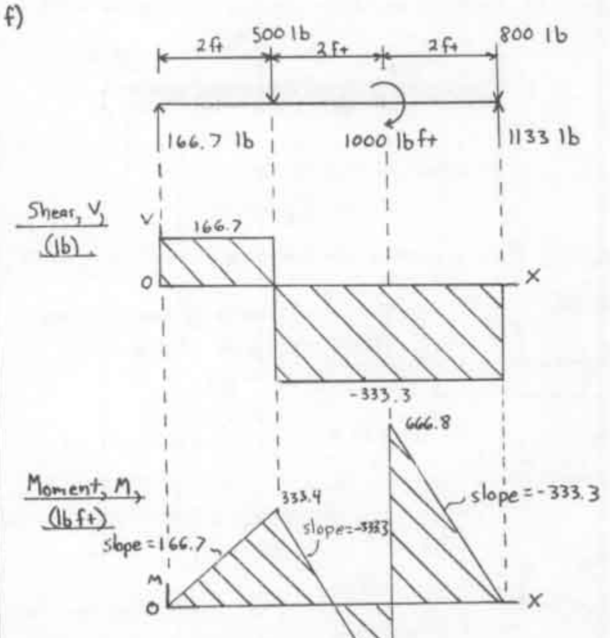


Figure f

g)

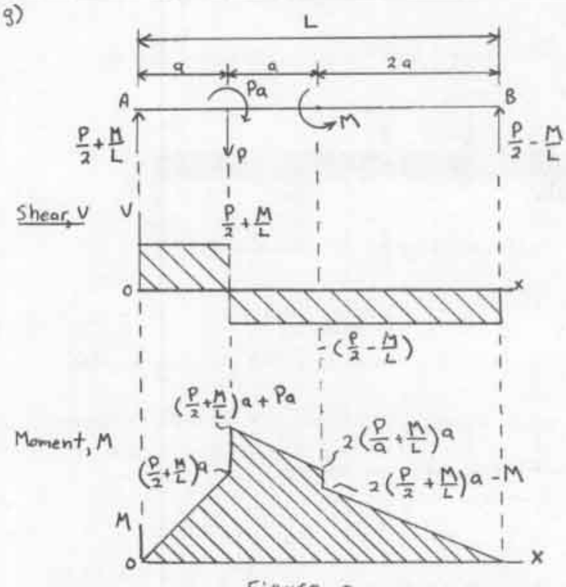


Figure g

11.36

Given: The beam loaded as shown in Fig. a. Neglect the weight of the beam.

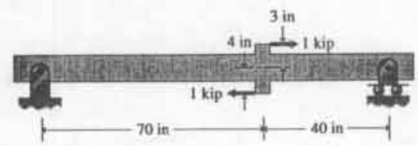


Figure a

Determine: The support reactions and construct the shear and the bending-moment diagrams.

Solution:

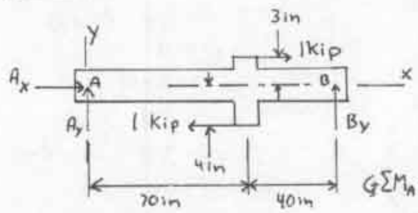


Figure b

By Fig. b,

$\Sigma F_x = 1 - 1 + A_x = 0$

$\underline{A_x = 0}$

$\circlearrowleft \Sigma M_A = -1000(3) - 1000(4) + B_y(110) = 0$

$\underline{B_y = 63.64 \text{ lb}}$

$\Sigma F_y = A_y + B_y = 0$

$\underline{A_y = -63.64 \text{ lb}}$

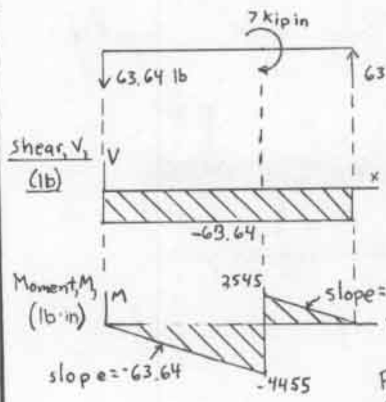


Figure c

The shear and bending-moment diagrams are shown in Fig. c.

The formulas for the shear and moment follow:

For $0 < x < 70$ in

$V = -63.64$ (lb)

$M = -63.64x$ (lb·in)

For $70 < x < 110$ in

$V = -63.64$ (lb); $M = 7000 - 63.64x$ (lb·in)

11.37

Given: The beam loaded as shown in Fig. a.

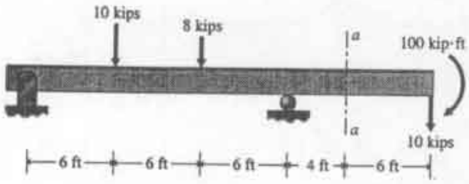


Figure a

Find: Construct the shear and the bending-moment diagrams.

Solution: Figure b is the free-body diagram of the beam.

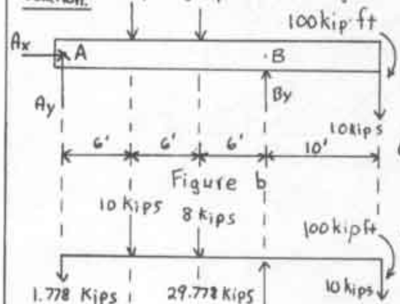


Figure b

By Fig. b,

$\Sigma F_x = A_x = 0$

$\circlearrowleft \Sigma M_A = -10(6) - 8(12) + B_y(18) - 10(28) - 100 = 0$

$B_y = 29.778$ Kips

$\Sigma F_y = A_y - 10 - 8 - 10 + B_y = 0$

$A_y = -1.778$ Kips

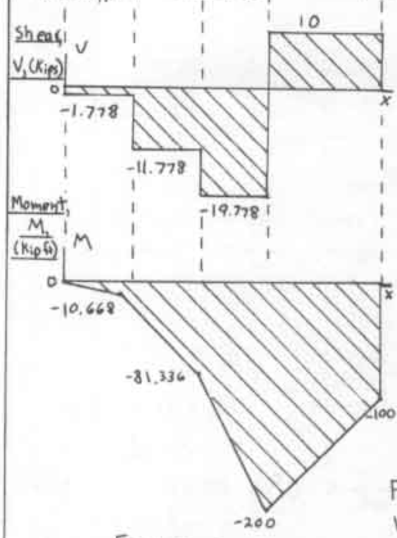


Figure c

The shear and bending-moment diagrams are shown in Fig. c. The formulas for the shear and the moment are:

For $0 < x < 6$ ft,

$V = -1.778$ (kips)

$M = -1.778x$ (kip·ft)

For $6 < x < 12$ ft,

$V = -11.778$ (kips)

$M = 60 - 11.778x$ (kip·ft)

For $12 < x < 18$ ft,

$V = -19.778$ (kips)

$M = 156 - 19.778x$ (kip·ft)

For $18 < x < 28$ ft,

$V = 10$ (kips)

$M = 10x - 380$ (kip·ft)

11.38

Given: The cantilever beam loaded as shown Fig. a (see also Fig. P11.3)

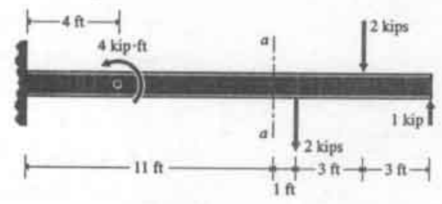


Figure a

Find: Construct the shear and the bending-moment diagrams.

Solution:

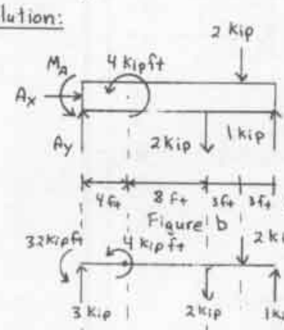


Figure b

By Fig. b, the free-body diagram of the beam

$\Sigma F_x = A_x = 0$

$\Sigma F_y = A_y - 2 - 2 + 1 = 0$

$A_y = 3$ Kips

$\circlearrowleft \Sigma M_A = M_A + 4 - 2(12) - 2(15) + 1(18) = 0$

$M_A = 32$ kip·ft

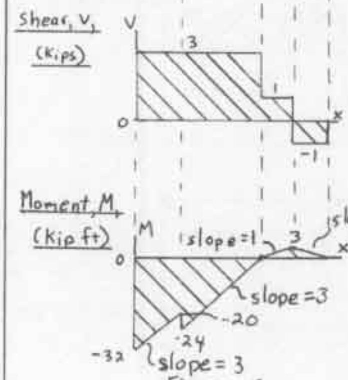


Figure c

The shear and bending-moment diagrams are shown in Fig. c. The formulas for the shear and moment are:

For $0 < x < 4$ ft,

$V = 3$ (kips)

$M = 3x - 32$ (kip·ft)

For $4 < x < 12$ ft,

$V = 3$ (kips)

$M = 3x - 36$ (kip·ft)

For $12 < x < 15$ ft,

$V = 1$ (kip)

$M = x - 12$ (kip·ft)

For $15 < x < 18$ ft,

$V = -1$ (kip)

$M = 18 - x$ (kip·ft)

11.39

Given: The cantilever beam loaded as shown in Fig. a.

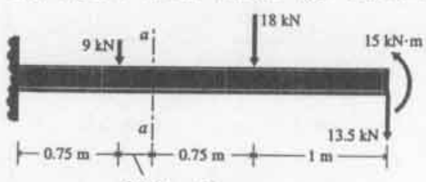


Figure a

Construct: The shear and the bending-moment diagrams

Solution:

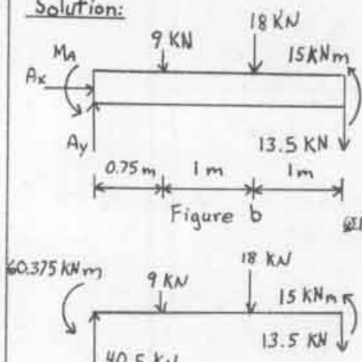


Figure b

Figure b is the free-body diagram of the beam.

By Fig. b,

$\Sigma F_x = A_x = 0$

$\Sigma F_y = A_y - 9 - 18 - 13.5 = 0$

$A_y = 40.5$ kN

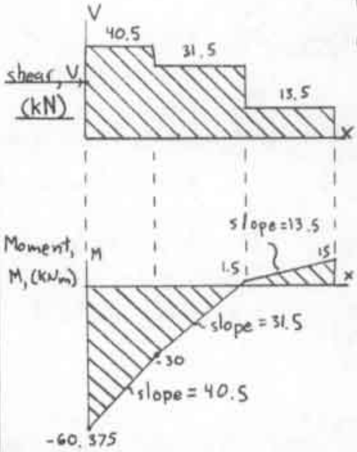
$\circlearrowleft \Sigma M_A = 0 = M_A + 15 - 9(0.75) - 18(1.75) - 13.5(2.75)$

$M_A = 60.375$ kN m

The shear and bending-moment diagrams are shown in Fig. C.

(Continued)

11.39 Cont.



The shear and bending-moment formulas are:

For $0 < x < 0.75$ m,

$V = 40.5$ (kN)

$M = 40.5x - 60.375$ (kN·m)

For $0.75 < x < 1.75$ m,

$V = 31.5$ (kN)

$M = 31.5x - 53.625$ (kN·m)

For $1.75 < x < 2.75$ m,

$V = 13.5$ (kN)

$M = 13.5x - 22.125$ (kN·m)

11.40

Given: The U-shaped beam shown in Fig. a.

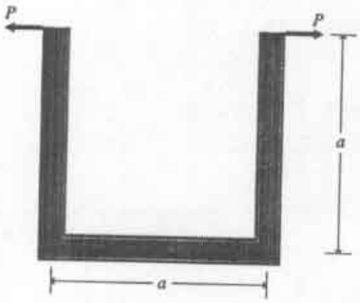


Figure a

Neglect the weight of the beam and construct the shear and bending-moment diagrams for the beam.

Solution:

By the sign convention of Fig. 11.6 of the text, starting at the top of the left leg, the positive shears and moments in the vertical and horizontal segments of the beam are shown in Fig. b at sections a-a, b-b, and c-c.

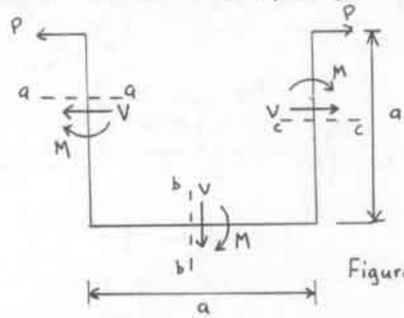


Figure b

Then, by inspection of Fig. b and the two rules given in section 11.4, we may construct the shear and bending-moment diagrams for the beam.

The shear in the beam is illustrated in Fig. c, and the moment in Fig. d, with the sign convention of Fig. 11.6.

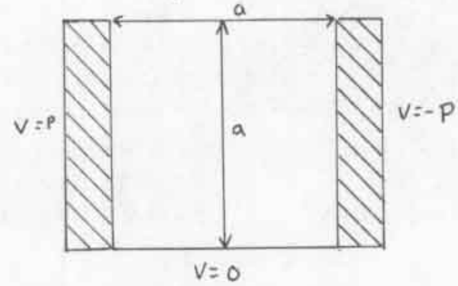


Figure c

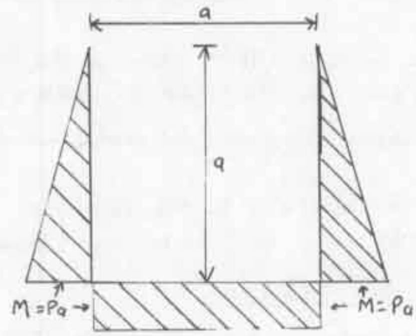


Figure d

11.41

Given: The beam loaded as shown in Fig. a

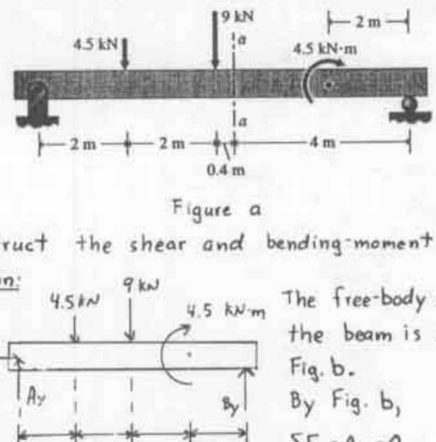


Figure a

Construct the shear and bending-moment diagrams

Solution:

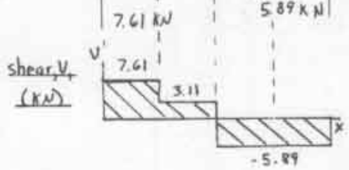


Figure b

The free-body diagram of the beam is shown in Fig. b.

By Fig. b,

$\Sigma F_x = A_x = 0$

$\circlearrowleft \Sigma M_A = -4.5(2) - 9(4) - 4.5 + B_y(8.4) = 0$

$B_y = 5.89$ kN

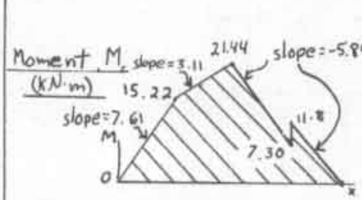
$\Sigma F_y = A_y - 4.5 - 9 + B_y = 0$

$A_y = 7.61$ kN

The shear and bending moments are shown in Fig. b.

(Continued)

11.41 Cont.



The formulas for the shear and the moment are:

For $0 < x < 2$ m,
$V = 7.61$ (kN)
$M = 7.61x$ (kN·m)

For $2 < x < 4$ m,
$V = 3.11$ (kN)
$M = 3.11x + 9$ (kN·m)

For $4 < x < 6.4$ m,
$V = -5.89$ (kN)
$M = -5.89x + 45$ (kN·m)

For $6.4 < x < 8.4$ m,
$V = -5.89$ (kN)
$M = -5.89x + 49.5$ (kN·m)

11.42

Given: The beams and loads shown in the following figures (see also the figure of Problem 11.6)

Find: Construct the shear and bending-moment diagrams

Solution:

The support reactions by the solution of Problem 11.6 are: $A_y = 4.5$ kN, $B_y = 5.375$ kN, $C_y = 9$ kN, and $D_y = 3.625$ kN

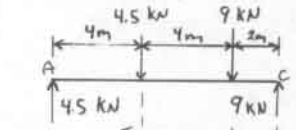


Figure a

For the beam AC (Fig. a), the shear and moment diagrams are shown in Fig. b.

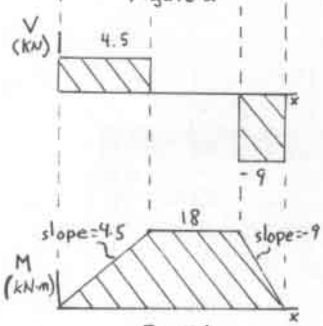


Figure b

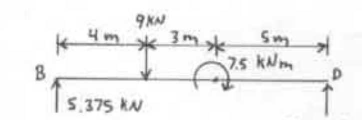


Figure c

For beam BD (Fig. c), the shear and moment diagrams are shown in Fig. d.

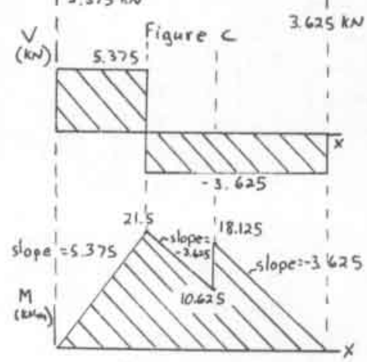


Figure d

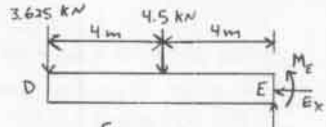


Figure e

By Fig. e,

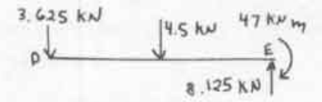
$\Sigma F_y = -4.5 - 3.625 + E_y = 0$

$E_y = 8.125$ kN

$\Sigma F_x: E_x = 0$

$\circlearrowleft \Sigma M_E = 3.625(8) + 4.5(4) + M_E = 0$

$M_E = -47$ kN m



The shear and moment diagrams for beam DE are shown in Fig. F

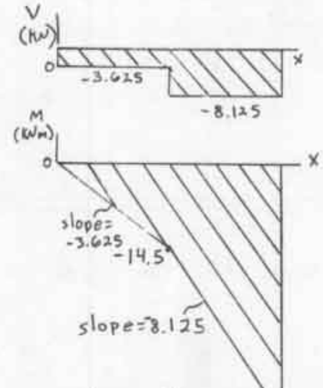


Figure f

11.43

Given: The cantilever beam loaded as shown in Fig. a

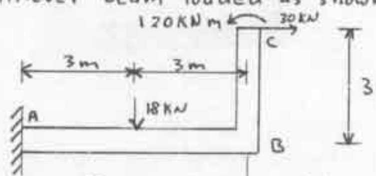


Figure a

Find:

a) Derive formulas for the shear for and the bending-moment in the beam.

b) Construct the shear and bending-moment diagrams

c) Construct the axial force diagram.

Solution:

a) By the free-body diagram of the beam (Fig. b),

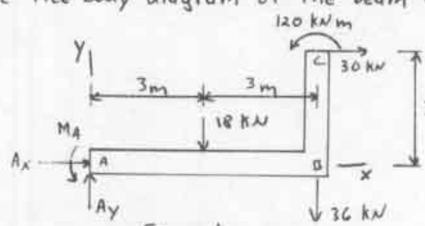


Figure b

$\Sigma F_x = A_x + 30 = 0$; $A_x = -30$ kN

$\Sigma F_y = A_y - 18 - 36 = 0$; $A_y = 54$ kN

$\circlearrowleft \Sigma M_A = -18(3) - 36(6) - 30(3) + 120 + M_A = 0$

$M_A = 240$ kN·m

(Continued)

11.43 Cont.

The free-body diagram of the beam element for $0 \le x \le 3$ m is shown in Fig. C.

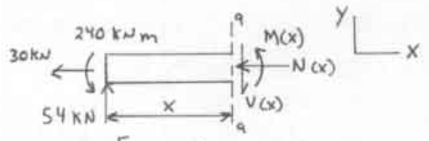


Figure c

By Fig. c,

$\Sigma F_x = -30 - N(x) = 0$; $N(x) = -30$ kN

$\Sigma F_y = 54 - V(x) = 0$; $\underline{\underline{V(x) = 54 \text{ kN}}}$

$\circlearrowleft \Sigma M_{a-a} = -54x + M(x) + 240 = 0$

$\underline{\underline{M(x) = 54x - 240}}$ (kN·m)

The free-body diagram of the beam element for $3 \le x < 6$ m is shown in Fig. d.

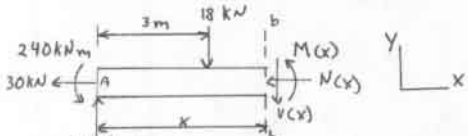


Figure d

By Fig. d,

$\Sigma F_x = -30 - N(x) = 0$; $N(x) = -30$ kN

$\Sigma F_y = 54 - 18 - V(x) = 0$; $\underline{\underline{V(x) = 36 \text{ kN}}}$

$\circlearrowleft \Sigma M_{b-b} = -54x + 18(x-3) + 240 + M(x) = 0$

$\underline{\underline{M(x) = 36x - 186}}$ (kN·m)

The free-body diagram for the vertical leg (Fig. b) is shown in Fig. e, (for $0 \le y \le 3$ m)

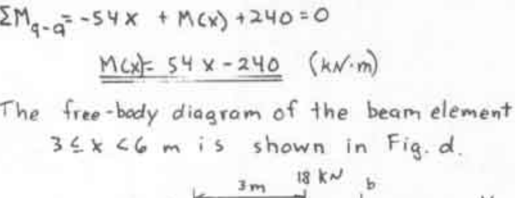


Figure e

By Fig. e,

$\Sigma F_x = V(y) - 30 = 0$; $\underline{\underline{V(y) = 30 \text{ kN}}}$

$\Sigma F_y = 54 - 18 - 36 - N(y) = 0$; $N(y) = 0$

$\circlearrowleft \Sigma M_{c-c} = 18(3) - 54(6) + 240 - 30y + M(y) = 0$

$\underline{\underline{M(y) = 30y + 30 \quad (\text{kN.m})}}$

b) The shear and bending-moment diagrams for the horizontal and vertical legs of the beam are shown in Fig. f and g, respectively using the sign convention of Fig. 11.6

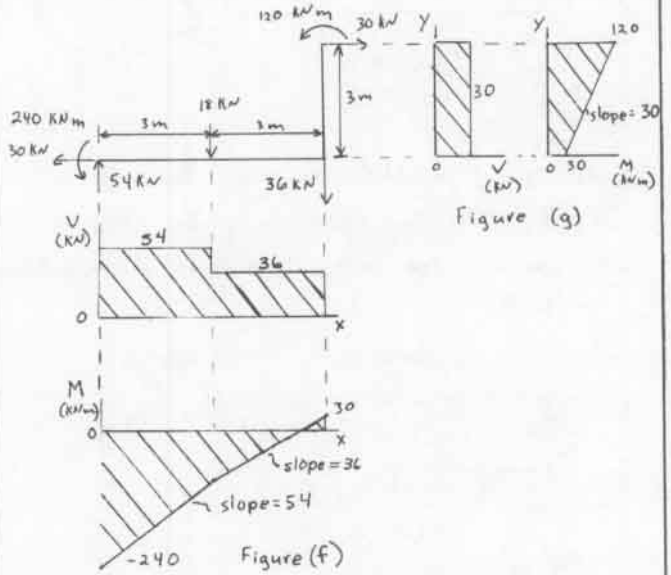


Figure (g)

Figure (f)

c) The axial force diagrams are shown in Figs. h and i

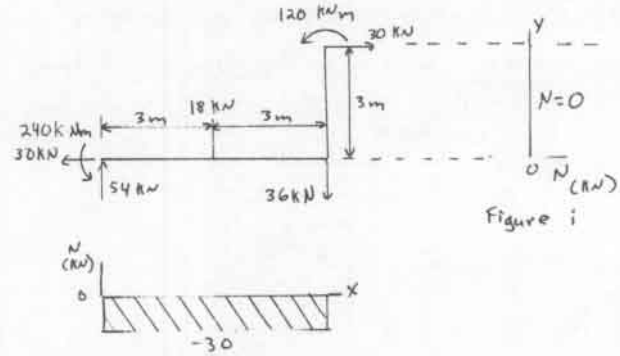


Figure i

Figure h

11.44

Given: The beam system loaded as shown in Fig. a

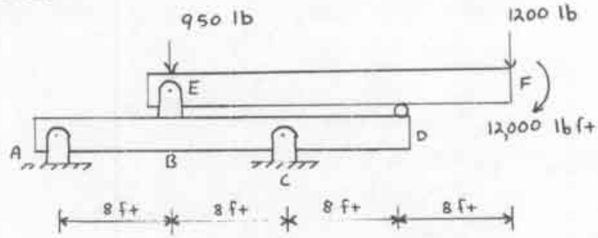


Figure a

Draw: The shear and bending-moment diagrams for beam ABCD.

(Continued)

11.44 Cont.

Solution: Consider first the free-body diagram of beam EF (Fig. b)

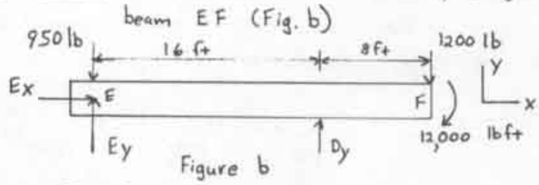


Figure b

By Fig. b,

$\Sigma F_x = E_x = 0$

$\circlearrowleft \Sigma M_E = D_y(16) - 1200(24) - 12{,}000 = 0$; $D_y = 2550$ lb

$\Sigma F_y = E_y - 950 + D_y - 1200 = 0$; $E_y = -400$ lb

Next, consider the free-body diagram of beam ABCD (Fig. c).

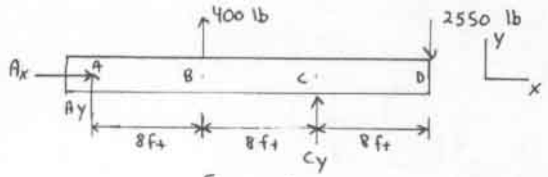


Figure c

By Fig. c,

$\Sigma F_x = A_x = 0$

$\circlearrowleft \Sigma M_A = 400(8) + C_y(16) - 2550(24) = 0$; $C_y = 3625$ lb

$\Sigma F_y = A_y + 400 + C_y - 2550 = 0$; $A_y = -1475$ lb

The shear and bending-moment diagrams of beam ABCD are shown in Fig. d,

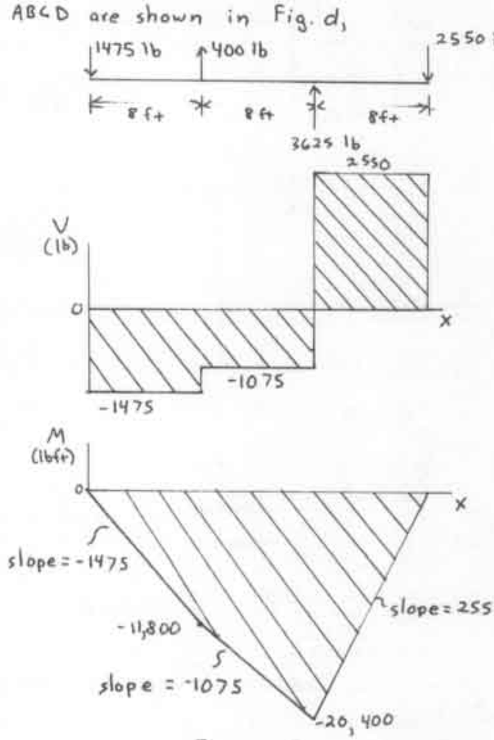


Figure d

The shear and bending-moment formulas are

For $0 < x < 8$ ft,
$V = -1475$ (lb)
$M = -1475x$ (lb·ft)

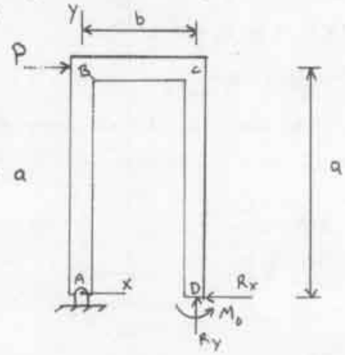
For $8 < x < 16$ ft,
$V = -1075$ (lb)
$M = -1075x - 3200$ (lb·ft)

For $16 < x < 24$ ft,
$V = 2550$ (lb)
$M = 2550x - 61200$ (lb·ft)

11.45

Given: For the bent of Problem 11.18 (see Fig. a) let $P = 1$ kip, $R_y = 0.5$ kip, $R_x = 0.6$ kip, and $a = 2b$

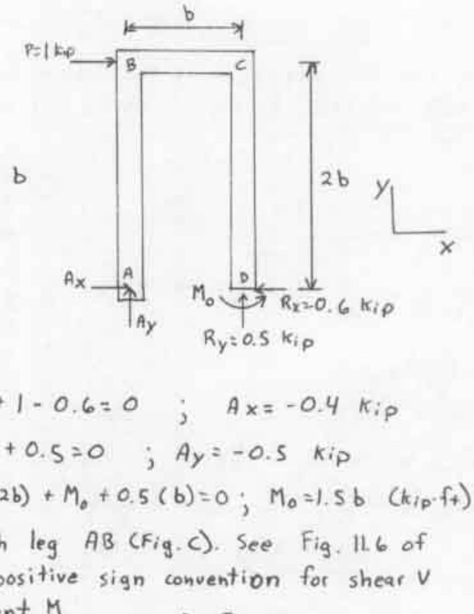
Figure a



Construct: The shear and bending-moment diagrams for each vertical leg and for the top member

Solution: The free-body diagram of the bent is shown in Fig. b.

Figure b



By Fig. b,

$\Sigma F_x = A_x + 1 - 0.6 = 0$; $A_x = -0.4$ kip

$\Sigma F_y = A_y + 0.5 = 0$; $A_y = -0.5$ kip

$\circlearrowleft \Sigma M_A = -1(2b) + M_o + 0.5(b) = 0$; $M_o = 1.5b$ (kip·ft)

Start with leg AB (Fig. C). See Fig. 11.6 of text for positive sign convention for shear V and moment M.

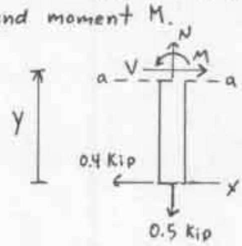


Figure C

By Fig. C,

$\circlearrowleft \Sigma M_{a\text{-}a} = M - 0.4y = 0$

or $M = 0.4y$; $0 \le y \le 2b$ (a)

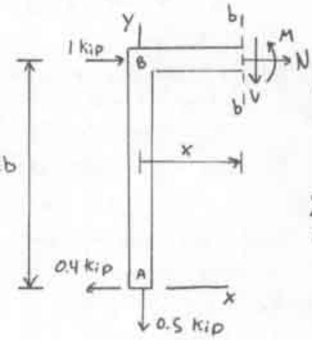
$\Sigma F_x = V - 0.4 = 0$

or $V = 0.4$ kip ; $0 \le y \le 2b$ (b)

(Continued)

11.45 Cont.

For the top member BC, by Fig. d,



$\circlearrowleft \sum M_{b-b} = M - 0.4(2b) + 0.5x = 0$

or $M = 0.8b - 0.5x$ (kip ft); $\quad$ (c)

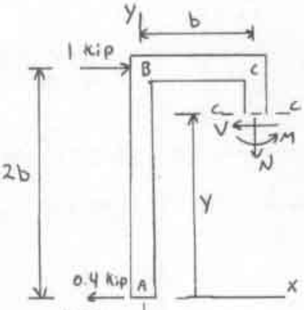
$0 \le x \le b$

$\sum F_y = -0.5 - V = 0$

or $V = -0.5$ kip; $0 \le x \le b \quad$ (d)

Figure d

For the right leg CD, by Fig. e



$\circlearrowleft \sum M_{c-c} = M - 1(2b - y) + 0.5b - 0.4y = 0$

or $M = 1.5b - 0.6y$ (kip ft); $\quad$ (e)

$0 \le y \le 2b$

$\sum F_x = 1 - V - 0.4 = 0$

or $V = 0.6$ kips; $0 \le y \le 2b \quad$ (f)

Figure e

By Eqs. (a), (c), and (e), the moment in the legs and the top is determined (see Fig. f)

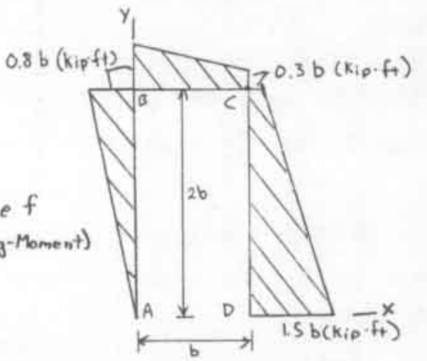


Figure f
(Bending-Moment)

By Eqs. (b), (d), and (f), the shear in the legs and top is determined (see Fig. g.)

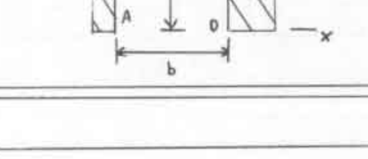


Figure g
(Shear)

11.46

Given: The split-ring beam shown in Fig. a.

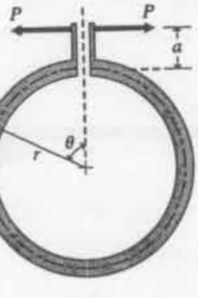


Figure a

Find: Construct shear and bending-moment diagrams as a function of P, and a, for $r = 5a$ and $0 \le \theta \le 180°$.

Solution:

Consider the free-body diagram of the element of beam from $\theta = 0$ to $\theta = \theta$ (section a-a in Fig. b).

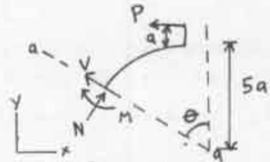


Figure b

By Fig. b,

$\circlearrowleft \sum M_{a-a} = -M + P(6a - 5a\cos\theta) = 0$

or $M = Pa(6 - 5\cos\theta) \quad$ (a)

$\sum F_x = -V\sin\theta + N\cos\theta - P = 0 \quad$ (b)

$\sum F_y = V\cos\theta + N\sin\theta = 0 \quad$ (c)

The solution of Eqs. (b) and (c) is

$V = -P\sin\theta \quad$ (d)

$N = P\cos\theta \quad$ (e)

The shear and moment diagrams are plotted in Figs. c and d, respectively

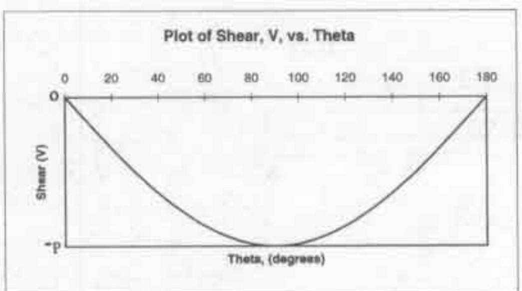


Figure c

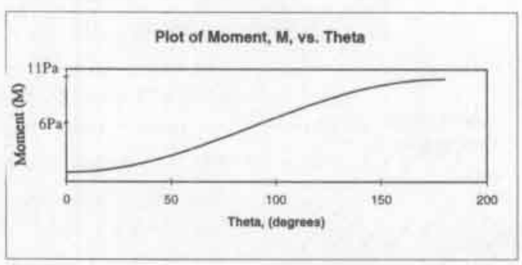


Figure d

11.47

Given: A truck that weighs W and has wheel span s travels over a single-span bridge of length L (Fig. a). The front wheels carry 2/3 W and the back wheels 1/3 W.

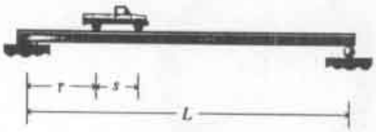


Figure a

Find:

a) For $s = 15$ ft, $L = 115$ ft, and $W = 7500$ lb, plot the bridge's shear and bending-moment diagrams due to the weight of the truck for $r = 42.5$ ft and for $r = 57.5$ ft, where r is the distance from the left support to the truck's rear wheels.

b) Plot the shear at the left support as a function of r for $0 \le r \le 100$ ft. For what value of r in this range is the shear a maximum?

c) Plot the bending-moment at mid span as a function of r for $0 \le r \le 42.5$ ft. For what value of r in this range is the bending-moment a maximum?

d) Compare the values of shear at the left support and bending-moment at the midspan obtained in parts b and c to those obtained in part a for $r = 42.5$ ft.

Solution:

a) For $r = 42.5$ ft, the free-body diagram and the shear and bending-moment diagrams are shown in Fig. b.

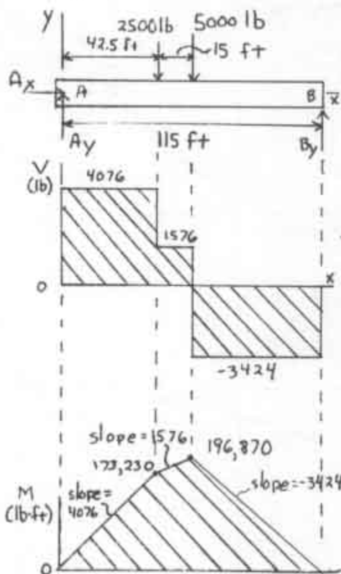


Figure b

By the free-body diagram,

$\Sigma F_x = A_x = 0$

$\circlearrowleft \Sigma M_A = -2500(42.5) - 5000(57.5) + B_y(115) = 0$

$B_y = 3423.9$ lb

$\Sigma F_y = A_y + B_y - 7500 = 0$

$A_y = 4076.1$ lb

The formulas for shear and moment are:

For $0 < x < 42.5$ ft,

$V = 4076$ (lb)

$M = 4076x$ (lb·ft)

For $42.5 < x < 57.5$ ft,

$V = 1576$ (lb)

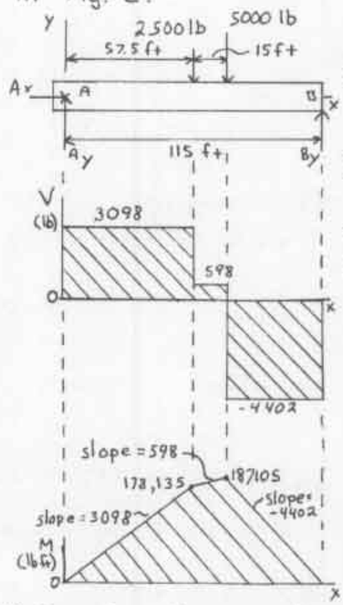
$M = 1576x + 106{,}250$ (lb·ft)

For $57.5 < x < 115$ ft,

$V = -3424$ (lb)

$M = -3424x + 393{,}760$ (lb·ft)

For $r = 57.5$ ft, the free-body diagram and the shear and bending-moment diagrams are shown in Fig. c.



By the free-body diagram,

$\Sigma F_x = A_x = 0$

$\circlearrowleft \Sigma M_A = -2500(57.5) - 5000(72.5) + B_y(115) = 0$

$B_y = 4402$ lb

$\Sigma F_y = A_y + B_y - 7500 = 0$

$A_y = 3098$ lb

The shear and moment formulas are:

For $0 < x < 57.5$ ft,

$V = 3098$ (lb)

$M = 3098x$ (lb·ft)

For $57.5 < x < 72.5$ ft,

$V = 598$ (lb)

$M = 598x + 143{,}750$ (lb·ft)

For $72.5 < x < 115$ ft,

$V = -4402$ (lb)

$M = -4402x + 506{,}250$ (lb·ft)

b) Consider the general case and free-body diagram shown in Fig. d.

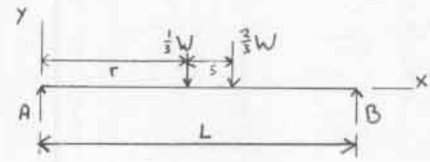


Figure d

By Fig. d,

$$\Sigma F_y = A + B - W = 0 \;;\quad A + B = W \qquad (a)$$

$$\circlearrowleft \Sigma M_B = -AL + \tfrac{2}{3}W(L - (r+s)) + \tfrac{1}{3}W(L - r) = 0$$

The solution of Eqs. (a) is

$$A = \frac{W}{L}\left(L - r - \tfrac{2}{3}s\right) \qquad (b)$$

$$B = \frac{W}{L}\left(r + \tfrac{2}{3}s\right) \qquad (c)$$

For $s = 15$ ft, $L = 115$ ft, and $W = 7500$ lb, Eq. (a) yields

$$A = 65.217\,(105 - r) \qquad (d)$$

Therefore, the shear at the left support as a function of r is,

$$A = V = 65.217\,(105 - r) \qquad (e)$$

The plot of V for $0 \le r \le 100$ ft is shown in Fig. e.

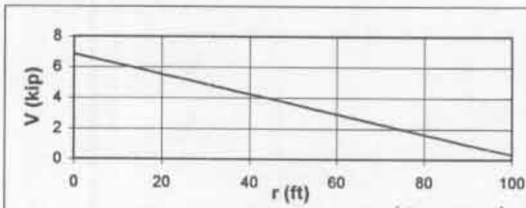


Figure e

(Continued)

11.47 Cont.

c) Consider the free-body diagram of the bridge section, for $r+s < L/2$, Fig. f. By Fig. f, taking moments about the midspan c, we have

$$\circlearrowleft\Sigma M_c = M + \tfrac{2}{3}W\left[\tfrac{L}{2}-(r+s)\right] + \tfrac{1}{3}W\left[\tfrac{L}{2}-r\right] - A\tfrac{L}{2} = 0 \quad (e)$$

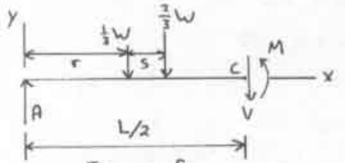


Figure f

By Eqs. (b) and (e), we find

$$M = \frac{W}{2}\left(r + \frac{2}{3}s\right) \quad (f)$$

For $W = 7500$ lb and $s = 15$ ft, Eq. (f) yields

$$M = 3750\,(r+10) \;;\quad 0 \le r \le 42.5 \text{ ft} \quad (g)$$

By Eq. (g), the plot of M is shown in Fig. (g)

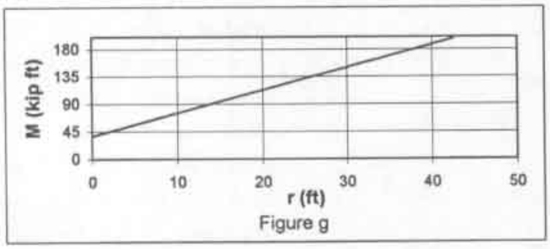


Figure g

d) By part b, for $r = 42.5$ ft, Eq. (e) yields $V = 4076$ lb. This agrees with the value of V of part a (see Fig. b).

By part c, for $r = 42.5$ ft, Eq. (g) yields $M = 196,875$ lb·ft. This agrees with the value of M of part a (see Fig. b)

11.48

Given: The beam loaded as shown in Fig. a

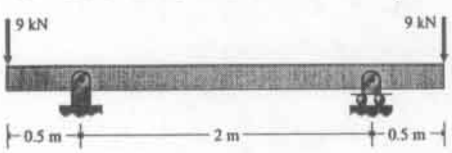


Figure a

Find: A concentrated load or couple that, when applied between supports, will reduce the bending-moment at midspan to zero.

Solution:

To do this, first find the bending-moment at midspan and apply a load or couple to make it zero.

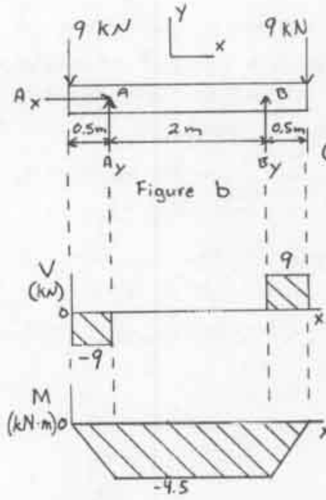


Figure c

Figure b is the free-body diagram of the beam

By Fig. b,

$\Sigma F_x = A_x = 0$

$\circlearrowleft\Sigma M_A = 9(0.5) + B_y(2) - 9(2.5) = 0$

$B_y = 9$ kN

$\Sigma F_y = A_y + B_y - 18 = 0$

$A_y = 9$ kN

The shear and bending-moment diagram of the beam is shown in Fig. c

For $0 < x < 0.5$ m

$V = -9$ (kN)

$M = -9x$ (kN·m)

For $0.5 < x < 2.5$ m,

$V = 0$

$M = -4.5$ (kN·m)

For $2.5 < x < 3$ m,

$V = 9$ (kN)

$M = 9x - 27$ (kN m)

To reduce the midspan moment $M = 4.5$ kN·m ↻ to zero, consider first a concentrated load Q at a distance x from the left support (Fig. d), between the supports.

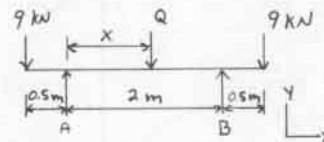


Figure d

By Fig. d,

$$\Sigma F_y = A + B - 18 - Q = 0 \quad (a)$$
$$\circlearrowleft\Sigma M_A = 9(0.5) - Q(x) + B(2) - 9(2.5) = 0$$

The solution of Eq. (a) is

$$A = 9 + Q\left(1 - \tfrac{1}{2}x\right) \quad (\text{kN})$$
$$B = 9 + \tfrac{1}{2}Qx \quad (\text{kN}) \quad (b)$$

Next consider the free-body diagram of one-half of the beam (Fig. e).

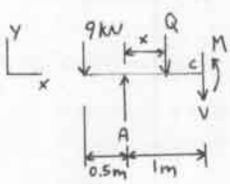


Figure e

By Fig. e,

$$\circlearrowleft\Sigma M_c = M + (1-x)Q - A(1) + 9(1.5) = 0 \quad (c)$$

By Eq. (c) and with A from Eq. (b), we find,

$$M = \tfrac{1}{2}Qx - 4.5 \quad (\text{kN·m}) \quad (d)$$

There fore, for $M = 0$, Eq. (d) requires that $Qx = 9$ kN m.

(Continued)

11.48 Cont.

Hence any load Q located at x, such that $Qx = 9$ kN·m, will reduce the moment at midspan to zero. For example, if $x = 1$ m, $Q = 9$ kN and it is located at the mid span of the beam.

Alternatively, consider a moment M_0 applied to the beam between the supports (Fig. f)

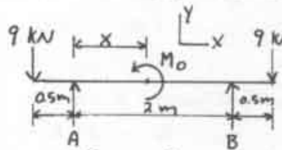


Figure f

By Fig. f,

$$\Sigma F_y = A + B - 18 = 0$$

$$\curvearrowleft \Sigma M_A = 9(0.5) + M_0 + B(2) - 9(2.5) = 0 \qquad (e)$$

The solution of Eqs. (e) is

$$A = 9 + \tfrac{1}{2} M_0 \quad (\text{kN}) \qquad (f)$$

$$B = 9 - \tfrac{1}{2} M_0 \quad (\text{kN}) \qquad (g)$$

Next consider the free-body diagram of one-half of the beam (Fig. g)

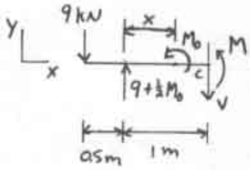


Figure g

By Fig. g,

$$\curvearrowleft \Sigma M_c = M + M_0 - (9 + \tfrac{1}{2} M_0) + 9(1.5) = 0$$

or $M = -4.5 - \frac{1}{2} M_0$ (kN·m) (h)

Therefore, for $M = 0$, Eq. (h) requires that $M_0 = -9$ kN·m or $M_0 = 9$ kN·m ↻. Since x does not occur in the solution for M_0, M_0 may be located anywhere between the supports.

Other solutions are possible; for example, a combination of a force and a couple.

11.49

Given: The beam loaded as shown in Fig. a.

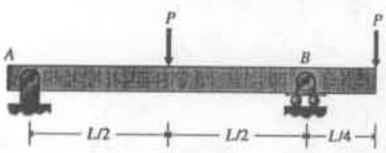


Figure a

Find: Relocate the support B so that the maximum positive moment in the beam equals the magnitude of the maximum negative moment.

Solution: As outlined in the suggested method, determine the support reactions for the beam as shown in Fig. a. Then, decide which way to move support B. Thus, the free-body diagram of the beam is shown in Fig. b,

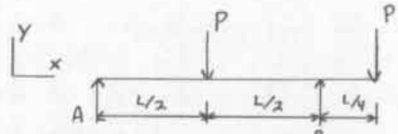


Figure b

By Figure b,

$$\Sigma F_y = A + B - 2P = 0$$

$$\curvearrowleft \Sigma M_B = -A(L) + P(\tfrac{L}{2}) - P(\tfrac{L}{4}) = 0 \qquad (a)$$

The solution of Eqs. (a) is

$$A = \frac{P}{4} \qquad B = \frac{7P}{4} \qquad (b)$$

The shear and moment diagrams are shown in Fig. c. The magnitude of the negative moment is twice that of the positive moment. Also note that the negative moment is also given by the force P at the right end of the beam times the distance L/4. To reduce the negative moment, the distance L/4 must be reduced; that is, the support B must be moved to the right. Therefore, let B be moved a distance x to the right of its present position (see Fig. d).

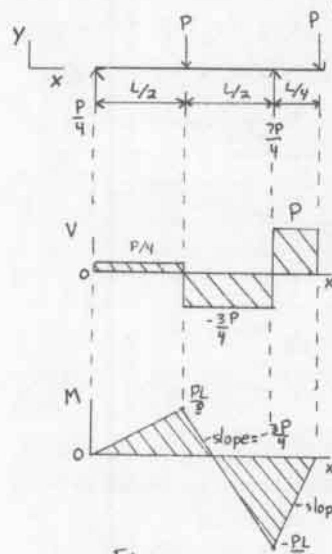


Figure c

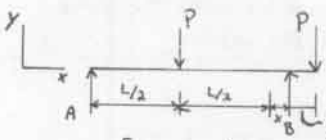


Figure d

By Fig. d,

$$\Sigma F_y = A + B - 2P = 0$$

$$\curvearrowleft \Sigma M_B = -A(L+x) + P(\tfrac{L}{2}+x) - P(\tfrac{L}{4}-x) = 0 \qquad (c)$$

The solution of Eqs. (c) is

$$A = \frac{P(L+8x)}{4(L+x)} \qquad B = \frac{7PL}{4(L+x)} \qquad (d)$$

With A and B known, we may construct the shear and moment diagrams (see Fig. e). Equating the magnitudes of the positive and negative moments we have

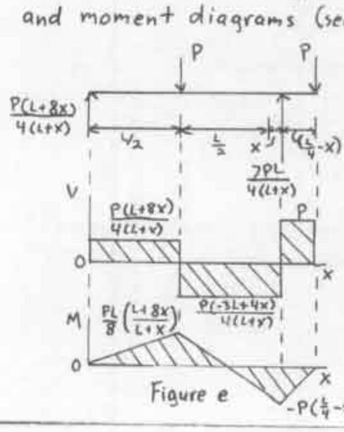


Figure e

$$\frac{PL}{8}\left(\frac{L+8x}{L+x}\right) = \frac{P}{4}(L - 4x)$$

or after simplification,

$$8\left(\frac{x}{L}\right)^2 + 14\left(\frac{x}{L}\right) - 1 = 0 \qquad (e)$$

The solution of Eq. (e) is

$\underline{\underline{x = 0.0687\,L}}$

Hence, support B must be moved a distance 0.0687L to the right

11.50

Given: The beam loaded as shown in Fig. a

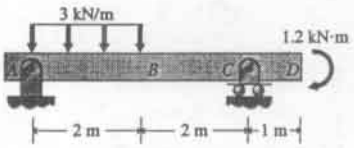


Figure a

Draw the shear and bending-moment diagrams with numerical values at A, B, C, and D labeled.

Solution: The free-body diagram of the beam is shown in Fig. b

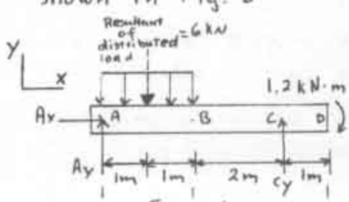


Figure b

By Fig. b,

$\Sigma F_x: A_x = 0$

$\circlearrowleft \Sigma M_A = -6(1) + C_y(4) + 1.2 = 0$

$C_y = 1.8$ (kN)

$\Sigma F_y: A_y - 6 + C_y = 0$

$A_y = 4.2$ (kN)

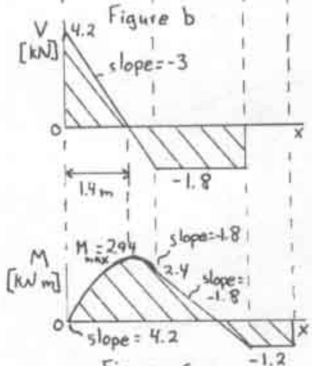


Figure c

The shear and bending-moment diagrams are shown in Fig. c. They were constructed using the rules given in sections 11.4 and 11.5. In addition, the equations for shear and moments are given below.

For $0 \le x \le 2$ m,

$V = 4.2 - 3x$ (kN)

$M = 4.2x - \frac{3}{2}x^2$ (kNm)

For $2 \le x \le 4$ m,

$V = -1.8$ (kN)

$M = 2.4 - 1.8(x-2)$

or

$M = 6.0 - 1.8x$ (kNm)

For $4 \le x \le 5$ m

$V = 0$

$M = -1.2$ (kN·m)

11.51

Given: The beam loaded as shown in Fig. a

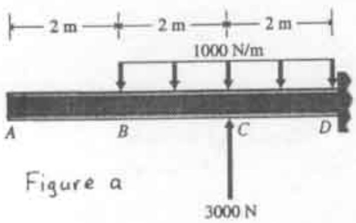


Figure a

Draw the shear and bending-moment diagrams and label numerical values at points A, B, C, and D.

Solution:

The free-body diagram of the beam is shown in Fig. b.

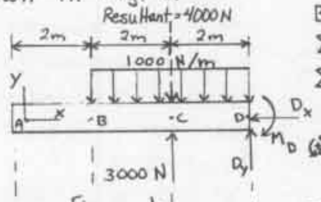


Figure b

By Fig. b,

$\Sigma F_x = D_x = 0$

$\Sigma F_y = D_y - 1000(4) + 3000 = 0$

$D_y = 1000$ N

$\circlearrowleft \Sigma M_D = 4000(2) - 3000(2) - M_D = 0$

$M_D = 2000$ N·m

By the rules given in sections 11.4 and 11.5, the shear and moment diagram (Fig. c) may be constructed. Also, in equation form, the shear and moment are given below.

For $0 \le x \le 2$ m,

$V = M = 0$

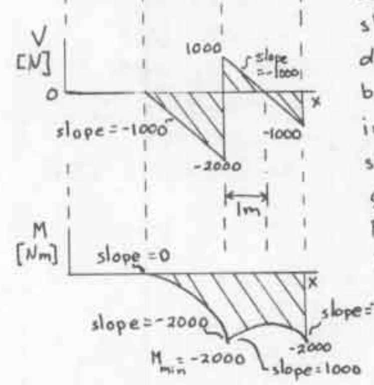


Figure c

For $2 \le x \le 4$ m

$V = -1000(x-2) = -1000x + 2000$ (N)

$M = -[1000(x-2)]\left(\frac{x-2}{2}\right) = -500(x-2)^2$ (kN·m)

For $4 \le x \le 6$ m,

$V = 5000 - 1000x$ (N)

$M = 5000x - 500x^2 - 14000$ (kN·m)

11.52

Given: The beam loaded as shown in Fig. a.

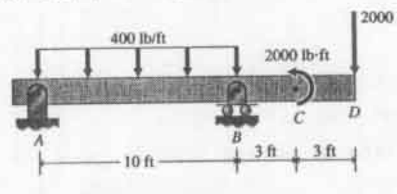


Figure a

Draw the shear and bending-moment diagrams and label numerical values for points A, B, C, and D.

Solution:

The free-body diagram of the beam is shown in Fig. b.

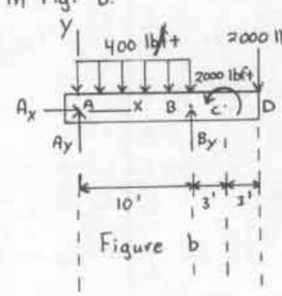


Figure b

By Fig. b,

$\Sigma F_x = A_x = 0$

$\circlearrowleft \Sigma M_A = -4000(5) + B_y(10) + 2000 - 2000(16) = 0$

$B_y = 5000$ lb

$\Sigma F_y = A_y + B_y - 4000 - 2000 = 0$

$A_y = 1000$ lb

(Continued)

11.52 Cont.

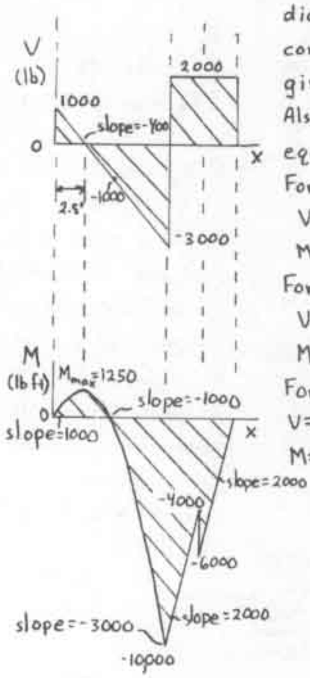


Figure c

The shear and moment diagrams (Fig. c) may be constructed using the rules given in sections 11.4 and 11.5. Also the shear and moment equations are given below.

For $0 \le x \le 10$ ft,

$V = 1000 - 400x$ (lb)

$M = 1000x - 200x^2$ (lb ft)

For $10 \le x < 13$ ft,

$V = 2000$ (lb)

$M = 2000x - 30000$ (lb ft)

For $13 < x \le 16$ ft,

$V = 2000$ (lb)

$M = -2000(16 - x)$ (lb ft)

11.53

Given: The cantilever beam loaded as shown in Fig. a.

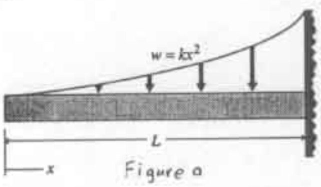


Figure a

a) Draw the free-body diagram of the beam and determine the support reactions.

b) Construct the shear and bending-moment diagrams.

c) Locate the sections where the shear and the bending-moment obtain the largest absolute values

d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

Solution:

a) The free-body diagram of the beam is shown in Fig. b.

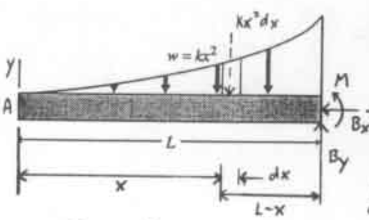


Figure b

By Fig. b,

$\Sigma F_x = \underline{B_x = 0}$

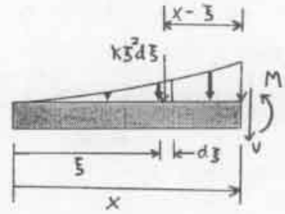
$\Sigma F_y = B_y - \int_0^L kx^2\,dx = 0$

or $\underline{B_y = \frac{kL^3}{3}}$

$\circlearrowleft \Sigma M_B = M + \int_0^L (kx^2 dx)(L - x) = 0$

or $\underline{\underline{M = -\frac{kL^4}{12}}}$

b) By Fig. c, the shear as a function of x is,



Greek: xi = ξ

Figure c

$$V = -\int_0^x k\xi^2 d\xi = -\frac{1}{3}kx^3 \qquad (a)$$

and the moment is

$$M = \int_0^x (k\xi^2)(x - \xi)\,d\xi = -\frac{1}{12}kx^4 \qquad (b)$$

The shear and bending-moment diagrams are plotted in Fig. d, using Eqs. (a) and (b).

c) The shear and moment attain the largest absolute values at section B (Fig. d)

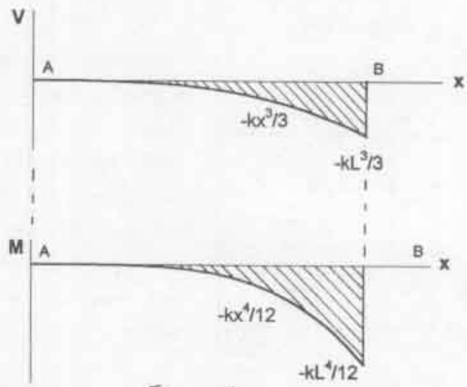


Figure d

d) No sudden changes in loads occur (Fig. a)

11.54

Given: The beam loaded as shown in Fig. a.

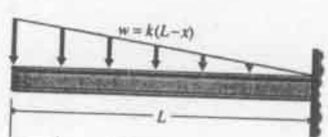


Figure a

a) Draw the free-body diagram of the beam and determine the support reactions.

b) Construct the shear and bending-moment diagrams.

c) Locate the sections where the shear and the bending-moment obtain the largest absolute values

d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

(Continued)

11.54 Cont.

Solution:

a) The free-body diagram of the beam is shown in Fig. b.

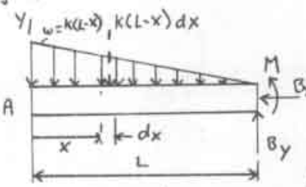


Figure b

By Fig. b,

$\Sigma F_x = \underline{B_x = 0}$

$\Sigma F_y = B_y - \int_0^L k(L-x)dx = 0$

or $\underline{B_y = \frac{kL^2}{2}}$

$\circlearrowleft \Sigma M_B = M + k\int_0^L (L-x)^2 dx = 0$

or $\underline{M = \frac{kL^3}{3}}$

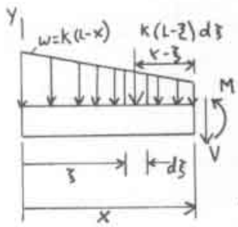


Figure c

b) By Fig. c, the shear as a function of x is

$$V = -\int_0^x k(L-\xi)d\xi = -k\left(Lx - \frac{x^2}{2}\right) \quad (a)$$

$$M = -\int_0^x k(L-\xi)(x-\xi)\,d\xi$$

or

$$M = -k\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) \quad (b)$$

The shear and bending-moment diagrams are plotted in Fig. d

c) The shear and moment attain the largest absolute values at section B (Fig. d)

d) No sudden changes in loads occur (Fig. a)

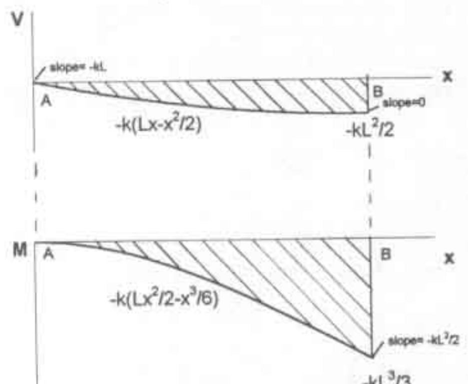


Figure d

d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

Solution:

a) The free-body diagram is shown in Fig. b.

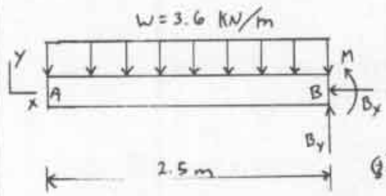


Figure b

By Fig. b,

$\Sigma F_x = \underline{B_x = 0}$

$\Sigma F_y = -(3.6)(2.5) + B_y = 0$

$\underline{B_y = 9 \text{ kN}}$

$\circlearrowleft \Sigma M_B = (3.6)(2.5)\left(\frac{2.5}{2}\right) + M = 0$

$\underline{M = -11.25 \text{ kNm}}$

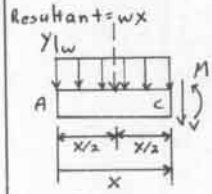


Figure c

b) By Fig. c,

$\Sigma F_y = -wx - V = 0$

or $\underline{V = -3.6x \quad (kN)}$ (a)

$\circlearrowleft \Sigma M_c = M + wx\left(\frac{x}{2}\right) = 0$

or $\underline{M = -1.8x^2 \quad (kNm)}$ (b)

c) The shear and bending-moment diagrams are shown in Fig. d

d) There is no sudden change in the load (Fig. a).

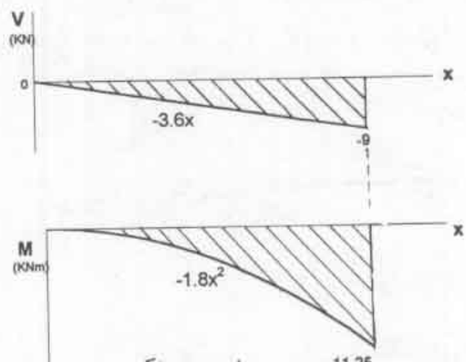


Figure d

11.55

Given: The cantilever beam loaded as shown in Fig. a.

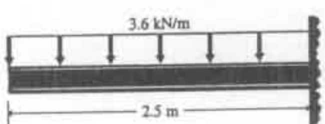


Figure a

a) Draw the free-body diagram of the beam and determine the support reactions.

b) Construct the shear and bending-moment diagrams.

c) Locate the sections where the shear and the bending-moment obtain the largest absolute values

11.56

Given: The cantilever beam loaded as shown in Fig. a

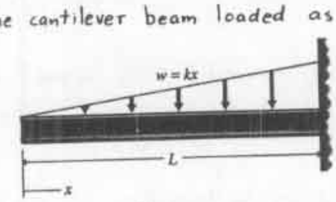


Figure a

a) Draw the free-body diagram of the beam and determine the support reactions.

b) Construct the shear and bending-moment diagrams.

(Continued)

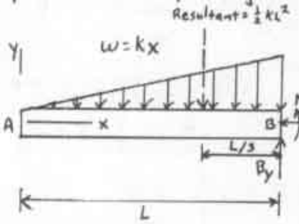
11.56 Cont.

c) Locate the sections where the shear and the bending-moment obtain the largest absolute values

d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

Solution:

a) By the free-body diagram of the beam (Fig. b),



$\Sigma F_x = B_x = 0$

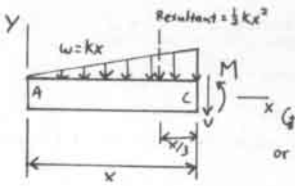
$\Sigma F_y = B_y - kL^2/2 = 0$

$B_y = \frac{kL^2}{2}$

$\circlearrowleft \Sigma M_B = \frac{kL^2}{2}(\frac{L}{3}) + M = 0$

$M = -\frac{kL^3}{6}$

b) Consider the free-body diagram of the beam element x=0 to x=x (Fig. C)



By Fig. C,

$\Sigma F_y = -\frac{1}{2}kx^2 - V = 0$; $V = -\frac{1}{2}kx^2$ (a)

$\circlearrowleft \Sigma M_c = (\frac{1}{2}kx^2)(\frac{x}{3}) + M = 0$

or $M = -\frac{1}{6}kx^3$ (b)

With Eqs. (a) and (b), the shear and bending-moment diagrams may be constructed (Fig. d).

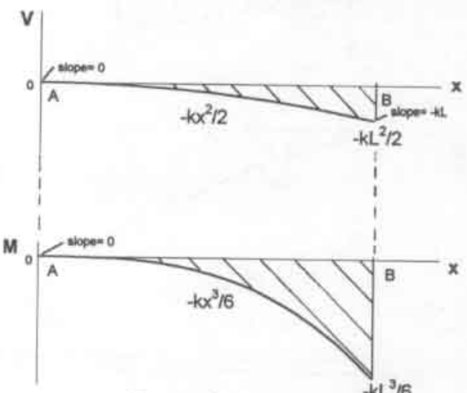


Figure d

c) By Fig. d, the largest absolute values of shear and moment are $|V| = \frac{1}{2}kL^2$ and $|M| = \frac{1}{6}kL^3$, respectively

d) No sudden changes in load occur (Fig. a)

11.57

Given: The beam loaded as shown in Fig. a

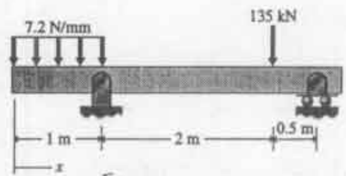


Figure a

a) Draw the free-body diagram of the beam and determine the support reactions.

b) Construct the shear and bending-moment diagrams.

c) Locate the sections where the shear and the bending-moment obtain the largest absolute values

d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

Solution:

a) By the free-body diagram of the beam (Fig. b)

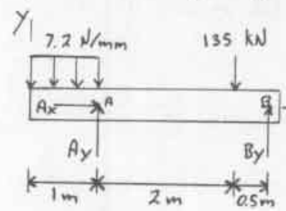


Figure b

$\Sigma F_x = A_x = 0$

$\circlearrowleft \Sigma M_A = 7.2(0.5) - 135(2) + B_y(2.5) = 0$

$B_y = 106.56$ kN

$\Sigma F_y = A_y - 7.2 - 135 + B_y = 0$

$A_y = 35.64$ kN

b) By the rules of sections 11.4 and 11.5, or by the following equations, the shear and bending-moment diagrams may be plotted (Fig. c)

For $0 < x < 1$m,

$V = -wx = -7.2x$ (kN)

$M = -\frac{1}{2}wx^2 = -3.6x^2$ (kNm)

For $1 < x < 3$ m,

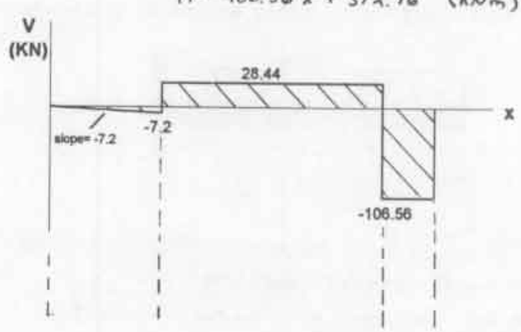
$V = 28.44$ (kN)

$M = 28.44x - 32.04$ (kNm)

For $3 < x < 3.5$ m,

$V = -106.56$ (kN)

$M = -106.56x + 372.96$ (kNm)



(Continued)

11.57 Cont.

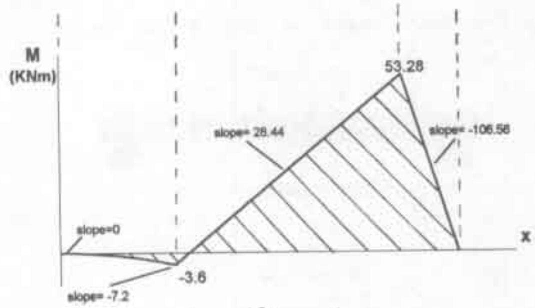


Figure C

c) The largest absolute value of shear occurs at $x = 3$ m, where $|V| = 106.56$ kN.
The largest absolute value of moment occurs at $x = 3$ m also, where $|M| = 53.28$ kN·m.

d) Sudden changes in loads occur at $x = 1$ m and $x = 3$ m (Fig. a)
At $x = 1$ m, $V = -7.2$ kN and $M = -3.6$ kN m

11.58

Given: The beam loaded as shown in Fig. a.

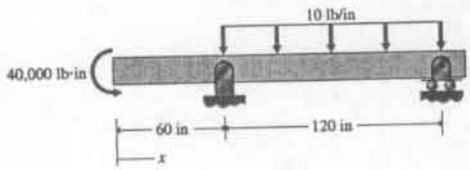


Figure a

a) Draw the free-body diagram of the beam and determine the support reactions.
b) Construct the shear and bending-moment diagrams.
c) Locate the sections where the shear and the bending-moment obtain the largest absolute values
d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

Solution:

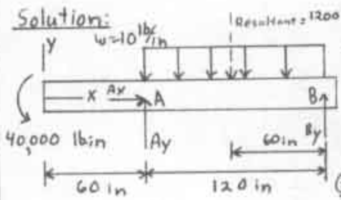


Figure b

a) By Fig. b, the free-body diagram of the beam

$\Sigma F_x = \underline{A_x = 0}$

$\circlearrowleft \Sigma M_A = -1200(60) + 40{,}000 + B_y(120) = 0$

$\underline{\underline{B_y = 266.6\bar{6} \text{ lb}}}$

$\Sigma F_y = A_y - 1200 + B_y = 0$

$\underline{\underline{A_y = 933.3\bar{3} \text{ lb}}}$

b) The shear and bending-moment diagrams may be constructed by the rules listed in sections 11.4 and 11.5, or by the following equations (see Fig. c)

For $0 < x < 60$ in,
$V = 0$
$M = -40{,}000$ (lb·in)

For $60 < x < 180$ in,
$V = 933.33 - 10(x - 60)$
or $V = -10x + 1533.33$ (lb)
$M = -5x^2 + 1533.33x - 114000$

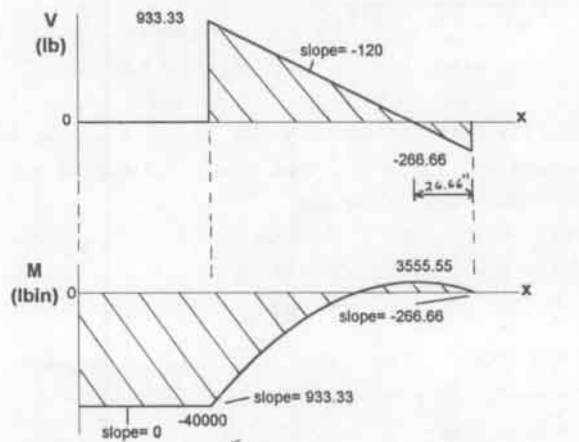


Figure c

c) The largest absolute value of shear and moment occur at $x = 60$ in (Fig. c). For the shear, $V = 933.3\bar{3}$ lb and for the moment, $|M| = 40{,}000$ lbin.

d) A sudden jump in load occurs at $x = 60$ in.
At $x = 60$ in, $V = 933.3\bar{3}$ lb and $M = -40{,}000$ lb·in

11.59

Given: The beam loaded as shown in Fig. a

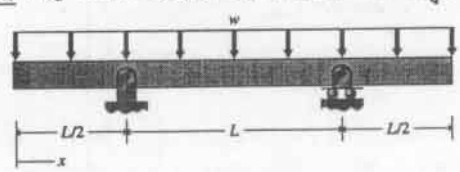


Figure a

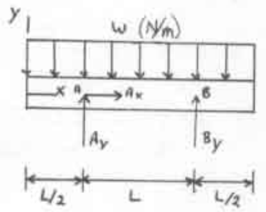
a) Draw the free-body diagram of the beam and determine the support reactions.
b) Construct the shear and bending-moment diagrams.
c) Locate the sections where the shear and the bending-moment obtain the largest absolute values
d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

(Continued)

11.59 Cont.

Solution:

a) The free-body diagram is shown in Fig. b



By Fig. b,

$\Sigma F_x = \underline{A_x = 0}$

$\circlearrowleft \Sigma M_A = -\frac{2wL^2}{2} + B_y(L) = 0$

$$\underline{\underline{B_y = wL}}$$

Figure b

$\Sigma F_y: A_y + B_y - 2wL = 0$

$$\underline{\underline{A_y = wL}}$$

b) By the rules in sections 11.4 and 11.5 or the following equations, the shear and moment diagrams may be constructed (Fig. c).

For $0 < x < L/2$

$V = -wx$

$M = -\frac{wx^2}{2}$

For $\frac{L}{2} < x < L$

$V = w(L-x)$

$M = -\frac{wx^2}{2} + wLx - \frac{wL^2}{2}$

For $x > L$, the shear and moment diagrams repeat themselves (Fig. c)

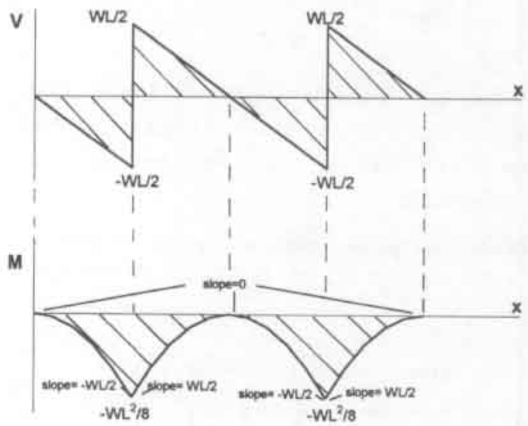


Figure c

c) The greatest absolute values of shear and moment occur at two values of x, namely $x = L/2$ and $x = 3L/2$ (Fig. c) where

$$|V| = \frac{wL}{2} \quad \text{and} \quad |M| = \frac{wL^2}{8}$$

d) Sudden changes in the load occur at $x = \frac{L}{2}$ and $x = \frac{3L}{2}$.

For $x = \frac{L}{2}(-)$, $V = -\frac{wL}{2}$; For $x = \frac{L}{2}(+)$, $V = \frac{wL}{2}$

For $x = \frac{3L}{2}(-)$, $V = -\frac{wL}{2}$; For $x = \frac{3L}{2}(+)$, $V = \frac{wL}{2}$

For $x = \frac{L}{2}$ and $x = \frac{3L}{2}$, $M = \frac{-wL^2}{8}$

11.60

Given: The beam loaded as shown in Fig. a

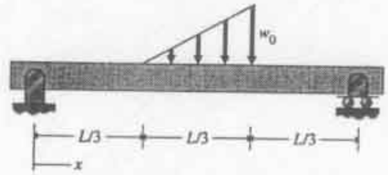


Figure a

a) Draw the free-body diagram of the beam and determine the support reactions.

b) Construct the shear and bending-moment diagrams.

c) Locate the sections where the shear and the bending-moment obtain the largest absolute values

d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

Solution:

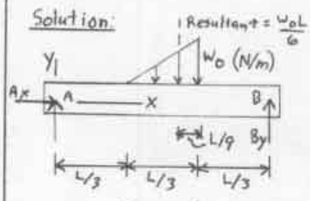


Figure b

a) The free-body diagram of the beam is shown in Fig. b.

By Fig. b,

$\Sigma F_x = \underline{\underline{A_x = 0}}$

$\circlearrowleft \Sigma M_A = B_y(L) - \left(\frac{w_0 L}{6}\right)\left(\frac{L}{3} + \frac{2}{3}\frac{L}{3}\right) = 0$

$$\underline{\underline{B_y = \frac{5}{54} w_0 L}}$$

$\Sigma F_y = A_y + B_y - \frac{w_0 L}{6} = 0$

$$\underline{\underline{A_y = \frac{2}{27} w_0 L}}$$

b) The shear and bending-moment diagrams may be constructed by the rules given in sections 11.4 and 11.5, or by the following equations (Fig. c)

For $0 < x < L/3$,

$V = \frac{2w_0 L}{27}$

$M = \left(\frac{2w_0 L}{27}\right)x$

For $L/3 < x < 2L/3$,

$V = \frac{2w_0 L}{27} - \frac{3w_0}{2L}\left(x - \frac{L}{3}\right)^2$

$M = \left(\frac{2w_0 L}{27}\right)x - \frac{w_0}{2L}\left(x - \frac{L}{3}\right)^3$

For $\frac{2L}{3} < x < L$

$V = -\frac{5}{4} w_0 L$

$M = \frac{5w_0 L}{54}(L - x)$

(Continued)

11.60 Cont.

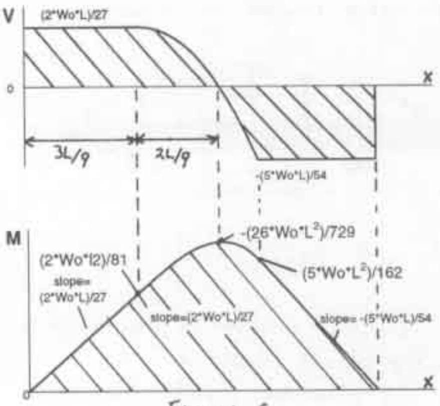


Figure c

c) The greatest absolute value of shear occurs at $x = 2L/3$, where $|V| = 5w_0L/54$.
The greatest absolute value of bending-moment occurs at $x = 5L/9$ (Fig. c), where $M = \frac{26}{729} w_0 L^2$.

d) There is a sudden change in load at $x = 2L/3$, where $V = -\frac{5w_0L}{54}$ and $M = \frac{5w_0L^2}{162}$.

11.61

Given: The L-shaped cantilever loaded as shown in Fig. a.

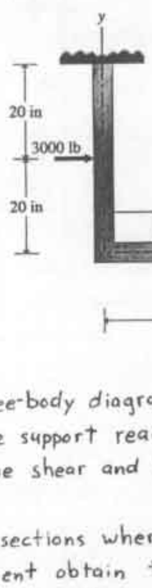


Figure a

a) Draw the free-body diagram of the beam and determine the support reactions.

b) Construct the shear and bending-moment diagrams.

c) Locate the sections where the shear and the bending-moment obtain the largest absolute values

d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

Solution:

a) The free-body diagram is shown in Fig. b.

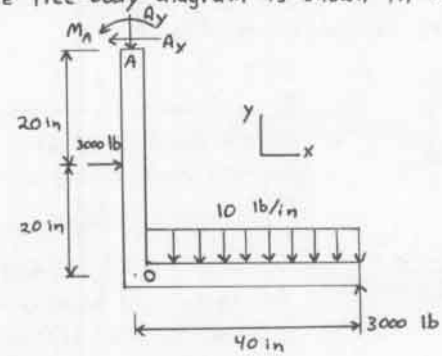


Figure b

By Fig. b,

$\Sigma F_x = -A_x + 3000 = 0$; $\underline{\underline{A_x = 3000 \text{ lb}}}$

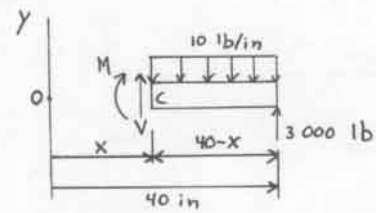
$\Sigma F_y = -A_y + 3000 - 10(40) = 0$; $\underline{\underline{A_y = 2600 \text{ lb}}}$

$\circlearrowleft \Sigma M_A = M_A + 3000(20) + 3000(40) - 10(40)(20) = 0$

$\underline{\underline{M_A = -172\,000 \text{ lb·in}}}$

b) The shear and bending-moment diagrams may be constructed using the rules given in sections 11.4 and 11.5. Alternatively, consider the horizontal leg of the beam (Fig. c)

Figure c



By Fig. c, for $0 < x < 40$ in,

$\Sigma F_y = V + 3000 - 10(40 - x) = 0$

or $V = -10x - 2600$ (lb) (a)

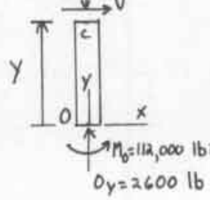
$\circlearrowleft \Sigma M_c = M + (10)(40-x)\left(\frac{40-x}{2}\right) - 3000(40-x) = 0$

or $M = -5(40-x)^2 + 3000(40-x)$ (b)

By Eqs. (a) and (b), the shear and bending-moment diagrams for the horizontal leg may be constructed (Fig. d on next page)

Next, consider the vertical leg of the beam (Fig. e) for $0 < y < 20$ in, $x = 0$.

By Eqs. (a) and (b), the force at $y = 0$ is $O_y = 2600$ lb and the moment is $M_o = 112{,}000$ lbin $\circlearrowright$



By Fig. e, For $0 < y < 20$ in,

$\Sigma F_x = V = 0$ (c)

$\Sigma F_y = 2600 - N = 0$; $N = 2600$ lb

$\circlearrowleft \Sigma M_c = M - 112{,}000 = 0$

or $M = 112\,000$ lb·in (d)

(Continued)

Figure e

11.61 Cont.

For $20 < y < 40$, the free-body diagram of the vertical leg is shown in Fig. f.

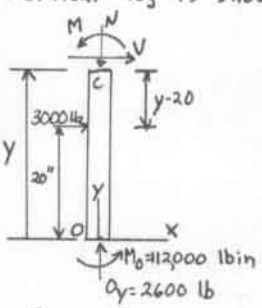


Figure f

By Fig. f,

$$\Sigma F_x = V + 3000 = 0;\ V = -3000 \text{ lb} \quad (e)$$

$$\Sigma F_y = 2600 - N = 0;\ N = 2600 \text{ lb}$$

$$\circlearrowleft \Sigma M_c = M + 112000 + 3000(y-20) = 0$$

or

$$M = -3000y - 52000,\ y > 20" \quad (f)$$

By Eqs. (c), (d), (e), and (f), The shear and bending-moment diagrams for the vertical leg may be constructed (Fig. g).

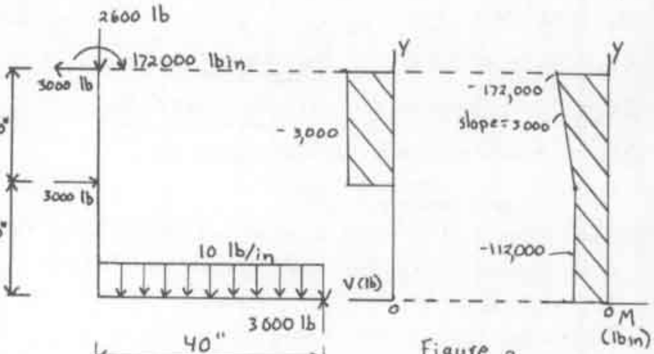


Figure g

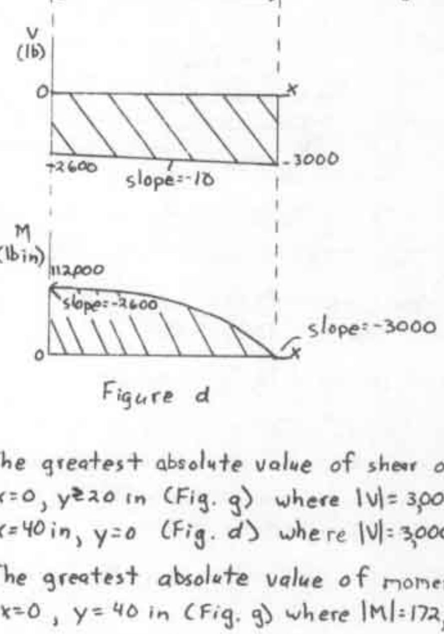


Figure d

c) The greatest absolute value of shear occurs at $x=0$, $y \geq 20$ in (Fig. g) where $|V| = 3{,}000$ lb and at $x = 40$ in, $y=0$ (Fig. d) where $|V| = 3{,}000$ lb.

The greatest absolute value of moment occurs at $x=0$, $y = 40$ in (Fig. g) where $|M| = 172{,}000$ lbin

d) There is a sudden change in load at the following points:

$x=0$, $y = 20$ in (Fig. g); $V = -3{,}000$ lb, $M = 112{,}000$ lbin

and

$x=0$, $y=0$ (Fig. d); $V = -2600$ lb, $M = 112{,}000$ lbin

11.62

Given: The beam loaded as shown in Fig. a.

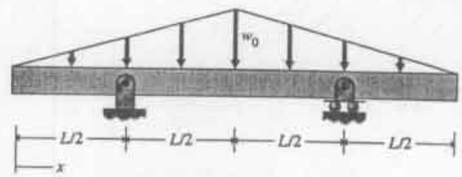


Figure a

a) Draw the free-body diagram of the beam and determine the support reactions.

b) Construct the shear and bending-moment diagrams.

c) Locate the sections where the shear and the bending-moment obtain the largest absolute values

d) Label numerical values of shear and moment at sections where the load undergoes a sudden change.

Solution:

a) The free-body diagram of the beam is shown in Fig. b

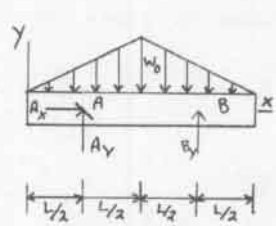


Figure b

By Fig. b

$$\Sigma F_x = \underline{A_x = 0}$$

$$\circlearrowleft \Sigma M_A = -(\tfrac{1}{2} w_0)(2L)(\tfrac{L}{2}) + B_y(L) = 0$$

or $\underline{\underline{B_y = \frac{w_0 L}{2}}}$

$$\Sigma F_y = A_y + B_y - \tfrac{1}{2}(w_0)(2L) = 0$$

or $\underline{\underline{A_y = \frac{w_0 L}{2}}}$

b) By the rules given in Section 11.4 and 11.5, the shear and bending-moment diagrams may be constructed (Fig. c). Alternatively, they may be constructed using the following shear and bending-moment equations.

For $0 < x < L/2$ (Fig. d)

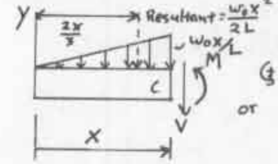


Figure d

$$\Sigma F_y = -\frac{w_0 x^2}{2L} - V = 0;\ V = -\frac{w_0 x^2}{2L} \quad (a)$$

$$\circlearrowleft \Sigma M_c = M + \left(\frac{w_0 x^2}{2L}\right)\left(\frac{x}{3}\right) = 0$$

or

$$M = \frac{w_0 x^3}{6L} \quad (b)$$

For $L/2 < x < L$ (Fig. e)

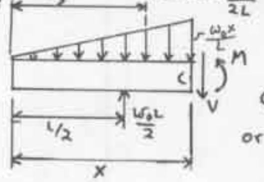


Figure e

$$\Sigma F_y = \frac{w_0 L}{2} - \frac{w_0 x^2}{2L} - V = 0$$

or

$$V = \frac{w_0}{2L}(L^2 - x^2) \quad (c)$$

$$\circlearrowleft \Sigma M_c = M + \left(\frac{w_0 x^2}{2L}\right)\left(\frac{x}{3}\right) - \left(\frac{w_0 L}{2}\right)\left(x - \frac{L}{2}\right) = 0$$

or

$$M = -\frac{w_0 x^3}{6L} + \frac{w_0 L}{2}\left(x - \frac{L}{2}\right) \quad (d)$$

(Continued)

11.62 Cont.

The shear and moment diagrams may be constructed by Eqs (a), (b), (c), and (d), with symmetry conditions (Fig. c)

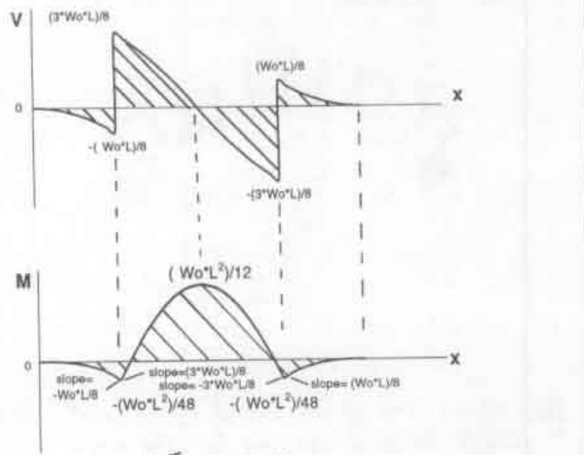


Figure C

c) The greatest absolute values of shear occurs at $x = L/2$ and $x = 3L/2$, where $|V| = 3w_oL/8$.
The greatest absolute value of bending-moment occurs at $x = L$, where $M = w_oL^2/12$.

d) Sudden changes in load occur at $x = L/2$ and $x = 3L/2$. See Fig. c for the corresponding values of V and M.

11.63

Given: The wall frame shown in Fig. a

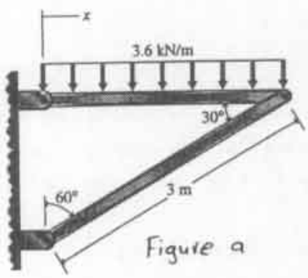


Figure a

a) Express the shear, bending-moment, and axial forces in the top member as a function of x.

b) Construct the shear, bending-moment, and axial force diagrams for the top member.

Solution:

a) The free-body diagram of the top member is shown in Fig. b.

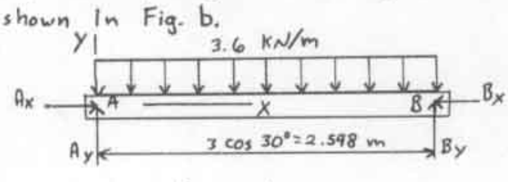


Figure b

By Fig. b,

$\Sigma F_x = A_x - B_x = 0$; $A_x = B_x$

By symmetry,

$A_y = B_y = \dfrac{(3.6)(2.598)}{2} = 4.676$ kN (a)

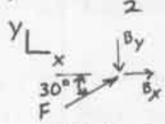


Figure C

By Fig. c, the free-body diagram of pin B,

$\Sigma F_x = B_x + F\cos 30° = 0$

$\Sigma F_y = F\sin 30° - 4.676 = 0$

$\therefore F = 9.352$ kN

$A_x = B_x = -8.099$ kN (b)

Using Eqs. (a) and (b), we may draw the free-body diagram of the element of the top member for $0 < x < 2.598$ m (Fig. d)

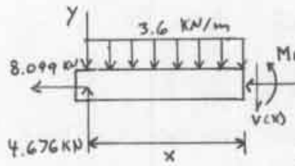


Figure d

By Fig. d,

$\Sigma F_x = -N(x) - 8.099 = 0$ (c)

$\underline{N(x) = -8.099\text{ kN} = 8.099\text{ kN (Tension)}}$

$\Sigma F_y = -V(x) + 4.676 - 3.6(x) = 0$

$\underline{V(x) = -3.6x + 4.676 \text{ (kN)}}$ (d)

$\circlearrowleft \Sigma M_{cut} = -4.676x + \dfrac{3.6}{2}x^2 + M(x) = 0$

$\underline{\underline{M(x) = -1.8x^2 + 4.676 \text{ (kN·m)}}}$ (e)

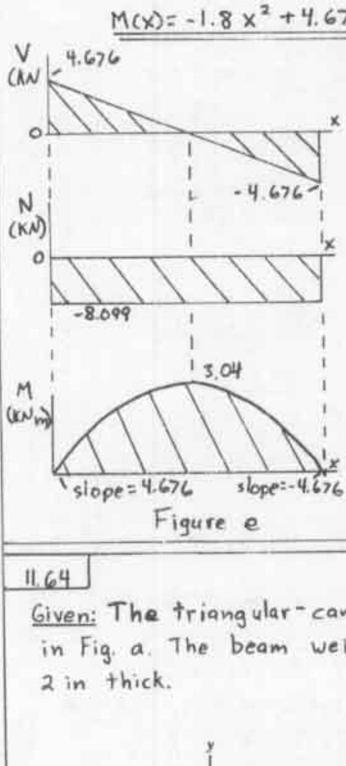


Figure e

b) Using either Eqs (c), (d), and (e) or the rules given in Sections 11.4 and 11.5, we may construct the shear, normal force, and bending-moment diagrams (Fig. e)

11.64

Given: The triangular-cantilever beam shown in Fig. a. The beam weighs 180 lb/ft³ and is 2 in thick.

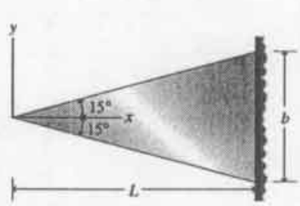


Figure a

(continued)

11.64 Cont.

a) Derive expressions for the shear force V_z, and the bending-moment M_y of the beam as a function of x.

b) Plot the shear and bending-moment for the beam.

Solution:

a) The side view of the beam is shown in Fig. b. Let L and b (Fig. a) be measured in feet.

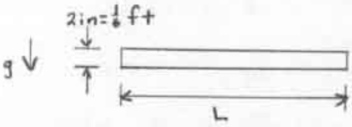


Figure b

The volume of the beam is (see Figs. a and b)

$$\text{Volume} = \frac{1}{2}(b)(L)\left(\frac{1}{6}\right) = \frac{1}{12}bL\ ft^3$$

Hence, the weight is

$$W = (180)\left(\frac{1}{12}bL\right) = 15\ bL\ \ lb$$

Likewise, the weight of an element x ft long is (see Fig. c)

$$w = (180)\left(\frac{1}{2}\right)(2x\tan 15^\circ)(x)\left(\frac{1}{6}\right) = 30x^2\tan 15^\circ$$

The weight w acts at G (Fig. c)

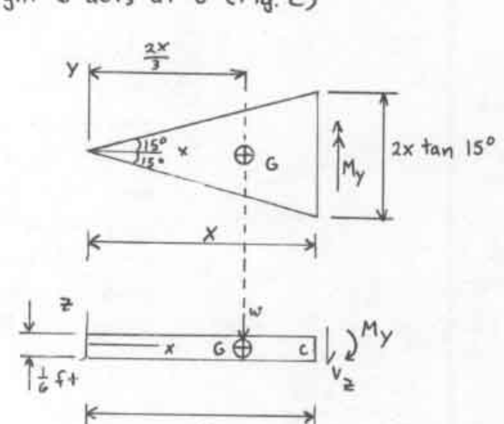


Figure c

By Fig. c,

$$\Sigma F_z = -w - V_z = 0\ ;\ \text{or}\ \ V_z = -30x^2\tan 15^\circ = -8.038x^2\ \ lb \quad (a)$$

$$\circlearrowleft + \Sigma M_c = M_y - \frac{wx}{3} = 0\ ;\ \text{or}\ \ M_y = 10x^3\tan 15^\circ = 2.679x^3 \quad (b)$$

The shear and moment diagrams may be constructed using Eqs. (a) and (b); see Fig. d.

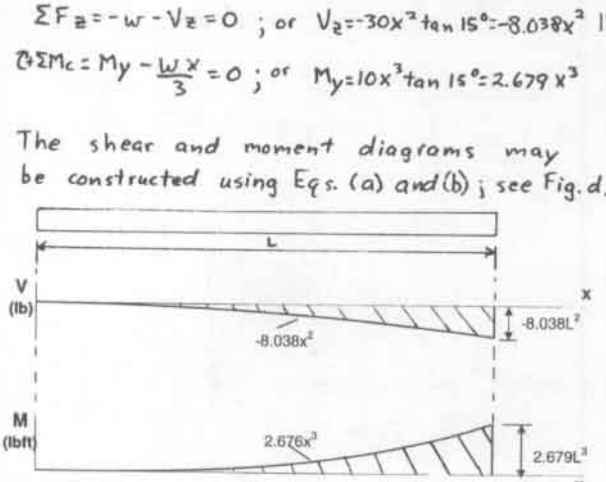


Figure d

11.65

Given: The airplane fuselage ring shown in Fig. (a) is subjected to a uniform pressure p [F/L]. The thickness h of the ring is small compared to its radius a.

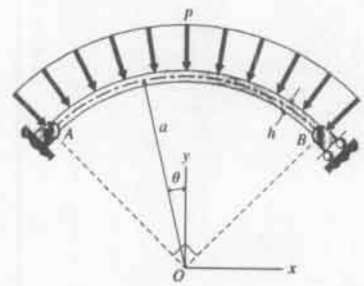


Figure a

Find: Draw the shear, bending-moment, and normal force diagrams for the ring.

Solution: Consider the free-body diagram of the ring (Fig. b)

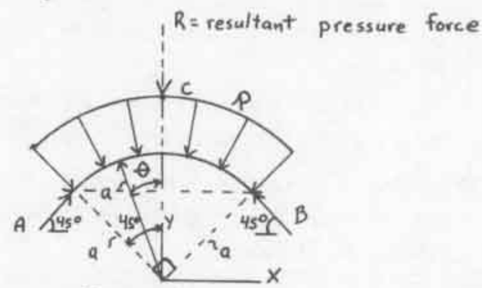


Figure b

By Theorem 9.5, the resultant vertical pressure force is

$$R = p(2a\sin 45^\circ) = \sqrt{2}\,pa \qquad (a)$$

Since the resultant horizontal pressure force is zero (by symmetry), to determine the reactions we may treat the ring as a three-force member.

Hence, by Figs. (b) and (c),

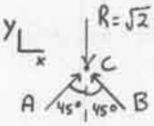


Figure c

$$\Sigma F_y = A\cos 45^\circ + B\cos 45^\circ - \sqrt{2}\,pa = 0$$
$$\Sigma F_x = A\sin 45^\circ - B\sin 45^\circ = 0 \qquad (b)$$

The solution of Eqs. (b) is,

$$A = B = pa \qquad (c)$$

(Continued)

11.65 Cont.

Now, consider the free-body diagram of an element AC of the ring (Fig. d)

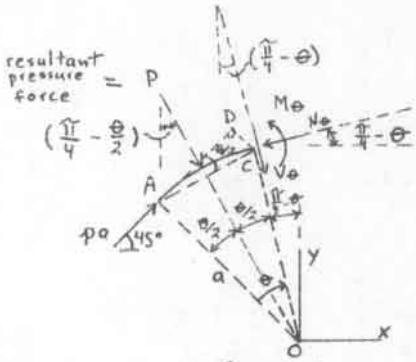


Figure d

By Fig. (d), $AC = 2[a \sin(\theta/2)]$. Hence, by Theorem 9.5 the resultant pressure force on element AC is

$$P = p\,AC = 2pa \sin(\theta/2) \qquad (c)$$

Also, by Fig. d, $CD = AC \sin(\theta/2)$. Then,

$$\circlearrowleft \Sigma M_C = 0 = M_\theta + [2pa \sin(\theta/2)][a \sin(\theta/2)] - pa[2a\sin(\tfrac{\theta}{2})][\sin(\tfrac{\theta}{2})]$$

or

$$\underline{M_\theta = 0} \qquad (d)$$

Also, by Fig. d,

$$\Sigma F_x = pa\frac{\sqrt{2}}{2} + 2pa[\sin(\tfrac{\theta}{2})]\sin(\tfrac{\pi}{4} - \tfrac{\theta}{2}) - N_\theta \cos(\tfrac{\pi}{4} - \theta) + V_\theta \sin(\tfrac{\pi}{4} - \theta) = 0 \qquad (e)$$

$$\Sigma F_y = pa\frac{\sqrt{2}}{2} + 2pa[\sin(\tfrac{\theta}{2})]\cos(\tfrac{\pi}{4} - \tfrac{\theta}{2}) - N_\theta \sin(\tfrac{\pi}{4} - \theta) - V_\theta \cos(\tfrac{\pi}{4} - \theta) = 0$$

The solution of Eqs. (e) is

$$\underline{\underline{N_\theta = pa \quad \text{(compression)}}} \qquad (f)$$

$$\underline{V_\theta = 0}$$

Alternatively, we could have solved for N_θ and V_θ, by using Theorem 9.5 and Fig. (e)

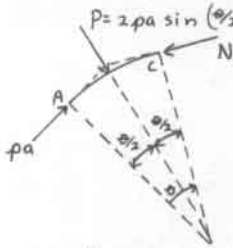


Figure e

Since the element AC is a three-force member the reaction at C is $N_\theta = pa$ for equilibrium. (see also Figs. b and c.) Therefore $M = V = 0$ in the ring.

Thus, the shear and moment diagrams are zero for all θ, and the normal force diagram is a constant compression for all θ (Fig. f)

Figure f

11.66

Given: The L-beam shown in Fig. a

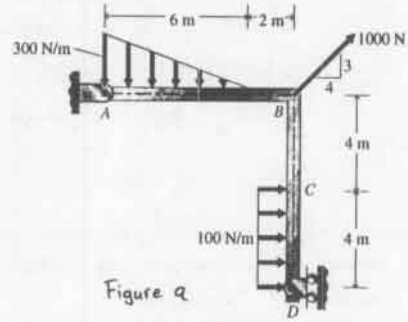


Figure a

Draw the shear, bending-moment, and normal force diagrams and label numerical values at A, B, C, and D

Solution: The free-body diagram of the beam is shown in Fig. b

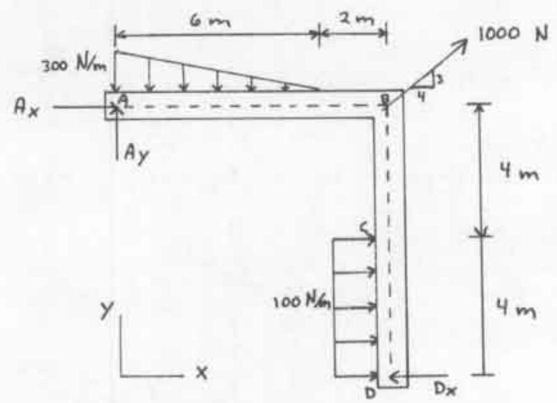


Figure b

By Fig. b,

$$\Sigma F_y = -\tfrac{1}{2}(300)(6) + A_y + \tfrac{3}{5}(1000) = 0$$

$$A_y = 300 \text{ N}$$

$$\circlearrowleft \Sigma M_A = -D_x(8) + 100(4)(6) + 1000(\tfrac{3}{5})(8) - \tfrac{1}{2}(300)(6)(2) = 0$$

$$D_x = 675 \text{ N}$$

$$\Sigma F_x = A_x - D_x + 100(4) + \tfrac{4}{5}(1000) = 0$$

$$A_x = -525 \text{ N}$$

The shear, bending-moment, and normal force diagrams may be drawn using the rules of Sections 11.4 and 11.5 or by deriving equations for the vertical and horizontal legs of the beam as follows.

(Continued)

11.66 Cont.

For the vertical leg, by Fig. c, for $0 \le y \le 4$ m,

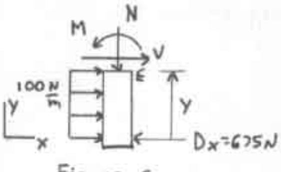


Figure c

$\Sigma F_x = V + 100(y) - 675 = 0$

$V = 675 - 100y$ (N); $0 \le y \le 4$ m (a)

$\Sigma F_y = -N = 0$; $N = 0$; $0 \le y \le 4$ m (b)

$\circlearrowleft \Sigma M_E = M + 100\,\frac{y^2}{2} - 675\,y = 0$

$M = 675y - 50y^2$ (N·m); $0 \le y \le 4$m (c)

For $4 \le y \le 8$ m, by Fig. d,

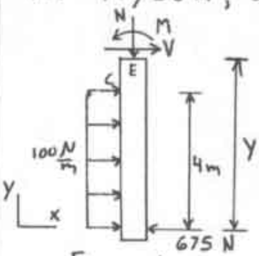


Figure d

$\Sigma F_x = V + 100(4) - 675 = 0$

$V = 275$ N; $4 \le y \le 8$ m (d)

$\Sigma F_y = -N = 0$; $N = 0$; $4 \le y \le 8$ m (e)

$\circlearrowleft \Sigma M_E = M + 100(4)(y-2) - 675\,y = 0$

$M = 275y + 800$ (N·m); $4 \le y \le 8$m (f)

By Eqs. (a), (b), (c), (d), (e), and (f), the shear, bending-moment, and normal force diagrams for the vertical leg may be constructed (see Fig. e)

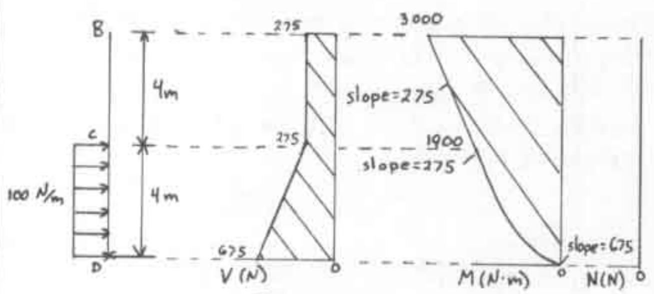


Figure e

For the horizontal leg, by Fig. f, for $6 \le x \le 8$ m

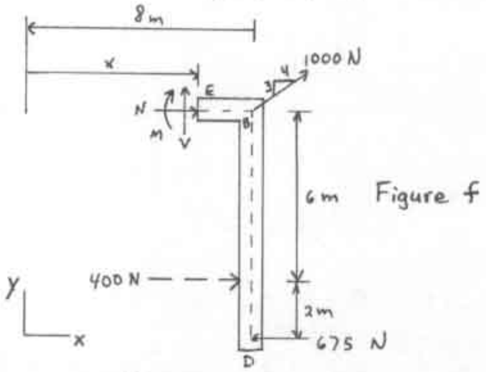


Figure f

$\Sigma F_x = N + 1000\left(\frac{4}{5}\right) + 400 - 675 = 0$

$N = -525$ N; $6 \le x \le 8$ m (g)

$\Sigma F_y = V + 1000\left(\frac{3}{5}\right) = 0$; $V = -600$ N; $6 \le x \le 8$ m (h)

$\circlearrowleft \Sigma M_E = M - 1000\left(\frac{3}{5}\right)(8-x) - 400(6) + 675(8) = 0$

$M = 1800 - 600x$ (N·m); $6 \le x \le 8$ m (i)

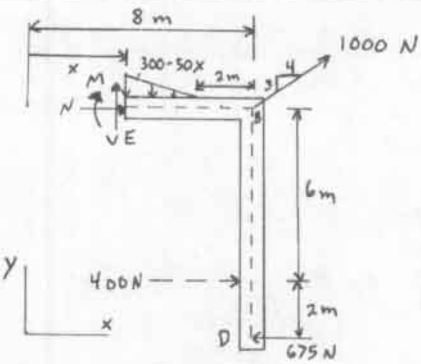


Figure g

For $0 \le x \le 6$ m, by Fig. g,

$\Sigma F_x = N + 1000\left(\frac{4}{5}\right) + 400 - 675 = 0$

$N = -525$ (N); $0 \le x \le 6$ m (j)

$\Sigma F_y = V + 1000\left(\frac{3}{5}\right) - \frac{1}{2}(300 - 50x)(6-x) = 0$

$V = 300 - 300x + 25x^2$ (N); $0 \le x \le 6$m (k)

$\circlearrowleft \Sigma M_E = M + \frac{1}{2}(300-50x)(6-x)\left(\frac{6-x}{3}\right) - 600(8-x) - 400(6) + 675(8) = 0$

$M = -\frac{1}{6}(300-50x)(6-x)^2 + 600(8-x) - 3000$ (N·m) (ℓ)

$0 \le x \le 6$m

By Eqs. (g), (h), (i), (j), (k), and (ℓ), the shear, bending-moment, and normal force diagrams for the horizontal leg may be constructed (see Fig. h).

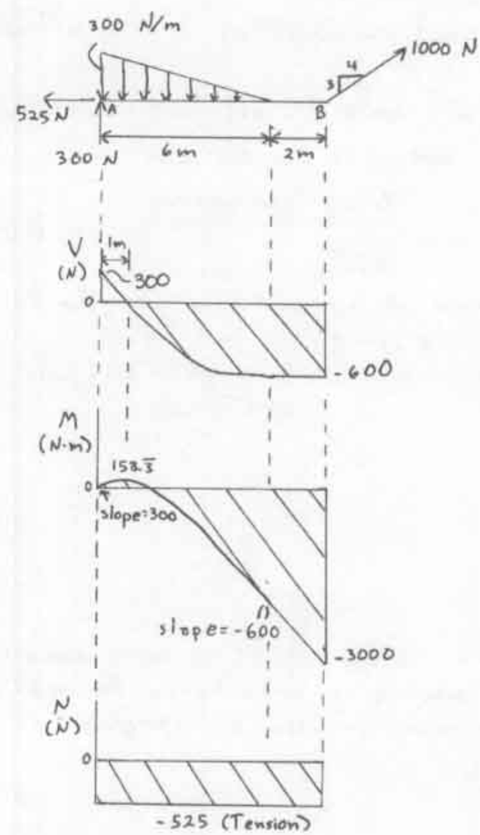


Figure h

11.67

Given: The airplane wing loaded as shown in Fig. a.

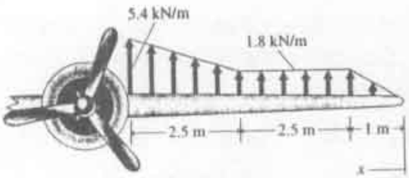


Figure a

a) Derive expressions for the shear and bending-moment in terms of x.

b) Construct the shear and bending-moment diagrams for the wing

Solution: Start at $x = 6$ m (the wing tip). The free-body diagram of the wing section for $5 < x \le 6$ m is shown in Fig. b

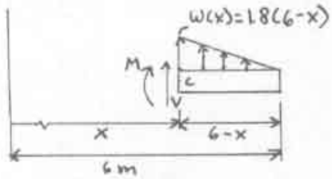


Figure b

By Fig. b,

$$\Sigma F_y = V + \frac{(1.8)(6-x)(6-x)}{2} = 0$$

or

$$V = -0.90(6-x)^2 \text{ (kN)}; \; 5 < x \le 6 \text{ m} \qquad \text{(a)}$$

$$\circlearrowleft + \Sigma M_c = \tfrac{1}{2}[(1.8)(6-x)(6-x)]\left(\frac{6-x}{3}\right) - M = 0$$

or $M = 0.30(6-x)^3$ (kN·m); $5 < x \le 6$ m (b)

The free-body diagram of the wing section for $2.5 \le x \le 5$ m is shown in Fig. c.

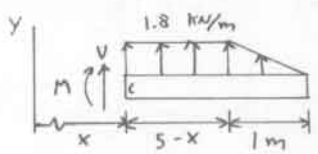


Figure c

By Fig. c,

$$\Sigma F_y = V + 1.8(5-x) + \tfrac{1}{2}(1.8)(1) = 0$$

$$V = 1.8x - 9.9 \text{ (kN)}; \; 2.5 \le x \le 5 \text{ m} \qquad \text{(c)}$$

$$\circlearrowleft + \Sigma M_c = \tfrac{1}{2}(1.8)(1)(5-x+\tfrac{1}{3}) + 1.8(5-x)\left(\frac{5-x}{2}\right) - M = 0$$

$$M = 0.9[(5-x)^2 + (5-x)] + 0.3 \text{ (kN·m)}; \; 2.5 \le x \le 5 \text{ m} \qquad \text{(d)}$$

The free-body diagram of the wing section for $0 \le x \le 2.5$ m is shown in Fig. d.

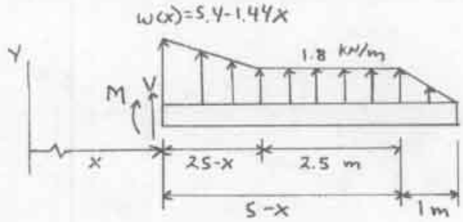


Figure d

By Fig. d,

$$\Sigma F_y = V + \tfrac{1}{2}[(5.4-1.44x)-1.8](2.5-x) + (1.8)(5-x) + \tfrac{1}{2}(1.8)(1) = 0$$

or

$$V = -(1.8-0.72x)(2.5-x) - (1.8)(5-x) - 0.9 \text{ (kN)} \qquad \text{(e)}$$

$$0 \le x \le 2.5 \text{ m}$$

$$\circlearrowleft \Sigma M_c = (1.8-0.72x)(2.5-x)\left(\frac{2.5-x}{3}\right) + (1.8)(5-x)\left(\frac{5-x}{2}\right) + 0.9(5-x+\tfrac{1}{3}) - M = 0$$

or

$$M = (0.6-0.24x)(2.5-x)^2 + 0.9[(5-x)^2 + (5-x)] + 0.30 \text{ (kN·m)} \qquad \text{(f)}$$

$$0 \le x \le 2.5 \text{ m}$$

By Eqs. (a), (b), (c), (d), (e), and (f), we may construct the shear and moment diagrams of the wing (see Fig. e).

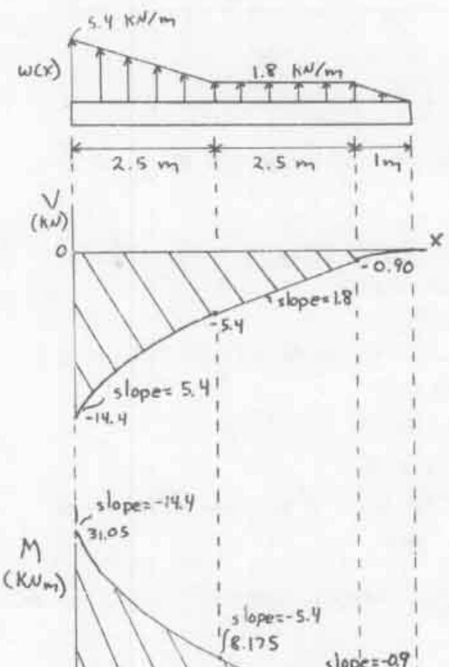


Figure e

11.68

Given: The airplane wing loaded as shown in Fig. a.

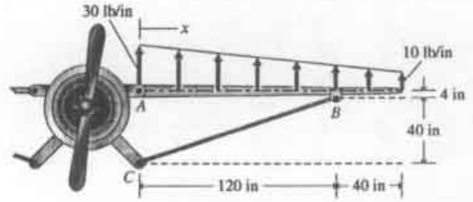


Figure a

a) Express the shear, bending-moment, and the normal force in the wing in terms of x.

b) Construct the shear, bending-moment, and normal force diagrams for the wing.

Solution:

a) By the free-body diagram of the wing (Fig. b) and (Fig. a),

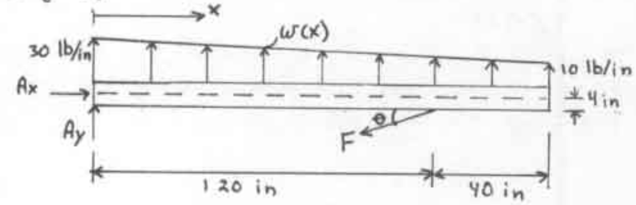


Figure b

$\tan \theta = \frac{40}{120} = \frac{1}{3}$

so, $\sin \theta = \frac{1}{\sqrt{10}} = 0.3162$ and $\cos \theta = \frac{3}{\sqrt{10}} = 0.9487$

also, by Fig. b,

$\Sigma M_A = -F\cos\theta(4) - F\sin\theta(120) + \frac{1}{2}(30-10)(160)(\frac{1}{3})(160) + 10(160)(80) = 0$

$F = 5110.8$ lb

$\Sigma F_y = -5110.8 \sin\theta + A_y + \frac{1}{2}(30-10)(160) + 10(160) = 0$

$A_y = -1583.8$ lb

$\Sigma F_x = -F\cos\theta + A_x = 0$

$A_x = 4848.6$ lb

The equation for $w(x)$ (Fig. b) is determined by the conditions

$w(x) = ax + b$

$w(0) = 30$ lb/in Hence, $w(x) = 30 - 0.125x$ (a)

$w(160) = 10$ lb/in

With Eq. (a), the free-body diagram of the wing element $0 \le x \le 120$ in is shown in Fig. c

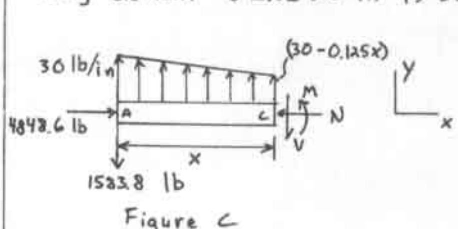


Figure c

By Fig. c,

$\Sigma F_y = \frac{1}{2}[30-(30-0.125x)](x) + (30-0.125x)(x) - 1583.8 - V = 0$

or

$V = -0.0625x^2 + 30x - 1583.8$ (lb); $0 \le x \le 120$ in (b)

$\Sigma M_C = M - \frac{1}{2}[30-(30-0.125x)](x)(\frac{2x}{3}) - (30-0.125x)(x)(\frac{x}{2}) + 1583.8(x) = 0$

or

$M = -0.02083x^3 + 15x^2 - 1583.8x$ (lb·in); $0 \le x \le 120$ in (c)

$\Sigma F_x = 4848.6 - N = 0$; or $N = 4848.6$ lb; $0 \le x \le 120$ in (d)

Alternatively M may be determined by

$M = \int_0^x V(\xi)\,d\xi = \int_0^x [-0.0625\xi^2 + 30\xi - 1583.8]\,d\xi$

or

$M = -0.02083x^3 + 15x^2 - 1583.8x$ (lb·in)

For $120 \le x \le 160$ in, it is convenient to consider a free-body diagram of the right section of the wing (Fig. d) for $x > 120$ in.

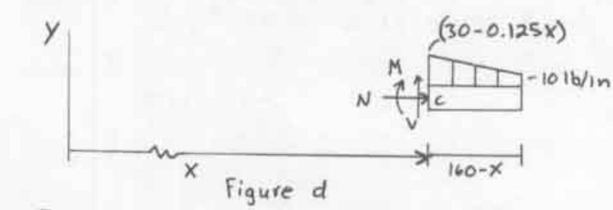


Figure d

By Fig. d,

$\Sigma F_y = V + \frac{1}{2}[(30-0.125x)-10](160-x) + (10)(160-x) = 0$

or $V = -(20-0.0625x)(160-x)$ (lb); $120 \le x \le 160$ in (e)

$\Sigma M_C = M - \frac{1}{2}[(30-0.125x)-10](160-x)(\frac{160-x}{3}) - 10(160-x)(\frac{160-x}{2}) = 0$

or

$M = (8.333-0.02083x)(160-x)^2$ (lb·in); $120 \le x \le 160$ in (f)

$\Sigma F_x = N = 0$; $120 \le x \le 160$ in (g)

b) The shear, bending-moment, and normal force diagrams may be constructed using Eqs. (b), (c), (d), (e), (f), and (g); see Fig. e

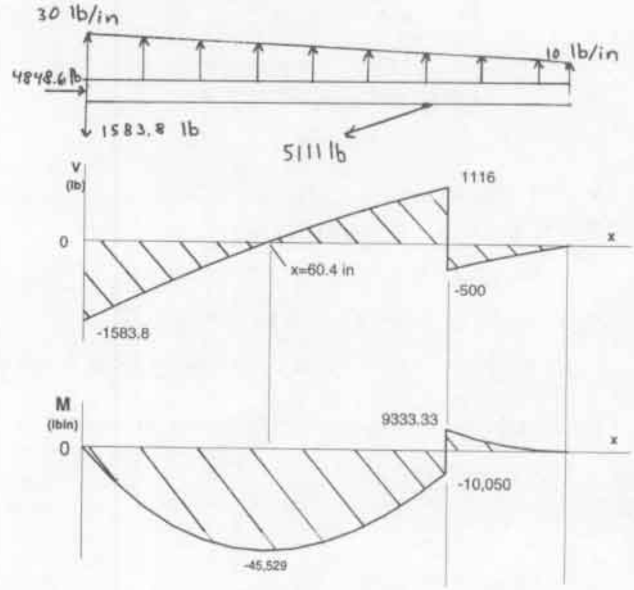


Figure e (Continued)

11.68 Cont.

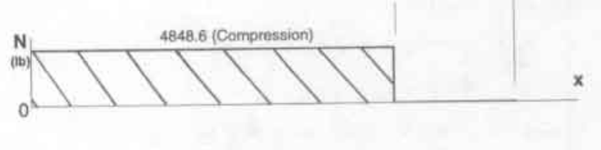


Figure e

11.69

Given: An army tank that weighs 20 metric tons and has a track span of 5 m moves slowly across an arch bridge in France (Fig. a)

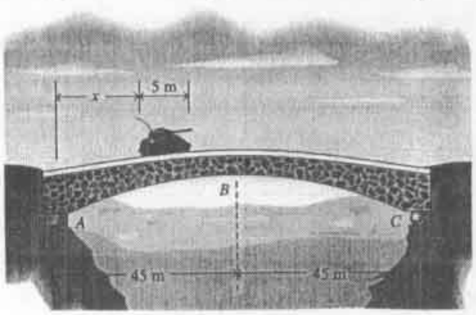


Figure a

a) Derive formulas for the support reactions due to the tank's weight at A and C as functions of x, the position of the tank from support A.

b) Derive formulas for the shear force at A and the bending-moment at B due to the tank's weight as functions of X, for $0 \le x \le 85$ m.

c) Plot the reactions at A and C, the shear force at A, and the bending-moment at B as functions of X, using the formulas derived in parts a and b.

d) The bridge can additionally support a maximum shear of 200 kN at A and a maximum bending-moment of 4 MN·m at B due to the tank's weight. Can the tank cross the bridge safely? Explain.

Solution:

a) Figure b is a simplified free-body diagram of the arch bridge (assumed to be flat since the arch is apparently small).

Note that

$$(20 \text{ metric ton})\left(\frac{1000 \text{ kg}}{1 \text{ metric ton}}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 196.2 \text{ kN}$$

Therefore,

$$w = \frac{(196.2)}{(5)} = 39.24 \text{ kN/m}$$

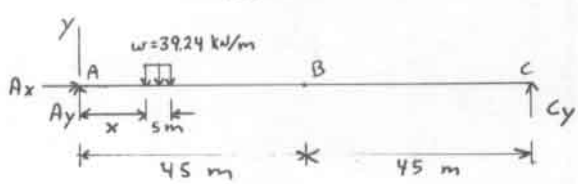


Figure b

Hence, the tank's weight is distributed uniformly ($w = 39.24$ kN/m) over a length of 5 m. By Fig. b, for $0 \le x \le 85$ m,

$$\Sigma F_x = A_x = 0$$

$$\Sigma F_y = A_y + C_y - (5)(39.24) = 0 \qquad (a)$$

$$\circlearrowleft \Sigma M_A = (5)(39.24)\left(x + \tfrac{5}{2}\right) - 90(C_y) = 0$$

The solution of Eqs. (a) is

$$\underline{A_x = 0},$$

$$\underline{\underline{A_y = 190.75 - 2.18x}} \quad (\text{kN}) \qquad (b)$$

$$\underline{\underline{C_y = 2.18x + 5.45}} \quad (\text{kN})$$

$$0 \le x \le 85 \text{ m}$$

b) To determine the shear at A as a function of x for $0 \le x \le 85$ m, consider the free-body diagram in Fig. c

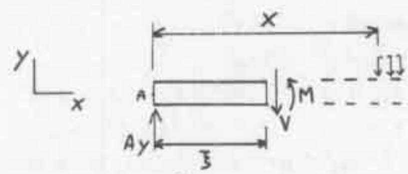


Figure c

$$\Sigma F_y = A_y - V = 0$$

or $V = A_y \qquad 0 \le \xi \le x$

Hence, with Eq. (b), at $\xi = 0$, $V = V_A$,

$$V_A = 196.2 - (2.18x + 5.45) \text{ kN}$$

or $\underline{V_A = 190.75 - 2.18x}$ (kN) (c)

To determine the bending moment at B as a function of x, consider first the free-body diagram of the left-half of the beam (Fig. d), for the range $0 \le x \le 40$ m,

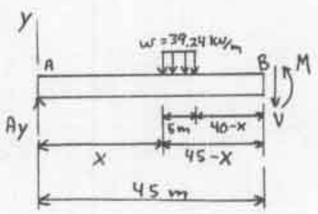


Figure d

By Fig. d,

$$\circlearrowleft \Sigma M_B = M + (5)(39.4)\left[(40-x) + \tfrac{5}{2}\right] - 45A_y = 0$$

Hence, with A_y given by Eq. (b),

$$\underline{M = 98.1x + 245.25} \ (\text{kN·m}); \ 0 \le x \le 40 \text{ m} \qquad (d)$$

Next consider the range $40 \le x \le 45$ m (Fig. e).

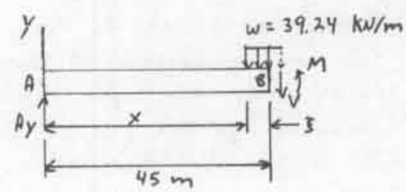


Figure e

(Continued)

11.69 Cont.

By Fig. e, $x + \bar{z} = 45$ m, or $\bar{z} = 45 - x$

Hence,

$\circlearrowleft \Sigma M_B = M + (39.24)(\bar{z})\left(\frac{\bar{z}}{2}\right) - 45 A_y = 0$

or

$M = 45\,A_y - \frac{1}{2}(39.24)(45 - x)^2$

with A_y given by Eq. (b), we obtain

$\underline{M = -19.62x^2 + 1667.7x - 31{,}147}$ (kN·m); $40 \le x \le 45$ m (e)

Finally, for the range $45 \le x \le 85$ m (Fig. f), the moment at B is given by

$\Sigma M_B = M + (5)(39.24)\left[(x - 45) + \frac{5}{2}\right] - 45 C_y = 0$

or $M = 45\,C_y - 196.2\,(x - 42.5)$

with C_y given by Eq. (b), we find

$$\underline{M = 8583.75 - 98.1x} \qquad (f)$$

Summary: By Eqs. (b) through (f),

$A_y = 190.75 - 2.18x$ (kN)

$C_y = 2.18x + 5.45$ (kN)

$V_A = 190.75 - 2.18x$ (kN); $0 \le x \le 85$ m

$M_B = 98.1x + 245.25$ (kN·m); $0 \le x \le 40$ m

$M_B = -19.62x^2 + 1667.7x - 31{,}147$ (kN·m); $40 \le x \le 45$ m

$M_B = -98.1x + 8583.75$ (kN·m); $45 \le x \le 85$ m

c)

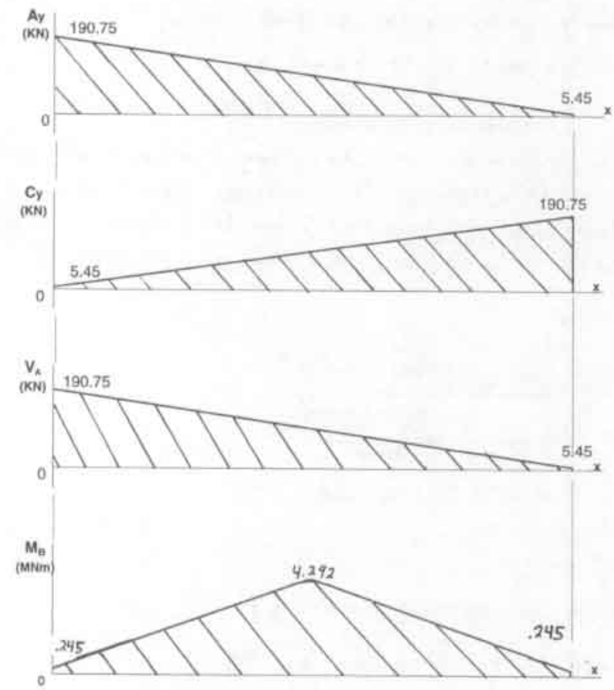


Figure f

d) By the plot of shear at A (also by Eq. b), we see that the maximum shear at A does not exceed 200 kN. Hence, the support at A will not fail. However, by the plot of the bending-moment, we see that the bending-moment exceeds 4 MN·m at midspan (B)

Also, by Eq. (e), the maximum moment occurs when

$\frac{dM}{dx} = -39.24x + 1667.7 = 0$

or when, $x = 42.5$ m

Hence with $x = 42.5$ m, Eq. (e) yields,

$M_{max} = 4292$ kN·m $= 4.292$ MNm > 4 MNm

Thus, the weight of the tank will cause the bridge to fail.

11.70

Given: The beam loaded as shown in Fig. a

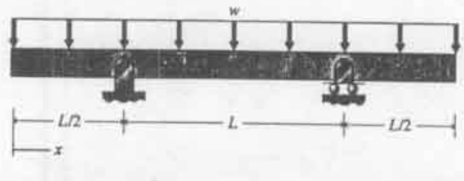


Figure a

Move the left support to the left end of the beam, and locate the right support so that the magnitudes of the maximum positive and negative bending-moments in the beam are equal

Solution:

First move the left support to the left end of the beam (Fig. b), determine the support reactions and plot the bending-moment diagram.

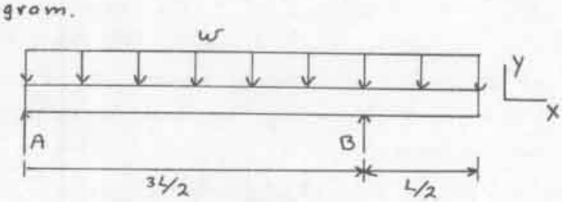


Figure b

By Fig. b,

$\Sigma F_y = A + B - w(2L) = 0$ (a)

$\circlearrowleft \Sigma M_A = B\left(\frac{3L}{2}\right) - w(2L)(L) = 0$

The Solution of Eqs. (a) is

$A = \frac{2}{3} wL$

$B = \frac{4}{3} wL$

For $0 \le x \le \frac{3L}{2}$, the bending-moment is given by (see Fig. c)

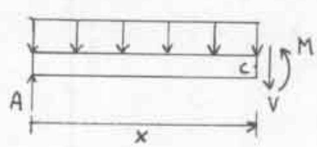


Figure c

(Continued)

11.70 Cont

$\circlearrowleft + \Sigma M_c = M + w\,x\left(\frac{x}{2}\right) - Ax = 0$

or $M = \frac{2}{3} wLx - \frac{wx^2}{2}$ (b)

For $x > \frac{3L}{2}$ (Fig. d), the bending-moment is given by

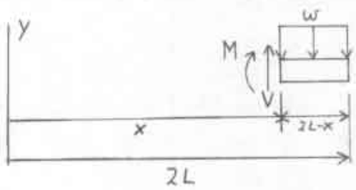


Figure d

$\circlearrowleft + \Sigma M_C = M + (w)(2L-x)\left(\frac{2L-x}{2}\right) = 0$

or $M = -\frac{1}{2} w\,(2L-x)$ (c)

By Eqs. (b) and (c), we may plot the bending-moment (Fig. e).

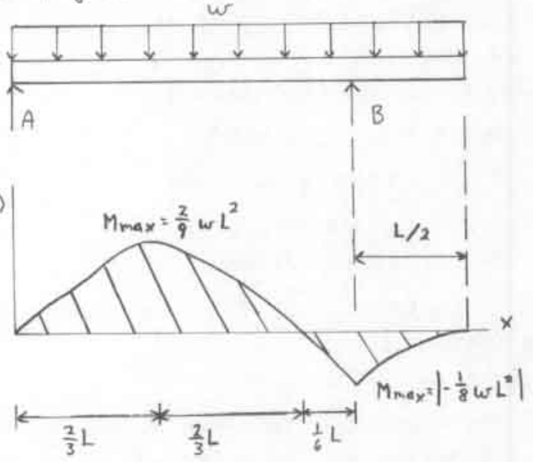


Figure e

Since the magnitude of the positive bending-moment is larger than the magnitude of the negative bending-moment, support B must be moved to the left to increase the negative bending-moment and decrease the positive bending-moment. Hence, let the support B be located at a distance $d < 3L/2$ from the left end of the beam (Fig. f)

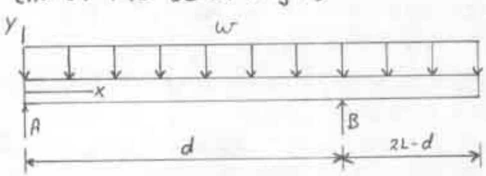


Figure f

By Fig. f,

$\Sigma F_y = A + B - 2wL = 0$

$\circlearrowleft \Sigma M_A = Bd - (2wL)(L) = 0$ (d)

The solution of Eqs. (d) is

$A = \frac{2wL}{d}(d-L)$

$B = \frac{2wL^2}{d}$ (e)

For $0 \le x \le d$, the bending-moment M is determined by (see Fig. g)

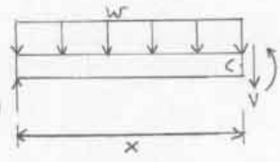


Figure g

$\circlearrowleft \Sigma M_C = M + wx\left(\frac{x}{2}\right) - Ax = 0$

Substitution for A by Eq. (e) yields

$M = \frac{2wL}{d}(d-L)x - \frac{1}{2}wx^2$ (kN·m); $0 \le x \le d$ (f)

For $d \le x \le 2L$, the bending-moment M is determined by (see Fig. h)

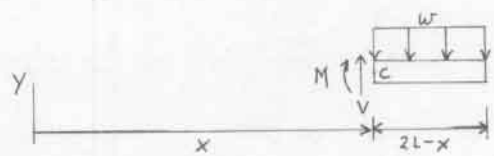


Figure h

$\circlearrowleft \Sigma M_c = M + w\,(2L-x)\frac{(2L-x)}{2} = 0$

or $M = -\frac{1}{2} w\,(2L-x)^2$ (kN·m); $d \le x \le 2L$ (g)

The maximum positive moment [Eq. (f)] occurs when $\frac{dM}{dx} = 0$ or when $x = \frac{2L}{d}(d-L)$. Hence, by Eq. (f)

$$M_{max_{(pos)}} = \frac{1}{2} w \left[\frac{2L(d-L)}{d}\right]^2 \quad (h)$$

and by Eq (g) the maximum negative moment occurs at $x = d$. Hence,

$$M_{max_{(neg)}} = -\frac{1}{2} w\,(2L-d)^2 \quad (i)$$

Equating the magnitudes of the maximum positive and negative moments, we obtain by Eqs. (h) and (i)

$$2L(d-L) = d\,(2L-d)$$

or

$$d = \sqrt{2}\,L$$

Hence, by Eq. (h),

$$M_{max_{(pos)}} = wL^2(\sqrt{2}-1)^2 = 0.1716\,wL^2$$

and by Eq. (h)

$$\left|M_{max_{(neg)}}\right| = wL^2(\sqrt{2}-1)^2 = 0.1716\,wL^2$$

The bending-moment diagram is shown in Fig. j.

(Continued)

11.70 Cont.

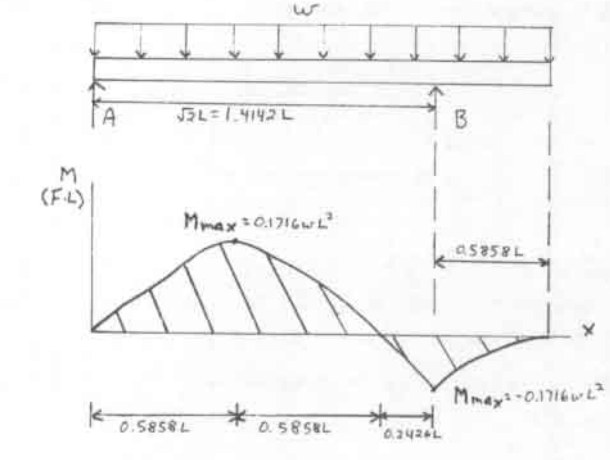


Figure j

11.71

Given: The airplane wing loaded as shown in Fig. a. The design engineer asks you to consider methods of reducing the moment at the root of the wing by 50%

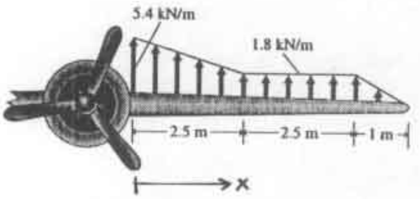


Figure a

a) Consider how to reduce the moment by 50% without changing the wing or the aerodynamic forces acting on the wing.

b) How would you implement your solution to part a.

Solution:

a) First, we need to determine the bending-moment with the load as shown in Fig. a. Also, we will assume that the loads shown are typical of the loads sustained during operation of the airplane. The free-body diagram of the wing is shown in Fig. b.

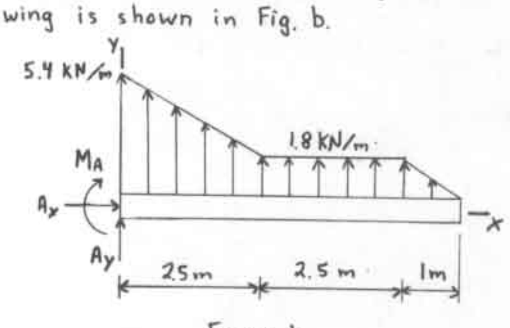


Figure b

By Fig. b,

$$\circlearrowleft + \Sigma M_A = -M_A + \tfrac{1}{2}(1.8)(1)(5\tfrac{1}{3}) + (1.8)(5)(2.5) + \tfrac{1}{2}(3.6)(2.5)(\tfrac{1}{3})(2.5) = 0$$

or

$$M_A = 31.05 \text{ kN·m}$$

Therefore, the moment must be reduced by 15.525 kNm.

Method 1.

For simplicity assume that you may place a weight W_1 at 2.5 m from A. Then, solve for W_1 from

$(2.5)(W_1) = 15.525$ kN·m. Thus, $W_1 = 6.21$ kN. This load is rather large and may require reinforcement of the wing to support the load

Method 2

Consider a uniformly distributed load w (kN/m) placed inside the wing starting at $x=0$ and extending to $x=5$ m. Then, w is determined by

$$15.525 \text{ kNm} = w(5)(\tfrac{5}{2})$$

or $$w = 1.242 \text{ kN/m}$$

Hence, the total required load is

$$W_2 = 5w = 6.21 \text{ kN}$$

This is the same as the total load required in Method 1

Method 3

Place a concentrated load W_3 as near the wing tip as possible, say at $x = 5.5$ m.

Then, $$(5.5)W_3 = 15.525 \text{ kNm}$$

or $$W_3 = 2.823 \text{ kN}$$

A reduction in the required load. It may be difficult to place this much load inside the wing near its tip.

b) Apparently, adding weight, as in Methods 1, 2, and 3, can reduce the moment by 50%. However, adding such weights to the two wings (left and right) increases fuel consumption and may induce control problems for the airplane.

One might consider adding a diagonal member (see Problem 11.68, Fig. P11.68). However, it does not appear feasible for this airplane's frame and may cause aerodynamic problems. A possible solution may be to reinforce the wing internally, near, and at the root, rather than reducing the bending-moment by 50%

(Continued)

11.71 Cont.

Finally, to implement the adding of weight to the wing, the skin of the wing would have to be removed and then replaced after the weight has been added. The choice of Methods 1, 2, or 3 would also have to be discussed with the design engineer.

11.72

Given: The maximum bending moment in the airplane wing shown in Fig. a is negative, has a magnitude of 45,530 lbin, and acts at 60.4 in to the right of support A. The design engineer asks you to redesign the wing supports so that the maximum moment is reduced to 35,000 lbin and occurs between the supports.

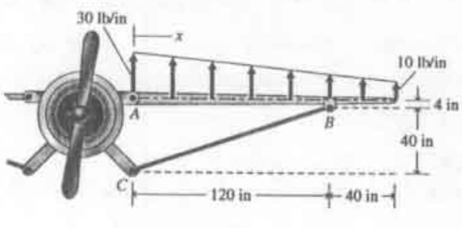


Figure a

Adjust the location of support B so that the design requirement is met.

Solution: Consider the free-body diagram of the wing (Fig. b), in which the support B is located a distance d from support A.

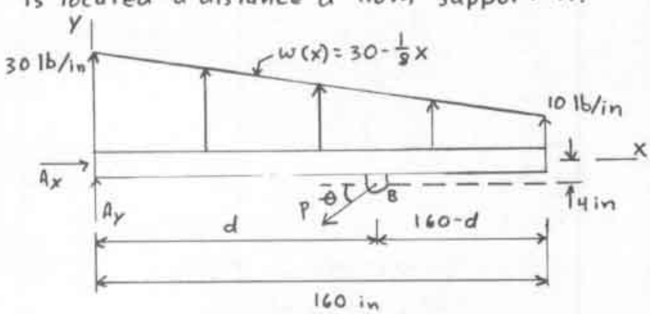
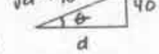


Figure b

By Figs. a and b, $\tan\theta = \frac{40}{d}$;



Hence, $\sin\theta = \frac{40}{\sqrt{d^2+40^2}}$ (a)

$\cos\theta = \frac{d}{\sqrt{d^2+40^2}}$

By Fig. b,

$\circlearrowleft \Sigma M_A = -(P\cos\theta)(4) - (P\sin\theta)d + \int_0^{160} [w(x)]x\,dx = 0$

with $w(x) = 30 - \frac{1}{8}x$ and Eqs (a), we have

$\frac{44Pd}{\sqrt{d^2+40^2}} = \int_0^{160} (30x - \frac{x^2}{8})dx = 213,333.\overline{3}$

or $P = \frac{4848.\overline{48}\sqrt{d^2+40^2}}{d}$ (lb) (b)

$\Sigma F_x = A_x - P\cos\theta = 0$

or with Eq. (b), $A_x = 4848.\overline{48}$ (lb)

$\Sigma F_y = A_y - P\sin\theta + \frac{(30+10)}{2}(160) = 0$

or $A_y = \frac{193939.\overline{39}}{d} - 3200$ (lb) (c)

Next consider the free-body diagram of the wing section for $0 \le x \le d$ (Fig. c)

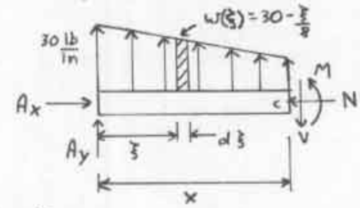


Figure c

By Fig. c,

$\circlearrowleft \Sigma M_c = M - \int_0^x [w(\xi)](x-\xi)d\xi - (A_y)(x) = 0$

or $M = (\frac{193939.\overline{39}}{d} - 3200)x + 15x^2 - \frac{x^3}{48}$ (d)

The moment M is a maximum when $\frac{dM}{dx} = 0$ Therefore, by Eq. (d),

$x^2 - 240x + 25,600 - \frac{1551515.\overline{15}}{d} = 0$ (e)

With $M = -35000$ lb·in, Eqs. (d) and (e) are two equations in x and d. The solution of Eqs. (d) and (e) is

$x = 52.24$ in
$d = 107.55$ in (f)

Therefore, the magnitude of the maximum moment between the supports is 35,000 lb·in at $x = 52.24$ in, with the strut connected to the wing at $d = 107.55$ in (Fig. d)

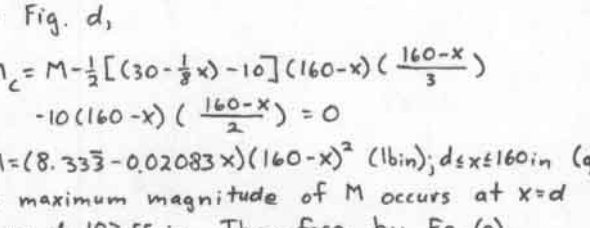


Figure d

By Fig. d,

$\circlearrowleft \Sigma M_c = M - \frac{1}{2}[(30 - \frac{1}{8}x) - 10](160-x)(\frac{160-x}{3}) - 10(160-x)(\frac{160-x}{2}) = 0$

or $M = (8.33\overline{3} - 0.02083x)(160-x)^2$ (lbin); $d \le x \le 160$ in (g)

The maximum magnitude of M occurs at $x = d$ where $d = 107.55$ in. Therefore, by Eq. (g),

$M_{max} = 16,762$ lb·in ; $x = d = 107.55$ in

Since 16,762 lb·in < 35,000 lb·in, the moment in the wing does not exceed 35,000 lbin. Hence, with the support at B located at $d = 107.55$ in from support A, the design requirement is satisfied.

11.78

Given: A pulley transmits a torque of 90 N·m to its shaft (Fig. a)

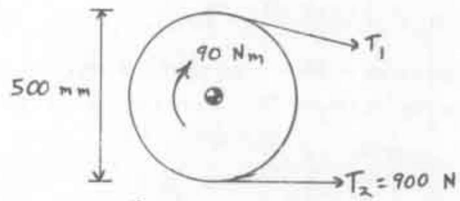


Figure a

Determine T_1

Solution:

By Fig. a, $T_1(0.250) - T_2(0.250) = 90$ N·m

Therefore,

$$T_1 = \frac{[(900)(0.250) + 90]}{0.250} = 1260 \text{ N}$$

11.79

Given: A semicircular cantilever beam in the xy plane is subject to a couple T_o (Fig. a)

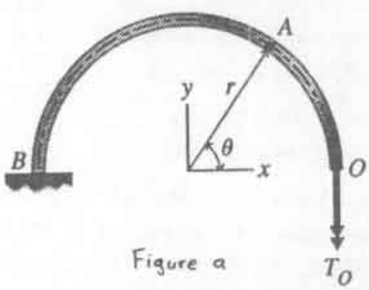


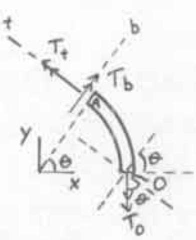
Figure a

Derive formulas for the bending-moment and the twisting moment in the beam in terms of θ.

Solution:

Consider the free-body diagram of the segment OA of the beam (Fig. b).

Figure b



By Fig. b,

$\Sigma M_b = T_b - T_o \sin\theta = 0$; $\underline{\underline{T_b = T_o \sin\theta}}$

$\Sigma M_t = T_t - T_o \cos\theta = 0$; $\underline{\underline{T_t = T_o \cos\theta}}$

11.80

Given: The cantilever L-bar ABC loaded as shown in Fig. a

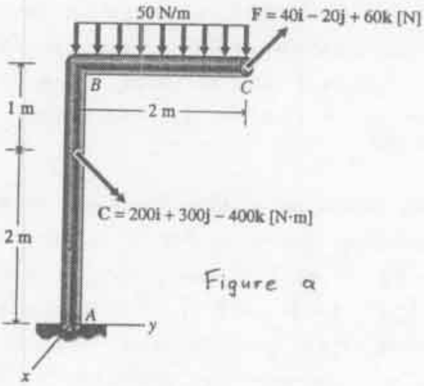


Figure a

a) Draw a free-body diagram of the L-bar ABC.

b) Draw shear, bending-moment, and normal force diagrams for components acting in the (yz) plane

Solution:

a) Figure b is the free-body diagram of the bar

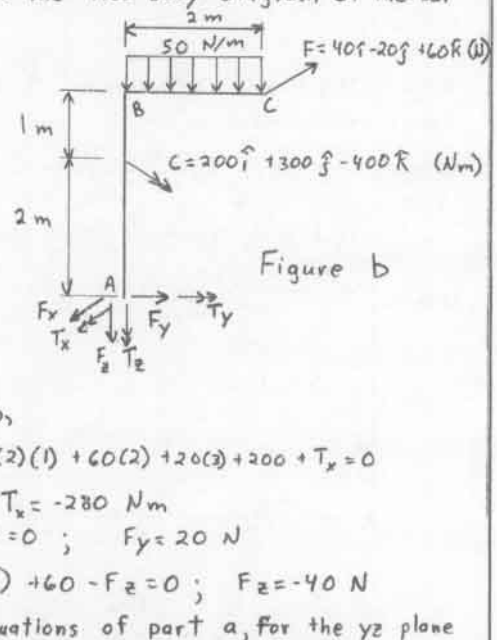


Figure b

By Fig. b,

$\circlearrowleft \Sigma M_{x_A} = -50(2)(1) + 60(2) + 20(3) + 200 + T_x = 0$

$T_x = -280$ Nm

$\Sigma F_y = F_y - 20 = 0$; $F_y = 20$ N

$\Sigma F_z = -50(2) + 60 - F_z = 0$; $F_z = -40$ N

b) By the equations of part a, for the yz plane the shear, bending-moment, and normal force diagrams are shown in Fig. c for the vertical leg and in Fig. d for the horizontal leg

(Continued)

11.80 Cont.

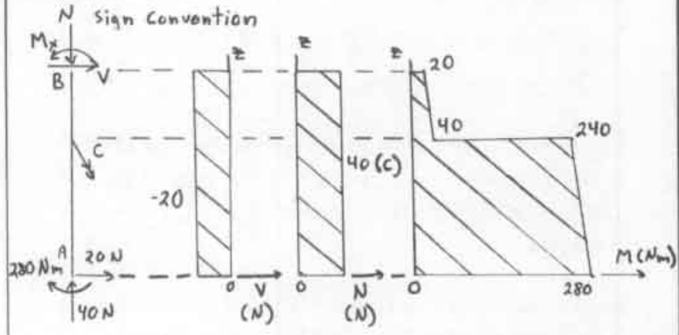


Figure c

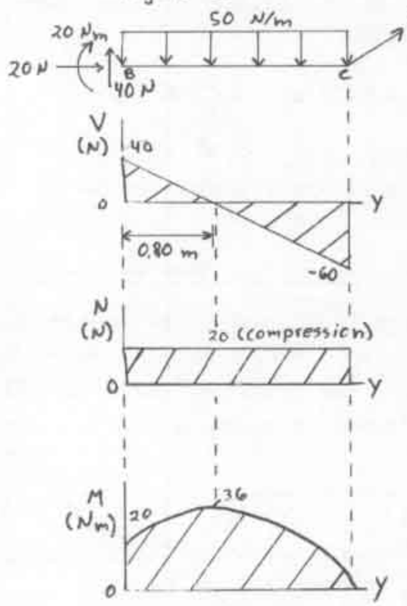


Figure d

12.1

Given: The work required to lift 6 people 100 m on an elevator is 408 kJ.

Find: The average mass and weight per person.

Solution: The work, by Eq (12.3) to overcome gravity is

$$U = \int_0^{100} mg\, dx = 100\, mg = 408 \text{ kJ}$$

Hence, the average mass per person is

$$\underline{\underline{m_{ave} = \tfrac{1}{6} m = (1/6)\left(\frac{408}{100 \times 9.81}\right) = 69.3 \text{ kg}}}$$

So the average weight per person is

$$\underline{\underline{W_{ave} = m_{ave}\, g = (69.3)(9.81) = 680 \text{ N}}}$$

12.2

Given: A 500 N force acts on a block that weighs 150 N (Fig a). The coefficient of sliding friction between the block and surface is 0.40.

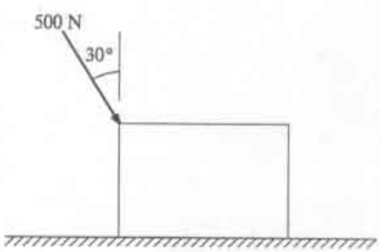


Figure a.

Find: The total work that all the forces perform on the block as it moves 0.5 m to the right.

Solution:

The free-body diagram of the block is shown in Fig b.

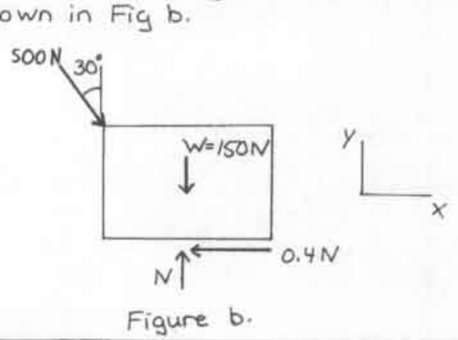


Figure b.

By Fig. b,

$$\Sigma F_x = (500)\sin 30° - 0.4N = ? \qquad (a)$$

$$\Sigma F_y = N - (500)\cos 30° - 150 = 0 \qquad (b)$$

Equation (a) does not provide useful information, since it is not clear that equilibrium in the x direction exists.

Equation (b) yields $N = 583$ N. Hence, $0.4N = (0.4)(583) = 233.2$ N. Then, by Fig. b and Eq (12.3), we have the work as

$$U = \int_0^{2.5} [500 \sin 30° - 233.2]\, dx$$

or

$$\underline{\underline{U = (250 - 233.2) \times \Big|_0^{2.5} = 42 \text{ J}}}$$

12.3

Given: Two tug boats pull a barge 200 m. The tensions in the lines are $T_{AC} = 13.57$ kN and $T_{BC} = 10$ kN. See sketch in Fig. a.

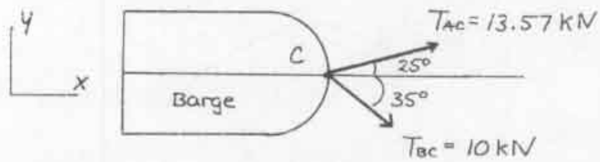


Figure a.

Find: The work done by the tug boats.

Solution:

By Fig a,

$$\Sigma F_y = T_{AC}(\sin 25°) - T_{BC}(\sin 35°) = 0 \text{ kN} \qquad (a)$$

$$\Sigma F_x = T_{AC}(\cos 25°) + T_{BC}(\cos 35°) = 20.49 \text{ kN} = F_x \qquad (b)$$

Hence, the net force is in the x direction. By Eqs (12.3) and (b),

$$U = \int_0^{200} F_x\, dx \quad \text{or} \quad \underline{U = 4098 \text{ kJ}}$$

12.4

Given: A block rests on a smooth horizontal plane and is attached to an unstretched spring with constant k (Fig. a).

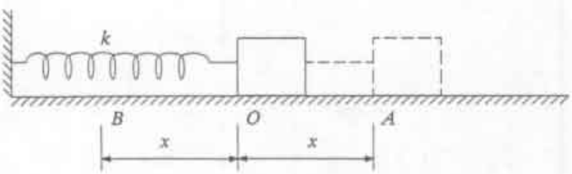


Figure a.

Find: a) The work performed on the block by the spring as the block moves from O to A.

b.) The work performed by the spring as the block moves from O to B.

c.) The work done as the block moves from O to A to B, back to O.

Solution:

a.) The free-body diagram of the block as it is moved from O to A is shown in Fig. b:

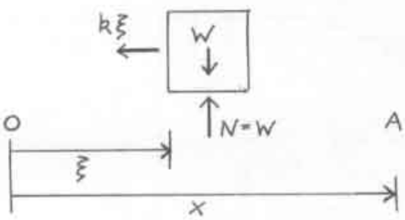


Figure b.

By Eq (12.3), the work performed by the spring from O to A is, with ξ+ to the right,

$$U_{OA} = \int_0^x (-k\xi)\,d\xi = -\tfrac{1}{2}kx^2 \qquad (a)$$

b.) The free-body diagram of the block as it is moved from O to B is shown in Fig. c.

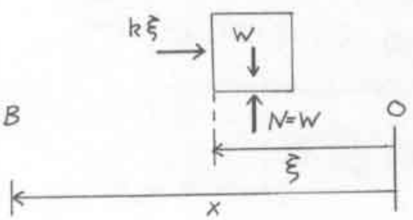


Figure c

By Eq (12.3), the work performed by the spring from O to B is, with ξ+ to the left,

$$U_{OB} = \int_0^x (-k\xi)\,d\xi = -\tfrac{1}{2}kx^2 \qquad (b)$$

c.) Since the block returns to O after moving from O to A to B to O, the net work performed by the spring is zero.

This result is verified as follows:

The work of the spring as the block goes from O to A is given by Eq. (a).

As the block moves from A to O, its free-body diagram is shown in Fig. d.

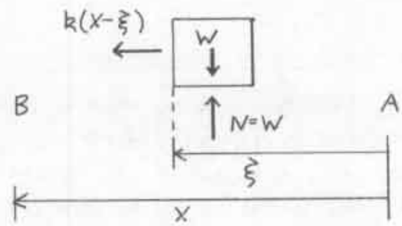


Figure d.

The work done by the spring is, with ξ+ to the left,

$$U_{AO} = \int_0^x k(x-\xi)\,d\xi = -\tfrac{1}{2}k(x-\xi)^2 \Big|_0^x$$

or

$$U_{AO} = \tfrac{1}{2}kx^2 \qquad (c)$$

The work of the spring as the block goes from O to B is given by Eq (b).

As the block moves from B to O, its free-body diagram is shown in Fig. e.

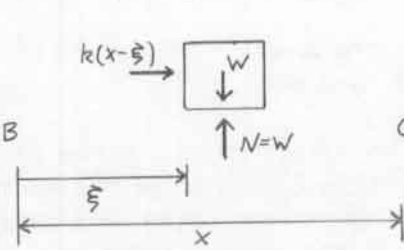


Figure e.

(Continued)

12.4 cont.

The work done by the spring as the block moves from B to O is, with ξ^+ to the right,

$$U_{BO} = \int_0^x k(x-\xi)\,d\xi = -\tfrac{1}{2}k(x-\xi)^2 \Big|_0^x$$

or $\quad U_{BO} = \tfrac{1}{2}kx^2 \qquad$ (d)

The net work from O to A, A to B and B to O is the sum of Eqs. (a), (b), (c, and (d). Thus,

$$U_{O\text{-}O} = U_{OA} + U_{AO} + U_{OB} + U_{BO}$$

or $\underline{\underline{U_{O\text{-}O} = -\tfrac{1}{2}kx^2 + \tfrac{1}{2}kx^2 - \tfrac{1}{2}kx^2 + \tfrac{1}{2}kx^2 = 0}}$

12.5

Given: A linear spring that is hung vertically in a gravity field has a mass m attached to its lower end, causing the spring to stretch a distance d. (Fig. a)

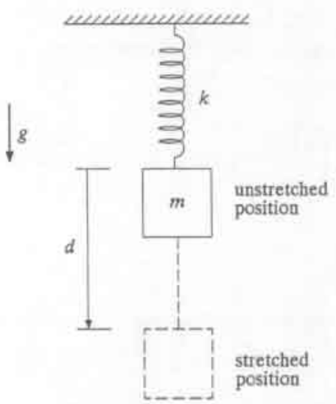


Figure a.

Find: The spring constant k in terms of m, g, and d.

Solution: The system starts at rest at the unstretched position of the spring and ends up at rest at the stretched position of the spring. Hence, the net work on the body is zero. The free-body diagram of the mass displaced a distance x is shown in Fig. b.

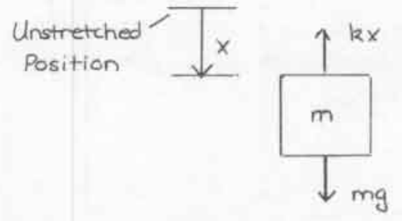


Figure a.

By Eq (12.3) and Fig a., the work done on the body from $x=0$ to $x=d$ is given as

$$U = \int_0^d (mg - kx)\,dx = mgd - \tfrac{1}{2}kd^2 = 0$$

Hence, $\quad \underline{\underline{k = \dfrac{2mg}{d}}}$

12.6

Given: The graph in Fig. a shows the pull of the propellers of an airplane during take-off.

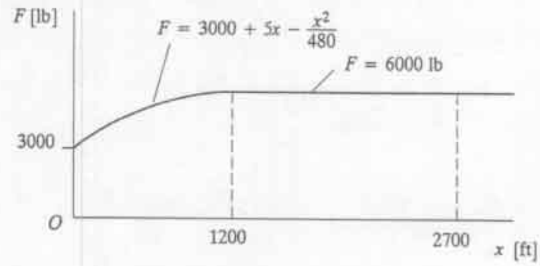


Figure a.

Find: the work done during take-off.

Solution: The work is obtained as two seperate integrals from 0 to 1200 ft. and from 1200 ft to 2700 ft. Thus,

$$U = \int_{x_1}^{x_2} F_1\,dx + \int_{x_2}^{x_3} F_2\,dx$$

$$= \int_0^{1200} \left(3000 + 5x - \frac{x^2}{480}\right) dx + \int_{1200}^{2700} 6000\,dx$$

$$= \left(3000x + \tfrac{5}{2}x^2 - \frac{x^3}{1440}\right)\Big|_0^{1200}$$

$$+ (6000)(2700-1200)$$

Hence,

$$\underline{\underline{U = 1.5 \times 10^7 \text{ ft-lb}}}$$

12.7

<u>Given</u>: Figure a, with the data shown, in which the force F pushes the crates 3 m to the right.

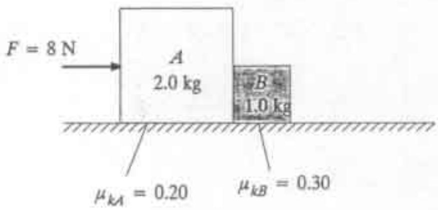


Figure a.

<u>Find:</u> the total work done on the crates

<u>Solution:</u> The free-body diagram of the crate system is shown in Fig b.

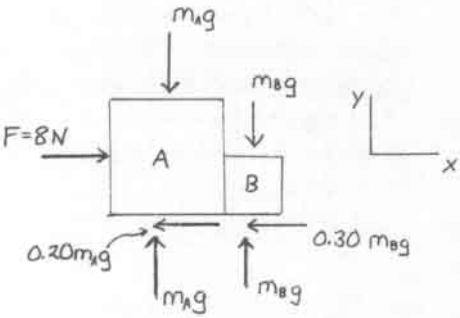


Figure b

$$\Sigma F_x = 8 - (0.20)(2.0)(9.81) - (0.30)(1.0)(9.81)$$

or $\Sigma F_x = 1.133\ N$ (a)

Hence, by Eqs (12.3) and (a), the work done is

$$U = \int_0^3 (\Sigma F_x)\, dx = (1.133)x \Big|_0^3$$

or $\underline{\underline{U = 3.40\ N\cdot m}}$

12.8

<u>Given:</u> A wheel barrow of weight W is subjected to a force F that slowly inclines it to an angle 15° above the horizontal. See Fig. P12.8

<u>Find:</u> the work done by each force in terms of W and F.

<u>Solution:</u> The free-body diagram of the wheel barrow is shown in Fig a.

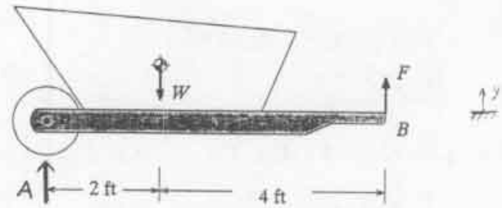


Figure a.

For the work done by the weight:

$$U_W = \int_0^{h_W} F\,dy = \int_0^{2\sin 15°} (-W)\,dy$$

$$\underline{\underline{U_W = -2W\sin 15 = -0.518\,W}}$$

For the work done by the force:

$$U_F = \int_0^{h_F} F\,dy = \int_0^{6\sin 15} F\,dy$$

$$\underline{\underline{U_F = 6F\sin 15° = 1.553\ F}}$$

Force A does no work since it does not undergo a displacement.

12.9

<u>Given:</u> a force in Newtons pulls a body along a floor. The force varies as shown in Fig a.

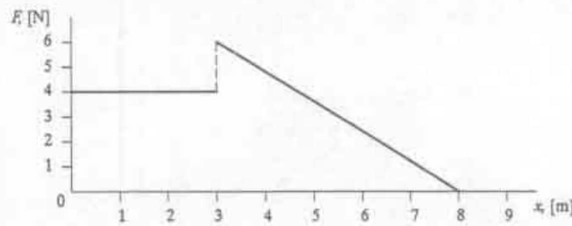


Figure a.

<u>Find</u>: the total work done on the body.

(Continued)

12.9 cont.

Solution: Since the work is equal to the area under the force-distance graph, by Fig. a,

$$U = (3\times 4) + \tfrac{1}{2}(5\times 6)$$

$$\underline{\underline{U = 27\ \text{J}}}$$

Alternatively, by Eq (12.3) and Fig (a),

$$U = \int_0^3 4\,dx + \int_0^8 \left(-\tfrac{6}{5}x + \tfrac{48}{5}\right)dx$$

$$= 12 + \left(\frac{-6x^2}{10} + \frac{48}{5}x\right)\Big|_3^8$$

-or $\underline{\underline{U = 12 + 15 = 27\ \text{J}}}$

12.10

Given: A 30-N crate slides 3 m down a ramp with an incline of 50° and a coefficient of friction $\mu_k = 0.50$ (See Fig a.)

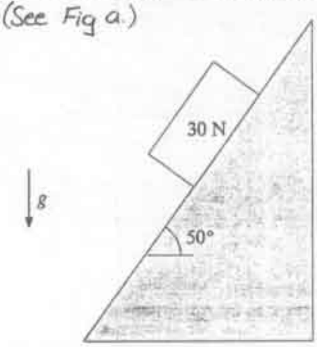


Figure a

Find: The net work done on the crate.

Solution: The free-body diagram of the crate is shown in Fig. b.

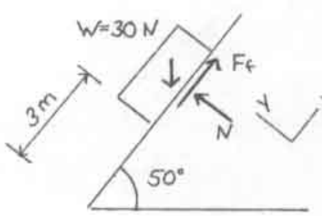


Figure b

By Fig. b,

$$\Sigma F_y = N - 30(\cos 50°) = 0; \quad N = 19.284\ N \quad (a)$$

$$F_f = 0.50\,N; \quad F_f = 9.642\ N \quad (b)$$

The net force that acts on the crate in the +x direction is (See Fig b. and Eq (b).)

$$F = 9.642 - 30\sin 50° = -13.339\ N \quad (c)$$

Hence, by Eqs. (12.3) and (c),

$$U = \int_0^{-3} F\,dx = (-13.339)\,x\Big|_0^{-3}$$

or $\underline{\underline{U = 40.0\ \text{J}}}$

12.11

Given: Two identical bar magnets that weigh 9 N each and have a force (N) of attraction $F = 22\,500/x^2$ between their poles rest on a wooden bench (Fig a.). The coefficients of friction are $\mu_s = \mu_k = 0.40$. Magnet B is cemented to the bench. Magnet A is pushed slowly toward Magnet B, until it starts sliding toward B.

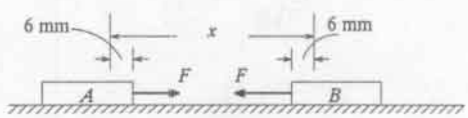


Figure a.

Find The work (in N·mm) performed on magnet A during the sliding.

Solution: The free-body diagram of Magnet A is shown in Fig b:

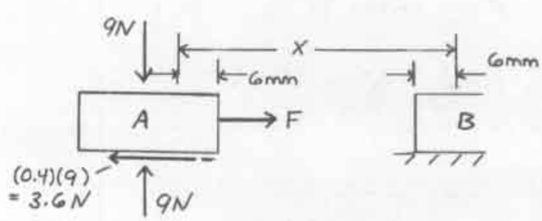


Figure b.

(Continued)

12.11 cont.

At impending motion of magnet A (see Fig B)

$$\sum F_x = 3.6 - F = 0$$

Therefore,

$$F = \frac{22\,500}{x^2} = 3.6 \quad \rightarrow \quad x = 79.06 \text{ mm}$$

The work that magnet B performs on magnet A is the work of the force F exerted on A by B from $x = 79.06$ mm to 12 mm (where A strikes B).

Thus, by Fig b,

$$U_B = -\int_{79.06}^{12} F\,dx = -\int_{79.06}^{12} \frac{22500}{x^2}\,dx$$

or

$$U_B = \frac{22500}{x^2}\Bigg|_{79.06}^{12} = 1590 \text{ N·m}$$

NOTE: Friction also does work on A.

12.12

Given: Three crates resting on a horizontal plane are connected and all have a coefficient of sliding friction $\mu_k = 0.3$. The crates move under the action of an 18 N force (Fig a)

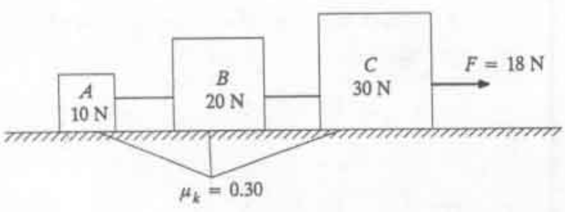


Figure a.

Find: a.) The work done on each crate
b.) The net work done on the system as the force pulls the crates 4 m.

Solution:

a.) The free-body diagram of the crates is shown in Figure b.

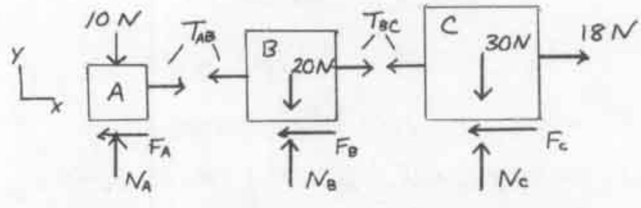


Figure b.

By Fig b,

$$F_A = \mu_k N_A = 3N, \qquad F_B = \mu_k N_B = 6\,N$$

$$F_c = \mu_k N_c = 9\,N.$$

For the entire system, ΣF_x yields:

$$\Sigma F_x = 18\,N - F_A - F_B - F_c = 0$$

By the free-body diagrams of the 10 N and 30 N crates, using $\Sigma F_x = 0$ we find:

$$T_{AB} = F_A = 3\,N, \qquad T_{BC} = 18\,N - F_c = 9\,N.$$

Hence, the work performed on each crate is.

$$U_A = \int_0^4 F\,dx = \int_0^4 (T_{AB} - F_A)\,dx\,; \qquad \underline{\underline{U_A = 0}}$$

$$U_B = \int_0^4 F\,dx = \int_0^4 (T_{BC} - T_{AB} - F_B)\,dx\,; \qquad \underline{\underline{U_B = 0}}$$

$$U_c = \int_0^4 F\,dx = \int_0^4 (18 - T_{BC} - F_c)\,dx\,; \qquad \underline{\underline{U_c = 0}}$$

b.) The work done on the system is

$$U_T = U_A + U_B + U_c\,; \qquad \underline{\underline{U_T = 0}}$$

12.13

Given: A constant force $\vec{F} = 3\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ [lb] acts on a particle, where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors along (x, y, z) axes.

Find: The work performed by $\vec{F}$ as the particle undergoes a displacement (in feet) from $(0,0,0)$ to $(0,2,4)$

(Continued)

12.13 cont.

Solution: By Eq (12.5), the work of $\vec{F}$ is, since $\vec{F}$ is constant;

$$U = \int \vec{F}\cdot d\vec{r} = \vec{F}\cdot\int d\vec{r} = \vec{F}\cdot\vec{r}$$

where $\vec{r} = 2\hat{j} + 4\hat{k}$. Hence,

$$\underline{\underline{U = (3\hat{i}+2\hat{j}+3\hat{k})\cdot(2\hat{j}+4\hat{k}) = 4+12 = 16 \text{ ft}\cdot\text{lb}}}$$

12.14

Given: A force $\vec{F} = -4t\hat{i} + 5\hat{j}$ (where t denotes time in seconds and $\hat{i}, \hat{j}$ are unit vectors in the x, y directions), acts on a particle that moves in the (x,y) plane such that its coordinates are:

$$x = 2.0 - 0.5t^3; \quad y = 1.6t^2$$

Find: The work done by the force on the particle during the time interval $0 \le t \le 3$ s.

Solution: By Eq (12.6), the work done by $\vec{F}$ is:

$$U = \int_{t_1}^{t_2} \left(F_x \frac{dx}{dt} + F_y \frac{dy}{dt} + F_z \frac{dz}{dt}\right) dt$$

or

$$U = \int_0^3 \left[-4t(-1.5t^2) + 5(3.2t)\right] dt$$

$$= \frac{6t^4}{4} + \frac{16.0t^2}{2} \Big|_0^3$$

or

$$\underline{\underline{U = 193.5 \text{ J}}}$$

12.15

Given: A particle P moves on the path

$$x = t, \quad y = t^2, \quad z = t^3 \text{ [m]} \qquad \text{(a)}$$

where t is a parameter. The motion of P is restricted by a force (N)

$$F_x = -3\frac{dx}{dt}, \quad F_y = -3\frac{dy}{dt}, \quad F_z = -3\frac{dz}{dt} \qquad \text{(b)}$$

Find: The work performed on P by $\vec{F}$ for the interval $0 \le t \le 3$.

Solution: By Eq (12.6), the work of $\vec{F}$ is:

$$U = \int_{t_1}^{t_2} \left(F_x \frac{dx}{dt} + F_y \frac{dy}{dt} + F_z \frac{dz}{dt}\right) dt \qquad \text{(c)}$$

where by Eq (a),

$$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = 2t, \quad z = 3t^2 \qquad \text{(d)}$$

By Eqs (b), (c), and (d), with $t_1 = 1$ and $t_2 = 3$,

$$U = -3\int_1^3 (1 + 4t^2 + 9t^4)\, dt$$

$$= -3\left(t + \tfrac{4}{3}t^3 + \tfrac{9}{5}t^5\right)\Big|_1^3$$

or

$$\underline{\underline{U = -1416.8 \text{ J}}}$$

12.16

Given: A 6 kg block rests on a frictionless horizontal surface (Fig a.). In this equilibrium position, the block is attached to an unstretched, non-linear spring, whose force-extension relation is:

$$F = 400 \sin 10x \quad \text{[N]} \qquad \text{(a)}$$

The block is moved a distance $x = 0.10$ m to the right and released.

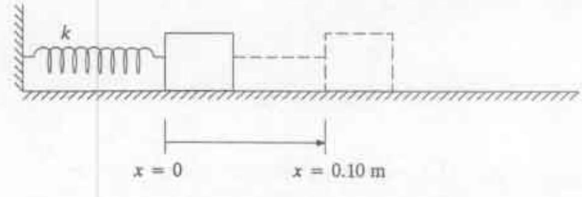


Figure a.

Find: a.) The work done on the block by the spring during the time that it takes the block to move back to its equilibrium position.

b.) The work done by the spring on the block as it moves from $x = 0$ to $x = -0.10$ m

(Continued)

12.16 cont.

Solution:

a.) The free-body diagram of the block as it moves from $x=0$ to $x=0.10\,m$ is shown in Fig. b.

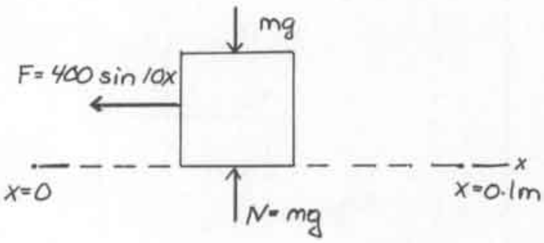


Figure b.

The work done by the spring is, with Eq (a) and Fig b.,

$$U=\int_{x=0.10}^{x=0}(-F)\,dx$$

$$=-\int_{0.10}^{0}400\sin 10x\,dx$$

$$=\frac{400}{10}\cos 10x\,\Big|_{0.10}^{0}$$

or $\underline{\underline{U=18.39\ J}}$

b.) The free-body diagram of the block as it moves from $x=0$ to $x=-0.10\,m$ is shown in Fig (c).

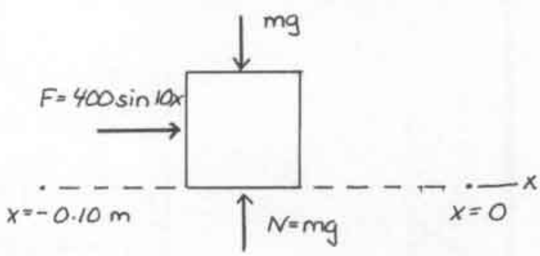


Figure c.

The work done by the spring is, with Eq (a) and Fig. (c),

$$U=\int_{0}^{-0.10}(400\sin 10x)\,dx$$

$$=-\frac{400}{10}\cos 10x\,\Big|_{0}^{-0.10}\quad \text{or}\quad \underline{\underline{U=18.39\ J}}$$

12.17

Given: A mechanism part that weighs 50 N slides on a circular rod under gravity (Fig a)
Its motion is resisted by a force

$$F=18\cos\theta\quad [N]$$

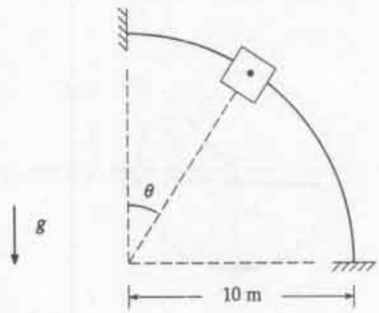


Figure a.

Find: The work done on the part as θ increases from $\theta=30°$ to $\theta=90°$.

Solution: The free-body diagram of the part is shown in Fig. b.

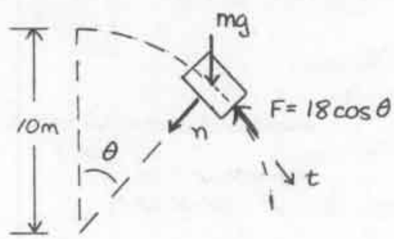


Figure b

The work done on the part is, with Fig. b and Eq (12.4)

$$U=\int F_t\,ds=\int_{30°}^{90°}(50\sin\theta-18\cos\theta)(10\,d\theta)$$

$$=(-500\cos\theta-180\sin\theta)\,\Big|_{30°}^{90°}$$

or

$\underline{\underline{U=343\ J}}$

12.18

Given: A particle moves on a circle under the action of a force $\vec{F}$, where

$$F_x = -ky, \quad F_y = kx \qquad (a)$$

and k = constant, (x, y) = coordinates of the particle. (Fig. a.)

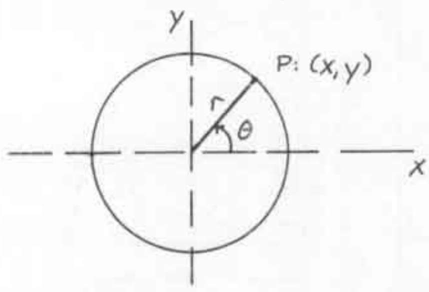


Figure a

The particle travels once around the circle in the counterclockwise direction.

Show that the work done by $\vec{F}$ is $U = 2kA$, where $A = \pi r^2$ is the area of the circle.

Solution:

By Fig (a),

$$x = r\cos\theta, \qquad y = r\sin\theta \qquad (b)$$

and by Eqs (a) and (b),

$$F_x = -kr\sin\theta \qquad F_y = kr\cos\theta \qquad (c)$$

By Eqs (b), (c), and (12.6), with $t = \theta$, the work done by $\vec{F}$ in the interval $\theta = 0$ to $\theta = 2\pi$ rad is:

$$U = \int_0^{2\pi} \left(F_x \frac{dx}{d\theta} + F_y \frac{dy}{d\theta}\right) d\theta$$

$$= \int_0^{2\pi} (kr^2\sin^2\theta + kr^2\cos^2\theta)\, d\theta$$

or $U = 2\pi k r^2$

Since $A = \pi r^2$, Eq (d) yields

$$\underline{\underline{U = 2kA}}$$

12.19

Given: A particle P that moves in the (x, y) plane is attracted to the origin by a force of magnitude $F = k/x$ (Fig. a.)

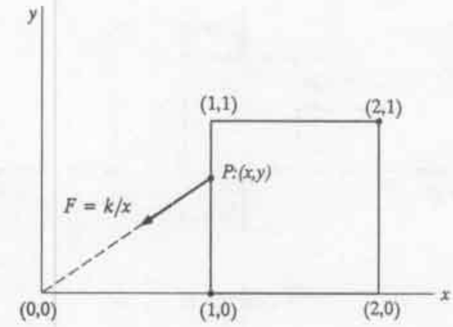


Figure a.

Find: a.) The work done by the force as the particle moves from $(1,0)$ to $(2,1)$ along the straight lines $(1,0)$ to $(1,1)$ and $(1,1)$ to $(2,1)$.

b.) Repeat part a. for the case where the particle moves from $(1,0)$ to $(2,1)$ along the straight lines $(1,0)$ to $(2,0)$ and $(2,0)$ to $(2,1)$.

c.) Does the work done depend on the paths from $(1,0)$ to $(2,1)$?

Solution:

a.) From $(1,0)$ to $(1,1)$, by Fig. b,

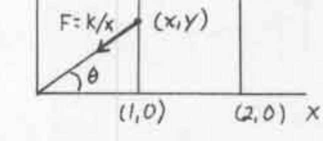


Figure b.

$x = 1$, $dx = 0$;

$y = \tan\theta$

$dy = \dfrac{d\theta}{\cos^2\theta}$;

$F_x = -k\cos\theta$,

$F_y = -k\sin\theta$

Hence, by Eq. (12.5)

$$U = \int_{(1,0)}^{(1,1)} F_x\,dx + F_y\,dy = 0 - k\int_0^{\pi/4} \sin\theta \left(\frac{d\theta}{\cos^2\theta}\right)$$

or

$$U_{(1,0)}^{(1,1)} = -k\left(\frac{1}{\cos\theta}\right)_0^{\pi/4}$$

(Continued)

12.19 cont.

$$U = -k(\sqrt{2}-1) = -0.4142\,k \qquad (a)$$

From (1,1) to (2,1) Fig c. gives:

$$x = x \qquad y = 1$$

$$dx = dx \qquad dy = 0$$

$$F_x = -\frac{k}{x}\cos\theta \qquad F_y = -\frac{k}{x}\sin\theta$$

$$\cos\theta = x/\sqrt{1+x^2} \qquad \sin\theta = 1/\sqrt{1+x^2}$$

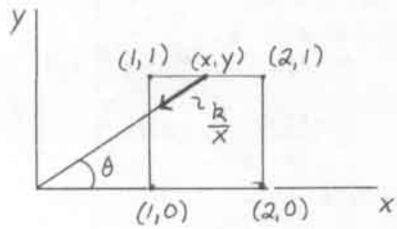


Figure c

Hence, by Eq (12.5)

$$U_{(1,1)}^{(2,1)} = \int_1^2 F_x\,dx + \int_1^1 F_y\,dy$$

$$= -k\int_1^2 \frac{(\cos\theta)}{x}\,dx = -k\int_1^2 \frac{dx}{\sqrt{1+x^2}}$$

or

$$U_{(1,1)}^{(2,1)} = -k\,\ln\left(x+\sqrt{1+x^2}\right)\Big|_1^2$$

Therefore,

$$U_{(1,1)}^{(2,1)} = -0.5622\,k \qquad (b)$$

By Eqs (a) and (b)

$$U_{(1,1)}^{(2,1)} = U_{(1,0)}^{(1,1)} + U_{(1,1)}^{(2,1)} = -(0.4142+0.5622)k$$

or

$$\underline{\underline{U_{(1,0)}^{(2,1)} = -0.9764\,k}} \qquad (c)$$

b.) From (1,0) to (2,0) see Fig. b,

$$\theta = 0; \quad x = x, \quad dx = dx$$

$$y = 0, \quad dy = 0;$$

$$F_x = -k/x \qquad F_y = 0$$

Therefore, by Eq (12.5),

$$U_{(1,0)}^{(2,0)} = \int_1^2 F_x\,dx + \int_0^0 F_y\,dy = \int_1^2 F_x\,dx + 0$$

$$= -\int_1^2 \frac{k}{x}\,dx = -k(\ln x)\Big|_1^2$$

$$= -0.6931\,k \qquad (d)$$

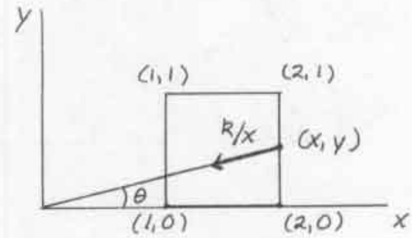


Figure d

From (2,0) to (2,1), see Fig. d.,

$$x = 2, \quad dx = 0,$$

$$y = 2\tan\theta \qquad dy = \left(\frac{2}{\cos^2\theta}\right)d\theta;$$

$$F_x = -\frac{k}{2}\cos\theta, \qquad F_y = -\frac{k}{2}\sin\theta$$

Hence, by Eq. (12.5) and Fig d,

$$U_{(2,0)}^{(2,1)} = \int_2^2 F_x\,dx + \int_0^1 F_y\,dy$$

$$= 0 + (-k)\int_0^{\tan^{-1}(1/2)} \frac{\sin\theta}{\cos^2\theta}\,d\theta$$

or

$$U_{(2,0)}^{(2,1)} = -k\left(\frac{1}{\cos\theta}\right)\Big|_0^{0.4636\text{ rad}} = -0.1180\,k \qquad (e)$$

By Eqs (d) and (e)

$$U_{(1,0)}^{(2,1)} = U_{(1,0)}^{(2,0)} + U_{(2,0)}^{(2,1)} = -(0.6931+0.1180)k$$

or

$$\underline{\underline{U_{(1,0)}^{(2,1)} = -0.8111\,k}} \qquad (f)$$

c.) Comparing Eqs (c) and (f), we see that

$$-0.9764\,k \neq -0.8111\,k$$

Hence, the work done by F depends on the path of the particle from (1,1) to (2,1).

12.20

Given: A particle P is constrained to move on a circular path (Fig a). The particle is attracted toward A by a force $F = kr$. At point B, $F = F_B$.

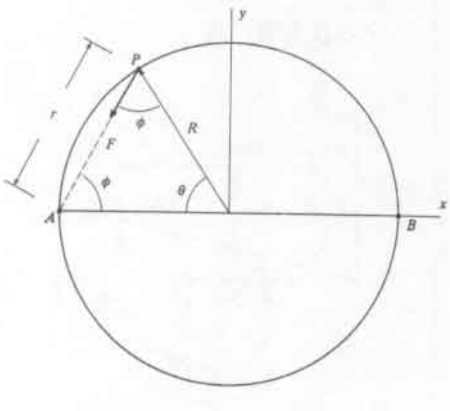


Figure a

Find: The work done on the particle by F, in terms of F_B and R, from point A to point B.

Solution:

By Fig. a,

$x = -R\cos\theta, \qquad dx = R\sin\theta\, d\theta$

$y = R\sin\theta, \qquad dy = R\cos\theta\, d\theta$

$F_x = -F\cos\phi = -F\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = -F\sin\frac{\theta}{2}$

$F_y = -F\sin\phi = -F\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = -F\cos\frac{\theta}{2}$

$r = 2R\sin\left(\frac{\theta}{2}\right)$

For $\theta = \pi$, $r = 2R$ and

$F = 2kR = F_B; \qquad k = F_B/2R$

Hence, by Eq (12.5),

$$U_A^B = \int F_x\, dx + \int F_y\, dy$$

$$= -F_B R \int_0^\pi \left[\sin\left(\frac{\theta}{2}\right)\sin\theta + \cos\left(\frac{\theta}{2}\right)\cos\theta\right]\sin\left(\frac{\theta}{2}\right) d\theta$$

Noting that $\cos\theta\cos\frac{\theta}{2} + \sin\theta\sin\frac{\theta}{2}$

$= \cos(\theta - \theta/2) = \cos\theta/2$

we may rewrite the above equation as:

$$U_A^B = -F_B R \int_0^\pi \sin\theta/2 \cos\theta/2\, d\theta$$

$$U_A^B = -F_B R \int_0^\pi \tfrac{1}{2}\sin\left(\frac{\theta}{2} + \frac{\theta}{2}\right) d\theta$$

$$= -\frac{F_B R}{2}\int_0^\pi \sin\theta\, d\theta = \frac{F_B R}{2}\cos\theta\Big|_0^\pi$$

or $\underline{\underline{U_A^B = -F_B R}}$

Alternatively,

$$F = kr = \left(\frac{F_B}{2R}\right)(2R)\sin\left(\frac{\theta}{2}\right) = F_B\sin\left(\frac{\theta}{2}\right)$$

$$U_A^B = -\int F_t\, ds = -\int_0^\pi \left(F\cos\frac{\theta}{2}\right) R\, d\theta$$

$$= -F_B R\int_0^\pi \sin\frac{\theta}{2}\cos\frac{\theta}{2}\, d\theta$$

$$= -\frac{F_B R}{2}\int_0^\pi \sin\theta\, d\theta = -\frac{F_B R}{2}(-\cos\theta)_0^\pi$$

or $\underline{\underline{U_A^B = -F_B R}}$

12.21

Given: Particle P moves on a circular path of radius r. It is acted on by $F = as + b$; a and b are constants and s is the arc length measured from point A. The particle has weight W. F acts tangent to the circular path at all times (Fig a).

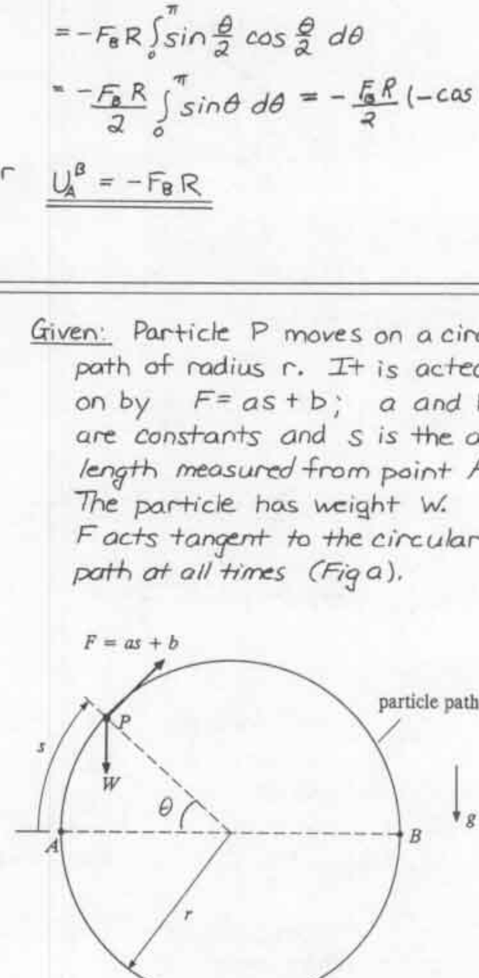


Figure a

Find: The work done on the particle from A to B in terms of a, b, and W.

Solution By Fig a, the net applied force that does work at any given position is the tangential force.

$F_t = F - W\cos\theta = as + b - W\cos\theta$

(Continued)

12.21 cont.

Therefore, by Eq (12.5),

$$U = \int F_t\, ds = \int F_t\,(r\,d\theta) = \int_0^{\pi} [as + b - W\cos\theta]\, r\,d\theta$$

or, since $s = r\theta$,

$$U = \int_0^{\pi} [ar\theta + b - W\cos\theta]\, r\,d\theta$$

$$= [\tfrac{1}{2} ar\theta^2 + b\theta - W\sin\theta]\, r \Big|_0^{\pi}$$

so,

$$\underline{\underline{U = \tfrac{1}{2} a(\pi r)^2 + br\pi}}$$

12.22

Given: The body B, shown in Fig a., weighs 10 N, and it slides along the frictionless bar AO.

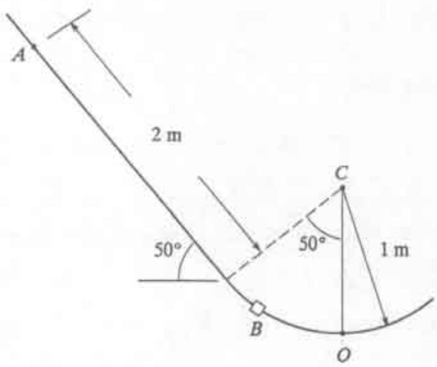


Figure a.

Find: The work required to raise the body from point O to point A.

Solution:

The work required to raise the body from point O to point A is the negative of the work done by the tangential component of the weight of the body.

Hence, by Figs. b and c, and Eq (12.5);

Figure b

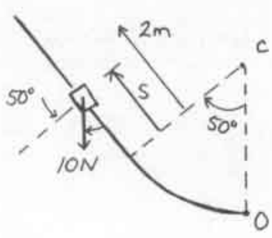


Figure c.

$$U_0^A = -\int F_t\, ds$$

$$-\int_0^{50°} (-10\sin\theta)(1)\,d\theta - \int_0^{2} (-10\sin 50°)(1)\,ds$$

or $U_0^A = -10\cos\theta \Big|_0^{50°} + (10\sin 50°)(s)\Big|_0^{2}$

Therefore,

$$\underline{\underline{U_0^A = 18.89 \ \text{N·m}}}$$

Alternatively, using the concept of work performed by gravity (since the body moves along the frictionless bar), the work required to raise B from O to A is the negative of the work of gravity from O to A. Thus,

$$U_0^A = -(-10)(\text{height from } O \text{ to } A)$$

or, by Fig. a,

$$U_0^A = 10\,[1\text{m} - (1\text{m})(\cos 50°) + 2\sin 50°]$$

So,

$$U_0^A = 10\,(0.3572 + 1.5321) = 18.89 \ \text{N·m}$$

12.23

Given: A cubical crate is set at a 30° angle and released from rest (Fig a.). The crate weighs 1260 N.

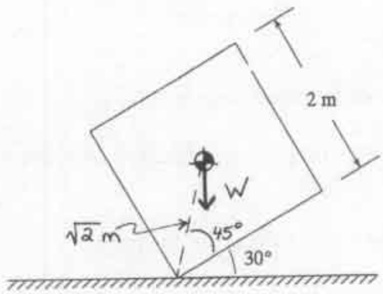


Figure a.

Find: The work performed by gravity from the time that the crate is released until the crate hits the floor.

(Continued)

12.23 cont.

Solution: The work performed by gravity equals the force of gravity times the change in height of the center of mass. Therefore,

$$U = W\left[(\sqrt{2})\sin 75^\circ - 1\right]$$
$$= 1260\,(0.3660)$$

or $\underline{U = 461.2\ \mathrm{J}}$

12.24

Given: Two water tanks are connected by a pipe (Fig. a). Tank A is 4 m wide and 6 m long (perpendicular to plane of Fig a.) and has 4 m of water in it. Tank B is empty and is 5 m wide and 6 m long. Water flows from A to B until the water level is the same in both tanks.

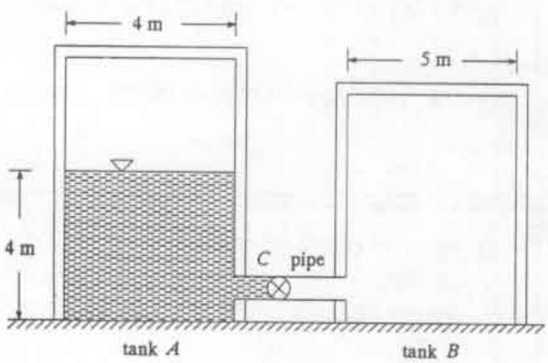


Figure a.

Find: The work done by gravity.

Solution: Initially, the volume of water in tank A is:

$$V = (4)(4)(6) = 96\ \mathrm{m^3}$$

The depth, h, will be the same in both tanks after water flows from tank A to tank B. Therefore,

$$V = (6)(h)(4) + (6)(h)(5) = 96\ \mathrm{m^3}$$

or $h = 1.778\ \mathrm{m}$

The work done is equal to the change in height of the center of mass times the force of gravity.

Therefore,

$$U = W\left[\tfrac{1}{2}(4 - 1.778)\right] = (9810\ \tfrac{\mathrm{N}}{\mathrm{m^2}})(96\ \mathrm{m^3})(1.11\ \mathrm{m})$$

$\underline{\underline{U = 1.046\ \mathrm{MJ}}}$

12.25

Given: The tank B in Problem 12.24 is lowered one meter.

Find: The work done by gravity.

Solution: As in Problem 12.24, the volume of water initially in tank A is

$$V = (4)(4)(6) = 96\ \mathrm{m^3}$$

After the valve is opened and the level of water is the same in tanks A and B,

$$V = 96\ \mathrm{m^3} = (4)(6)(h) + (5)(6)(h+1) \qquad (a)$$

Where h is the elevation of the water surface above the bottom of tank A. By Eq (a),

$$h = 1.222\ \mathrm{m}$$

The center of gravity of the water is now at the elevation (above the bottom of tank A)

$$W h_G = W_A h_{GA} + W_B h_{GB}$$

or
$$h_G = \frac{(4)(6)(h)(h/2) + (5)(6)(h+1)\left(\frac{h-1}{2}\right)}{96}$$
$$= \frac{(4)(6)(1.222)\left(\frac{1.222}{2}\right) + (5)(6)(2.222)\left(\frac{0.222}{2}\right)}{96}$$

or
$$h_G = \frac{17.925 + 7.407}{96} = 0.2639$$

Hence, the work done by gravity is

$$U = W(2 - 0.2639)$$
$$= (9810\ \tfrac{\mathrm{N}}{\mathrm{m^2}})(96\ \mathrm{m^3})(1.736\ \mathrm{m})$$

or

$\underline{\underline{U = 1.635\ \mathrm{MN{\cdot}m}}}$

12.26

Given: The uniform bar AB in Fig a. weighs 40 lb and is hinged at A. The disk weighs 18 lb.

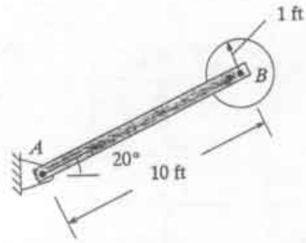


Figure a.

Find: The work performed by gravity as the bar and disk drop to the horizontal position from the position shown.

Solution: The work done on the bar (Fig a.) is:

$$U_{AB} = W_{AB}(5\sin 20°) = (40)(5\sin 20°)$$
$$= 68.404 \text{ ft}\cdot\text{lb} \qquad (a)$$

The work done on the disk (Fig a.) is

$$U_{DISK} = W_{DISK}(10\sin 20°) = (18)(10\sin 20°)$$
$$= 61.564 \text{ ft}\cdot\text{lb} \qquad (b)$$

Hence, the total work performed by gravity is, with Eqs. (a) and (b)

$$U = U_{AB} + U_{DISK} = 68.404 + 61.564$$

$$\underline{\underline{U = 129.97 \text{ ft}\cdot\text{lb}}}$$

12.27

Given: One end of a spring is attached to body B (weight = 10N) in Fig. P12.22 and the other end is fixed at C (see Fig. a). The undeformed length of the spring is 900 mm, and the spring constant is $k = 175$ N/m.

Find: The work that we must supply to lift the body from point O to point A.

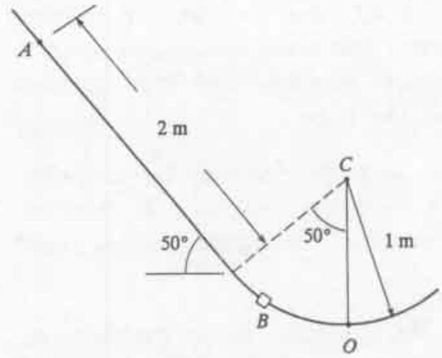


Figure a.

Solution: The initial stretch of the spring is $1\text{m} - 0.9\text{m} = 0.1\text{m}$. The final stretch of the spring is:

$$AC - 0.90 = \sqrt{1+2^2} - 0.90 = 1.336 \text{ m}$$

Hence, the work done in stretching the spring is:

$$U_{spring} = \int_{0.1}^{1.336} k e\, de = \int_{0.1}^{1.336} (175) e\, de$$
$$= \tfrac{1}{2}(175)e^2 \Big|_{0.1}^{1.336}$$

or $\quad U_{spring} = 155.30 \text{ N}\cdot\text{m} \qquad (a)$

The work done to lift B from O to A is (Fig. a):

$$U_B = 10\left[(1-\cos 50°) + 2\sin 50°\right]$$
$$= 18.89 \text{ N}\cdot\text{m} \qquad (b)$$

The total work performed is, with Eqs. (a) and (b),

$$U = U_{spring} + U_B$$
$$= 155.30 + 18.89$$
$$\underline{\underline{= 174.19 \text{ N}\cdot\text{m}}}$$

12.28 <u>Given:</u> A column of mercury weighing 270 N occupies a bent tube (Fig. a). The mercury is released from the position shown and begins to flow out the bottom of the tube.

<u>Find:</u> The work performed by gravity from the time the mercury is released to the time the right side passes point B.

<u>Solution:</u> The position BC of mercury is effectively transferred to the vertical leg of the tube below A. The total length of the mercury column is (Fig a),

$$L = (0.75)\left(\frac{\pi}{2}\right) + 1 = 2.178 \text{ m} \qquad (a)$$

Hence, the weight of mercury in part BC is, with Eq (a),

$$W_{BC} = 270\left(\frac{1}{2.178}\right) = 123.96 \text{ N} \qquad (b)$$

This part descends a distance of:

$$h = 0.750 + 0.50 = 12.50 \text{ m} \qquad (c)$$

Hence, by Eqs (b) and (c), the work done by gravity is:

$$\underline{\underline{U = W_{BC}h = (123.96)(1.250) = 154.95 \text{ N}\cdot\text{m}}}$$

12.29 <u>Given:</u> Water weighing 62.4 lb/ft^3 is pumped from a cylindrical well with a diameter of 20 ft into a cylindrical tank, 30 ft in diameter. The bottom of the tank is originally 80 ft above the surface of the water in the well.

<u>Find:</u> The total work performed against gravity to lift 10,000 ft^3 of water into the tank.

Assume the well doesn't refill during the pumping.

<u>Solution:</u> Figure a is a schematic diagram of the well and tank.

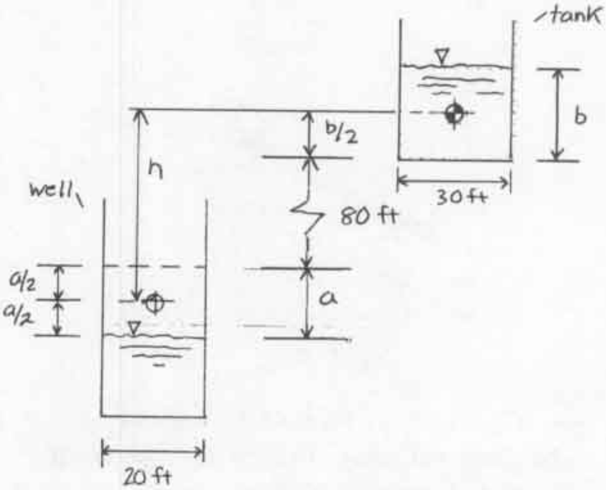


Figure a.

Let a be the distance that the water in the well is lowered. Hence, by Fig. a,

$$\pi r_{well}^2 a = 10{,}000$$

or $$a = \frac{10{,}000}{\pi(10)^2} = 31.831 \text{ ft.}$$

Let b be the depth of water in the tank. Therefore, by Fig a.,

$$\pi r_{tank}^2 b = 10{,}000$$

or $$b = \frac{10{,}000}{\pi(15)^2} = 14.147 \text{ ft}$$

Hence, the height that the water is raised is

$$h = \frac{a}{2} + 80 + \frac{b}{2}$$

$$= \frac{31.831}{2} + 80 + \frac{14.147}{2}$$

or $$h = 103.0 \text{ ft}$$

So, the work done against gravity is,

$$U = Wh = (10{,}000 \text{ ft}^3)(62.4 \tfrac{lb}{ft^3})(103.0 \text{ ft})$$

or

$$\underline{\underline{U = 64{,}265{,}000 \text{ ft}\cdot\text{lb}}}$$

12.30

<u>Given:</u> A horizontal trough of rectangular cross section is 80 ft long and 2 ft wide and it contains liquid of specific weight 70 lb/ft^3. At a particular instant, the liquid's surface has a wavy profile defined by

$$y = 1.5 \sin(\pi x/10) \quad [\text{ft}]$$

where x is a lengthwise coordinate [ft] measured from the left end of the trough, and y is a vertical coordinate.

<u>Find:</u> The amount of work performed by gravity until the waves die out.

<u>Solution:</u> Consider the sketch of the profile of the waves (Fig a.), where h denotes the mean depth of the liquid.

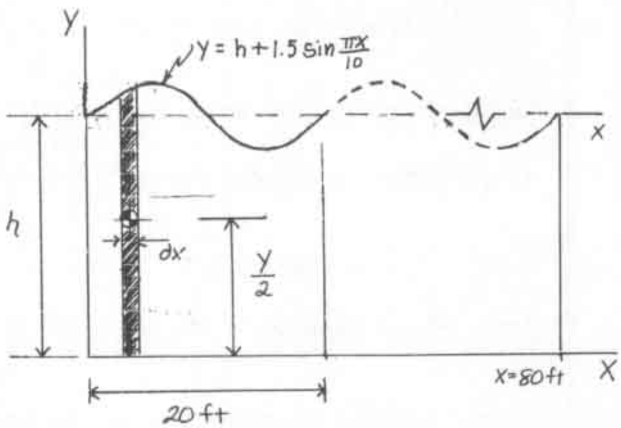


Figure a

The moment of the shaded area about the X axis is

$$M_x = \int \left(\tfrac{1}{2} Y\right) dA = \int_0^{80} \left(\tfrac{1}{2} Y\right) Y\,dx \qquad \text{(a)}$$

where

$$Y = h + 1.5 \sin \frac{\pi x}{10} \qquad \text{(b)}$$

By Eqs. (a) and (b),

$$M_x = \frac{1}{2}\int_0^{80} \left(h + 1.5 \sin \frac{\pi x}{10}\right)^2 dx$$

Integration yields

$$M_x = 40h^2 + 45 \qquad \text{(c)}$$

The area of the shaded part of Fig. a is

$$A = \int_0^{80} Y\,dx = \int_0^{80} \left(h + 1.5 \sin \frac{\pi x}{10}\right) dx$$

or $\quad A = 80\,h \qquad \text{(d)}$

The initial height of the center of gravity of the liquid is, with Eqs. (c) and (d),

$$Y_{Gi} = \frac{M_x}{A} = \frac{40h^2 + 45}{80h} = \frac{h}{2} + \frac{45}{80h} \qquad \text{(e)}$$

The final height of the center of gravity of the liquid is (after the waves die out).

$$Y_{Gf} = \frac{h}{2} \qquad \text{(f)}$$

Hence, by Eqs. (e) and (f), the descent of the center of gravity, when the waves die out is,

$$\Delta Y_G = Y_{Gi} - Y_{Gf} = \frac{45}{80h} \qquad \text{(g)}$$

The total weight of the liquid is,

$$W = (80\,\text{ft})(2\,\text{ft})(h\,\text{ft})(70\,\text{lb/ft}^3)$$

or $\quad W = 11{,}200\,h\ \text{lb} \qquad \text{(h)}$

Therefore, by Eqs. (g) and (h), the amount of work done by gravity is;

$$U = W(\Delta Y_G) = (11{,}200\,h)\left(\frac{45}{80h}\right)$$

or

$$\underline{\underline{U = 6300\ \text{ft}\cdot\text{lb}}}$$

12.31

<u>Given:</u> The cube shown in Fig. a rotates clockwise 45° in the xy plane from position A to position B.

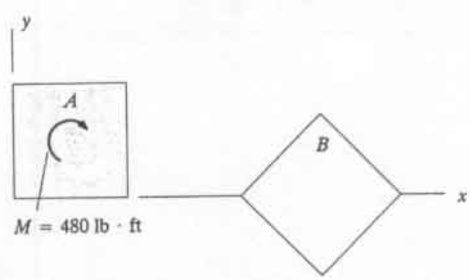


Figure a.

(Continued)

12.31 cont.

Find: The work performed by the couple during this displacement.

Solution: The work performed by a constant couple of magnitude M that acts on a body through a rotation θ (rad) in the same sense as the couple is:

$$U = M\theta \qquad (a)$$

Hence, with $M = 480$ lb·ft and $\theta = 45° = \pi/4$ radians, Eq. (a) yields:

$$\underline{\underline{U = (480)\left(\frac{\pi}{4}\right) = 376.99 \text{ ft·lb}}}$$

12.32

Given: A torque with magnitude

$$M = a\theta^2 + b\theta \qquad (a)$$

is applied to a rotating shaft, where θ is the angular displacement of the shaft and a and b are constants. The sense of the torque is the same as the positive sense of θ.

Find: The work U performed by the torque in terms of θ_0 and θ_1, the initial and final angles of rotation.

Solution: By Eq (12.8) and Eq. (a), the work U is:

$$U = \int_{\theta_0}^{\theta_1} M\,d\theta = \int_{\theta_0}^{\theta_1} (a\theta^2 + b\theta)\,d\theta \qquad (b)$$

Integration of Eq. (b) yields:

$$U = \frac{a\theta^3}{3} + \frac{b\theta^2}{2} \Big|_{\theta_0}^{\theta_1}$$

$$\underline{\underline{U = \frac{a}{3}(\theta_1^3 - \theta_0^3) + \frac{b}{2}(\theta_1^2 - \theta_0^2)}}$$

12.33

Given: A couple is defined by

$$\vec{M} = 5t\hat{\imath} + 16t^2\hat{\jmath} + 12\hat{k} \quad [FL] \quad (a)$$

where t is a parameter. The couple acts on a rigid body that undergoes a rotation

$$\vec{\theta} = e^{-t}\hat{\imath} + \left(\frac{1}{t}\right)\hat{\jmath} + t^{1/2}\hat{k} \quad [\text{rad}] \quad (b)$$

Find: The work performed on the body by the couple over the range $0 \le t \le 10$.

Solution: By Eq (12.9), the work done by $\vec{M}$ is:

$$U = \int \vec{M} \cdot d\vec{\theta} \qquad (c)$$

By Eqs. (a), (b), and (c), we find

$$U = \int_0^{10} (5t\hat{\imath} + 16t^2\hat{\jmath} + 12\hat{k}) \cdot \left(-e^{-t}\hat{\imath} - \frac{1}{t^2}\hat{\jmath} + \frac{1}{2t^{1/2}}\hat{k}\right)dt$$

or

$$U = \int_0^{10} (-5te^{-t} - 16 + 6t^{-1/2})\,dt \qquad (d)$$

Integration of Eq. (d) yields

$$U = \left[-5(-te^{-t} - e^{-t}) - 16t + 12t^{1/2}\right] \Big|_0^{10}$$

So,

$$U = -127.05 \ [LF]$$

The units of U depend on the units of t.

12.34

Given: The crank ABC in Fig a is free to rotate about the frictionless pin B.

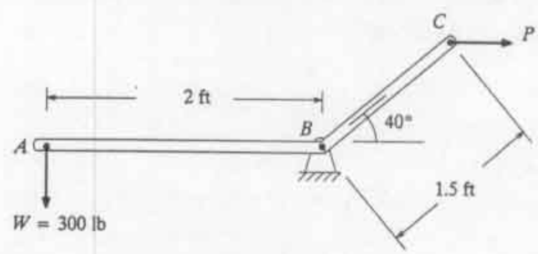


Figure a.

Find: The magnitude P of the horizontal force at C required to hold the crank in equilibrium.

(Continued)

12.34 Cont.

Solution: By the principle of virtual work, let the crank ABC undergo a virtual clockwise rotation $\delta\theta$. Then, the virtual work is:

$$\delta U = (P\sin 40^\circ)(1.5)\delta\theta - (300)(2)\delta\theta$$

or $$\delta U = [P(1.5\sin 40^\circ) - 600]\delta\theta \qquad (a)$$

If δU is negative for $^{+}\delta\theta$, it becomes positive for $^{-}\delta\theta$. Consequently, δU cannot be negative for all $\delta\theta$. However, $\delta U = 0$ if $P(1.5\sin 40^\circ) - 600 = 0$. Then, by Theorem 12.2, the crank is in equilibrium for

$$\underline{\underline{P = 600/(1.5\sin 40^\circ) = 622.29 \text{ lb}}}$$

12.35

Given: The bell crank ABC shown in Fig. a.

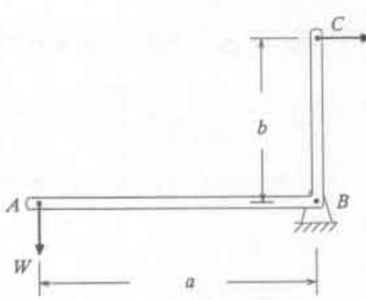


Figure a.

Find: a.) The magnitude of force P, in terms of a, b, and W, required for equilibrium of the bell crank. Use the principle of virtual work.

b.) Check your result using the condition of equilibrium of moments.

Solution: Let the bell crank undergo a virtual rotation $\delta\theta$, clockwise. Then, the virtual work is.

$$\delta U = Pb\,\delta\theta - Wa\,\delta\theta = (Pb - Wa)\delta\theta$$

If δU is negative for positive $\delta\theta$, then δU is positive for negative $\delta\theta$. Hence, δU cannot be negative for all $\delta\theta$.

However, if $Pb - Wa = 0$, $\delta U = 0$, and by Theorem 12.2, the bell crank is in equilibrium. Then

$$\underline{\underline{P = \frac{a}{b}W}} \qquad (a)$$

b.) Consider the free-body diagram of the bell crank (Fig b).

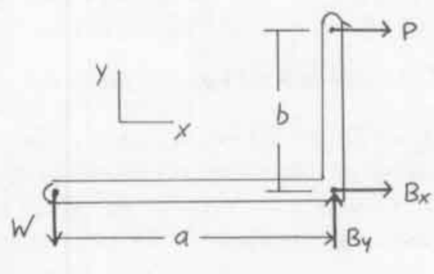


Figure b.

By summation of moments about B, we have for equilibrium,

$$\circlearrowleft\!+ \; \Sigma M_B = Pb - Wa = 0$$

or $$\underline{\underline{P = \frac{a}{b}W}} \qquad (b)$$

Equation (b) agrees with Eq. (a).

12.36

Given: A pulley-crank mechanism that is used to raise the 400 lb weight (Fig a.).

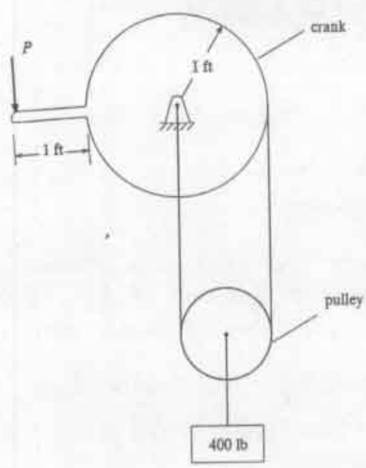


Figure a.

(Continued)

12.36 cont.

Find: The force P, using the principle of virtual work. Neglect the weight of the pulley.

Solution: Let $\delta\theta$ be the counterclockwise virtual rotation of the crank. Then, the force P undergoes the displacement $2\delta\theta$, and the weight undergoes the displacement $\frac{1}{2}\delta\theta$. Hence, the virtual work is:

$$\delta U = P(2)\delta\theta - (400)(\tfrac{1}{2})\delta\theta$$

For equilibrium (that is, for moving the load slowly or at constant velocity), $\delta U = 0$. (See discussion in Sec. 12.5 and solutions to problems 12.34 and 12.35.)

$$\therefore 2P - (400)(\tfrac{1}{2}) = 0$$

or $\underline{\underline{P = 100 \text{ lb}}}$

12.37

Given: The weight of the block shown in Fig. a. is W_B. The frictionless system is held in equilibrium by the weight W_A.

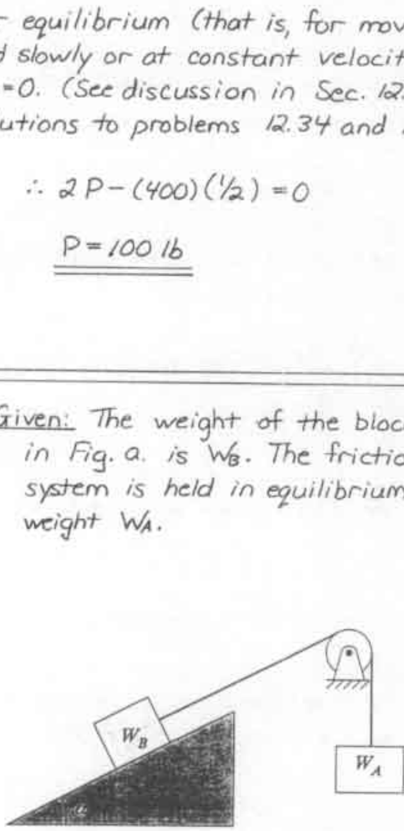


Figure a.

Find: The ratio W_A/W_B by the principle of virtual work.

Solution: Let W_A recieve a virtual displacement (down) of δy. Then, the virtual work of the system is

$$\delta U = W_A \delta y - W_B (\sin\alpha)\delta y$$
$$= (W_A - W_B \sin\alpha)\delta y$$

For equilibrium (Theorem 12.2), $\delta U = 0$, Therefore:

$$\underline{\underline{\frac{W_A}{W_B} = \sin\alpha}} \qquad (a)$$

Check: The free-body diagrams of W_A and W_B are shown in Fig. b.

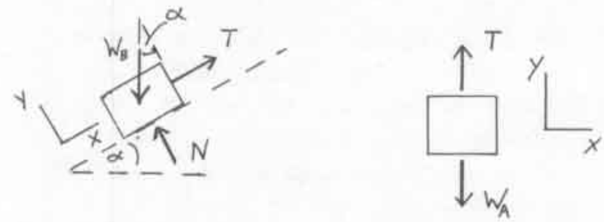


Figure b

By the free-body diagram of W_A

$$\Sigma F_y = T - W_A = 0; \quad T = W_A \qquad (b)$$

By the free-body diagram of W_B

$$\Sigma F_x = T - W_B \sin\alpha = 0 \qquad (c)$$

By Eqs (b) and (c), $W_A = W_B \sin\alpha$ or,

$$\frac{W_A}{W_B} \sin\alpha \qquad (d)$$

Equations (a) and (d) are identical.

12.38

Given: The frictionless mechanism shown in Fig a. is in equilibrium. The weights of the bars are negligible.

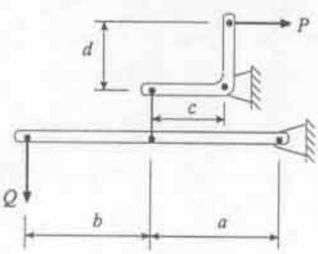


Figure a.

(Continued)

12.38 cont.

Find: The relation between forces P and Q, in terms of a, b, c, and d, by the principle of virtual work.

Solution: Let the bell crank rotate clockwise through the virtual angle $\delta\theta$. Then, the horizontal bar rotates clockwise through the vertical angle $\delta\phi$, where $c\delta\theta = a\delta\phi$ or $\delta\phi = \frac{c}{a}\delta\theta$.

Hence, the virtual work of the mechanism is:

$$\delta U = P d \delta\theta - Q(a+b)\frac{c}{a}\delta\theta$$
$$= \left[Pd - \frac{(a+b)c}{a}Q\right]\delta\theta \qquad (a)$$

For equilibrium (Theorem 12.2), $\delta U \leq 0$. Since $\delta U \nless 0$ for all $\delta\theta$, $\delta U = 0$ and by Eq (a),

$$Pd = \frac{(a+b)c}{a}Q$$

or $$\frac{P}{Q} = \frac{c}{d}\left(1+\frac{b}{a}\right)$$

12.39

Given: The frictionless frame ABCD shown in Fig. a. is in equilibrium.

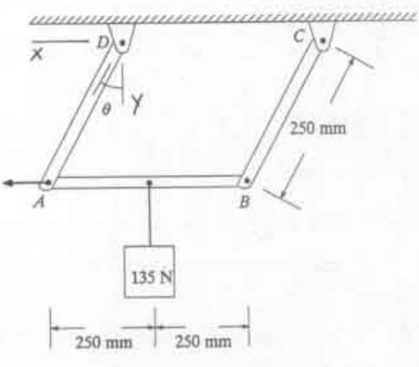


Figure a.

Find: a) The magnitude P of the required force, by the principle of virtual work. Neglect the weight of the bars of the frame.
b) Plot of P versus θ for $0 \leq \theta \leq \pi/2$

Solution:

a.) Let the bar AB undergo a horizontal virtual displacement δx to the left (See Fig a.). Then, the 135 N load undergoes a vertical displacement δy. By Fig. a,

$$x = 250\sin\theta, \quad y = 250\cos\theta$$

so,

$$\delta x = (250)(\cos\theta)\delta\theta, \quad \delta y = (-250)(\sin\theta)\delta\theta$$

Hence, the virtual work is

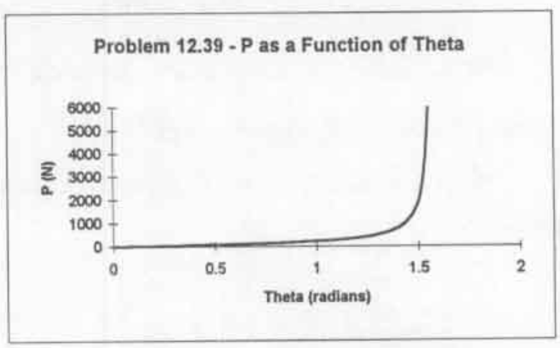
$$\delta U = P\delta x + (135)\delta y$$
$$= [P(250)(\cos\theta) - (135)(250)(\sin\theta)]\delta\theta$$

Therefore, by Theorem 12.2,

$$P(250)(\cos\theta) - (135)(250)\sin\theta = 0$$

or $$P = 135\tan\theta$$

b.) See the plot of P vs. θ for $0 \leq \theta \leq \frac{\pi}{2}$. Note that $P \to \infty$ as $\theta \to \pi/2$.



12.40

Given: Rotational springs are inserted at joints D and C in the frame of problem 12.39. The springs resist rotation of bars AD and BC with moments $M = -4.2\theta$ [N·m] for any rotation θ.

Find: The magnitude of the horizontal force required to hold the frame at an angle $\theta = 30°$. Use the principle of virtual work.

12.40 cont.

Solution: As in Problem 12.39, let the bar AB undergo a horizontal virtual displacement. Then, the virtual work of P is (see the solution of Problem 12.39):

$$\delta U_P = P\delta x = P(250)(\cos\theta)\delta\theta \qquad (a)$$

and of the 135 N load,

$$\delta U_{135} = -(135)(250)(\sin\theta)\delta\theta \qquad (b)$$

The net virtual work of the springs at joints D and C is:

$$\delta U_{springs} = 2M\delta\theta = -(2)(4.2)(\theta)\delta\theta \qquad (c)$$

The total virtual work of the system is, with Eqs (a), (b), and (c),

$$\delta U = \delta U_P + \delta U_{135} + \delta U_{springs}$$

or

$$\delta U = [250\,P\cos\theta - (135)(250)\sin\theta - (2)(4200)\theta]\,\delta\theta$$

For equilibrium (see Theorem 12.2), $\delta U = 0$ for all $\delta\theta$. Therefore,

$$250\,P\cos\theta - (135)(250)\sin\theta - (2)(4200)\theta = 0$$

For $\theta = 30°$ the above yields

$$P[250(0.8660)] = (135)(250)(0.50) + (2)(4200)\left(\tfrac{\pi}{6}\right)$$

or

$$\underline{\underline{P = 98.26\ \text{N}}}$$

12.41

Given: A rotational spring is inserted at joint B of the bell crank shown in Fig. P12.35 (See Fig a. below).

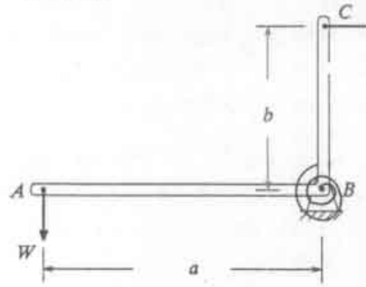


Figure a.

The spring exerts a moment $M = -k\theta$ on the crank where k is the rotational spring constant [FL] and θ is the rotation of the crank in radians.

Find: a) A formula for P for equilibrium in terms of W, a, b, k, and θ, by first expressing the work U done on the bell crank in a rotation θ in terms of θ. Then, form δU by differentiation.

b.) Check the formula for P by equilibrium of moments about B.

c.) For $W = 500$ N, $a = 600$ mm, $b = 500$ mm, and $k = 90$ N·m, calculate P for $\theta = 0°, 30°$, and $45°$

Solution: The work done by P, W, and M under a rotation θ is (See Fig. a)

$$U = P(b\sin\theta) - W(a\sin\theta) - \tfrac{1}{2}k\theta^2$$

Hence, the virtual work is

$$\delta U = \delta[Pb\sin\theta - Wa\sin\theta - \tfrac{1}{2}k\theta^2]$$

or, by differentiation, $\delta[\,] = \frac{\delta[\,]}{\delta\theta}\delta\theta$

$$\therefore \quad \delta U = Pb(\cos\theta)\delta\theta - Wa(\cos\theta)\delta\theta - k\theta\delta\theta$$

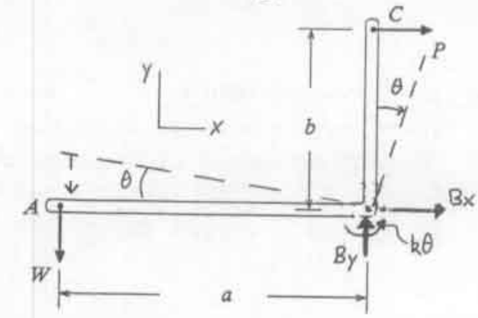
or

$$\delta U = (Pb\cos\theta - Wa\cos\theta - k\theta)\delta\theta \qquad (a)$$

For equilibrium (since δU cannot be negative for all $\delta\theta$), by Theorem 12.2, $\delta U = 0$. Thus, by Eq (a),

$$\underline{\underline{P = W\frac{a}{b} + \frac{k\theta}{b\cos\theta}}} \qquad (b)$$

b.) The free-body diagram of the bell crank is shown in Fig. b.



12.41 cont.

Give the crank a rotation θ.

Then, summation of the moments about joint B yields

$$\Sigma M_B = Pb\cos\theta - Wa\cos\theta - k\theta = 0$$

$$P = W\frac{a}{b} + \frac{k\theta}{b\cos\theta} \qquad (c)$$

Equation (c) is identical to Eq (b).

c.) For $W = 500$ N, $a = 600$ mm, $b = 500$ mm, $k = 90$ N·m, find P for $\theta = 0°, 30°, 45°$.

For $\theta = 0°$, $P = 500\left(\frac{600}{500}\right) = \underline{\underline{600\text{ N}}}$

For $\theta = 30°$, $P = 600 + \frac{90{,}000\,(\pi/6)}{(500)(0.8660)}$

$= \underline{\underline{708.8\text{ N}}}$

For $\theta = 45°$ $P = 600 + \frac{90{,}000\,(\pi/4)}{(500)(0.7071)}$

$= \underline{\underline{799.9\text{ N}}}$

12.42

Given: The steel body shown in Fig. a consists of a solid rectangular prism welded to a solid semicylindrical roller that rests on a horizontal surface. Friction prevents sliding.

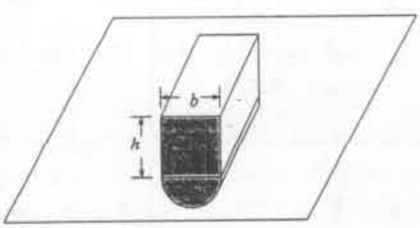


Figure a.

Find: Show that if $b/h < \sqrt{6}$ the upright position is unstable.

Solution: Let the body undergo a clockwise rotation θ (Fig. b). By Fig. b,

$$y_1 = \frac{b}{2} - \frac{2b}{3\pi}\cos\theta$$
$$y_2 = \frac{b}{2} + \frac{h}{2}\cos\theta \qquad (a)$$

Hence, the work performed by gravity under the rotation θ is, with Eqs. (a) and Fig. b,

$$U = W_1\left[\left(\frac{b}{2} - \frac{2b}{3\pi}\right) - y_1\right] + W_2\left[\left(\frac{b}{2} + \frac{h}{2}\right) - y_2\right] \qquad (b)$$

also,

$$W_1 = \frac{\pi}{2}\left(\frac{b}{2}\right)^2 L\gamma$$
$$W_2 = bhL\gamma \qquad (c)$$

where L is the length of the body and γ is the specific weight.

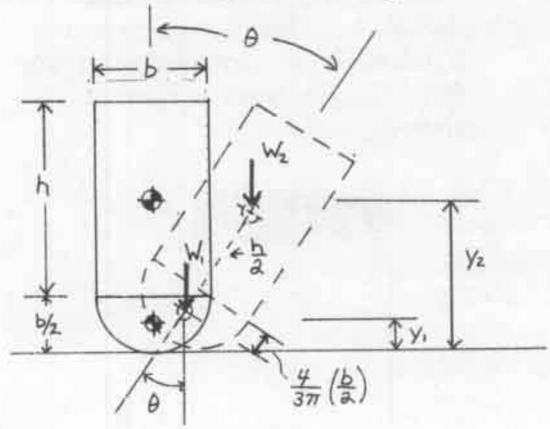


Figure b.

Substituting Eqs (a) and (c) into Eq (b) and simplifying, we obtain

$$U = \frac{bL\gamma}{2}\left(h^2 - \frac{b^2}{6}\right)(1 - \cos\theta) \qquad (d)$$

Hence, the virtual work due to a virtual rotation $\delta\theta$ is, by differentiation of Eq (d),

$$\delta U = \frac{\delta U}{\delta\theta}\delta\theta = \frac{bL\gamma}{2}\left(h^2 - \frac{b^2}{6}\right)(\sin\theta)\delta\theta \qquad (e)$$

where $\delta U/\delta\theta$ denotes the derivative of U with respect to θ.

(Continued)

12.42 cont.

By Theorem 12.3, the body is stable, if, and only if, $\delta U < 0$ for every small virtual rotational displacement $\delta\theta$. Thus, by Eq (e), the body is stable provided $h^2 - b^2/6 < 0$. It is unstable if $h^2 - b^2/6 > 0$. Hence, the body is unstable if

$$\frac{b}{h} < \sqrt{6}$$

12.43

Given: A solid homogeneous body made up of a cylinder of height h cemented to a hemisphere of radius r. The body rests on a table in an upright position (Fig. a). Friction prevents sliding.

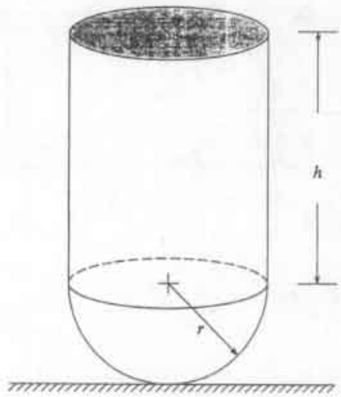


Figure a.

Determine the range of the ratio h/r for which the body is stable.

Solution: Let the body undergo a clockwise rotation (Fig. b). By Fig b,

$$y_1 = r - \frac{3r}{8}\cos\theta$$
$$y_2 = r + \frac{h}{2}\cos\theta \qquad (a)$$

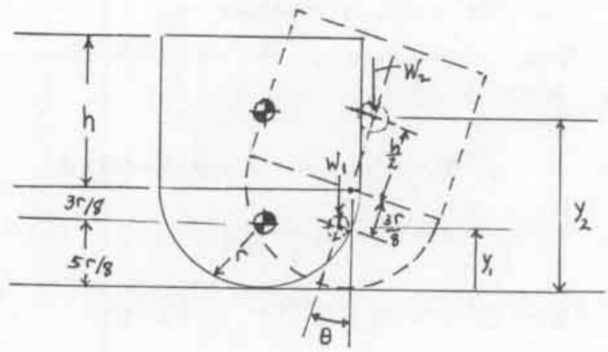


Figure b

Hence, the work performed by gravity under the rotation θ is, with Eqs. (a) and Fig. b,

$$U = W_1\left(\frac{5r}{8} - y_1\right) + W_2\left(r + \frac{h}{2} - y_2\right) \qquad (b)$$

where

$$W_1 = \frac{1}{2}\left(\frac{4\pi}{3}r^2\right)\gamma = \frac{2\pi}{3}r^2\gamma$$
$$W_2 = \pi r^2 h\gamma \qquad (c)$$

and γ = specific weight.

Substituting Eqs (a) and (c) into Eq (b) and simplifying, we find

$$U = \frac{\pi}{4}r^2\gamma(2h^2 - r^2)(1-\cos\theta) \qquad (d)$$

Hence, the virtual work due to a virtual rotation $\delta\theta$ is, by differentiation of Eq. d.

$$\delta U = \frac{\delta U}{\delta\theta}\delta\theta = \frac{\pi}{4}r^2\gamma(2h^2 - r^2)(\sin\theta)\delta\theta \qquad (e)$$

where $\delta U/\delta\theta$ denotes the derivative of U with respect to θ.

By theorem 12.3, the body is stable if, and only if, $\delta U < 0$ for all small virtual rotations $\delta\theta$. Thus, by Eq (e), the body is stable provided $2h^2 - r^2 < 0$. It is unstable if $2h^2 - r^2 \geq 0$. Hence, the body is stable when h/r lies in the range

$$0 \leq \frac{h}{r} \leq \frac{1}{\sqrt{2}}$$

12.44

<u>Given</u>: Two uniform identical rods are hinged together, rest against a 45° inclined plane, and are constrained to remain in a vertical plane (Fig. a).

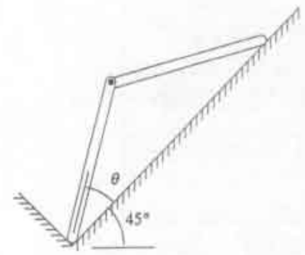


Figure a.

<u>Find</u>: (a) The smallest nonzero angle θ for which equilibrium is possible.

(b) Whether or not the equilibrium state in part a is stable.

<u>Solution</u>:

a) The free-body diagram of the system is shown in Fig. b. The work performed by gravity as the rods are raised from the horizontal (the X axis) to the position shown is

$$U = -Wy_1 - Wy_2 \qquad (a)$$

where by Fig. b,

$$y_1 = \frac{L}{2}\sin(45^\circ+\theta)$$

$$y_2 = L\sin(45^\circ+\theta) + \frac{L}{2}\sin(45^\circ-\theta) \qquad (b)$$

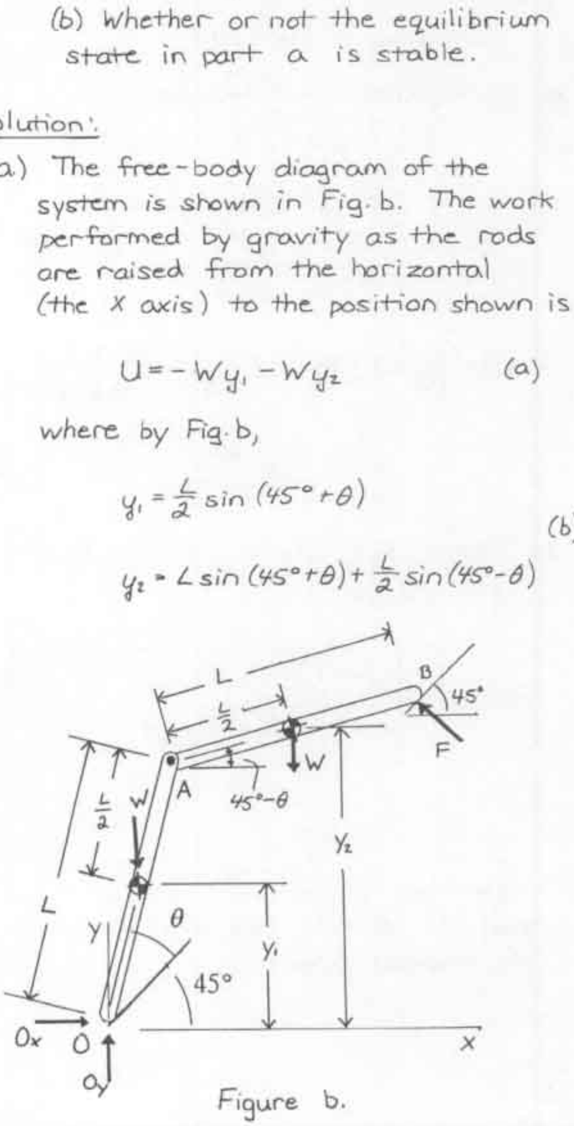


Figure b.

Substitution of Eqs. (b) into Eq (a) yields:

$$U = -\frac{WL}{2}\left[3\sin(45^\circ+\theta) + \sin(45^\circ-\theta)\right] \qquad (c)$$

Using the formulas for the sine of a sum and a difference of two angles (Appendix B) and simplifying Eq (c), we obtain

$$U = -WL(\sin 45^\circ)(2\cos\theta + \sin\theta) \qquad (d)$$

Hence, the virtual work due to a virtual rotation $\delta\theta$ is, by differentiation of Eq. (d),

$$\delta U = \frac{\delta U}{\delta\theta}\delta\theta = -WL(\sin 45^\circ)(-2\sin\theta + \cos\theta)\delta\theta \qquad (e)$$

where $\frac{\delta U}{\delta\theta}$ denotes the derivative of U with respect to θ.

By Theorem 12.2, since δU in Eq. (e) can not be negative for all $\delta\theta$ (+ and −), for equilibrium $\delta U = 0$. Hence, by Eq (e),

$$\cos\theta - 2\sin\theta = 0$$

$$\tan\theta = 1/2; \qquad \underline{\underline{\theta = 26.56^\circ}}$$

b.) By Theorem 12.3, for stable equilibrium $\delta U < 0$. For $\theta = 26.56^\circ$, by Eq. (e), $\delta U = 0$. Hence, the equilibrium state is unstable.

12.45

<u>Given</u>: A rectangular frame of rigid bars hinged at their ends (Fig a.). The four hinges contain springs that each produce a restoring moment $k\theta$, where θ is the relative angular displacement between two bars. The forces of magnitude F remain directed along the diagonals of the parallelogram when the vertical bars are rotated through an angle θ.

(Continued)

12.45 cont

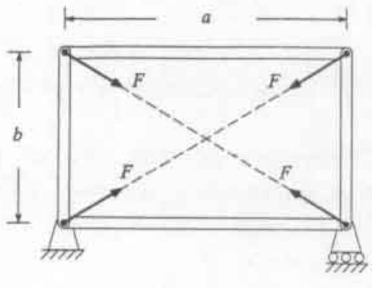


Figure a.

Show that the critical or buckling load is (see Example 12.18)

$$F_{cr} = 2kc^3a^{-2}b^{-2}; \quad c^2 = a^2+b^2 \qquad (a)$$

Solution: To calculate the work done by the forces when the vertical bars undergo rotation θ (Fig. b), we must determine the work done by the 4 forces of magnitude F and the work done by the 4 springs at A, B, C, and D.

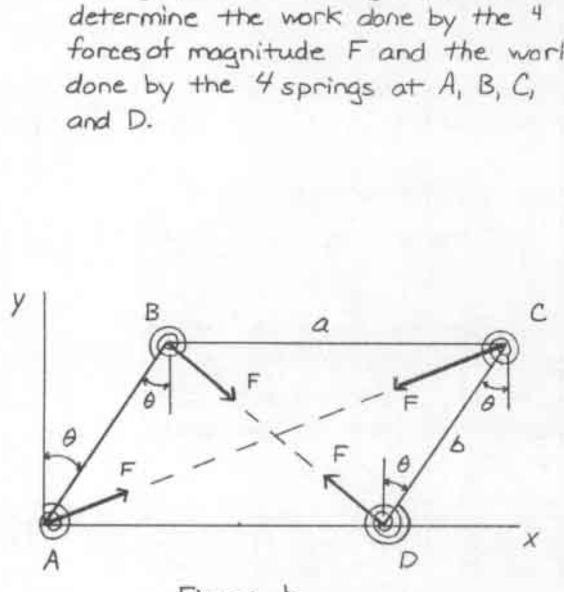


Figure b.

The work performed by the springs is (See Fig. b)

$$U_{springs} = -4\int k\theta \, d\theta = -2k\theta^2 \qquad (b)$$

The forces at A and D do no work since points A and D are fixed. The force at B is always directed toward the fixed point D and the force at C is directed toward the fixed point A.

Hence, by the results of Example 12.7 [Eq (d)], the work done by these forces is (see Figs a and b)

$$\begin{aligned} U_B &= F(r_1-r_2) = F(c-\overline{BD}) \\ U_C &= F(r_1-r_2) = F(c-\overline{AC}) \end{aligned} \qquad (c)$$

where $c^2 = a^2+b^2$ and

$$\begin{aligned} \overline{BD} &= \sqrt{a^2+b^2-2ab\sin\theta} \\ &= \sqrt{c^2-2ab\sin\theta} \end{aligned} \qquad (d)$$

and

$$\begin{aligned} \overline{AC} &= \sqrt{a^2+b^2+2ab\sin\theta} \\ &= \sqrt{c^2+2ab\sin\theta} \end{aligned} \qquad (e)$$

Hence, the total work is, with Eqs. (b), (c), (d), and (e),

$$U = U_{springs} + U_B + U_C$$

or

$$U = -2k\theta^2 - F\sqrt{c^2-2ab\sin\theta} - F\sqrt{c^2+2ab\sin\theta} + \text{constant} \qquad (f)$$

Hence, the virtual work due to a virtual rotation $\delta\theta$ is, by differentiation of Eq. (f),

$$\delta U = \frac{\delta U}{\delta\theta}\delta\theta = \left[-4k\theta + \frac{Fab\cos\theta}{\sqrt{c^2-2ab\sin\theta}} - \frac{Fab\cos\theta}{\sqrt{c^2+2ab\sin\theta}}\right]\delta\theta \qquad (g)$$

By Theorem 12.3, $\delta U < 0$ for stability. Hence, by Eq. (g),

$$Fabc\cos\theta\left[\frac{\sqrt{1+\frac{2ab}{c^2}\sin\theta} - \sqrt{1-\frac{2ab}{c^2}\sin\theta}}{c^2\sqrt{1-\frac{4a^2b^2}{c^4}\sin\theta}}\right] < 4k\theta \qquad (h)$$

To determine the critical value of F, we must let $\theta \to 0$. For small θ, by the binomial expansion:

(Continued)

12.45 cont.

$$\sqrt{1+\frac{2ab}{c^2}\sin\theta} = 1+\frac{1}{2}\left(\frac{2ab}{c^2}\right)\sin\theta - \cdots$$

$$\sqrt{1-\frac{2ab}{c^2}\sin\theta} = 1-\frac{1}{2}\left(\frac{2ab}{c^2}\right)\sin\theta + \cdots$$

Then Eq. (h) reduces to

$$\frac{2Fa^2b^2\cos\theta}{c^3}\cdot\frac{1}{\sqrt{1-\frac{4a^2b^2}{c^4}\sin\theta}} < 4k\frac{\theta}{\sin\theta}$$

As $\theta\to 0$, $\cos\theta\to 1$, $\sin\theta\to 0$, and $\theta/\sin\theta \to 1$. Hence, for stability, the above yields:

$$F < 2kc^3a^{-2}b^{-2}$$

and the critical (buckling) value of F is:

$$\underline{\underline{F_{cr} = 2kc^3a^{-2}b^{-2}}}$$

12.46

Given: The forces F in Problem 12.45 (see Fig. a) remain parallel to their original directions when the frame buckles.

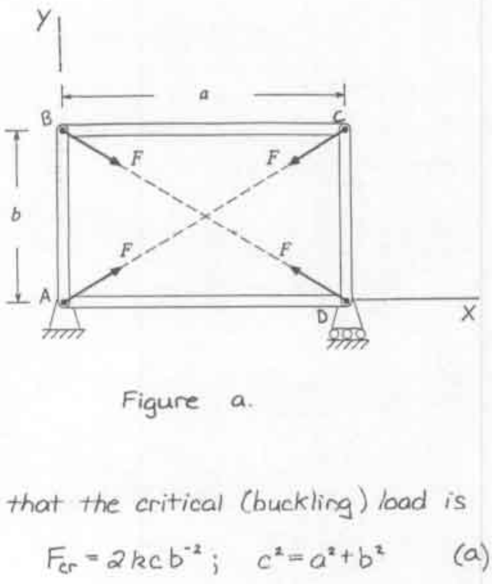


Figure a.

Show that the critical (buckling) load is

$$F_{cr} = 2kcb^{-2};\quad c^2 = a^2+b^2 \qquad (a)$$

Solution: By Fig. a, the (x, y) projections of the forces at B and D follow:

Force at B: $c^2 = a^2+b^2$

$$F_{xB} = \frac{a}{c}F \rightarrow$$
$$F_{yB} = -\frac{b}{c}F = \frac{b}{c}F\downarrow \qquad (b)$$

Force at C:

$$F_{xC} = -\frac{a}{c}F = \frac{a}{c}F\leftarrow$$
$$F_{yC} = -\frac{b}{c}F = \frac{b}{c}F\downarrow$$

By Fig. b, the work performed by the forces at B and C are

$$U_B = \left(\frac{a}{c}\right)F\,b\sin\theta + \left(\frac{b}{c}\right)F\,(b-b\cos\theta)$$

or $$U_B = \left(\frac{ab}{c}\right)F\sin\theta + \left(\frac{b^2}{c}\right)F\,(1-\cos\theta) \qquad (c)$$

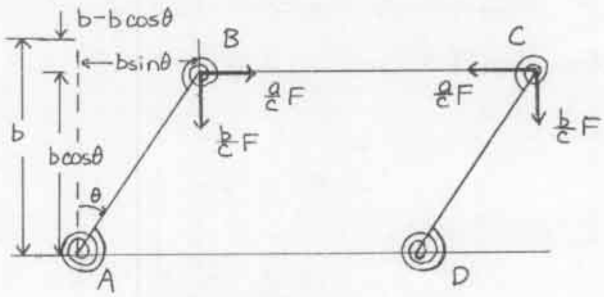


Figure b

$$U_C = -\left(\frac{a}{c}\right)F\,b\sin\theta + \left(\frac{b}{c}\right)F(b-b\cos\theta)$$

or $$U_C = -\left(\frac{ab}{c}\right)F\sin\theta + \left(\frac{b^2}{c}\right)F\,(1-\cos\theta) \qquad (d)$$

The work of the forces at A and D is zero, since points A and D are fixed.

As in Problem 12.45, the work of the springs is

$$U_{springs} = -4\int k\theta\,d\theta = -2k\theta^2 \qquad (e)$$

Hence, the total work is, with Eqs (c), (d), and (e):

$$U = U_B + U_C + U_{springs}$$

or $$U = 2\left(\frac{b^2}{c}\right)F\,(1-\cos\theta) - 2k\theta^2 \qquad (f)$$

(Continued)

12.46 cont.

Then, the virtual work due to a virtual rotation θ is, by differentiation of Eq. (f),

$$\delta U = \frac{\delta U}{\delta \theta}\delta\theta = \left[2\left(\frac{b^2}{c}\right)F\sin\theta - 4k\theta\right]\delta\theta \qquad (g)$$

By Theorem 12.3, $\delta U < 0$ for stability at $\theta = 0$.

Hence, by Eq (g),

$$F < 2kcb^{-2}\left(\frac{\theta}{\sin\theta}\right)\bigg|_{\theta\to 0} = 2kcb^{-2} \qquad (h)$$

since $\theta/\sin\theta \to 1$ as $\theta \to 0$. The critical buckling load is (see Example 12.18).

$$\underline{\underline{F = 2kcb^{-2}; \quad c^2 = a^2 + b^2}} \qquad (i)$$

Equation (i) verifies Eq. (a)

C.1

Derive an expression for the moment of inertia with respect to the centroidal axis x, of the right triangle (Fig. a)

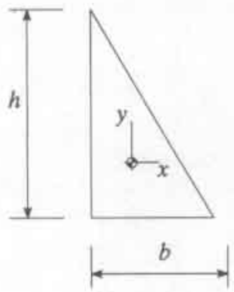


Figure a

Solution: Refer to Fig. b. The equation of line AB is

$$x = -\frac{b}{h}y + \frac{1}{3}b = \frac{1}{3}\frac{b}{h}(h-3y) \qquad (a)$$

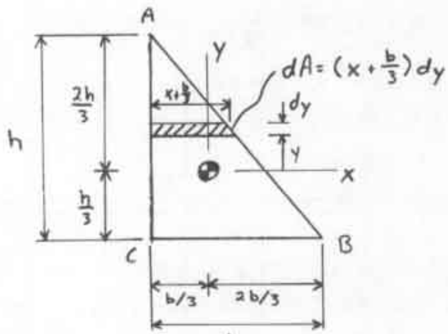


Figure b

The moment of inertia with respect to the x-axis is, with Eq. (a),

$$I_x = \int y^2 dA = \int_{-h/3}^{2h/3} y^2 \left(x + \frac{b}{3}\right) dy$$

or

$$I_x = \int_{-h/3}^{2h/3} y^2 \left[\frac{b}{3h}(2-3y)\right] dy$$

$$= \frac{2b}{3h}\frac{y^3}{3} - \frac{b}{h}\frac{y^4}{4} \Big|_{-h/3}^{2h/3}$$

or

$$\underline{\underline{I_x = \frac{bh^3}{36}}}$$

C.2

Derive an expression for the product of inertia with respect to the centroidal axes (x, y) of the right triangle in Fig. a.

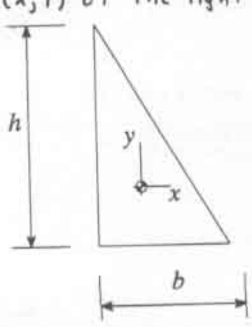


Figure a

Solution: By Eq. (C.4) and Fig. b,

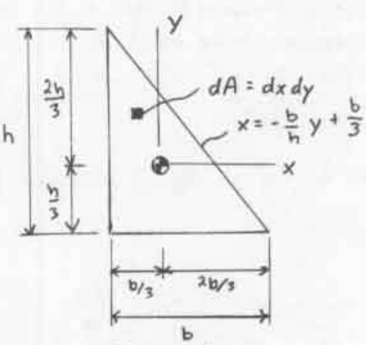


Figure b

$$I_{xy} = \int xy\, dA$$

$$I_{xy} = \int_{-\frac{h}{3}}^{\frac{2h}{3}} \int_{-\frac{b}{3}}^{-\frac{b}{h}y+\frac{b}{3}} xy\, dx\, dy = \int_{-\frac{h}{3}}^{\frac{2h}{3}} y\left[\frac{x^2}{2}\right]\Bigg|_{x=-\frac{b}{3}}^{x=-\frac{b}{h}y+\frac{b}{3}}$$

$$I_{xy} = \int_{-h/3}^{2h/3} y\left[\frac{(-\frac{b}{h}y+\frac{b}{3})^2}{2} - \frac{(-\frac{b}{3})^2}{2}\right] dy$$

$$I_{xy} = \int_{-h/3}^{2h/3} y\left[\frac{b^2 y^2}{2h^2} - \frac{b^2 y}{3h}\right] dy = \left(\frac{b^2y^4}{8h^2} - \frac{b^2y^3}{9h}\right)\Big|_{-h/3}^{2h/3}$$

$$I_{xy} = \frac{b^2(\frac{2h}{3})^4}{8h^2} - \frac{b^2(\frac{2h}{3})^3}{9h} - \frac{b^2(-\frac{h}{3})^4}{8h^2} + \frac{b^2(-\frac{h}{3})^3}{9h}$$

$$I_{xy} = \frac{2b^2h^2}{81} - \frac{8b^2h^2}{243} - \frac{b^2h^2}{648} - \frac{b^2h^2}{243}$$

$$\underline{\underline{I_{xy} = -\frac{27b^2h^2}{1944} = -\frac{b^2h^2}{72}}}$$

C.3

Derive an expression for the polar moment of inertia with respect to the center O of a circular cross section of radius R. Use polar coordinates (r, θ) and an infinitesimal area $dA = (r\,d\theta)(dr)$

Solution: Refer to Fig. a. By Eq. (C.3) and Fig. a,

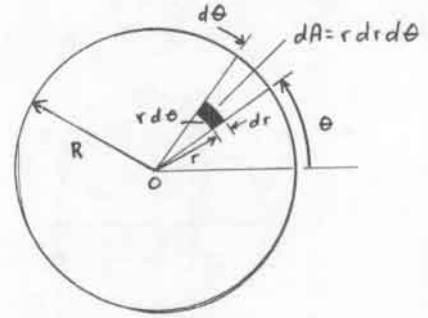


Figure a

$$J_O = \int r^2 dA = \int_0^R \int_0^{2\pi} r^2 (r\,dr)\,d\theta$$

Integration of Eq. (a) yields

$$\underline{\underline{J_O = (2\pi)\left(\frac{r^4}{4}\right)\Big|_0^R = \frac{\pi R^4}{2}}}$$

C.4

Show that the polar moment of inertia with respect to the centroid of the cross section of a hollow circular tube with outer radius b and inner radius a is

$$J_o = \frac{\pi (b^4 - a^4)}{2}$$

Solution: Refer to Fig. a. By Eq. (C.3) and Fig. a,

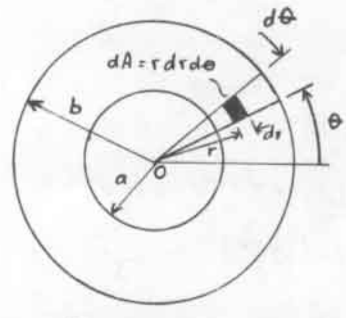


Figure a

$$J_o = \int r^2 dA = \int_a^b \int_0^{2\pi} r^2 (r dr d\theta) \qquad (a)$$

Integration of Eq. (a) yields

$$\underline{\underline{J_o = (2\pi)\left(\frac{r^4}{4}\right)\Big|_a^b = \frac{\pi}{2}(b^4 - a^4)}}$$

C.5

Derive an expression for the moment of inertia I_y for the semicircular cross section shown in Fig. a, with respect to the centroidal axis y.

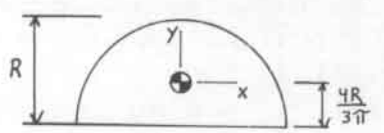


Figure a

Solution: Refer to Fig. b. By Eq. (C.2) and Fig. b,

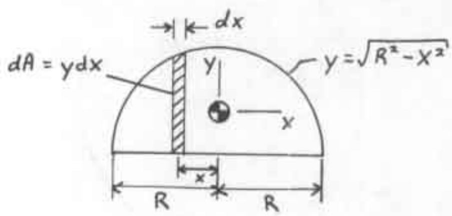


Figure b

$$I_y = \int x^2 dA = \int_{-R}^{R} x^2 y\, dx = \int_{-R}^{R} x^2 \sqrt{R^2 - x^2}\, dx$$

Integration yields

$$I_y = \left[-\frac{x}{4}\sqrt{(R^2 - x^2)^3} + \frac{R^2}{8}\left(x\sqrt{R^2 - x^2} + R^2 \sin^{-1}\frac{x}{R}\right)\right]\Big|_{-R}^{R}$$

Therefore

$$I_y = \frac{R^4}{8}\left[\sin^{-1}(1) - \sin^{-1}(-1)\right] = \frac{R^4}{8}\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

or $\underline{I_y = \frac{\pi}{8} R^4}$

C.6

For the semielliptical cross section shown in Fig. a, derive an expression for the moment of inertia I_x with respect to the diametral axis x.

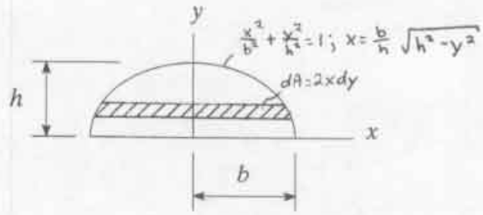


Figure a

Solution: Refer to Fig. a. By Eq. (C.1) and Fig. a,

$$I_x = \int y^2 dA = \int y^2 (2x dy) = \frac{2b}{h}\int_0^h y^2 \sqrt{h^2 - y^2}\, dy \qquad (a)$$

Integration of Eq. (a) yields

$$I_x = \frac{2b}{h}\left[-\frac{y}{4}\sqrt{(h^2 - y^2)^3} + \frac{h^2}{8}\left(y\sqrt{h^2 - y^2} + h^2 \sin^{-1}\left(\frac{y}{h}\right)\right)\right]\Big|_0^h$$

$$I_x = \left(\frac{2b}{h}\right)\left(\frac{h^4}{8}\right)\left[\sin^{-1}(1) - \sin^{-1}(0)\right]$$

$$I_x = \frac{bh^3}{4}\left(\frac{\pi}{2} - 0\right)$$

or $\underline{\underline{I_x = \frac{\pi}{8} bh^3}}$

C.7

Refer to Problem C.4. For a thin-walled, hollow tube with circular cross section, show that the polar moment of inertia is approximately

$$J_o = 2\pi R^3 t \qquad (a)$$

where $R = (a+b)/2$ and t = wall thickness = b-a ($t \ll R$). Then determine the percent error in J_o as t increases from 0.001 R to 0.2 R.

Solution: By Problem (C.4)

$$J_o = \frac{\pi}{2}(b^4 - a^4) = \frac{\pi}{2}(b^2 - a^2)(b^2 + a^2)$$

$$J_o = \frac{\pi}{2}(b+a)(b-a)(b^2 + a^2)$$

or

$$J_o = \pi (R)(t)(b^2 + a^2) \qquad (a)$$

But $b = R + \frac{t}{2}$, $a = R - \frac{t}{2}$

Hence,

$$b^2 + a^2 = 2R^2 + \frac{t^2}{2} \qquad (b)$$

Equations (a) and (b) yield

$$J_o = \pi R t \left(2R^2 + \frac{t^2}{2}\right)$$

or since $t \ll R$,

$$\underline{J_o \approx 2\pi R^3 t} \qquad (c)$$

In general, the percent error is

$$\%\ \text{Error} = \left[\frac{J_o(\text{exact}) - J_o(\text{approx.})}{J_o(\text{exact})}\right] \times 100 \qquad (d)$$

For $t = 0.001 R$:

$$b = R + \frac{t}{2} = 1.0005 R$$
$$a = R - \frac{t}{2} = 0.9995 R \qquad (e)$$

(Continued)

C.7 Cont.

Therefore, with Eqs. (e)

$$J_o(\text{exact}) = \frac{\pi}{2}(b^2 - a^2) = 0.0020000005\,\pi R^4$$

For J_o (approx), with $t = 0.001R$ and Eq. (c)

$$J_o(\text{approx}) = 2\pi R^3(0.001R) = 0.002\,\pi R^4$$

Therefore,

$$\%\,\text{Error} = \left[\frac{0.0020000005 - 0.002}{0.0020000005}\right]\frac{\pi R^4}{\pi R^4} \times 100$$

or

$$\underline{\underline{\%\,\text{Error} = 2.5 \times 10^{-5}\ \%}} \qquad (f)$$

For $t = 0.2R$:

$$b = R + \frac{t}{2} = 1.1$$
$$a = R - \frac{t}{2} = 0.9 \qquad (g)$$

with Eqs. (g),

$$J_o(\text{exact}) = \frac{\pi}{2}(b^4 - a^4) = 0.404\,\pi R^4$$

with $t = 0.2R$ and Eq. (c)

$$J_o(\text{approx}) = 2\pi R^3(0.2R) = 0.4\,\pi R^4$$

Therefore,

$$\%\,\text{Error} = \frac{(0.404 - 0.4)\pi R^4}{0.404\,\pi R^4} \times 100$$

or $\underline{\underline{\%\,\text{Error} = 0.99\%}}$

Hence, Eq. (a) is a fairly accurate approximation for $t \le 0.2R = (0.1)(a+b)$

C.8

Calculate the moment of inertia of the T-beam shown in Fig. a with respect to A-A

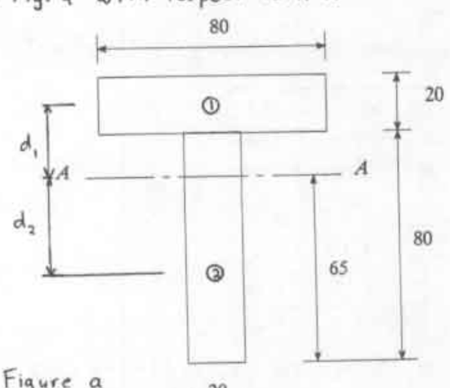


Figure a

Solution: The cross sectional properties are listed in Table P.C.8

Table P.C.8: Moment of inertia with Respect to A-A

i	I_x	A_i	d_i^2	$A_i d_i^2$
1	53 333.3	1600	625	10^6
2	853 333.3	1600	625	10^6
Σ	906 666.6			2×10^6

$$I_{A\text{-}A} = \Sigma I_x + \Sigma A_i d_i^2 = 2\ 906\ 666.67\ \text{mm}^4$$

$$\underline{I_{A\text{-}A} = 2.907 \times 10^6\ \text{mm}^4}$$

C.9

Determine the moments of Inertia I_x, I_y, J_o for the cross section of the regular hexagonal shaft shown in Fig. a

Solution: Divide the hexagon into 5 parts (see Fig. a). Thus, the hexagon is divided into 4 right triangles and a rectangle (see Table D.2). The calculations are listed in Table PC.9.

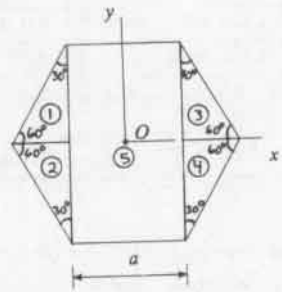


Figure a

Table PC.9: Moments of inertia for Figure (a)

i	I_{x_i}	I_{y_i}	J_{o_i}	A	$(dx_i)^2$	$(dy_i)^2$	Adx_i^2	Ady_i^2	$A\sqrt{dx_i^2+dy_i^2}$
1	$0.009\,02a^4$	$0.003a^4$	$0.012a^4$	$0.217a^2$	$0.444a^2$	$0.08\bar{3}a^2$	$0.096a^4$	$0.0181a^4$	$0.158a^4$
2	$0.009\,02a^4$	$0.003a^4$	$0.012a^4$	$0.217a^2$	$0.444a^2$	$0.08\bar{3}a^2$	$0.096a^4$	$0.0181a^4$	$0.158a^4$
3	$0.009\,02a^4$	$0.003a^4$	$0.012a^4$	$0.217a^2$	$0.444a^2$	$0.08\bar{3}a^2$	$0.096a^4$	$0.0181a^4$	$0.158a^4$
4	$0.009\,02a^4$	$0.003a^4$	$0.012a^4$	$0.217a^2$	$0.444a^2$	$0.083a^2$	$0.096a^4$	$0.0181a^4$	$0.158a^4$
5	$0.433a^4$	$0.144a^4$	$0.577a^4$	$1.73a^2$	0	0	0	0	0
Σ	$0.469a^4$	$0.156a^4$	$0.625a^4$				$0.386a^4$	$0.072\,3a^4$	$0.631a^4$

$$I_x = \Sigma I_{x_i} + \Sigma A dy_i^2 = 0.469a^4 + 0.072\,3a^4 = \underline{\underline{0.541a^4}}\ [L^4]$$

$$I_y = \Sigma I_{y_i} + \Sigma A dx_i^2 = 0.156a^4 + 0.386a^4 = \underline{\underline{0.542a^4}}\ [L^4]$$

$$J_o = \Sigma J_{o_i} + \Sigma A\sqrt{dx_i^2 + dy_i^2} = 0.625a^4 + 0.631a^4 = \underline{\underline{1.256a^4}}\ [L^4]$$

C.10

a) Determine the coordinate $\overline{y}$ of the centroid of the trapezoid shown in Fig. a. The calculations are given in Table PC10 a

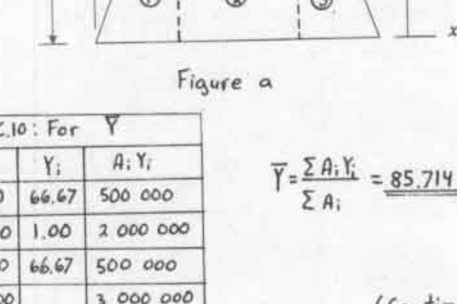


Figure a

Table PC.10: For $\overline{Y}$

i	A_i	Y_i	$A_i Y_i$
1	7 500	66.67	500 000
2	20 000	1.00	2 000 000
3	7 500	66.67	500 000
Σ	35 000		3 000 000

$$\overline{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \underline{85.714\ \text{mm}}$$

(Continued)

C.10 Cont.

b) Determine the moment of inertia with respect to A-A. The calculations are tabulated in Table PC.10 b

i	I_{x_i}	$\bar{Y}_i = \bar{Y} - y_i$	$A\bar{Y}_i^2$
1	16 666 666.7	19.048	2 721 197
2	66 666 666.7	-14.286	4 081 796
3	16 666 666.7	19.048	2 721 197
Σ	83 333 333.$\bar{3}$		9 524 191

Table PC.10b: For moment of inertia with repect to A-A

$I_{A-A} = \Sigma I_{x_i} + \Sigma A\bar{y}_i^2 = 83\,333\,333 + 9\,524\,191$

or $\underline{I_{A-A} = 9.286 \times 10^7 \text{ mm}^4 \text{ or } 9.286 \times 10^{-5} \text{ m}^4}$

C.11

Determine the moment of inertia of the cross section of the slotted beam shown in Fig. a with respect to the x axis. The calculations are tabulated in Table PC.11.

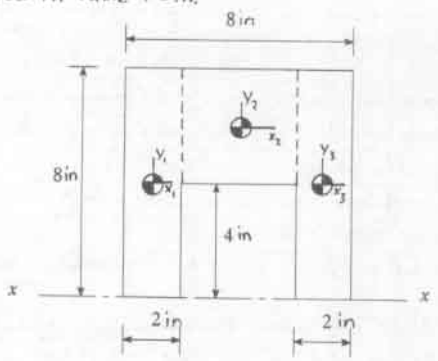


Figure a

Solution:

i	I_{x_i}	A_i	$\bar{Y}_i$	$A_i\bar{Y}_i^2$
1	85.33	16	4	256
2	21.33	16	6	576
3	85.33	16	4	256
Σ	192			1088

Table PC.11: For the moment of inertia with repect to the x axis

$I_x = \Sigma I_{x_i} + \Sigma A_i \bar{Y}_i^2 = 192 + 1088 = \underline{1280 \text{ in}^4}$

Alternatively, consider the cross section of the slotted beam as two square cross sections, one with positive area and one with negative area (Fig. b)

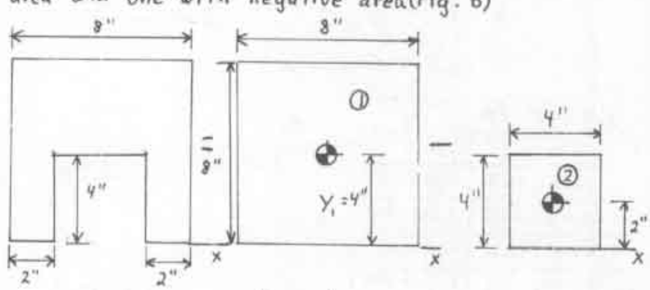


Figure b

By Fig. b,

$I_x = \bar{I}_1 + A_1 y_1^2 - I_2 - A_2 \bar{y}_2$

$I_x = \frac{1}{12}(8)(8)^3 + (8)(8)(4)^2 - \frac{1}{12}(4)(4)^3 - (4)(4)(2)^2$

or $\underline{\underline{I_x = 1280 \text{ in}^4}}$

C.12

For the cross section of the bulb T-beam (Fig. a)

a) Determine the y coordinate of the centroid.

b) Determine the moment of inertia with respect to the y axis

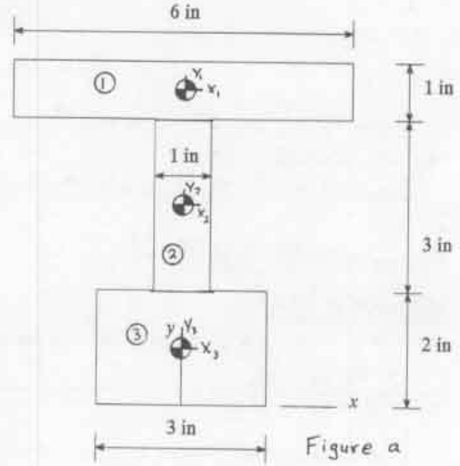


Figure a

Solution:

a) The properties are listed in table PC.12a

i	A_i	Y_i	A_iY_i
1	6	5.5	33.0
2	3	3.5	10.5
3	6	1	6.0
Σ	15		49.5

Table PC12a: For Y Coordinate of the centroid

$\bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{49.5}{15} = \underline{\underline{3.3 \text{ in}}}$

b) Because of the symmetry of the cross section with respect to the Y axis, the parallel axis theorem reduces to $I_y = \Sigma I_{y_i}$

i	$I_{y_i} = \frac{1}{12}bh^3$
1	$\frac{1}{12}(1)(6)^3 = 18.0$
2	$\frac{1}{12}(3)(1)^3 = 0.25$
3	$\frac{1}{12}(2)(3)^3 = 4.5$
Σ	$I_y = 22.75 \text{ in}^4$

Table PC.12 b

Hence, by Table PC.12 b, we find,

$\underline{\underline{I_y = 22.75 \text{ in}^4}}$

C.13

a) Determine I_x of the area shown in Fig. (a) with respect to the x-axis (using Table D.2)

b) Determine I_y of Fig. (a) with respect to the Y-axis.

c) Locate the centroid of the area in Fig. (a) and find I_y', where y' is a vertical axis through the centroid.

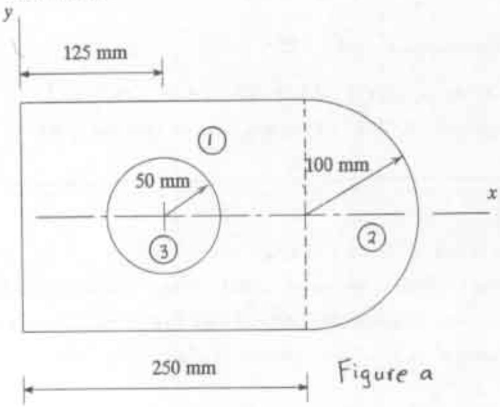


Figure a

Solution:

a) In Fig. a, area 1 is a rectangle 250 mm by 200 mm, area 2 is a semicircular area of radius r=100 mm, and area 3 is a negative circular area of radius 50 mm.

Hence, $I_{x1} = \frac{1}{12}bh^3 = \frac{1}{12}(250)(200)^3 = 166\,666\,667 \text{ mm}^4$

$I_{x2} = \frac{\pi}{8}R^4 = \frac{\pi}{8}(100)^4 = 39\,269\,908 \text{ mm}^4$

$I_{x3} = -\frac{\pi}{4}R^4 = -\frac{\pi}{4}(50)^4 = -4\,908\,739 \text{ mm}^4$

$\therefore I_x = I_{x1} + I_{x2} + I_{x3}$

$I_x = 201\,027\,836 \text{ mm}^4 = 2.01\times 10^8 \text{ mm}^4$

or $\underline{\underline{I_x = 2.01\times 10^{-4} \text{ m}^4}}$

b) The calculations are tabulated in Table PC.13a

Table PC.13a (For I_y)

i	$\bar{I}_{y_i}$ (m⁴)	A_i (m²)	dx_i^2 (m²)	$A\,dx_i^2$ (m⁴)
1	2.604×10^{-4}	0.05	0.0156	0.000781
2	1.098×10^{-5}	0.0157	0.0855	0.00134
3	-4.91×10^{-6}	-0.00785	0.0156	-0.000123
Σ	2.66×10^{-4}			0.001998

$I_y = \Sigma \bar{I}_{y_i} + \Sigma A dx_i^2 = 2.66\times10^{-4} + 0.001998$

$\therefore \underline{\underline{I_y = 0.00226 \text{ m}^4 \text{ or } 2.26\times10^9 \text{ mm}^4}}$

c) The calculations are tabulated in Table PC.13b and Table PC.13c.

Table PC.13b: (For centroid)

i	A_i (m²)	x_i (m)	y_i (m)	$A_i x_i$ (m³)	$A_i y_i$ (m³)
1	0.05	0.125	0	0.00625	0
2	0.0157	0.292	0	0.00458	0
3	-0.00785	0.125	0	-0.00098	0
Σ	0.05785			0.00985	0

$\bar{x}' = \frac{\Sigma A x_i}{A_i} = \frac{0.00985}{0.05785} = \underline{\underline{0.170 \text{ m}}}$

$\bar{y}' = \frac{\Sigma A y_i}{A_i} = \underline{\underline{0 \text{ m}}}$

Table PC.13c: (For I_y')

i	$\bar{I}_{y_i}$ (m⁴)	A_i (m²)	$(d\bar{x}_i)^2$ (m²)	$A_i (d\bar{x}_i)^2$ (m⁴)
1	2.604×10^{-4}	0.05	0.002025	0.000101
2	1.098×10^{-5}	0.0157	0.01499	0.000235
3	-4.91×10^{-6}	-0.00785	0.002025	-0.000016
Σ	2.66×10^{-4}			0.000321

$I_y' = \Sigma \bar{I}_{y_i} + \Sigma A_i (d\bar{x}_i)^2 = 2.66\times10^{-4} + 3.21\times10^{-4}$

$\underline{\underline{I_y' = 5.87\times10^{-4} \text{ m}^4 \text{ or } 5.87\times10^8 \text{ mm}^4}}$

The relationship between I_y from part b and I_y' is

$I_y = I_y' + A(\bar{x}')^2$

or

$I_y = 2.26\times10^{-3} \text{ m}^4 = 5.87\times10^{-4} + (0.05785)(0.170)^2$

$I_y = 2.26\times10^{-3} \text{ m}^4$

C.14

Locate the centroid for the L-shaped cross section shown in Fig. a and find I_x, I_y, and I_{xy} about the centroidal axes (x, y)

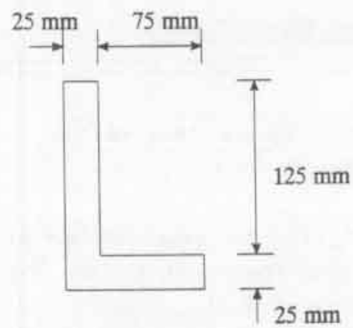


Figure a

Solution: Divide the L-shaped cross section into parts ① and ② (Fig. b). The properties are shown in Table PC.14a.

(Continued)

C.14 Cont.

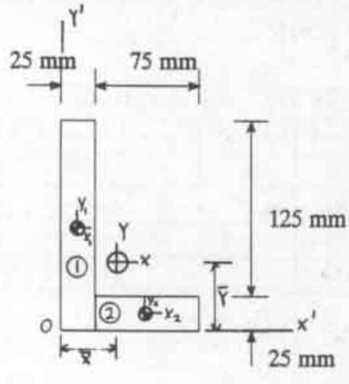


Figure b

Table PC.14a - Centroid of L-shaped Area					
i	A_i (mm²)	x_i (mm)	y_i (mm)	$A_i x_i$ (mm³)	$A_i y_i$ (mm³)
1	3750	12.5	75	46875	281250
2	1875	62.5	12.5	117187.5	23437.5
Σ	5625			164062.5	304687.5

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{164\ 062.5}{5625} = \underline{\underline{29.17\ \text{mm}}}$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{304\ 687.5}{5625} = \underline{\underline{54.17\ \text{mm}}}$$

The computations for the second moments of inertia are tabulated in Table PC.14b.

Table PC.14 b - Second moments of Inertia (mm⁴)								
i	I_{x_i}	I_{y_i}	I_{xy_i}	$\bar{x}_i$ (mm)	$\bar{y}_i$ (mm)	$A_i x_i^2$	$A_i (y_i)^2$	$A_i x_i y_i$
1	7031250	195312	0	16.67	-20.83	1042083	1627083	-1302135
2	97656	878906	0	-33.33	41.67	2082917	3255729	-2604115
Σ	7128906	1074218	0			3125000	4882812	-3906250

$$I_x = \Sigma I_{x_i} + \Sigma A y_i^2 = 7\ 128\ 906 + 4\ 882\ 812 = \underline{\underline{1.201 \times 10^7\ \text{mm}^4}}$$

$$I_y = \Sigma I_{y_i} + \Sigma A x_i^2 = 1\ 074\ 218 + 3\ 125\ 000 = \underline{\underline{4.20 \times 10^6\ \text{mm}^4}}$$

$$I_{xy} = \Sigma A x_i y_i = \underline{\underline{-3.906 \times 10^6\ \text{mm}^4}}$$

C.15

Prove that for a square cross section, any centroidal axis is a principal axis.

Solution:

Let (x, y) be principal axes located at the centroid of the square cross section (Fig. a)

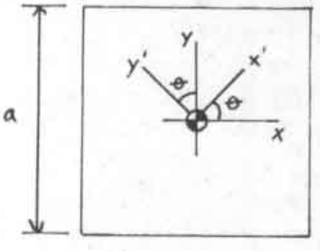


Figure a

Thus, for a square (see Table D.2)

$$I_x = \frac{1}{12}(a)(a)^3 = I_y \quad \text{and } I_{xy} = 0$$

By Eq. (C.11c), the product of inertia relative to axes (x', y') is

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta \qquad \text{(a)}$$

Since $I_x = I_y$ and $I_{xy} = 0$, Eq. (a) yields

$$I_{x'y'} = 0$$

independent of θ

Hence, any axis through the centroid of a square cross section is a principal axis

C.16

Determine the orientation of the principal axes at the centroid and the corresponding principal moments of inertia for an area composed of two semicircles (Fig. a).

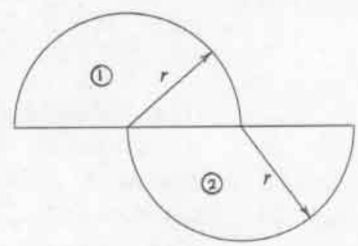


Figure a

Solution: First to determine the centroid of the area, select axes (X, Y) at the origin O of the left semicircle ① (Fig. b).

By definition,

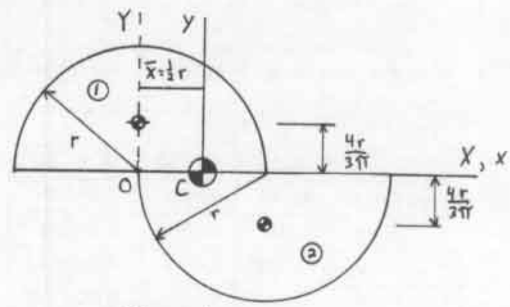


Figure b

$$\bar{X} = \frac{A_1\bar{X}_1 + A_2\bar{X}_2}{A_1 + A_2} = \frac{(\frac{1}{2}\pi r^2)(0) + (\frac{1}{2}\pi r^2)(r)}{(\frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2)}$$

$$\bar{X} = \frac{1}{2} r \qquad \text{(a)}$$

$$\bar{Y} = \frac{(\frac{1}{2}\pi r^2)(\frac{4r}{3\pi}) + (\frac{1}{2}\pi r^2)(-\frac{4r}{3\pi})}{\pi r^2} = 0 \qquad \text{(b)}$$

(Continued)

C.16 Cont.

Thus, the centroid of the area is located at C (Figs. b and c).

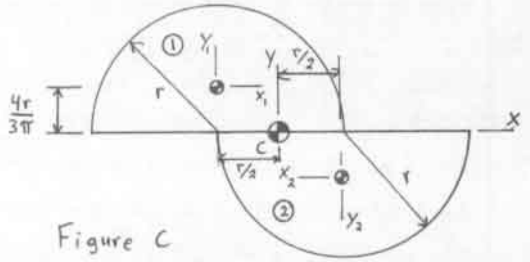


Figure C

With Fig. C and Table D.2, we list cross sectional properties in Table PC.16

Table PC.16: Cross-Sectional Properties

i	A_i	$\bar{x}_i$	$\bar{y}_i$	I_{x_i}	I_{y_i}	I_{xy_i}	$A\bar{x}_i^2$	$A\bar{y}_i^2$	$A\bar{x}_i\bar{y}_i$
1	$\frac{\pi r^2}{2}$	$\frac{r}{2}$	$-\frac{4r}{3\pi}$	$\pi r^4(\frac{1}{8}-\frac{8}{9\pi^2})$	$\frac{\pi r^4}{8}$	0	$\frac{\pi r^4}{8}$	$\frac{16r^4}{18\pi}$	$-\frac{r^4}{3}$
2	$\frac{\pi r^2}{2}$	$\frac{r}{2}$	$-\frac{4r}{3\pi}$	$\pi r^4(\frac{1}{8}-\frac{8}{9\pi^2})$	$\frac{\pi r^4}{8}$	0	$\frac{\pi r^4}{8}$	$\frac{16r^4}{18\pi}$	$-\frac{r^4}{3}$
Σ				$0.0699\pi r^2$	$\frac{\pi r^4}{4}$	0	$\frac{\pi r^4}{4}$	$\frac{16r^4}{9\pi}$	$-\frac{2r^4}{3}$

$$I_x = \Sigma I_{x_i} + \Sigma A\bar{y}_i^2 = 0.0699\pi r^4 + \frac{16r^4}{9\pi} = 0.7855 r^4$$

$$I_y = \Sigma I_{y_i} + \Sigma A\bar{x}_i^2 = \frac{\pi r^4}{4} + \frac{\pi r^4}{4} = \frac{\pi r^4}{2} = 1.571 r^4$$

$$I_{xy} = \Sigma I_{xy} + \Sigma A\bar{x}_i\bar{y}_i = -\frac{2r^4}{3} = -0.6667 r^4$$

Since $I_{xy} \neq 0$, axes (x, y) are not principal axes By Eq. (C.12), the orientation θ of the principal axes at C is given by

$$\tan 2\theta = -\frac{2I_{xy}}{I_x - I_y} = -\frac{(2)(-0.6667r^4)}{(0.7855-1.571)r^4}$$

$$\tan 2\theta = -1.697; \quad \underline{\underline{\theta = -29.75°, 60.25°}}$$

or (see Fig. d)

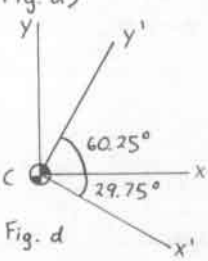


Fig. d

Hence, by Eqs. (C.13)

$$I_{x'} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$I_{x'} = \left[\frac{0.7855+1.571}{2} + \sqrt{\left(\frac{0.7855-1.57}{2}\right)^2 + 0.667^2}\right] r^4$$

or

$$\underline{\underline{I_{x'} = 1.17825 r^4 + 0.77378 r^4 = 1.952 r^4}}$$

$$\underline{\underline{I_{y'} = 1.17825 r^4 - 0.77378 r^4 = 0.40447 r^4}}$$

$$\underline{\underline{I_{x'y'} = 0}}$$

C.17

Determine the orientation of the axes and the principal moments of inertia for an area composed of two triangles (Fig. a)

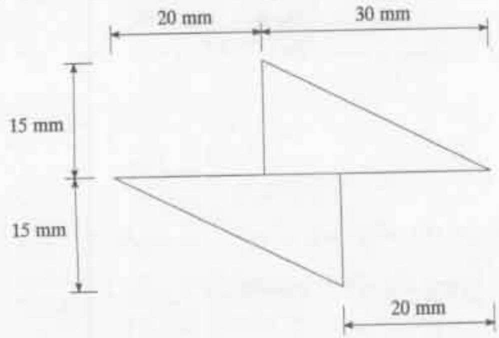


Figure a

Due to rotational symmetry Point O is the centroid. (see also Prob. C.16 for method of computing the centroid.) The computations for the moments of inertia are tabulated in Table PC.17 (see Fig. b and Table D.2)

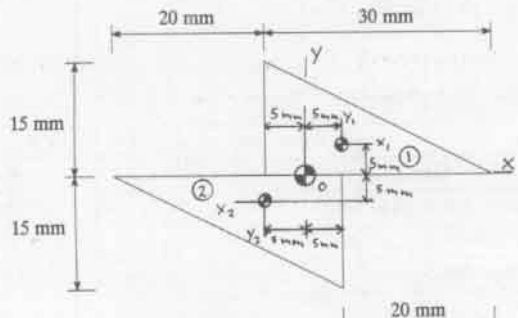


Figure b

Table PC.17: Cross-sectional Properties

i	A_i (mm²)	$\bar{x}_i$ (mm)	y_i (mm)	I_{x_i} (mm⁴)	I_{y_i} (mm⁴)	$I_{x_iy_i}$ (mm⁴)	$A\bar{x}_i^2$ (mm⁴)	$A\bar{y}_i^2$ (mm⁴)	$A\bar{x}_i\bar{y}_i$ (mm⁴)
1	225	-5	-5	2812.5	11250	-2812.5	5625	5625	5625
2	225	-5	-5	2812.5	11250	-2812.5	5625	5625	5625
Σ				5625	22500	-5625	11250	11250	11250

$$I_x = \Sigma I_{x_i} + \Sigma A\bar{y}_i^2 = 5625 + 11250 = 16875 \text{ mm}^4$$

$$I_y = \Sigma I_{y_i} + \Sigma A\bar{x}_i^2 = 22500 + 11250 = 33750 \text{ mm}^4$$

$$I_{xy} = \Sigma I_{xy} + \Sigma A\bar{x}_i\bar{y}_i = -5625 + 11250 = 5625 \text{ mm}^4$$

Since $I_{xy} \neq 0$, axes (x, y) are not principal axes The orientation of the principal axes (x', y') is given by Eq. (C.12)

$$\tan 2\theta = \frac{-2(5625)}{(16875-33750)} = +0.6667$$

$$\underline{\underline{\theta = 16.85°, 106.85°}}$$

see Fig. c.

(continued)

C.17 Cont.

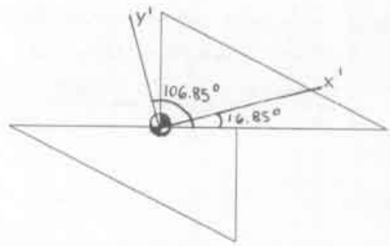


Figure c

By Eq. (C.13), the principal moments of inertia are

$$I_{x'} = \frac{16875 + 33750}{2} + \sqrt{\left(\frac{16875 - 33750}{2}\right)^2 + 5625^2}$$

$$\underline{I_{x'} = 35453 \text{ mm}^4}$$

$$I_{y'} = \frac{16875 + 33750}{2} - \sqrt{\left(\frac{16875 - 33750}{2}\right)^2 + 5625^2}$$

$$\underline{\underline{I_{y'} = 15172 \text{ mm}^4}}$$

C.18

a) Locate the centroid of the area. (see Fig. a and Table PC.18a)

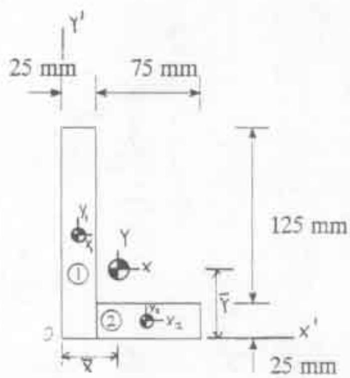


Figure a

Table PC.18a - Centroid of L-shaped Area					
i	A_i (mm²)	x_i (mm)	y_i (mm)	$A_i x_i$ (mm³)	$A_i y_i$ (mm³)
1	3750	12.5	75	46875	281250
2	1875	62.5	12.5	117187.5	23437.5
Σ	5625			164062.5	304687.5

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{164\ 062.5}{5625} = \underline{\underline{29.17 \text{ mm}}}$$

$$\bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{304\ 687.5}{5625} = \underline{\underline{54.17 \text{ mm}}}$$

b) Determine the orientation of the principal Axes (see Fig. **b** and Table PC.18b)

Table PC.18 b - Second moments of Inertia (mm⁴)								
i	I_{x_i}	I_{y_i}	I_{xy_i}	$\bar{x}_i$ (mm)	$\bar{y}_i$ (mm)	$A_i x_i^2$	$A_i (y_i)^2$	$A_i x_i y_i$
1	7031250	195312	0	16.67	-20.83	1042083	1627083	-1302135
2	97656	878906	0	-33.33	41.67	2082917	3255729	-2604115
Σ	7128906	1074218	0			3125000	4882812	-3906250

$$I_x = \Sigma I_{x_i} + \Sigma A y_i^2 = 7\ 128\ 906 + 4\ 882\ 812 = 1.201 \times 10^7 \text{ mm}^4$$

$$I_y = \Sigma I_{y_i} + \Sigma A x_i^2 = 1\ 074\ 218 + 3\ 125\ 000 = 4.20 \times 10^6 \text{ mm}^4$$

$$I_{xy} = \Sigma A x_i y_i = -3.906 \times 10^6 \text{ mm}^4$$

Using (Eq. C12)

$$\tan 2\theta = \frac{-2(-3.906 \times 10^6)}{[(1.2012 \times 10^7) - (4.199 \times 10^6)]} = 1.000$$

$$\underline{\underline{\theta = 22.5°,\ 112.5°}}$$

See Fig. b

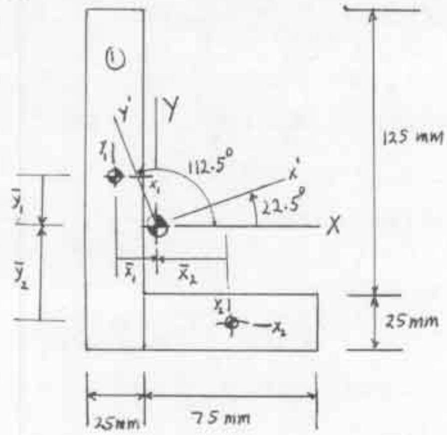


Figure b

c) Determine the principal moments of inertia. Using Eq. (C13),

$$I_{x'} = \left[\frac{(1.2012 \times 10^7) + (4.199 \times 10^6)}{2}\right] + \sqrt{\left[\frac{(1.2012 \times 10^7) - (4.199 \times 10^6)}{2}\right]^2 + (3.906 \times 10^6)^2}$$

$$I_{x'} = 8.1055 \times 10^6 + 5.524 \times 10^6$$

or $\underline{\underline{I_{x'} = 1.363 \times 10^7 \text{ mm}^4}}$

$$I_{y'} = \left[\frac{(1.2012 \times 10^7) + (4.199 \times 10^6)}{2}\right] - \sqrt{\left[\frac{(1.2012 \times 10^7 - (4.199 \times 10^6)}{2}\right]^2 + (3.906 \times 10^6)^2}$$

$$I_{y'} = 8.1055 \times 10^6 - 5.524 \times 10^6$$

or $\underline{\underline{I_{y'} = 2.582 \times 10^6 \text{ mm}^4}}$

C.19

a) Locate the centroid of the area shown in Fig. a

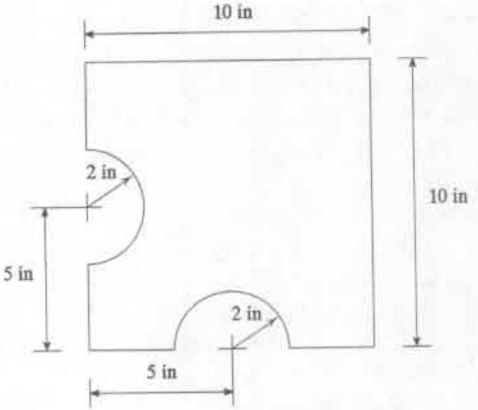


Figure a

Solution: Consider the area to consist of three parts; part 1, a square 10×10 in, part 2, a semicircle of negative area and a radius of 2 in on the y axis, and part 3, a semicircle of negative area and a radius of 2 in on the x axis. The area properties are listed in Table PC.19a (see Table D.2 and Fig. b)

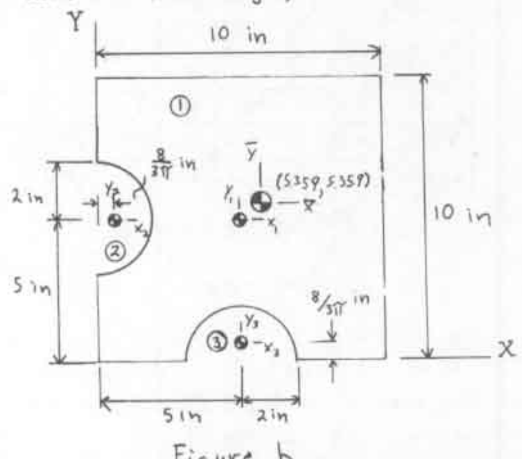


Figure b

Table PC.19a : (Centroid)					
i	A_i (in²)	x_i (in)	y_i (in)	$A_i x_i$ (in³)	$A_i y_i$ (in³)
1	100	5	5	500	500
2	-6.2832	0	5	0	-31.416
3	-6.2832	5	0	-31.416	0
Σ	87.4336			468.58	468.58

$$\bar{X} = \frac{\Sigma A_i X_i}{\Sigma A_i} = \frac{468.58}{87.4336} = \underline{\underline{5.359 \text{ in}}}$$

$$\bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{468.58}{87.4336} = \underline{\underline{5.359 \text{ in}}}$$

b) Determine the orientation of the principal axes (see Table PC.19 b)

Table PC.19b: (Principal Axes)									
i	A_i (in²)	$\bar{x}_i$ (in)	$\bar{y}_i$ (in)	I_{x_i} (in⁴)	I_{y_i} (in⁴)	I_{xy_i} (in⁴)	$A\bar{x}_i^2$ (in⁴)	$A\bar{y}_i^2$ (in⁴)	$A\bar{x}_i\bar{y}_i$
1	100	0.359	0.359	833.$\bar{3}$	833.$\bar{3}$	0	12.89	12.89	12.89
2	-6.283	4.510	0.359	-6.283	-1.756	0	-127.80	-0.810	-10.17
3	-6.283	0.359	4.510	-1.756	-6.283	0	-0.810	-127.80	-10.17
Σ				825.29	825.29	0	-115.72	-115.72	-7.45

$I_x = \Sigma I_{x_i} + \Sigma A y_i^2 = 825.29 - 115.72 = 709.6 \text{ in}^4$

$I_y = \Sigma I_{y_i} + \Sigma A x_i^2 = 825.29 - 115.72 = 709.6 \text{ in}^4$

$I_{xy} = -7.45 \text{ in}^4$

By Eq. (C.12), the orientation θ of the principal axes is given by

$$\tan 2\theta = \frac{-2(-7.45)}{[709.6 - 709.6]} = \infty \therefore 2\theta = 90°$$

$$\theta = 45°$$

c) Determine the principal moments of inertia of the area.

By Eq. (C.13),

$$I_{x'} = \frac{709.6 + 709.6}{2} + \sqrt{\left(\frac{709.6 - 709.6}{2}\right)^2 + (-7.45)^2}$$

$I_{x'} = 709.6 + 7.45$

or, $\underline{\underline{I_{x'} = 717.05 \text{ in}^4}}$

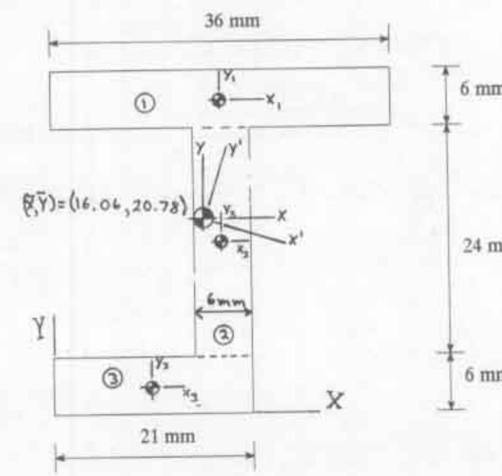
$$I_{y'} = \frac{709.6 + 709.6}{2} - \sqrt{\left(\frac{709.6 - 709.6}{2}\right)^2 + (-7.45)^2}$$

$I_{y'} = 709.6 - 7.45$

or, $\underline{\underline{I_{y'} = 702.2 \text{ in}^4}}$

C.20

a) Locate the centroid of the area shown in Fig. a. Divide the T cross section into three rectangles 1, 2, and 3 (Fig. a) Then the cross section properties are tabulated in Table PC.20 a.



(Continued)

C.20 Cont.

Table PC.20a: Centroid					
i	A_i (mm^2)	x_i (mm)	y_i (mm)	$A_i x_i$ (mm^3)	$A_i y_i$ (mm^3)
1	216	18	33	3888	7128
2	144	18	18	2592	2592
3	126	10.5	3	1323	378
Σ	486			7803	10098

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{7803}{486} = \underline{\underline{16.0556 \text{ mm}}}$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{10098}{486} = \underline{\underline{20.778 \text{ mm}}}$$

b) Determine the orientation of the principal axes (see Table D.2 and Fig. a)

Table PC.20b: (Principal Axes)									
i	A_i (mm^2)	$\bar{x}_i$ (mm)	$\bar{y}_i$ (mm)	I_{x_i} (mm^4)	I_{y_i} (mm^4)	$I_{x_i y_i}$ (mm^4)	$A\bar{x}_i^2$ (mm^4)	$A\bar{y}_i^2$ (mm^4)	$A\bar{x}_i\bar{y}_i$ (mm^4)
1	216	-1.944	-12.222	648	23328	0	816.29	32265	5132.1
2	144	-1.944	2.778	6912	432	0	544.20	1111.29	-777.66
3	126	5.556	17.778	378	46305	0	3889.5	39823	12445.6
Σ				7938	283905	0	5250	73199	16800

$I_x = \Sigma I_{x_i} + \Sigma A_i \bar{y}_i^2 = 7938 + 73199 = 81\,137 \text{ mm}^4$

$I_y = \Sigma I_{y_i} + \Sigma A_i \bar{x}_i^2 = 283905 + 5250 = 33\,640 \text{ mm}^4$

$I_{xy} = \Sigma A \bar{x}_i \bar{y}_i = 16800 \text{ mm}^4$

By Eq. (C.12), the orientation θ of the principal axes is

$$\tan 2\theta = -\frac{2(16800)}{(81137-33640)} = -0.7074$$

$$\underline{\underline{\theta = -17.64°,\ 72.36°}} \quad \text{(see Fig. a)}$$

c) Determine the principal moments of inertia

$$I_{x'} = \frac{81137+33640}{2} + \sqrt{\left[\frac{81137-33640}{2}\right]^2 + 16800^2}$$

$$= 57388.5 + 29090.0$$

or $\quad \underline{\underline{I_{x'} = 86479 \text{ mm}^4}}$

$$I_{y'} = \frac{81137+33640}{2} - \sqrt{\left[\frac{81137-33640}{2}\right]^2 + 16800^2}$$

$$= 57388.5 - 29090.0$$

$$\underline{\underline{I_{y'} = 28\,298 \text{ mm}^4}}$$

C.21

a) Locate the centroid of the area shown in Fig. a.

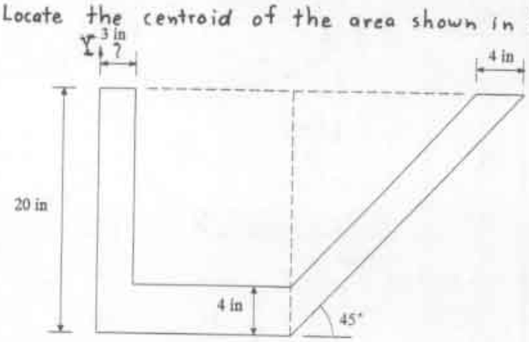


Figure a

Solution:

The cross section may be considered to be composed of 4 parts (Fig. b). part 1, a rectangle 16 in × 3 in, part 2, a rectangle 16 in × 4 in, part 3, a right triangle ADE, and part 4, a triangle ABC of negative area. The calculations are tabulated in Table PC.21a.

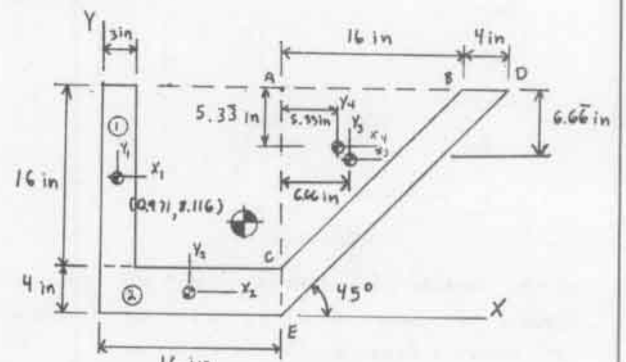


Figure b

Table PC.21a: Centroid					
i	A_i (in^2)	x_i (in)	y_i (in)	$A_i x_i$ (in^3)	$A_i y_i$ (in^3)
1	48	1.5	12	72	576
2	64	8	2	512	128
3	200	$22.\bar{6}$	$13.\bar{3}$	$4533.\bar{3}$	$2666.\bar{6}$
4	-128	$21.\bar{3}$	$14.\bar{6}$	$-2730.\bar{6}$	$-1877.\bar{3}$
Σ	184			$2386.\bar{6}$	1493.33

$$\bar{X} = \frac{2386.\bar{6}}{184} = 12.971 \text{ in}$$

$$\bar{y} = \frac{1493.33}{184} = 8.116 \text{ in}$$

see Fig. b.

b) Determine the orientation of the principal axes of the area. (See Table PC.21b for the tabulation of the solution.)

Table PC.21b: Principal Axes									
i	A_i (in^2)	$\bar{x}_i$ (in)	$\bar{y}_i$ (in)	I_{x_i} (in^4)	I_{y_i} (in^4)	$I_{x_i y_i}$ (in^4)	$A_i \bar{x}_i^2$ (in^4)	$A_i \bar{y}_i^2$ (in^4)	$A_i \bar{x}_i \bar{y}_i$ (in^4)
1	48	11.471	-3.884	1024	36	0	6316.0	724.1	-2138.56
2	64	4.971	6.116	$85.\bar{3}$	$1365.\bar{3}$	0	1581.5	2393.9	1945.77
3	200	-9.695	-5.217	$4444.\bar{4}$	$4444.\bar{4}$	$-2222.\bar{2}$	18 798.6	5443.4	10115.76
4	-128	-8.362	-6.551	$-1820.\bar{4}$	$-1820.\bar{4}$	$910.\bar{2}$	-8950.1	-5493.2	-7011.77
Σ	184			$3733.\bar{3}$	$4025.\bar{3}$	-1312	17 746	3068.2	2911.2

$I_x = \Sigma I_{x_i} + \Sigma A_i \bar{y}_i^2 = 3733.\bar{3} + 3068.2$

or $\quad \underline{\underline{I_x = 6801.5 \text{ in}^4}}$

$I_y = \Sigma I_{y_i} + \Sigma A_i \bar{x}_i^2 = 4025.\bar{3} + 17746$

or $\quad \underline{\underline{I_y = 21771 \text{ in}^4}}$

$I_{xy} = \Sigma I_{x_i y_i} + \Sigma A_i \bar{x}_i \bar{y}_i = -1312 + 2911.2$

or $\quad I_{xy} = 1599.2 \text{ in}^4$

(Continued)

C.21 Cont.

By Eq. (C.12),

$$\tan 2\theta = -\frac{2I_{xy}}{I_x - I_y} = -\frac{2(1599.2)}{(6801.5-21771)} = 0.2137$$

$$\underline{\underline{\theta = 12.06^\circ,\ 102.06^\circ}}$$

c) Determine the principal moments of inertia of the area.

By Eqs. (C.13),

$$I_{x'} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{6801.5+21771}{2} + \sqrt{\left(\frac{6801.5-21771}{2}\right)^2 + (1599.2)^2}$$

$$= 14286.25 + 7653.69$$

or $\underline{\underline{I_{x'} = 21940 \text{ in}^4}}$

and $I_{y'} = 14286.25 - 7653.69$

or $\underline{\underline{I_{y'} = 6633 \text{ in}^4}}$